Is partitional information always correct?

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1. Extended abstract

Traditional theory implicitly assumes that information is modeled as a partition of the state space. Starting from Geanokoplos (1989) and Bacharach (2005) a number of papers have examined situations with nonpartitional information functions. Relaxing this specific assumption leads to a number of anomalies and misperceptions in the analysis of the information structure and consequently the functionality of the knowledge operators, which have been basically dealt with models of bounded rationality. Board (2003) presents a variant of the well known absentminded driver's paradox with nonpartitional information. Following simple logical reasoning one is able to easily deduce further information that is not consistent with the information function and ultimately reduce the problem to an ordinary situation with partitional structure. Would this conclusion imply that nonpartitional information functions are contradictory?

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In the first part of the present paper we study the epistemic conditions of models with nonpartitional information structure. We see that properties, such as positive and negative introspection, which hold under the standard assumptions, could well be violated. Therefore we implicitly relax the assumption that decision makers know what they know and what they do not know, allowing thus for models of bounded rationality where partitionality is not a requirement.

The second, and most substantial, part of the paper studies decision making under nonpartitional information structures. In order to analyze this problem we divide our focus on two separate tasks: determining a behavioral strategy before the beginning of the game when an overall view is possible (planning stage) and deciding on a mixed strategy after having arrived at some node (action stage).

In the first case, the whole problem is transformed into an ordinary problem with partitional information. Even though it is rather trivial designing an optimal strategy in one-player decision problems, things get a little complicated when dealing with games, where equilibrium existence is not ensured. The reason behind this phenomenon is the existence of absentmindedness in some games, rather than the generalized information structure *per se*. Isbell (1957) and Alpern (1988) extensively study this kind of games.

Decision making in the action stage is based on a quite different mechanism. The individual chooses the optimal action given the strategy he anticipates himself to implement at the other nodes, implying that he optimizes given his belief function. Then we define the Nash equilibrium of the multi-agent normal form game as a strategy vector that optimizes the expected payoff at every node given what is played outside the current information set. Unlike the planning stage, Nash equilibria not always do they exist. Then we show that this could be the case only under nonpartitional information structure. On the other hand a partitional function ensures the existence of equilibrium points in the multi-agent normal form game.

A further complication in this kind of models arises due to the fact that a number of equilibria in the multi-agent normal form game seem to be rather unreasonable. Similarly then to the mainstream analysis of normal form games, we introduce the trembling hand perfect equilibrium of the multi-agent game, we study its properties and we connect it to the optimal behavioral strategy of the original extensive form decision problem.