In Bargaining We Trust*

Rene Saran†

July 6, 2006

Abstract

This paper studies the influence of trust in a bilateral trading problem by introducing trustworthy types of players. It shows that the effects of *degree* and *distribution* of trust are notably different in direct mechanisms vis-à-vis k-double auctions. If either the *degree of trust* increases or the *distribution of trust* changes so that high-surplus types are now more likely among trustworthy types, then we can design direct mechanisms with higher probability of trade. In fact, with a high enough *degree of trust*, it is possible to construct direct mechanisms that are ex-post efficient. None of these results are true for k-double auctions.

JEL Classification Number: C78

Keywords: Bargaining, Trust, Double Auctions, Incomplete Information, Probability of Trade, Ex-post Efficiency

1 Introduction

Trust is a significant element in almost every economic interaction. Its importance is based on the complexity of today's economic environment in which individuals meet and undertake decisions and the high transaction costs associated with writing and enforcing complete contracts. Uncertainty about the future, asymmetric information (either adverse selection or moral hazard) make it impossible to define a complete contract. Even when a potential complete contract can be written, high fees of lawyers and cost of litigation make it very expensive to write and enforce such a contract. In the absence of trust among the interested individuals, the typical incomplete contracts that will be agreed upon under such situations will be inefficient. A common perception is that if the individuals trust each other then such inefficiencies will not be observed. Al-Najjar and Casadesus-Masanell [2] show that other

*I am indebted to Roberto Serrano for his guidance and advice throughout the project—and the title. I am also grateful to Pedro Dal Bó and Glenn Loury for their comments and suggestions.

†Department of Economics, Brown University, Providence, R.I. 02912. Email: rene_saran@brown.edu

incomplete but more efficient contracts may endogenously arise in complex environments when there is enough trust.

Similar issues arise within the context of bargaining. Private information about the valuations lead to uncertainty regarding the gains from trade that can be achieved through negotiations. Most of the times this private information is not verifiable (e.g., valuation of an object) or even if verifiable there is no court that has the authority to adjudicate the matter (e.g., in international disputes over territory). If then players make claims (either explicitly or implicitly though body language or other posturing) about their valuations or if they have a reputation to be of a particular type, it is not always the case that others will disregard such claims, often leading to disastrous consequences.

> "...Barak's actions led to a classic case of misaddressed messages: the intended recipients of his tough statements—the domestic constituency he was seeking to carry with him—barely listened, while their unintended recipients—Palestinians he would sway with his final offer—listened only too well...In short, everything Barak saw as evidence that he was serious, the Palestinians considered to be evidence that he was not."[1]

Inability to monitor the actions of players also raises the question of whether to trust the other player with respect to, for example, implementation of the agreement—will she cut the cake as she had agreed to? Similarly, uncertainty concerning the future (e.g., possibility of strong opposition against an international treaty at home forcing the government to renege on the treaty) will influence the strategies players adopt in bargaining. All these considerations affect negotiations through the beliefs that players hold regarding them; neither do they always trust nor are they entrenched pessimists.

I study the influence of *degree* and *distribution* of trust among traders in a specific bargaining problem, a bilateral trading problem. A buyer and a seller of an indivisible object engage in a trading mechanism, which determines whether they trade and at what price. A player has incomplete information about the valuation type of the other player, that is, a buyer does not know the valuation of the seller, she only knows its distribution. Similarly, the seller only knows the distribution of buyer's valuation. Each player can be of either trustworthy disposition or strategic disposition but players do not know the true disposition type of each other. The probability that a player is trustworthy type reflects the *degree of trust* of the other player in the former. The distribution of valuation of a trustworthy type reflects the *distribution of trust*, that is, which valuation type is more or less likely among trustworthy types. This paper studies how the *degree* and *distribution* of trust affects the probability of trade among strategic types.

I use the conventional definition of trustworthy disposition. A player is trustworthy with respect to a claim made by her if what she claims is true to the best of her knowledge. A player is trustworthy with respect to an action that positively affects truster's welfare if she can be relied upon to successfully perform that action.[2]

---

[1]Aga and Malley (2001).

[2]Dasgupta (2000) and Gambetta (2000) define trust as a belief over actions of others. Good (2000) says that trust is based on claims made by individuals. Hardin (2002) stresses the relationship between trust and trustworthiness of an individual.

In a direct trading mechanism players are required to submit a report about their types. I assume that trustworthy types are trustworthy with respect to their reports and hence report their type truthfully. A k-double auction is another trading mechanism that has been studied both theoretically and experimentally due to its similarity to real world bargaining procedures.[3] In a k-double auction, players simultaneously submit sealed bids. For this trading mechanism, I assume that trustworthy types are trustworthy with respect to bidding truthfully, that is, they bid equal to their valuation. In fact, these assumptions about trustworthy types are motivated by behavior of players observed in experiments on 1/2-double auctions conducted by Valley et al. (2002) and McGinn, Thompson and Bazerman (2003). In these experiments, some players truthfully reveal their valuation when they are allowed to communicate, for instance, McGinn, Thompson and Bazerman (2003) find over 50% of individual communications in which players revealed truthfully. McGinn, Thompson and Bazerman (2003) also find players bidding truthfully both when allowed to communicate (30% of individual bids) and when not allowed to communicate (44% of individual bids) before the double auction.

A common perception is that trust among individuals involved in an interaction is better for their welfare. This paper shows that this is indeed the case when we can use direct mechanisms to solve the bilateral trading problem. For any given *distribution of trust*, if there is an increase in the *degree of trust*, then we can design incentive compatible[4] and individually rational direct mechanisms with at least as high probability of trade among strategic types as before. However, this is not true for k-double auctions. If valuations of all types are distributed uniformly on $[0, 1]$, then the probability of trade among strategic types in any equilibrium outcome of any k-double auction when there is a positive *degree of trust* is strictly less than the highest achievable probability of trade using k-double auctions when there is no trust. Hence, trust is not necessarily an elixir that, if present, can improve the welfare of individuals in the society. Unless all the individuals involved in an interaction are trustworthy, those who are strategic will take advantage of such trust to further their self interest, which might prove detrimental to well-being of the society.'...if trust exists only unilaterally cooperation may also fail, and if it is blind it may constitute rather an incentive to deception' (Gambetta (2000), p. 219).

In fact, for any *distribution of trust*, there exist ex-post efficient, incentive compatible and individually rational direct mechanisms if and only if at least one player has high enough *degree of trust* in the other player. Also, the threshold on the *degree of trust* of a player necessary to achieve ex-post efficiency decreases as the *degree of trust* of the other player increases.

For the bilateral trading problem with only strategic types of players, Myerson and Satterthwaite (1983) prove that, under some mild assumptions about the distribution of

[3]For theoretical analysis see Chatterjee and Samuelson (1983), Farell and Gibbons (1989), Mathews and Postlewaite (1989), Leininger, Linhart and Radner (1989) and Satterthwaite and Williams (1989). Radner and Schotter (1989), Valley et al. (2002) and McGinn, Thompson and Bazerman (2003) conduct experiments on a k-double auction.

[4]Incentive compatibility for trustworthy types is trivially satisfied since trustworthy types report truthfully.

valuations, there does not exist an ex-post efficient mechanism that is incentive compatible and individually rational. They show that any ex-post efficient and incentive compatible mechanism will require some minimum ex-ante subsidy to ensure voluntary participation by all valuation types of players. This subsidy can be provided exogenously by a third player, like the mechanism designer or a broker. This paper shows that this ex-ante subsidy can also be generated endogenously if there is high enough *degree of trust* and therefore, we get the above mentioned positive result. Trustworthy types provide the subsidy through the ex-ante gains from trade they generate by being truthful in communicating their type. In the ex-post efficient direct mechanisms, these ex-ante gains of trustworthy players are used to make lump-sum transfers to strategic types in a manner that ensures individual rationality for all types and truth-telling by strategic types in a Bayesian-Nash equilibrium of the mechanism.

With only strategic types of players in the trading problem, Myerson and Satterthwaite (1983) result implies that any k-double auction with or without any form of pre-play communication is ex-post inefficient. This paper shows that for any *distribution of trust*, any k-double auction with or without pre-play communication is ex-post inefficient irrespective of the *degree of trust* among players (except when at least one player is for sure trustworthy type). Unlike the direct mechanisms that can be constructed to achieve ex-post efficiency when there is positive *degree of trust*, the k-double auction with or without pre-play communication cannot use the ex-ante gains from trade of trustworthy types to subsidize the strategic types. These gains can be transferred to strategic types only if the trustworthy type of buyer (seller) bids above (below) her valuation in the bidding stage, which cannot happen. Hence, the real world trading mechanisms where the final stage is a k-double auction are not well designed from the perspective of achieving ex-post efficiency by adequately "using" the trust among players.

The *distribution of trust* also affects trading outcomes since it is a belief that a player holds regrading which valuation type is more or less likely given that the other player is trustworthy type. I prove that for any *degree of trust*, it is possible to design a direct mechanism with at least as high probability of trade among strategic types as before if we change the *distribution of trust* so that high surplus types (high valuation types of buyer and low valuation types of seller) are more likely among trustworthy types. With this change in *distribution of trust*, there are more ex-ante gains from trade generated by trustworthy types and hence at least as much subsidy as before can be provided to strategic types.

However, this result does not hold for k-double auctions. I consider two bilateral trading problems with the same uniform distribution of valuations of strategic types but with different *distributions of trust*. In the first trading problem, the valuations of trustworthy types are also distributed uniformly. In this case, as mentioned above, the probability of trade among strategic types in any equilibrium of any k-double auction when there is positive *degree of trust* is lower than the highest achievable probability of trade when players do not trust each other. In the second trading problem, I change the *distribution of trust* so that high surplus types are *less* likely among trustworthy types. In this case, however, there exists an equilibrium with probability of trade among strategic types higher than the highest achievable probability of trade when players do not trust each other.

For a bilateral trading problem with only strategic types and valuations distributed uniformly on $[0, 1]$, Myerson and Sattethwaite (1983) show that a 1/2-double auction is an optimal mechanism since when players play the Chatterjee and Samuelson (1983) linear strategies (C-S equilibrium), the outcome of the 1/2-double auction attains both the maximum ex-ante gains and ex-ante probability of trade relative to any other equilibrium outcome of any trading mechanism that satisfies incentive compatibility and individual rationality. Satterthwaite and Williams (1989), however, prove that with only strategic types, k-double auctions are not optimal for generic distributions of valuations. In a similar vein, the results of this paper show that the set of outcomes attainable using direct mechanisms differ from the the set of outcomes of k-double auctions if we perturb the disposition type of players.

Saran (2006) shows that when valuations of strategic types are uniformly distributed on $[0, 1]$, then by adding behavioral types whose valuations are also uniformly distributed on $[0, 1]$ and who play the Chatterjee and Sameulson (1983) linear strategies if they hear the "right" message, it is possible to achieve a higher probability of trade among strategic types in a 1/2-double auction with pre-play communication than the maximum attainable without such behavioral types (As mentioned above, this maximum value equals the probability of trade in the C-S equilibrium). However, the probability of such behavioral types has to be at least 11% to do better than the C-S equilibrium. Similarly, the equilibrium of the 1/2-double auction in section 4.2, in which high-surplus types less likley among trustworthy types, also achieves a higher probability to trade among strategic types than the C-S equilibrium value for all *degrees of trust* less than 50%.

This paper is related to the literature on reputation beginning with the seminal papers of Kreps et al. (1982), Kreps and Wilson (1982), Milgrom and Roberts (1982). Sobel (1985) studies the effect of introducing honest type of sender in a model of strategic information transmission and Dasgupta (2000) studies the influence of honest type of salesman in the market for lemons. However, it must be emphasized that this paper does not focus on the issue of strategic types building a reputation of being trustworthy type in a repeated interaction.

The next section outlines the bilateral trading problem and the assumptions about trustworthy disposition. It then characterizes the set of direct mechanisms satisfying incentive compatibility and individual rationality. The third section presents the results related to the *degree of trust* while the fourth section presents the results related to the *distribution of trust*. I conclude in the the final section. Tables pertaining to the 1/2-double auction with high-surplus types less likely among trustworthy types are in the appendix.

2 Bilateral Trading Problem

A buyer (denoted by b) and a seller (denoted by s) engage in a trading mechanism to trade an indivisible good. Each player can have two possible dispositions (d_i), trustworthy (tr) and strategic (st). The probability that a buyer is trustworthy type is $\epsilon_b \in [0, 1]$, which is independent of seller's valuation and disposition. Similarly, the probability that a seller is trustworthy type is $\epsilon_s \in [0, 1]$ which is also independent of buyer's valuation and disposition.

Valuations of strategic and trustworthy types of buyer are distributed on some interval $[\underline{a}_b, \bar{a}_b]$ independently of the seller's valuation and disposition. Valuations of strategic and trustworthy types of seller are distributed on $[\underline{a}_s, \bar{a}_s]$ independently of the buyer's valuation and disposition. Let F_{d_i}, $i = b, s$ & $d_i = tr, st$, be the distribution of valuations. The associated density functions and f_{d_i} are continuous and positive on their respective domains. Players know only their own type (v_i, d_i). All the other information is common knowledge. $(\epsilon_i, F_{st_i}, F_{tr_i})_{i=b,s}$ defines a bilateral trading problem.

An outcome of a trading mechanism specifies the following for all pairs of valuation types (v_b, v_s):

1. Probability of trade:
   - if both players are strategic, $p_{(st,st)}(v_b, v_s)$.
   - if only the buyer is strategic, $p_{(st,tr)}(v_b, v_s)$.
   - if only the seller is strategic, $p_{(tr,st)}(v_b, v_s)$.
   - if both players are trustworthy, $p_{(tr,tr)}(v_b, v_s)$.
2. Payment from the buyer to the seller:
   - if both players are strategic, $x_{(st,st)}(v_b, v_s)$.
   - if only the buyer is strategic, $x_{(st,tr)}(v_b, v_s)$.
   - if only the seller is strategic, $x_{(tr,st)}(v_b, v_s)$.
   - if both players are trustworthy, $x_{(tr,tr)}(v_b, v_s)$.

Trustworthy types communicate truthfully or act in ways that is not detrimental to the welfare of the truster. Hence, in the context of trading, it is natural to interpret trustworthiness as a behavior that facilitates trade. This interpretation is the motivation behind the following assumptions about trust.

Assumption 2.1 ***Assumptions about Trustworthy Types****:*

1. *in any equilibrium outcome of any trading mechanism, the expected payoff of any valuation type of trustworthy player is non-negative, that is, individual rationality for trustworthy types.*

2. *if players are asked about their type (valuation or disposition), then a trustworthy type answers truthfully. Therefore, in any direct trading mechanism in which each player is asked to report her type, a trustworthy type of player will report truthfully.*

3. *if players have no agreements prior to submitting their final bids, then a trustworthy type bids equal to her valuation. Shading by a buyer or exaggeration by a seller reduces the likelihood of trade and hence has a negative consequence on the truster's welfare. Therefore, in any k-double auction trustworthy types bid equal to their valuation.*

Define the following for all types (v_i, d_i),[5]

$$\bar{p}_{(b,d_b)}(v_b) \equiv (1-\epsilon_s)\int_{[\underline{a}_s,\bar{a}_s]} p_{(d_b,st)}(v_b,v_s)f_{st_s}dv_s + \epsilon_s \int_{[\underline{a}_s,\bar{a}_s]} p_{(d_b,tr)}(v_b,v_s)f_{tr_s}dv_s$$
$$\bar{p}_{(s,d_s)}(v_s) \equiv (1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]} p_{(st,d_s)}(v_b,v_s)f_{st_b}dv_b + \epsilon_b \int_{[\underline{a}_b,\bar{a}_b]} p_{(tr,d_b)}(v_b,v_s)f_{tr_b}dv_b$$
$$\bar{x}_{(b,d_b)}(v_b) \equiv (1-\epsilon_s)\int_{[\underline{a}_s,\bar{a}_s]} x_{(d_b,st)}(v_b,v_s)f_{st_s}dv_s + \epsilon_s \int_{[\underline{a}_s,\bar{a}_s]} x_{(d_b,tr)}(v_b,v_s)f_{tr_s}dv_s$$
$$\bar{x}_{(s,d_s)}(v_s) \equiv (1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]} x_{(st,d_s)}(v_b,v_s)f_{st_b}dv_b + \epsilon_b \int_{[\underline{a}_b,\bar{a}_b]} x_{(tr,d_s)}(v_b,v_s)f_{tr_b}dv_b$$

The payoffs of the players for an outcome of a trading mechanism are

$$U_{(b,d_b)}(v_b) = v_b\bar{p}_{(b,d_b)}(v_b) - \bar{x}_{(b,d_b)}(v_b)$$
$$U_{(s,d_b)}(v_s) = \bar{x}_{(s,d_b)}(v_s) - v_s\bar{p}_{(s,d_b)}(v_s)$$

Definition 2.2 *IC^*: An outcome of a trading mechanism is incentive compatible* for strategic types if*

$$\forall\ v_b, v_b',\ \ U_{(b,st)}(v_b) \geq v_b\bar{p}_{(b,st)}(v_b') - \bar{x}_{(b,st)}(v_b')$$
$$\forall\ v_s, v_s',\ \ U_{(s,st)}(v_s) \geq \bar{x}_{(s,st)}(v_s') - v_s\bar{p}_{(s,st)}(v_s')$$

An IC^* outcome is such that no valuation type of strategic player will prefer to imitate the strategy of another valuation type of that strategic player. The following lemma gives an important necessary condition that any IC^* outcome of a trading mechanism must satisfy.

Lemma 2.3 *For any IC^* outcome of a trading mechanism it must be that $\bar{p}_{(b,st)}$ is weakly increasing, $\bar{p}_{(s,st)}$ is weakly decreasing and*

$$\begin{aligned}
&(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right] - \left[v_s + \frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right) p_{(st,st)}(v_b,v_s)f_{st_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(v_b - \left[v_s + \frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right) p_{(tr,st)}(v_b,v_s)f_{st_s}f_{tr_b}dv_sdv_b\\
&+\epsilon_s(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right] - v_s\right) p_{(st,tr)}(v_b,v_s)f_{tr_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b\epsilon_s\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}(v_b-v_s)p_{(tr,tr)}(v_b,v_s)f_{tr_s}f_{tr_b}dv_sdv_b\\
=&(1-\epsilon_b)U_{(b,st)}(\underline{a}_b) + (1-\epsilon_s)U_{(s,st)}(\bar{a}_s) + \epsilon_b\int_{[\underline{a}_b,\bar{a}_b]}U_{(b,tr)}(v_b)f_{tr_b}dv_b + \epsilon_s\int_{[\underline{a}_s,\bar{a}_s]}U_{(s,tr)}(v_s)f_{tr_s}dv_s
\end{aligned} \tag{1}$$

[5] I sometimes drop the argument v_i of the function f_{d_i} to simplify notation.

Proof: IC^* implies that for all v_b and v_b'

$$U_{(b,st)}(v_b) = v_b\bar{p}_{(b,st)}(v_b) - \bar{x}_{(b,st)}(v_b) \geq v_b\bar{p}_{(b,st)}(v_b') - \bar{x}_{(b,st)}(v_b')$$

and

$$U_{(b,st)}(v_b') = v_b'\bar{p}_{(b,st)}(v_b') - \bar{x}_{(b,st)}(v_b') \geq v_b'\bar{p}_{(b,st)}(v_b) - \bar{x}_{(b,st)}(v_b)$$

Therefore we must have,

$$(v_b - v_b')\bar{p}_{(b,st)}(v_b) \geq U_{(b,st)}(v_b) - U_{(b,st)}(v_b') \geq (v_b - v_b')\bar{p}_{(b,st)}(v_b')$$

Hence if $v_b > v_b'$ then it must be that $\bar{p}_{(b,st)}(v_b) \geq \bar{p}_{(b,st)}(v_b')$. This then implies that $\frac{dU_{(b,st)}}{dv_b} = \bar{p}_{(b,st)}(v_b)$ at almost all v_b. Therefore we get

$$U_{(b,st)}(v_b) = U_{(b,st)}(\underline{a}_b) + \int_{(\underline{a}_b, v_b]} \bar{p}_{(b,st)}(y_b)dy_b.$$

Similarly we can show that if $v_s > v_s'$ then it must be that $\bar{p}_{(s,st)}(v_s) \leq \bar{p}_{(s,st)}(v_s')$ and $U_{(s,st)}(v_s) = U_{(s,st)}(\bar{a}_s) + \int_{[v_s,\bar{a}_b)} \bar{p}_{(s,st)}(y_s)dy_s$.

Any mechanism that satisfies IC^* for the strategic players must be such that

$$\begin{aligned}
&(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}(v_b - v_s)p_{(st,st)}(v_b,v_s)f_{st_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}(v_b - v_s)p_{(tr,st)}(v_b,v_s)f_{st_s}f_{tr_b}dv_sdv_b\\
&+\epsilon_s(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}(v_b - v_s)p_{(st,tr)}(v_b,v_s)f_{tr_s}f_{st_s}dv_sdv_b\\
&+\epsilon_b\epsilon_s\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}(v_b - v_s)p_{(tr,tr)}(v_b,v_s)f_{tr_s}f_{tr_b}dv_sdv_b\\
=&(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}U_{(b,st)}(v_b)f_{st_b}dv_b + \epsilon_b\int_{[\underline{a}_b,\bar{a}_b]}U_{(b,tr)}(v_b)f_{tr_b}dv_b\\
&+(1-\epsilon_s)\int_{[\underline{a}_s,\bar{a}_s]}U_{(s,st)}(v_s)f_{st_s}dv_s + \epsilon_s\int_{[\underline{a}_s,\bar{a}_s]}U_{(s,tr)}(v_s)f_{tr_s}dv_s\\
=&(1-\epsilon_b)\left(U_{(b,st)}(\underline{a}_b) + \int_{[\underline{a}_b,\bar{a}_b]}\int_{(\underline{a}_b,v_b]}\bar{p}_{(b,st)}(y_b)dy_bf_{st_b}dv_b\right)\\
&+(1-\epsilon_s)\left(U_{(s,st)}(\bar{a}_s) + \int_{[\underline{a}_s,\bar{a}_s]}\int_{[v_s,\bar{a}_s)}\bar{p}_{(s,st)}(y_s)dy_sf_{st_s}dv_s\right)\\
&+\epsilon_b\int_{[\underline{a}_b,\bar{a}_b]}U_{(b,tr)}(v_b)f_{tr_b}dv_b + \epsilon_s\int_{[\underline{a}_s,\bar{a}_s]}U_{(s,tr)}(v_s)f_{tr_s}dv_s
\end{aligned}$$

$$
\begin{aligned}
=&(1-\epsilon_b)U_{(b,st)}(\underline{a}_b)+(1-\epsilon_s)U_{(s,st)}(\bar{a}_s)\\
&+\epsilon_b\int_{[\underline{a}_b,\bar{a}_b]}U_{(b,tr)}(v_b)f_{tr_b}dv_b+\epsilon_s\int_{[\underline{a}_s,\bar{a}_s]}U_{(s,tr)}(v_s)f_{tr_s}dv_s\\
&+(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\bar{p}_{(b,st)}(v_b)(1-F_{st_b}(v_b))dv_b+(1-\epsilon_s)\int_{[\underline{a}_s,\bar{a}_s]}\bar{p}_{(s,st)}(v_s)F_{st_s}(v_s)dv_s
\end{aligned}\tag{2}
$$

However,

$$
\begin{aligned}
&(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\bar{p}_{(b,st)}(v_b)(1-F_{st_b}(v_b))dv_b+(1-\epsilon_s)\int_{[\underline{a}_s,\bar{a}_s]}\bar{p}_{(s,st)}(v_s)F_{st_s}(v_s)dv_s\\
=&(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}p_{(st,st)}(v_b,v_s)(1-F_{st_b}(v_b))f_{st_s}dv_sdv_b\\
&+(1-\epsilon_b)\epsilon_s\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}p_{(st,tr)}(v_b,v_s)(1-F_{st_b}(v_b))f_{tr_s}dv_sdv_b\\
&+(1-\epsilon_s)(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}p_{(st,st)}(v_b,v_s)F_{st_s}(v_s)f_{st_b}dv_sdv_b\\
&+(1-\epsilon_s)\epsilon_b\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}p_{(tr,st)}(v_b,v_s)F_{st_s}(v_s)f_{tr_b}dv_sdv_b
\end{aligned}
$$

By subtracting the above equation from (2) we get,

$$
\begin{aligned}
&(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)p_{(st,st)}(v_b,v_s)f_{st_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(v_b-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)p_{(tr,st)}(v_b,v_s)f_{st_s}f_{tr_b}dv_sdv_b\\
&+\epsilon_s(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-v_s\right)p_{(st,tr)}(v_b,v_s)f_{tr_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b\epsilon_s\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}(v_b-v_s)p_{(tr,tr)}(v_b,v_s)f_{tr_s}f_{tr_b}dv_sdv_b\\
=&(1-\epsilon_b)U_{(b,st)}(\underline{a}_b)+(1-\epsilon_s)U_{(s,st)}(\bar{a}_s)+\epsilon_b\int_{[\underline{a}_b,\bar{a}_b]}U_{(b,tr)}(v_b)f_{tr_b}dv_b+\epsilon_s\int_{[\underline{a}_s,\bar{a}_s]}U_{(s,tr)}(v_s)f_{tr_s}dv_s
\end{aligned}
$$

■

Definition 2.4 *IC: An outcome of a trading mechanism is incentive compatible for strategic types if it is incentive compatible* and*

$$
\begin{aligned}
\forall\, v_b, v_b',\quad U_{(b,st)}(v_b) \geq v_b\bar{p}_{(b,tr)}(v_b')-\bar{x}_{(b,tr)}(v_b')\\
\forall\, v_s, v_s',\quad U_{(s,st)}(v_s) \geq \bar{x}_{(s,tr)}(v_s')-v_s\bar{p}_{(s,tr)}(v_s')
\end{aligned}
$$

By *Revelation Principle*, any Bayesian-Nash equilibrium outcome of a trading mechanism must satisfy IC, otherwise some valuation type of a strategic player will prefer to deviate and imitate another type of that player. Moreover, to ensure voluntary participation, all types of players must get non-negative payoffs. This is termed as individual rationality.

Definition 2.5 *IR: An outcome of a trading mechanism is individually rational for all players if*

$$\forall\ (v_i, d_i),\ \ U_{(i,d_i)}(v_i) \geq 0$$

Definition 2.6 *NUT: An outomce of a trading mechanism satisfies no undesirable trade if*

$$\forall\ (d_b, d_s),\ v_b < v_s \implies p_{(d_b,d_s)}(v_b, v_s) = 0$$

2.1 Characterization of Direct Mechanisms satisfying IC and IR

A direct mechanism is such that each player is asked to report her type and for each reported pair of type, it specifies an outcome. Hence, we can identify a direct mechanism with its outcome. By definition of trustworthy disposition, trustworthy types of players send truthful reports. Therefore, any direct mechanism that satisfies IC is such that truth-telling by all types of players is a Bayesian-Nash equilibrium. The next theorem characterizes the set of direct mechanisms that satisfy IC and IR for any (ϵ_b, ϵ_s).

Theorem 2.7 *Suppose $p_{(st,st)}, p_{(st,tr)}, p_{(tr,st)}, p_{(tr,tr)}$ are functions from $[\underline{a}_b, \bar{a}_b] \times [\underline{a}_s, \bar{a}_s]$ to $[0,1]$. Then there exist functions $x_{(st,st)}, x_{(st,tr)}, x_{(tr,st)}, x_{(tr,tr)}$ such that $(p_{(d_b,d_s)}, x_{(d_b,d_s)})_{d_b=st,tr;\ d_s=st,tr}$ is a direct mechanism that satisfies IC and IR if and only if $\bar{p}_{(b,st)}$ is weakly increasing, $\bar{p}_{(s,st)}$ is weakly decreasing and*

$$\begin{aligned}
&(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right] - \left[v_s + \frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right) p_{(st,st)}(v_b,v_s) f_{st_s} f_{st_b}\, dv_s dv_b \\
&+ \epsilon_b(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(v_b - \left[v_s + \frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right) p_{(tr,st)}(v_b,v_s) f_{st_s} f_{tr_b}\, dv_s dv_b \\
&+ \epsilon_s(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right] - v_s\right) p_{(st,tr)}(v_b,v_s) f_{tr_s} f_{st_b}\, dv_s dv_b \\
&+ \epsilon_b\epsilon_s\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}(v_b - v_s) p_{(tr,tr)}(v_b,v_s) f_{tr_s} f_{tr_b}\, dv_s dv_b \\
&\geq\ 0
\end{aligned} \tag{3}$$

Proof: From lemma 2.3, we get that any direct mechanism that satisfies IC must be such that $\bar{p}_{(b,st)}$ is weakly increasing, $\bar{p}_{(s,st)}$ is weakly decreasing and it satisfies condition (1). Since the direct mechanism also satisfies IR, the left-hand side of condition (1) must be non-negative, which proves the if part.

To prove the only if part, consider the following cases:

1. $\epsilon_b < 1$ and $\epsilon_s < 1$. Define

$$\begin{aligned}
x_{(st,st)}(v_b, v_s) =& \frac{1}{1-\epsilon_s} \int_{[\underline{a}_b, v_b]} y_b d\bar{p}_{(b,st)}(y_b) + \frac{1}{1-\epsilon_b} \int_{[\underline{a}_s, v_s]} y_s d\bar{p}_{(s,st)}(y_s) \\
&+ \frac{1}{1-\epsilon_s} \underline{a}_b \bar{p}_b(\underline{a}_b) - \frac{1}{1-\epsilon_b} \int_{[\underline{a}_s, \bar{a}_s]} y_s (1 - F_{st_s}(y_s)) d\bar{p}_{(s,st)}(y_s) \\
&- \frac{\epsilon_s}{(1-\epsilon_b)(1-\epsilon_s)} \int_{[\underline{a}_s, \bar{a}_s]} y_s \bar{p}_{(s,tr)}(y_s) f_{tr_s}(y_s) dy_s \\
x_{(st,tr)}(v_b, v_s) =& \frac{1}{1-\epsilon_b} v_s \bar{p}_{(s,tr)}(v_s) \\
x_{(tr,st)}(v_b, v_s) =& \frac{1}{1-\epsilon_s} v_b \bar{p}_{(b,tr)}(v_b) \\
x_{(tr,tr)}(v_b, v_s) =& 0
\end{aligned}$$

It is easy to calculate that:

$$\begin{aligned}
\bar{x}_{(b,st)}(v_b) =& \int_{[\underline{a}_b, v_b]} y_b d\bar{p}_{(b,st)}(y_b) + \frac{1-\epsilon_s}{1-\epsilon_b} \int_{[\underline{a}_s, \bar{a}_s]} \int_{[\underline{a}_s, v_s]} y_s f_{st_s}(v_s) d\bar{p}_{(s,st)}(y_s) dv_s \\
&+ \underline{a}_b \bar{p}_b(\underline{a}_b) - \frac{1-\epsilon_s}{1-\epsilon_b} \int_{[\underline{a}_s, \bar{a}_s]} y_s (1 - F_{st_s}(y_s)) d\bar{p}_{(s,st)}(y_s) \\
&- \frac{\epsilon_s}{1-\epsilon_b} \int_{[\underline{a}_s, \bar{a}_s]} y_s \bar{p}_{(s,tr)}(y_s) f_{tr_s}(y_s) dy_s + \frac{\epsilon_s}{1-\epsilon_b} \int_{[\underline{a}_s, \bar{a}_s]} v_s \bar{p}_{(s,tr)}(v_s) f_{tr_s}(v_s) dv_s \\
=& \int_{[\underline{a}_b, v_b]} y_b d\bar{p}_{(b,st)}(y_b) + \frac{1-\epsilon_s}{1-\epsilon_b} \int_{[\underline{a}_s, \bar{a}_s]} y_s (1 - F_{st_s}(y_s)) d\bar{p}_{(s,st)}(y_s) \\
&+ \underline{a}_b \bar{p}_b(\underline{a}_b) - \frac{1-\epsilon_s}{1-\epsilon_b} \int_{[\underline{a}_s, \bar{a}_s]} y_s (1 - F_{st_s}(y_s)) d\bar{p}_{(s,st)}(y_s) \\
=& \int_{[\underline{a}_b, v_b]} y_b d\bar{p}_{(b,st)}(y_b) + \underline{a}_b \bar{p}_b(\underline{a}_b)
\end{aligned}$$

$$\begin{aligned}
\bar{x}_{(s,st)}(v_s) =& \frac{1-\epsilon_b}{1-\epsilon_s} \int_{[\underline{a}_b, \bar{a}_b]} \int_{[\underline{a}_b, v_b]} y_b f_{st_b}(v_b) d\bar{p}_{(b,st)}(y_b) dv_b + \int_{[\underline{a}_s, v_s]} y_s d\bar{p}_{(s,st)}(y_s) \\
&+ \frac{1-\epsilon_b}{1-\epsilon_s} \underline{a}_b \bar{p}_b(\underline{a}_b) - \int_{[\underline{a}_s, \bar{a}_s]} y_s (1 - F_{st_s}(y_s)) d\bar{p}_{(s,st)}(y_s) \\
&- \frac{\epsilon_s}{1-\epsilon_s} \int_{[\underline{a}_s, \bar{a}_s]} y_s \bar{p}_{(s,tr)}(y_s) f_{tr_s}(y_s) dy_s
\end{aligned}$$

$$\bar{x}_{(b,tr)}(v_b) = v_b \bar{p}_{(b,tr)}(v_b)$$

$$\bar{x}_{(s,tr)}(v_s) = v_s \bar{p}_{(s,tr)}(v_s)$$

Payoffs of strategic players:

$$
\begin{aligned}
U_{(b,st)}(v_b) =& v_b\bar{p}_{(b,st)}(v_b) - \int_{[\underline{a}_b, v_b]} y_b d\bar{p}_{(b,st)}(y_b) - \underline{a}_b\bar{p}_b(\underline{a}_b) \\
U_{(s,st)}(v_s) =& - v_s\bar{p}_{(s,st)}(v_s) + \frac{1-\epsilon_b}{1-\epsilon_s}\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_b,v_b]} y_b f_{st_b}(v_b) d\bar{p}_{(b,st)}(y_b) dv_b + \int_{[\underline{a}_s,v_s]} y_s d\bar{p}_{(s,st)}(y_s) \\
&+ \frac{1-\epsilon_b}{1-\epsilon_s}\underline{a}_b\bar{p}_b(\underline{a}_b) - \int_{[\underline{a}_s,\bar{a}_s]} y_s(1-F_{st_s}(y_s)) d\bar{p}_{(s,st)}(y_s) \\
&- \frac{\epsilon_s}{1-\epsilon_s}\int_{[\underline{a}_s,\bar{a}_s]} y_s\bar{p}_{(s,tr)}(y_s) f_{tr_s}(y_s) dy_s
\end{aligned}
$$

Individual rationality for trustworthy players:

$$
\begin{aligned}
U_{(b,tr)}(v_b) = v_b\bar{p}_{(b,tr)}(v_b) - \bar{x}_{(b,tr)}(v_b) = v_b\bar{p}_{(b,tr)}(v_b) - v_b\bar{p}_{(b,tr)}(v_b) = 0 \\
U_{(s,tr)}(v_s) = \bar{x}_{(s,tr)}(v_s) - v_s\bar{p}_{(s,tr)}(v_s) = v_s\bar{p}_{(s,tr)}(v_s) - v_s\bar{p}_{(s,tr)}(v_s) = 0
\end{aligned}
$$

Incentive compatibility for strategic players:
First we check that no strategic player would lie about his valuation while she announces her disposition type truthfully. For all $v_b' < v_b$, we have (similar argument works if $v_b' > v_b$)

$$
\begin{aligned}
& v_b\bar{p}_{(b,st)}(v_b) - \bar{x}_{(b,st)}(v_b) - (v_b\bar{p}_{(b,st)}(v_b') - \bar{x}_{(b,st)}(v_b')) \\
&= v_b(\bar{p}_{(b,st)}(v_b) - \bar{p}_{(b,st)}(v_b')) - \int_{[v_b',v_b]} y_b d\bar{p}_{(b,st)}(y_b) \\
&= \int_{[v_b',v_b]} (v_b - y_b) d\bar{p}_{(b,st)}(y_b) \geq 0
\end{aligned}
$$

For all $v_s' > v_s$, we have (similar argument works if $v_s' < v_s$)

$$
\begin{aligned}
& \bar{x}_{(s,st)}(v_s) - v_s\bar{p}_{(s,st)}(v_s) - (\bar{x}_{(s,st)}(v_s') - v_s\bar{p}_{(s,st)}(v_s')) \\
&= \int_{[\underline{a}_s,v_s]} y_s d\bar{p}_{(s,st)}(y_s) - v_s\bar{p}_{(s,st)}(v_s) - \int_{[\underline{a}_s,v_s']} y_s d\bar{p}_{(s,st)}(y_s) + v_s\bar{p}_{(s,st)}(v_s') \\
&= v_s(\bar{p}_{(s,st)}(v_s') - \bar{p}_{(s,st)}(v_s)) - \int_{[v_b,v_b']} y_s d\bar{p}_{(s,st)}(y_s) \\
&= \int_{[v_s,v_s']} (v_s - y_s) d\bar{p}_{(s,st)}(y_s) \geq 0
\end{aligned}
$$

Therefore, the constructed mechanism satisfies (1). Since the functions $p_{(st,st)}, p_{(st,tr)}, p_{(tr,st)}, p_{(tr,tr)}$ satisfy (3), $U_{(b,st)}(\underline{a}_b) = 0$, $U_{(b,tr)}(v_b) = 0 \ \forall \ v_b$ and $U_{(s,tr)}(v_s) = 0 \ \forall \ v_s$, it implies that $U_{(s,tr)}(\bar{a}_s) \geq 0$. It is easy to see that $U_{(b,st)}(v_b)$ is non-decreasing and $U_{(s,st)}(v_s)$ is non-increasing, the mechanism satisfies individual rationality for strategic players.

Finally, since all trustworthy players' expected payoff is 0, no strategic player can do better by announcing herself as trustworthy type.

2. $\epsilon_b = 1$ and $\epsilon_s < 1$. Define

$$\begin{aligned}x_{(tr,st)}(v_b, v_s) =& \int_{[\underline{a}_s, v_s]} y_s d\bar{p}_{(s,st)}(y_s) - \int_{[\underline{a}_s, \bar{a}_s]} y_s(1 - F_{st_s}(y_s)) d\bar{p}_{(s,st)}(y_s) \\ &- \frac{\epsilon_s}{1-\epsilon_s} \int_{[\underline{a}_s, \bar{a}_s]} y_s \bar{p}_{(s,tr)}(y_s) f_{tr_s}(y_s) dy_s + \frac{1}{1-\epsilon_s} v_b \bar{p}_{(b,tr)}(v_b) \\ x_{(tr,tr)}(v_b, v_s) =& v_s \bar{p}_{(s,tr)}(v_s)\end{aligned}$$

3. $\epsilon_b < 1$ and $\epsilon_s = 1$. Define

$$\begin{aligned}x_{(st,tr)}(v_b, v_s) =& \int_{[\underline{a}_b, v_b]} y_b d\bar{p}_{(b,st)}(y_b) - \int_{[\underline{a}_b, \bar{a}_b]} y_b(1 - F_{st_b}(y_b)) d\bar{p}_{(b,st)}(y_b) \\ &- \frac{\epsilon_b}{1-\epsilon_b} \int_{[\underline{a}_b, \bar{a}_b]} y_b \bar{p}_{(b,tr)}(y_b) f_{tr_b}(y_b) dy_b + \frac{1}{1-\epsilon_b} v_s \bar{p}_{(s,tr)}(v_s) \\ x_{(tr,tr)}(v_b, v_s) =& v_b \bar{p}_{(b,tr)}(v_b)\end{aligned}$$

4. $\epsilon_b = \epsilon_s = 1$. Trivial. ■

In the following sections, I study the influence of the *degree of trust*, (ϵ_b, ϵ_s), and *distribution of trust*, (F_{tr_b}, F_{tr_s}) on the ex-ante probability of trade among strategic types. Precisely, for any given bilateral trading problem, I am interested in direct mechanisms that solve the following problem:[6]

$$\begin{aligned}&\max \int_{[\underline{a}_b, \bar{a}_b]} \int_{[\underline{a}_s, \bar{a}_s]} p_{(st,st)} f_{st_s} f_{st_b} dv_s dv_b \\ &\text{subject to,} \\ &IC,\ IR,\ NUT \text{ and Assumption 2.1.}\end{aligned} \tag{4}$$

I then compare these optimal direct mechanisms with k-double auctions.

3 *Degree of Trust* and Trade

In this section, I fix the *distribution of trust*, $(F_{tr_i})_{i=b,s}$, and ask the following questions: first, is higher *degree of trust* always better? Second, do there exist trading mechanisms that achieve ex-post efficiency if we allow for positive *degree of trust*?

[6] It is easy to show that a direct mechanism solves this problem only if it is ex-post incentive efficient in the ex-post event that both players are strategic within the set of direct mechanisms satisfying IR, NUT and Assumption 2.1. Also, none of the results will change if instead the objective function is the ex-ante gains from trade in the event that both players are strategic.

3.1 Is More Trust Always Better?

This subsection proves that the answer depends on the trading mechanism. Proposition 3.1 proves that if we use direct mechanisms to solve the trading problem, then an increase in the *degree of trust* is weakly better in the sense that we can design a new direct mechanism that will have at least as high probability of trade among strategic types as before. In contrast, section 3.1.1 shows that for a bilateral trading problem with uniform distribution of valuations of all disposition types, any positive *degree of trust* leads to a lower probability of trade among strategic types in any equilibrium outcome of any k-double auction compared to the highest achievable probability of trade using k-double auctions when the *degree of trust* is 0.

Proposition 3.1 *Suppose for the bilateral trading problem* $(\epsilon_b, \epsilon_s, F_{st_b}, F_{st_s}, F_{tr_b}, F_{tr_s})$*, the direct mechanism* $(p_{(d_b,d_s)}, x_{(d_b,d_s)})_{d_b=st,tr;\, d_s=st,tr}$ *satisfies IC, IR, NUT and Assumption 2.1. Consider any* $(\epsilon_b', \epsilon_s') \geq (\epsilon_b, \epsilon_s)$*. Then there exists a direct mechanism* $(p'_{(d_b,d_s)}, x'_{(d_b,d_s)})_{d_b=st,tr;\, d_s=st,tr}$ *for the bilateral trading problem* $(\epsilon_b', \epsilon_s', F_{st_b}, F_{st_s}, F_{tr_b}, F_{tr_s})$ *satisfying IC, IR, NUT and Assumption 2.1 and such that*

$$\int_{[\underline{a}_b,\bar{a}_b]} \int_{[\underline{a}_s,\bar{a}_s]} p'_{(st,st)}(v_b, v_s) f_{st_s} f_{st_b} dv_s dv_b \geq \int_{[\underline{a}_b,\bar{a}_b]} \int_{[\underline{a}_s,\bar{a}_s]} p_{(st,st)}(v_b, v_s) f_{st_s} f_{st_b} dv_s dv_b$$

Proof: Without loss of generality we can assume that

$$p_{(tr,tr)}(v_b, v_s) = \begin{cases} 1 & \text{if } v_b \geq v_s \\ 0 & \text{if } v_b < v_s \end{cases}$$

For all (v_b, v_s), define the following:

$$\begin{aligned}
p'_{(st,st)}(v_b, v_s) &= p_{(st,st)}(v_b, v_s) \\
p'_{(tr,tr)}(v_b, v_s) &= p_{(tr,tr)}(v_b, v_s) \\
p'_{(st,tr)}(v_b, v_s) &= \frac{\epsilon_s(1-\epsilon_s')}{\epsilon_s'(1-\epsilon_s)} p_{(st,tr)}(v_b, v_s) \leq p_{(st,tr)}(v_b, v_s) \\
p'_{(tr,st)}(v_b, v_s) &= \frac{\epsilon_b(1-\epsilon_b')}{\epsilon_b'(1-\epsilon_b)} p_{(tr,st)}(v_b, v_s) \leq p_{(tr,st)}(v_b, v_s)
\end{aligned}$$

It follows that

$$\begin{aligned}
\bar{p}'_{(s,st)}(v_s) &= (1-\epsilon_b') \int_{[\underline{a}_b,\bar{a}_b]} p_{(st,st)}(v_b, v_s) f_{st_b} dv_b + \epsilon_b' \int_{[\underline{a}_b,\bar{a}_b]} \frac{\epsilon_b(1-\epsilon_b')}{\epsilon_b'(1-\epsilon_b)} p_{(tr,st)}(v_b, v_s) f_{tr_b} dv_b \\
&= \frac{1-\epsilon_b'}{1-\epsilon_b} \left((1-\epsilon_b) \int_{[\underline{a}_b,\bar{a}_b]} p_{(st,st)}(v_b, v_s) f_{st_b} dv_b + \epsilon_b \int_{[\underline{a}_b,\bar{a}_b]} p_{(tr,st)}(v_b, v_s) f_{tr_b} dv_b \right) \\
&= \frac{1-\epsilon_b'}{1-\epsilon_b} \bar{p}_{(s,st)}(v_s)
\end{aligned}$$

Similarly, $\bar{p}'_{(b,st)}(v_b) = \frac{1-\epsilon'_s}{1-\epsilon_s}\bar{p}_{(b,st)}(v_b)$. Therefore, $\bar{p}'_{(b,st)}(v_b)$ is weakly increasing and $\bar{p}'_{(s,st)}(v_s)$ is weakly decreasing. So we only need to check that $(p'_{(d_b,d_s)})_{d_b=st,tr\, d_s=st,tr}$ satisfies condition (3). To simplify notation, define

$$\gamma_{(st,st)} = (1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right] - \left[v_s + \frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right) p_{(st,st)}(v_b,v_s) f_{st_s} f_{st_b} dv_s dv_b$$

$$\gamma_{(st,tr)} = (1-\epsilon_b)\epsilon_s\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right] - v_s\right) p_{(st,tr)}(v_b,v_s) f_{tr_s} f_{st_b} dv_s dv_b$$

$$\gamma_{(tr,st)} = (1-\epsilon_s)\epsilon_b\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(v_b - \left[v_s + \frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right) p_{(tr,st)}(v_b,v_s) f_{st_s} f_{tr_b} dv_s dv_b$$

$$\gamma_{(tr,tr)} = \epsilon_b\epsilon_s\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}(v_b - v_s) p_{(tr,tr)}(v_b,v_s) f_{tr_s} f_{tr_b} dv_s dv_b$$

With $(p'_{(d_b,d_s)})_{d_b=st,tr\, d_s=st,tr}$, condition (3) simplifies to

$$\begin{aligned}
&\frac{(1-\epsilon'_b)(1-\epsilon'_s)}{(1-\epsilon_b)(1-\epsilon_s)}\left(\gamma_{(st,st)} + \gamma_{(st,tr)} + \gamma_{(tr,st)}\right) + \epsilon'_b\epsilon'_s\gamma_{(tr,tr)} \\
&\geq \frac{(1-\epsilon'_b)(1-\epsilon'_s)}{(1-\epsilon_b)(1-\epsilon_s)}\left(\gamma_{(st,st)} + \gamma_{(st,tr)} + \gamma_{(tr,st)}\right) + \epsilon_b\epsilon_s\gamma_{(tr,tr)} \\
&\geq \frac{(1-\epsilon'_b)(1-\epsilon'_s)}{(1-\epsilon_b)(1-\epsilon_s)}\left(\gamma_{(st,st)} + \gamma_{(st,tr)} + \gamma_{(tr,st)} + \epsilon_b\epsilon_s\gamma_{(tr,tr)}\right) \\
&\geq 0,
\end{aligned}$$

where the first inequality follows from $\gamma_{(tr,tr)} \geq 0$ and the second inequality uses $\frac{(1-\epsilon'_b)(1-\epsilon'_s)}{(1-\epsilon_b)(1-\epsilon_s)} < 1$. ■

3.1.1 k-Double Auction with Uniform *Distribution of Trust*

For a bilateral trading problem with only strategic types and valuations distributed uniformly and independently on $[0,1]$, Myerson and Sattethwaite (1983) prove that the highest ex-ante probability of trade achievable in any equilibrium of any trading mechanism equals 9/32. This upper bound is achieved in a 1/2-double auction when players play the Chatterjee-Samuelson linear strategies (C-S equilibrium). This section proves that when the valuations of all types are distributed uniformly and independently on $[0,1]$, then for any positive *degree of trust*, any Bayesian-Nash equilibrium outcome of any k-double auction will have lower ex-ante probability of trade between strategic types than C-S equilibrium. Thus higher *degree of trust* is not necessarily better for the set of k-double auction trading mechanisms.

So now $[\underline{a}_s,\bar{a}_s] = [\underline{a}_b,\bar{a}_b] = [0,1]$. Also, $\epsilon_b = \epsilon_s = \epsilon$ and $F_{st_b} = F_{st_s} = F_{tr_b} = F_{tr_s}$ are uniform on $[0,1]$. The trading mechanism is a k-double auction in which both players simultaneously submit sealed bids. If the buyer's bid p_b is greater than or equal to seller's bid p_s, then trade takes place at price $kp_b + (1-k)p_s$, where $k \in [0,1]$; otherwise, there is no trade and no payment by the buyer to the seller. By assumption, the trustworthy types bid truthfully. Without loss of generality, assume that bids of the players lie in the interval $[0, 1]$. For $\epsilon > 0$, let Γ^ϵ denote the game defined by this trading mechanism.

Unless otherwise noted, t_i is the bid of player i, $\sigma_i(v_i)$ is the equilibrium strategy (mixed or pure) of strategic type of player i with valuation type v_i, $Supp(\sigma_i(v_i))$ the support of $\sigma_i(v_i)$ and G_i is the distribution of bids induced by σ_i. $\underline{t}_s = \sup\{t_i | G_i(t_i) = 0\}$ and $\bar{t}_i = \inf\{t_i | G_i(t_i) = 1\}$.

The next lemma is a monotonicity property of bids of strategic types in any equilibrium of Γ^ϵ. It says that any bid in the support of the equilibrium strategy of a valuation type of strategic player is at least as high as any bid in the support of the equilibrium strategy of any lower valuation type of that strategic player.

Lemma 3.2 *In any equilibrium of* Γ^ϵ*, the strategy of the strategic players,* (σ_b, σ_s)*, are such that*

1. $v_b > \hat{v}_b$ *then* $t_b \in Supp(\sigma_b(v_b))$ *and* $\hat{t}_b \in Supp(\sigma_b(\hat{v}_b)) \implies t_b \geq \hat{t}_b$.
2. $v_s < \hat{v}_s$ *then* $t_s \in Supp(\sigma_s(v_s))$ *and* $\hat{t}_s \in Supp(\sigma_s(\hat{v}_s)) \implies t_s \leq \hat{t}_s$.

Proof: To prove this, pick a $v_b > \hat{v}_b$ and let $t_b \in Supp(\sigma_b(v_b))$ and $\hat{t}_b \in Supp(\sigma_b(\hat{v}_b))$. Then the following inequalities are true:

$$\begin{aligned}(1-\epsilon)\int_{[\underline{t}_s, t_b]}(v_b - (kt_b + (1-k)t_s))dG_s + \epsilon\int_{[0,t_b]}(v_b - (kt_b + (1-k)v_s))dF_{tr_s} \geq \\ (1-\epsilon)\int_{[\underline{t}_s, \hat{t}_b]}(v_b - (k\hat{t}_b + (1-k)t_s))dG_s + \epsilon\int_{[0,\hat{t}_b]}(v_b - (k\hat{t}_b + (1-k)v_s))dF_{tr_s}\end{aligned} \tag{5}$$

$$\begin{aligned}(1-\epsilon)\int_{[\underline{t}_s, \hat{t}_b]}(\hat{v}_b - (k\hat{t}_b + (1-k)t_s))dG_s + \epsilon\int_{[0,\hat{t}_b]}(\hat{v}_b - (k\hat{t}_b + (1-k)v_s))dF_{tr_s} \geq \\ (1-\epsilon)\int_{[\underline{t}_s, t_b]}(\hat{v}_b - (kt_b + (1-k)t_s))dG_s + \epsilon\int_{[0,t_b]}(\hat{v}_b - (kt_b + (1-k)v_s))dF_{tr_s}\end{aligned} \tag{6}$$

Multiplying (6) by -1 and adding it to (5) we get,

$$(v_b - \hat{v}_b)((1-\epsilon)G_s(t_b) + \epsilon F_{tr_s}(t_b)) \geq (v_b - \hat{v}_b)((1-\epsilon)G_s(\hat{t}_b) + \epsilon F_{tr_s}(\hat{t}_b)) \tag{7}$$

Since $v_b > \hat{v}_b$, then as a consequence of (7), it must be true that $(1-\epsilon)G_s(t_b) + \epsilon F_{tr_s}(t_b) \geq (1-\epsilon)G_s(\hat{t}_b) + \epsilon F_{tr_s}(\hat{t}_b)$. But, G_s is non-decreasing and F_{tr_s} is strictly increasing on $[0, 1]$, and therefore, $t_b \geq \hat{t}_b$. ■

Now, I define a set of direct mechanisms that contains any equilibrium outcome of Γ^ϵ when $k \in (0, 1)$.

Definition 3.3 *Let* $\mathcal{M}^\epsilon$ *be the set of direct mechanisms that satisfy* IC^**, individual rationality for strategic types and*

- *R1:* $\forall (v_b, v_s)$ *and* (d_i, d_j), $0 \le x^{\epsilon}_{(d_i,d_j)}(v_b, v_s) \le 1$.
- *R2:*

$$\sup_{v_b} \int_{[0,1]} p^{\epsilon}_{(st,tr)}(v_b, v_s)dv_s < 1, \quad \sup_{v_b} \int_{[0,1]} x^{\epsilon}_{(st,tr)}(v_b, v_s)dv_s < 1$$
$$\sup_{v_s} \int_{[0,1]} p^{\epsilon}_{(tr,st)}(v_b, v_s)dv_b < 1, \quad \sup_{v_s} \int_{[0,1]} x^{\epsilon}_{(tr,st)}(v_b, v_s)dv_b < 1$$

- *R3:*
$\int_{[0,1]} \int_{[0,1]} p^{\epsilon}_{(st,st)}(v_b, v_s)dv_s dv_b \le \min\left\{\int_{[0,1]} \int_{[0,1]} p^{\epsilon}_{(st,tr)}(v_b, v_s)dv_s dv_b, \int_{[0,1]} \int_{[0,1]} p^{\epsilon}_{(tr,st)}(v_b, v_s)dv_s dv_b\right\}$
- *R4: Define,* $\bar{p}^{\epsilon}_b(v_b) = \int_{[0,1]} p^{\epsilon}_{(st,st)}(v_b, v_s)\, dv_s$ *and* $\bar{p}^{\epsilon}_s(v_s) = \int_{[0,1]} p^{\epsilon}_{(st,st)}(v_b, v_s)\, dv_b$. $\bar{p}^{\epsilon}_b$ *is non-decreasing and* $\bar{p}^{\epsilon}_s$ *is non-increasing.*

Lemma 3.4 *Fix* $k \in (0,1)$. *Suppose that for some* $\epsilon > 0$, *the strategy pair* (σ_b, σ_s) *is a Bayesian-Nash equilibrium of the game* Γ^{ϵ}. *Then there exists a direct mechanism* $M^{\epsilon} \in \mathcal{M}^{\epsilon}$ *that generates the same outcome as* (σ_b, σ_s).

Proof: Consider the outcome $(p^{\epsilon}_{(d_b,d_s)}, x^{\epsilon}_{(d_b,d_s)})_{d_b=st,tr;\, d_s=st,tr}$ generated by the given equilibrium pair of strategies for the strategic types, (σ_b, σ_s). It is straightforward to check IC^* and individual rationality for the strategic types. R1 follows from the assumption that bids of players are in the interval $[0,1]$. Since $Supp(\sigma^{\epsilon}_b(v_b)) \subseteq [0, v_b]$ and $Supp(\sigma^{\epsilon}_s(v_s)) \subseteq [1, v_s]$ and because $Supp(\sigma^{\epsilon}_b(1)) \subset [0,1)$ and $Supp(\sigma^{\epsilon}_s(0)) \subset [1,0)$ (follows from the fact that the trustworthy types always bid equal to their valuation and lemma 3.11), restrictions R2 and R3 are also satisfied. Finally, R4 follows from lemma 3.2. ■

We need one more lemma before showing that for any $\epsilon > 0$ any equilibrium of Γ^{ϵ} has a lower probability of trade among the strategic types compared to the probability of trade in the C-S equilibrium.

Lemma 3.5 *Suppose for some* $\epsilon > 0$, $M^{\epsilon} \in \mathcal{M}^{\epsilon}$. *Then there exists a* $\hat{\epsilon} < \epsilon$ *and a mechanism* $M^{\hat{\epsilon}} \in \mathcal{M}^{\hat{\epsilon}}$, *such that the ex-ante probability of trade in the event that at least one player is strategic is higher in* $M^{\hat{\epsilon}}$ *than in* M^{ϵ}.

Proof: Let

$$M^{\epsilon} = \{(p^{\epsilon}_{(st,st)}, p^{\epsilon}_{(st,tr)}, p^{\epsilon}_{(tr,st)}, p^{\epsilon}_{(tr,tr)}), (x^{\epsilon}_{(st,st)}, x^{\epsilon}_{(st,tr)}, x^{\epsilon}_{(tr,st)}, x^{\epsilon}_{(tr,tr)})\}.$$

Define the following:

1. $x^{\epsilon-h}_{(st,st)}(v_b, v_s) = \frac{1-\epsilon}{1-\epsilon+h} x^{\epsilon}_{(st,st)}(v_b, v_s)$ and $p^{\epsilon-h}_{(st,st)}(v_b, v_s) = \frac{1-\epsilon}{1-\epsilon+h} p^{\epsilon}_{(st,st)}(v_b, v_s)$.
2. $x^{\epsilon-h}_{(st,tr)}(v_b, v_s) = \frac{\epsilon}{\epsilon-h} \int_{[0,1]} x^{\epsilon}_{(st,tr)}(v_b, v_s)dv_s$ and $p^{\epsilon-h}_{(st,tr)}(v_b, v_s) = \frac{\epsilon}{\epsilon-h} \int_{[0,1]} p^{\epsilon}_{(st,tr)}(v_b, v_s)dv_s$.
3. $x^{\epsilon-h}_{(tr,st)}(v_b, v_s) = \frac{\epsilon}{\epsilon-h} \int_{[0,1]} x^{\epsilon}_{(tr,st)}(v_b, v_s)dv_b$ and $p^{\epsilon-h}_{(tr,st)}(v_b, v_s) = \frac{\epsilon}{\epsilon-h} \int_{[0,1]} p^{\epsilon}_{(tr,st)}(v_b, v_s)dv_b$.

4. $x^{\epsilon-h}_{(tr,tr)}(v_b, v_s) = x^{\epsilon}_{(tr,tr)}(v_b, v_s)$ and $p^{\epsilon-h}_{(tr,tr)}(v_b, v_s) = p^{\epsilon}_{(tr,tr)}(v_b, v_s)$.

So, for all (i, v_i), we now have $U^{\epsilon-h}_{(i,st)}(v_i) = U^{\epsilon}_{(i,st)}(v_i)$. Since $M^\epsilon \in \mathcal{M}^\epsilon$, there exists a small enough $h^* > 0$ such that $M^{\epsilon-h^*} \in \mathcal{M}^{\epsilon-h^*}$. Let $\hat{\epsilon} = \epsilon - h^*$.

The ex-ante probability of trade in the event that at least one player is strategic in $M^{\hat{\epsilon}}$ is defined as,

$$\frac{1}{(1-\hat{\epsilon})^2 + 2\hat{\epsilon}(1-\hat{\epsilon})} \int_{[0,1]} \int_{[0,1]} \{(1-\hat{\epsilon})^2 p^{\hat{\epsilon}}_{(st,st)} + (1-\hat{\epsilon})\hat{\epsilon}(p^{\hat{\epsilon}}_{(st,tr)} + p^{\hat{\epsilon}}_{(tr,st)})\} dv_b dv_s$$

$$= \frac{1}{1+\hat{\epsilon}} \int_{[0,1]} \int_{[0,1]} \{(1-\epsilon) p^{\epsilon}_{(st,st)} + \epsilon(p^{\epsilon}_{(st,tr)} + p^{\epsilon}_{(tr,st)})\} dv_b dv_s$$

$$> \frac{1}{1+\epsilon} \int_{[0,1]} \int_{[0,1]} \{(1-\epsilon) p^{\epsilon}_{(st,st)} + \epsilon(p^{\epsilon}_{(st,tr)} + p^{\epsilon}_{(tr,st)})\} dv_b dv_s$$

$$= \frac{1}{(1-\epsilon)^2 + 2\epsilon(1-\epsilon)} \int_{[0,1]} \int_{[0,1]} (1-\epsilon)^2 p^{\epsilon}_{(st,st)} + (1-\epsilon)\epsilon(p^{\epsilon}_{(st,tr)} + p^{\epsilon}_{(tr,st)}) dv_b dv_s$$

The last expression is the ex-ante probability of trade in the event that at least one player is strategic in M^ϵ. ■

Finally, the next proposition proves that the probability of trade among strategic types in any equilibrium of Γ^ϵ is strictly lower than the probability of trade in C-S equilibrium.

Proposition 3.6 *$\forall\, \epsilon > 0$, the probability of trade in the event that both players are strategic in any equilibrium of Γ^ϵ is less than the the ex-ante probability of trade in the C-S equilibrium.*

Proof:

Case 1: $k = 0 \vee 1$.

Suppose $k = 0$. $\forall\, \epsilon > 0$, a strategic type of buyer strictly prefers to bid her valuation v_b than any other bid in Γ^ϵ. If she bids $t_b > v_b$, then she trades with all valuation types of trustworthy seller who bid in the interval (v_b, t_b) and ends up paying a price greater than her valuation on those trades. If she bids $t_b < v_b$, then she does not trade with all valuation types of trustworthy seller who bid in the interval (t_b, v_b) whereas by bidding equal to her valuation she would have traded with these valuation types of trustworthy seller without changing the price on any trade with any other type of seller.

So, from the point of view of strategic seller, the buyer's bid is uniformly distributed on $[0, 1]$ since both disposition types of buyer bid equal to their valuation. Then, it is straightforward to see that the strategic seller will bid $\frac{1}{2}(1 + v_s)$. Therefore, the ex-ante probability of trade in the event that both players are strategic type is $\frac{1}{4} < \frac{9}{32}$. A similar proof works if $k = 1$.

Case 2: $k \in (0, 1)$.

Suppose there exists an $\hat{\epsilon} > 0$ and an equilibrium which has at least as high a probability of trade in the event that both players are strategic than the ex-ante probability of trade in the C-S equilibrium. Consider the equivalent direct mechanism for that equilibrium,

$M^{\hat{\epsilon}} \in \mathcal{M}^{\hat{\epsilon}}$. By R3, the ex-ante probability of trade in the event that at least one player is strategic in $M^{\hat{\epsilon}}$ is also greater than or equal to 9/32. Using lemma 3.5, one can construct a sequence of direct mechanisms, $M^{\epsilon_n} \in \mathcal{M}^{\epsilon_n}$ with $\epsilon_n < \hat{\epsilon}$ and $\epsilon_n \to 0$, such that the ex-ante probability of trade in the event that at least one player is strategic in $M^{\epsilon_{n+1}}$ is greater than the corresponding ex-ante probability of trade in M^{ϵ_n}. Hence the sequence of these probabilities is increasing, which implies that it will converge to some number greater than 9/32. Therefore,

$$\lim_{\epsilon_n \to 0} \frac{1}{1+\epsilon_n} \int_{[0,1]} \int_{[0,1]} \{(1-\epsilon_n) p^{\epsilon_n}_{(st,st)} + \epsilon_n (p^{\epsilon_n}_{(st,tr)} + p^{\epsilon_n}_{(tr,st)})\} dv_b dv_s$$

$$= \lim_{\epsilon_n \to 0} \int_{[0,1]} \int_{[0,1]} p^{\epsilon_n}_{(st,st)} \, dv_b dv_s > \frac{9}{32}$$

Hence there exists a N and a small enough $\phi > 0$ such that for all $n \geq N$, $\int_{[0,1]} \int_{[0,1]} p^{\epsilon_n}_{(st,st)} \, dv_b dv_s \geq \frac{9}{32} + \phi$.

Let $X = \{(v_b, v_s) \mid 0 \leq v_b, v_s \leq 1, \ 2v_b - 2v_s - 1 \geq 0\}$ and $I_X(v_b, v_s)$ be the indicator function that takes the value 1 in the set X and 0 otherwise. Define the following:

$$q^{\epsilon_n}_{\theta}(v_b, v_s) = (1-\theta) p^{\epsilon_n}_{(st,st)}(v_b, v_s) + \theta I_X(v_b, v_s), \ \theta \in (0,1)$$

$$\forall\, v_b, \ \bar{q}^{\epsilon_n}_{(b,\theta)}(v_b) = \int_{[0,1]} q^{\epsilon_n}_{\theta}(v_b, v_s) \, dv_s$$

$$\forall\, v_s, \ \bar{q}^{\epsilon_n}_{(s,\theta)}(v_s) = \int_{[0,1]} q^{\epsilon_n}_{\theta}(v_b, v_s) \, dv_b.$$

Note that,

$$\int_{[0,1]} \int_{[0,1]} q^{\epsilon_n}_{\theta} \, dv_b dv_s = (1-\theta) \int_{[0,1]} \int_{[0,1]} p^{\epsilon_n}_{(st,st)} \, dv_b dv_s + \theta \int_{[0,1]} \int_{[0,1]} I_X \, dv_b dv_s$$

$$= (1-\theta) \int_{[0,1]} \int_{[0,1]} p^{\epsilon_n}_{(st,st)} \, dv_b dv_s + \frac{1}{8}\theta$$

Hence there exists a $\hat{\theta} > 0$ such that for all $n \geq N$ and $\theta < \hat{\theta}$, we have

$$\int_{[0,1]} \int_{[0,1]} q^{\epsilon_n}_{\theta} \, dv_b dv_s = (1-\theta) \int_{[0,1]} \int_{[0,1]} p^{\epsilon_n}_{(st,st)} \, dv_b dv_s + \frac{1}{8}\theta > \frac{9}{32}$$

I show that there exists a $n^* \geq N$ and $\theta^* < \hat{\theta}$ such that $\bar{q}^{\epsilon_{n^*}}_{(b,\theta^*)}$ is non-decreasing, $\bar{q}^{\epsilon_{n^*}}_{(s,\theta^*)}$ is non-increasing and

$$\int_{[0,1]} \int_{[0,1]} (2v_b - 2v_s - 1) q^{\epsilon_{n^*}}_{\theta^*}(v_b, v_s) \, dv_b dv_s \geq 0.$$

By Theorem 1 in Myerson and Satterthwaite [15], the above will imply that there exists a $x(v_b, v_s)$ such that $q^{\epsilon_{n^*}}_{\theta^*}$ along with $x(v_b, v_s)$ is an incentive compatible and individually rational direct mechanism for the bilateral trading problem *without trustworthy types* when valuations of players are uniformly distributed on $[0,1]$. However, this will contradict the fact that the direct mechanism corresponding to the C-S equilibrium has the highest ex-ante probability of trade in that problem.

Since M^{ϵ_n} satisfies R4, $\bar{p}^{\epsilon_n}_b$ is a non-decreasing while $\bar{p}^{\epsilon_n}_s$ is non-increasing. This is sufficient to show that $\bar{q}^{\epsilon_n}_{(b,\theta)}$ is non-decreasing and $\bar{q}^{\epsilon_n}_{(s,\theta)}$ is non-increasing for all n and θ.

If there exists a $\hat{n} \geq N$ such that

$$\int_{[0,1]}\int_{[0,1]} (2v_b - 2v_s - 1) p^{\epsilon_{\hat{n}}}_{(st,st)}(v_b, v_s)\, dv_b dv_s \geq 0$$

then take $n^* = \hat{n}$ and $\theta^* = 0$.

If not, then let $\theta(n)$ be such that

$$\int_{[0,1]}\int_{[0,1]} (2v_b - 2v_s - 1) q^{\epsilon_n}_{\theta(n)}(v_b, v_s)\, dv_b dv_s$$

$$= (1-\theta(n)) \int_{[0,1]}\int_{[0,1]} (2v_b - 2v_s - 1) p^{\epsilon_n}_{(st,st)}(v_b, v_s)\, dv_b dv_s + \frac{1}{24}\theta(n) = 0$$

Since M^{ϵ_n} satisfies IC^* and individual rationality for strategic types, we get the following inequality using condition (1) in lemma 2.3

$$\begin{aligned}
&(1-\epsilon_n)^2 \int_{[0,1]}\int_{[0,1]} (2v_b - 2v_s - 1) p^{\epsilon_n}_{(st,st)}(v_b, v_s)\, dv_b dv_s \\
&+\epsilon_n(1-\epsilon_n) \int_{[0,1]}\int_{[0,1]} (2v_b - v_s - 1) p^{\epsilon_n}_{(st,tr)}(v_b, v_s)\, dv_b dv_s \\
&+\epsilon_n(1-\epsilon_n) \int_{[0,1]}\int_{[0,1]} (v_b - 2v_s) p^{\epsilon_n}_{(tr,st)}(v_b, v_s)\, dv_b dv_s \\
&+\epsilon_n^2 \int_{[0,1]}\int_{[0,1]} (v_b - v_s) p^{\epsilon_n}_{(tr,tr)}(v_b, v_s)\, dv_b dv_s \\
&-\epsilon_n\{\int_{[0,1]} U^{\epsilon_n}_{(b,tr)}(v_b) dv_b + \int_{[0,1]} U^{\epsilon_n}_{(s,tr)}(v_s) dv_s\} \geq 0.
\end{aligned}$$

Taking the limit of the above expression as ϵ_n goes to 0, we get (using R1)

$$\lim_{\epsilon_n \to 0} \int_{[0,1]}\int_{[0,1]} (2v_b - 2v_s - 1) p^{\epsilon_n}_{(st,st)}(v_b, v_s)\, dv_b dv_s \geq 0.$$

Therefore, it must be the case that $\theta(n) \to 0$. Now, pick $n^* \geq N$ to be such that $\theta(n^*) < \hat{\theta}$ and let $\theta^* = \theta(n^*)$ and we are done. ■

3.2 Is It Possible To Achieve Ex-Post Efficiency?

An outcome of a mechanism is ex-post efficient for strategic types if trade occurs whenever the valuation type of the strategic buyer is greater than the valuation type of the strategic seller.

Definition 3.7 *EX^*: An outcome of a trading mechanism is ex-post efficient for strategic types if*

$$p_{(st,st)}(v_b, v_s) = \begin{cases} 1 & \text{if } v_b \geq v_s \\ 0 & \text{if } v_b < v_s \end{cases}$$

We can similarly define ex-post efficiency.

Definition 3.8 *EX: An outcome of a trading mechanism is ex-post efficient if*

$$p_{(d_b,d_s)}(v_b, v_s) = \begin{cases} 1 & \text{if } v_b \geq v_s \\ 0 & \text{if } v_b < v_s \end{cases}$$

Note that $EX \implies EX^*$.

The following theorem is a restatement of the theorem by Myerson and Satterthwaite (1983) which proves that when both players are only strategic type and the intersection of the intervals of players' valuation has a non-empty interior, then it is impossible to achieve ex-post efficiency in any outcome of any trading mechanism that satisfies IC and IR.[7]

Theorem 3.9 *Myerson and Satterthwaite (1983): If $\epsilon_b = \epsilon_s = 0$ and $(\underline{a}_b, \bar{a}_b) \cap (\underline{a}_s, \bar{a}_s) \neq \Phi$, then there does not exist an outcome of any trading mechanism that satisfies EX, IC and IR.*

Myerson and Satterthwaite (1983) showed that under the conditions of the theorem, any outcome of a trading mechanism that satisfies EX and IC will require an ex-ante subsidy of at least $\int_{[\underline{a}_b,\bar{a}_s]}(1 - F_{st_b}(y))F_{st_s}(y)dy$ amount to satisfy IR. Section 3.2.1 shows how this subsidy can be generated if and only if at least one player has a high enough *degree of trust*. The constructed optimal trading mechanism is a direct mechanism. Section 3.2.2, however, shows that k-double auctions without or without pre-play communication are not optimal even in this setup since they are ex-post inefficient for any *degree of trust*.

3.2.1 Achieving Ex-Post Efficiency Using Direct Mechanism

With high enough probabilities of trustworthy types, it is possible to generate ex-ante gains from trade greater than the required minimum subsidy to strategic types, $\int_{[\underline{a}_b,\bar{a}_s]}(1 - F_{st_b}(y))F_{st_s}(y)dy$, because of truthful revelation of trustworthy types in direct mechanisms. Notice that the payment functions constructed in the proof of theorem 2.7 are such that every valuation type of trustworthy players gets an expected-payoff of 0. Thus, by pushing the trustworthy types to their individual rationality constraints, we extract all the ex-ante

[7]Note that in this case, EX^* is equivalent to EX and IC^* is equivalent to IC.

gains from trade that they generate and then redistribute them using lump-sum transfers to the strategic types. Hence it becomes possible to get ex-post efficient outcome even while satisfying IC and IR. This is proved in the next proposition.

Proposition 3.10 *There exist weakly decreasing functions $\psi_i : [0,1] \to [0,1],\ i = b, s$ with $\psi_i(\epsilon_i) < 1\ \forall\, \epsilon_i > 0$, such that for all $(\hat{\epsilon}_i, \hat{\epsilon}_j) \geq (\epsilon_i, \psi(\epsilon_i))$ there exist direct mechanisms that satisfies IC, IR and EX (and hence, they also satisfy EX^*).*

Proof: It is easy to see that the functions $p_{(st,st)}, p_{(st,tr)}, p_{(tr,st)}, p_{(tr,tr)}$ satisfying EX are such that $\bar{p}_{(b,st)}$ is weakly increasing and $\bar{p}_{(s,st)}$ is weakly decreasing. Hence, only condition (3) is left to be checked.

For the functions satisfying EX, the left-hand side of (3) is

$$\begin{aligned}
&(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)f_{st_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}\left(v_b-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)f_{st_s}f_{tr_b}dv_sdv_b\\
&+\epsilon_s(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-v_s\right)f_{tr_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b\epsilon_s\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}(v_b-v_s)f_{tr_s}f_{tr_b}dv_sdv_b \qquad (8)
\end{aligned}$$

Myerson and Satterthwaite (1983) (p. 272) showed that

$$\begin{aligned}
&\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)f_{st_s}f_{st_b}dv_sdv_b\\
&=-\int_{[\underline{a}_b,\bar{a}_s]}(1-F_{st_b}(y))F_{st_s}(y)dy
\end{aligned}$$

Similarly, it is easy to show that

$$\begin{aligned}
&\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}\left(v_b-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)f_{st_s}f_{tr_b}dv_sdv_b\\
&=\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}\left(\left[v_b-\frac{1-F_{tr_b}(v_b)}{f_{tr_b}(v_b)}\right]-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)f_{st_s}f_{tr_b}dv_sdv_b\\
&\quad+\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}(1-F_{tr_b}(v_b))f_{st_s}dv_sdv_b\\
&=-\int_{[\underline{a}_b,\bar{a}_s]}(1-F_{tr_b}(y))F_{st_s}(y)dy+\int_{[\underline{a}_b,\bar{a}_b]}(1-F_{tr_b}(v_b))F_{st_s}(v_b)dv_b\\
&=\int_{[\bar{a}_s,\bar{a}_b]}(1-F_{tr_b}(y))dy\geq 0
\end{aligned}$$

and

$$\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-v_s\right)f_{tr_s}f_{st_b}dv_sdv_b=\int_{[\underline{a}_s,\underline{a}_b]}F_{tr_s}(y)dy\geq 0$$

Define,

$$\alpha_{(st,st)} = -\int_{[\underline{a}_b,\bar{a}_s]} (1 - F_{st_b}(y))F_{st_s}(y)dy$$
$$\alpha_{(st,tr)} = \int_{[\underline{a}_s,\underline{a}_b]} F_{tr_s}(y)dy$$
$$\alpha_{(tr,st)} = \int_{[\bar{a}_s,\bar{a}_b]} (1 - F_{tr_b}(y))dy$$
$$\alpha_{(tr,tr)} = \int_{[\underline{a}_b,\bar{a}_b]} \int_{[\underline{a}_s,\min\{v_b,\bar{a}_s\}]} (v_b - v_s)f_{tr_s}f_{tr_b}dv_s dv_b$$

Hence (8) can be written as

$$(1 - \epsilon_b)(1 - \epsilon_s)\alpha_{(st,st)} + \epsilon_b(1 - \epsilon_s)\alpha_{(tr,st)} + (1 - \epsilon_b)\epsilon_s\alpha_{(st,tr)} + \epsilon_b\epsilon_s\alpha_{(tr,tr)} \tag{9}$$

1. $\bar{a}_b \leq \underline{a}_s$: In this case, $\alpha_{(d_b,d_s)} = 0,\ \forall\,(d_b, d_s)$. Therefore, the left-hand side of (3) equals 0 irrespective of the value of ϵ_b and ϵ_s. Hence, in this case $\psi_i(\epsilon_i) = 0,\ \forall\,\epsilon_i \in [0, 1]$ for $i = b, s$.

2. $\bar{a}_s \leq \underline{a}_b$: In this case, $\alpha_{(st,st)} = 0$ but $\alpha_{(st,tr)}, \alpha_{(tr,st)}$ and $\alpha_{(tr,tr)}$ are positive. Thus, the left-hand side of (3) is positive irrespective of the value of ϵ_b and ϵ_s. Therefore, in this case $\psi_i(\epsilon_i) = 0,\ \forall\,\epsilon_i \in [0, 1]$ for $i = b, s$.

3. $[\underline{a}_b, \bar{a}_b] \cap [\underline{a}_s, \bar{a}_s]$ has a non-empty interior: In this case, $\alpha_{(st,st)} < 0$, $\alpha_{(tr,tr)} > 0$ and both $\alpha_{(st,tr)}$ and $\alpha_{(tr,st)}$ are non-negative. Define

$$\psi_b(\epsilon_b) = \max\left\{0, \frac{(1 - \epsilon_b)\alpha_{(st,st)} + \epsilon_b\alpha_{(tr,st)}}{(1 - \epsilon_b)(\alpha_{(st,st)} - \alpha_{(st,tr)}) + \epsilon_b(\alpha_{(tr,st)} - \alpha_{(tr,tr)})}\right\}$$

Note that $0 < \psi_b(0) \leq 1$ and $\psi_b(\epsilon_b) > 0 \implies \psi_b'(\epsilon_b) < 0$. Therefore, ψ_b is weakly decreasing and $\psi_b(\epsilon_b) < 1,\ \forall\,\epsilon_b > 0$.

It is easy to check that for any ϵ_b, the expression in (9) is non-negative for all $\epsilon_s \geq \psi_b(\epsilon_b)$. Also, $(\hat{\epsilon}_b, \hat{\epsilon}_s) \geq (\epsilon_b, \psi_b(\epsilon_b)) \implies (\hat{\epsilon}_b, \hat{\epsilon}_s) \geq (\hat{\epsilon}_b, \psi_b(\hat{\epsilon}_b))$.

Similarly, it is easy to show that

$$\psi_s(\epsilon_s) = \max\left\{0, \frac{(1 - \epsilon_s)\alpha_{(st,st)} + \epsilon_s\alpha_{(st,tr)}}{(1 - \epsilon_s)(\alpha_{(st,st)} - \alpha_{(tr,st)}) + \epsilon_s(\alpha_{(st,tr)} - \alpha_{(tr,tr)})}\right\}$$

■

3.2.2 Inefficiency of k-double auctions

If any $\epsilon_i = 1$, then it is straightforward to satisfy EX in a k-double auction. For instance, if $\epsilon_b = 1$, then take $k = 1$. For any strategic type of seller in this double auction, truthful bidding dominates any other bid. Also, all trustworthy types of both buyer and seller also bid truthfully, so we get an ex-post efficient outcome. Therefore, in what follows, I assume $(\epsilon_b, \epsilon_s) \ll 1$.

However, when $(\epsilon_b, \epsilon_s) \ll 1$, no k-double auction will satisfy EX^* and hence will also not satisfy EX. This follows from the following lemma, which to my knowledge has not been proved before. Lemma 4.3 in Leininger, Linhart and Radner (1989) proves that in a 1/2-double auction no buyer will bid more than her valuation and no seller will bid below her valuation except where the probability of trade is 0, which is weaker than the following lemma.

Lemma 3.11 *In any k-double auction, if $k < 1$ and $Probability(t_b > v_s) > 0$ for any strategic seller with valuation v_s, then $Supp(\sigma_s(v_s)) \subset (v_s, \infty)$. Similarly, if $k > 0$ and $Probability(t_s < v_b) > 0$ for any strategic buyer with valuation v_b, then $Supp(\sigma_b(v_b)) \subset (-\infty, v_b)$.*

Proof: It is sufficient to show that for a strategic seller bidding less than or equal to her valuation is dominated by some bid greater than her valuation. Since $k < 1$ and $Probability(t_b > v_s) > 0$, it is easy to see that for a strategic seller bidding equal to her valuation dominates bidding less than her valuation. Now, consider the difference between the payoffs of a strategic seller with valuation v_s from bidding $t_s > v_s$ and from bidding v_s,

$$\begin{aligned}
&\int_{[t_s,\bar{t}_b]} (kt_b + (1-k)t_s - v_s)\, d\hat{G}_b - \int_{(v_s,\bar{t}_b]} (kt_b + (1-k)v_s - v_s)\, d\hat{G}_b \\
&= (1-k)\int_{[t_s,\bar{t}_b]} (t_s - v_s)\, d\hat{G}_b - k\int_{(v_s,t_s)} (t_b - v_s)\, d\hat{G}_b \\
&\geq (t_s - v_s)\left((1-k)\int_{[t_s,\bar{t}_b]} d\hat{G}_b - k\int_{(v_s,t_s)} d\hat{G}_b\right),
\end{aligned}$$

where $\hat{G}_b$ is the distribution of buyer's bid and $\bar{t}_b = \inf\{t_b \mid \hat{G}_b(t_b) = 1\}$. There must exist a $t_s > v_s$ such that the last term is positive. If not, then

$$\begin{aligned}
&\lim_{t_s \searrow v_s} \left((1-k)\int_{[t_s,\bar{t}_b]} d\hat{G}_b - k\int_{(v_s,t_s)} d\hat{G}_b\right) = (1-k)(1-\hat{G}_b(v_s)) \leq 0 \\
&\Longrightarrow \hat{G}_b(v_s) \geq 1 \text{ because } k < 1,
\end{aligned}$$

which contradicts $Probability(t_b > v_s) > 0$. ■

Corollary 3.12 *If $(\epsilon_b, \epsilon_s) \ll 1$ and $(\underline{a}_s, \bar{a}_s) \cap (\underline{a}_b, \bar{a}_b) \neq \Phi$, then any Bayesian-Nash equilibrium outcome of any k-double auction does not satisfy EX^*, and hence, it also does not satisfy EX.*

Proof: Follows from lemma 3.11.[8] ■

The above inefficiency result applies to k-double auctions in which players do not communicate before submitting their bids. Players can reduce the uncertainty by communicating their types in equilibrium (See, Farell and Gibbons (1989)). Since incomplete information is

[8]Notice that this corollary follows directly from lemma 3.11 without using the assumption regarding trustworthy disposition. In fact, this result is true under any other dispositional assumption.

the reason for inefficiency, maybe allowing for pre-play communication can, therefore, generate ex-post efficient outcomes? To answer this question, let's consider a k-double auction with a pre-play communication stage, in which players communicate, sequentially or simultaneously, using some arbitrary message space before the final stage during which they play according to the rules of the k-double auction. In this case, assumption 2.1 is too strong a restriction on the behavior of trustworthy types. For example, trustworthy types may make pre-play non-binding verbal agreements to bid some value not equal to their valuation and then fulfilling such agreements in the bidding stage. Therefore, I instead assume that trustworthy types do not bid irrationally, that is, trustworthy buyer (seller) does not bid greater (less) than her valuation. Note that this assumption does not restrict the behavior of trustworthy types in the communication stage apart from not allowing such types to verbally agree to bid irrationally.

Assumption 3.13 *In a k-double auction with pre-play communication, trustworthy buyer does not bid more than her valuation and trustworthy seller does not bid less than her valuation. This assumption hence distinguishes trustworthy disposition from irrationality or altruism. Experimental studies on $1/2$-double auctions by Valley et al. (2002) and McGinn, Thompson and Bazerman (2003) find players fulfilling non-binding verbal agreements to bid a particular price but they do not find any instance of a buyer (seller) bidding above (below) her valuation.*[9]

The next proposition proves that under assumption 3.13 and, in particular, when the interval of valuations coincide, any k-double auction with pre-play communication is ex-post inefficient. By assumption, in the final bidding stage, the trustworthy type of buyer will bid less than or equal to her valuation and the trustworthy type of seller will bid greater than or equal to her valuation. Therefore, the ex-ante gains from trade that are generated in the event that both players are trustworthy type cannot be transferred to strategic types in order to subsidize the latter types.

Proposition 3.14 *Suppose assumption 3.13 holds. If $\underline{a}_b \leq \underline{a}_s$ and $\bar{a}_b \leq \bar{a}_s$, then for all $(\epsilon_b, \epsilon_s) \ll 1$, any Bayesian-Nash equilibrium outcome of a k-double auction with pre-play communication does not satisfy EX.*

Proof: Let $(p_{(d_b,d_s)}, x_{(d_b,d_s)})_{d_b=st,tr;\, d_s=st,tr}$ be a Bayesian-Nash equilibrium outcome of some k-double auction with any form of pre-play communication that satisfies EX. By lemma 2.3, it must satisfy condition (1), which after some simple manipulation can be re-written

[9]Note that the strategic buyer (seller) who trades with positive probability in any equilibrium of such a trading mechanism also does not bid more (less) than her valuation.

as,

$$
\begin{aligned}
&(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)p_{(st,st)}(v_b,v_s)f_{st_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(x_{(tr,st)}(v_b,v_s)-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]p_{(tr,st)}(v_b,v_s)\right)f_{st_s}f_{tr_b}dv_sdv_b\\
&+\epsilon_s(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]p_{(st,tr)}(v_b,v_s)-x_{(st,tr)}(v_b,v_s)\right)f_{tr_s}f_{st_b}dv_sdv_b\\
=&(1-\epsilon_b)U_{(b,st)}(\underline{a}_b)+(1-\epsilon_s)U_{(s,st)}(\bar{a}_s)
\end{aligned}
$$

Since the outcome is EX, $p_{((d_b,d_s)}(v_b,v_s)\in\{0,1\}$. Also, according to the rules of k-double auction, a buyer makes a payment to a seller if and only if they trade, that is, $x_{(d_b,d_s)}(v_b,v_s)=0$ if $p_{(d_s,d_b)}(v_b,v_s)=0$. Therefore, for all (d_b,d_s) and (v_b,v_s), we must have $x_{(d_b,d_s)}(v_b,v_s)=x_{(d_b,d_s)}(v_b,v_s)p_{(d_s,d_b)}(v_b,v_s)$. Finally, $x_{(tr,st)}(v_b,v_s)\le v_b$ and $x_{(st,tr)}(v_b,v_s)\ge v_s$. Substituting these in the above equality, we get

$$
\begin{aligned}
&(1-\epsilon_b)U_{(b,st)}(\underline{a}_b)+(1-\epsilon_s)U_{(s,st)}(\bar{a}_s)\\
\le&(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)p_{(st,st)}(v_b,v_s)f_{st_s}f_{st_b}dv_sdv_b\\
&+\epsilon_b(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(v_b-\left[v_s+\frac{F_{st_s}(v_s)}{f_{st_s}(v_s)}\right]\right)p_{(tr,st)}(v_b,v_s)f_{st_s}f_{tr_b}dv_sdv_b\\
&+\epsilon_s(1-\epsilon_b)\int_{[\underline{a}_b,\bar{a}_b]}\int_{[\underline{a}_s,\bar{a}_s]}\left(\left[v_b-\frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]-v_s\right)p_{(st,tr)}(v_b,v_s)f_{tr_s}f_{st_b}dv_sdv_b\\
=&-(1-\epsilon_b)(1-\epsilon_s)\int_{[\underline{a}_b,\bar{a}_s]}(1-F_{st_b}(y))F_{st_s}(y)dy+\epsilon_b(1-\epsilon_s)\int_{[\bar{a}_s,\bar{a}_b]}(1-F_{tr_b}(y))dy\\
&+(1-\epsilon_b)\epsilon_s\int_{[\underline{a}_s,\underline{a}_b]}F_{tr_s}(y)dy\\
<&\ 0,
\end{aligned}
$$

where the second step follows from the fact that the outcome satisfies EX (see proof of 3.10) and the final step uses the fact that $(\underline{a}_s,\underline{a}_b)=(\bar{a}_s,\bar{a}_b)=\Phi$ since $\underline{a}_b\le\underline{a}_s$ and $\bar{a}_b\le\bar{a}_b$. Therefore, the outcome does not satisfy IR. ■

Remark 3.15 *Two remarks regarding proposition 3.14.*

1. *It only proved that any equilibrium will not satisfy EX, which does not imply that any equilibrium will also not satisfy EX^*. However, it seems reasonable assume that the behavior of trustworthy types in a k-double auction with pre-play communication is such that if any pair of valuation types of strategic players for whom the gains from trade are non-negative, trade in equilibrium with probability* 1*, then that pair of valuation types must also trade in equilibrium with probability* 1 *if one or both players are trustworthy*

types. More precisely, in any equilibrium outcome of any trading mechanism, if for any (v_b, v_s)*, with* $v_b \geq v_s$*,* $p_{(st,st)}(v_b, v_s) = 1$ *, then* $p_{(tr,st)}(v_b, v_s) = p_{(st,tr)}(v_b, v_s) = p_{(tr,tr)}(v_b, v_s) = 1$*. With this assumption,* EX^* *is equivalent to* EX *and hence, the above result shows that no equilibrium of any* k*-double auction with pre-play communication will satisfy* EX^**.*

2. *A similar method of proof cannot be used if either* $\underline{a}_s < \underline{a}_b$ *or* $\bar{a}_s < \bar{a}_b$*. Then both* $\int_{[\bar{a}_s,\bar{a}_b]}(1 - F_{tr_b}(y))dy$ *and* $\int_{[\underline{a}_s,\underline{a}_b]} F_{tr_s}(y)dy$ *are positive and so the last inequality in the proof of the proposition does not follow. However, no communication structure comes to mind that, using the minimal assumptions on trustworthy types, generates an efficient Bayesian-Nash equilibrium outcome in this case.*

4 *Distribution of Trust* and Trade

In this section, we fix the *degree of trust*, $(\epsilon_i)_{i=b,s}$, and ask the following question: What *distribution of trust* is better from the perspective of maximizing the probability of trade among strategic types? Section 4.1 answers that with respect to direct mechanisms, it is better to have high-surplus types (high valuation type buyer and low valuation type seller) more likely among trustworthy types than low-surplus types (low valuation type buyer and high valuation type seller).

In section 4.2, I analyze a 1/2-double auction with valuations of strategic types uniformly distributed on $[0, 1]$ but with valuations of trustworthy types distributed so that for any positive *degree of trust*, high-surplus types are *less* likely among trustworthy types. I then find an equilibrium with ex-ante probability of trade between strategic types strictly greater than C-S equilibrium value for all *degrees of trust* less than or equal to 0.5. However, section 3.1.1 proved that if, instead, the *distribution of trust* is uniform, then no equilibrium outcome of any k-double auction can do better than C-S equilibrium outcome. Therefore, unlike direct mechanisms, for k-double auctions it is not necessarily better to have high-surplus types more likely among trustworthy types.

4.1 Direct Mechanisms

The next proposition proves that for any fixed *degree of trust*, if we change the *distribution of trust* from (F_{tr_b}, F_{tr_s}) to (F'_{tr_b}, F'_{tr_s}) so that F'_{tr_b} first-order stochastically dominates F_{tr_b} and F_{tr_s} first-order stochastically dominates F'_{tr_s}, then we can construct a direct mechanism with at least as high probability of trade among strategic types as in any mechanism before the change. The reason is that with this change, high-surplus types become more likely among trustworthy types. Hence, the ex-ante gains from trade that can be generated using trustworthy types increases, which in turn increases the ex-ante subsidy that can be offered to strategic types.

Proposition 4.1 *Suppose for the bilateral trading problem* $(\epsilon_b, \epsilon_s, F_{st_b}, F_{st_s}, F_{tr_b}, F_{tr_s})$, *the direct mechanism* $(p_{(d_b,d_s)}, x_{(d_b,d_s)})_{d_b=st,tr;\, d_s=st,tr}$ *satisfies IC, IR, NUT and Assumption 2.1. Consider any* (F'_{tr_b}, F'_{tr_s}) *such that:*

1. F'_{tr_b} *first-order stochastically dominates* F_{tr_b}.

2. F_{tr_s} *first-order stochastically dominates* F'_{tr_s}.

Then there exists a direct mechanism $(p'_{(d_b,d_s)}, \hat{x}_{(d_b,d_s)})_{d_b=st,tr;\, d_s=st,tr}$ *for the bilateral trading problem* $(\epsilon_b, \epsilon_s, F_{st_b}, F_{st_s}, F'_{tr_b}, F'_{tr_s})$ *satisfying IC, IR, NUT and Assumption 2.1 and such that*

$$\int_{[\underline{a}_b,\bar{a}_b]} \int_{[\underline{a}_s,\bar{a}_s]} p'_{(st,st)}(v_b, v_s) f_{st_s} f_{st_b} dv_s dv_b \geq \int_{[\underline{a}_b,\bar{a}_b]} \int_{[\underline{a}_s,\bar{a}_s]} p_{(st,st)}(v_b, v_s) f_{st_s} f_{st_b} dv_s dv_b$$

Proof: Without loss of generality we can assume that

$$p_{(tr,tr)}(v_b, v_s) = \begin{cases} 1 & \text{if } v_b \geq v_s \\ 0 & \text{if } v_b < v_s \end{cases}$$

NUT and F_{tr_s} first-order stochastically dominates F'_{tr_s} imply that,

$$\int_{[\underline{a}_s,\bar{a}_s]} p_{(st,tr)}(v_b, v_s) f_{tr_s} dv_s \leq F_{tr_s}(v_b) \leq \hat{F}_{tr_s}(v_b)$$

Hence there exists a $a_s(v_b) \leq v_b$ such that,

$$\int_{[\underline{a}_s,a_s(v_b)]} f'_{tr_s} dv_s = \int_{[\underline{a}_s,\bar{a}_s]} p_{(st,tr)}(v_b, v_s) f_{tr_s} dv_s$$

Similarly we have,

$$\int_{[\underline{a}_b,\bar{a}_b]} p_{(tr,st)}(v_b, v_s) f_{tr_b} dv_b \leq 1 - F_{tr_b}(v_s) \leq 1 - F'_{tr_b}(v_s)$$

and hence there exists a $a_b(v_s) \geq v_s$ such that,

$$\int_{[a_b(v_s),\bar{a}_b]} f'_{tr_b} dv_b = \int_{[\underline{a}_b,\bar{a}_b]} p_{(tr,st)}(v_b, v_s) f_{tr_b} dv_b$$

Define $(p'_{(d_b,d_s)})_{d_b=st,tr;\, d_s=st,tr}$ as follows:

$$p'_{(st,st)}(v_b, v_s) = p_{(st,st)}(v_b, v_s)$$
$$p'_{(st,tr)}(v_b, v_s) = \begin{cases} 1 & \text{if } (v_b, v_s) \in [\underline{a}_s, a_s(v_b)] \\ 0 & \text{otherwise} \end{cases}$$
$$p'_{(tr,st)}(v_b, v_s) = \begin{cases} 1 & \text{if } (v_b, v_s) \in [a_b(v_s), \bar{a}_b] \\ 0 & \text{otherwise} \end{cases}$$
$$p'_{(tr,tr)}(v_b, v_s) = p_{(tr,tr)}(v_b, v_s)$$

It is easy to see that $(p'_{(d_b,d_s)})_{d_b=st,tr;\,d_s=st,tr}$ satisfies NUT and Assumption 2.1. Also,

$$\begin{aligned}
\bar{p}'_{(b,st)}(v_b) =&(1-\epsilon_s)\int_{[\underline{a}_s,\bar{a}_s]} p'_{(st,st)}(v_b,v_s)f_{st_s}dv_s + \epsilon_s\int_{[\underline{a}_s,\bar{a}_s]} p'_{(st,tr)}(v_b,v_s)f'_{tr_s}dv_s \\
=&(1-\epsilon_s)\int_{[\underline{a}_s,\bar{a}_s]} p_{(st,st)}(v_b,v_s)f_{st_s}dv_s + \epsilon_s\int_{[\underline{a}_s,a_s(v_b)]} f'_{tr_s}dv_s \\
=&\bar{p}_{(b,st)}(v_b)
\end{aligned}$$

and similarly $\bar{p}'_{(s,st)}(v_s) = \bar{p}_{(s,st)}(v_s)$. Therefore, $\bar{p}'_{(b,st)}(v_b)$ is weakly increasing and $\bar{p}'_{(s,st)}(v_s)$ is weakly decreasing. We will be done if we show that $(p'_{(d_b,d_s)})_{d_b=st,tr;\,d_s=st,tr}$ satisfies condition (3).

Let

$$\begin{aligned}
h_{v_b}(v_s) &= \frac{p_{(st,tr)}(v_b,v_s)f_{(st,tr)}(v_b,v_s)}{\int_{[\underline{a}_s,\bar{a}_s]} p_{(st,tr)}(v_b,y_s)f_{(st,tr)}(v_b,y_s)dy_s} \qquad \text{and} \qquad H_{v_b}(v_s) = \int_{[\underline{a}_s,v_s]} h_{v_b}(y_s)dy_s \\
\hat{h}_{v_b}(v_s) &= \frac{p'_{(st,tr)}(v_b,v_s)f'_{(st,tr)}(v_b,v_s)}{\int_{[\underline{a}_s,\bar{a}_s]} p'_{(st,tr)}(v_b,y_s)f'_{(st,tr)}(v_b,y_s)dy_s} \qquad \text{and} \qquad \hat{H}_{v_b}(v_s) = \int_{[\underline{a}_s,v_s]} \hat{h}_{v_b}(y_s)dy_s
\end{aligned}$$

Then H_{v_b} first-order stochastically dominates $\hat{H}_{v_b}$. For all $v_s \geq a_s(v_b)$, $\hat{H}_{v_b}(v_s) = 1 \geq H_{v_b}(v_s)$. And for all $v_s \leq a_s(v_b)$

$$\begin{aligned}
\hat{H}_{v_b}(v_s) =&\frac{\int_{[\underline{a}_s,v_s]} f'_{(st,tr)}(v_b,y_s)dy_s}{\int_{[\underline{a}_s,\bar{a}_s]} p'_{(st,tr)}(v_b,y_s)f'_{(st,tr)}(v_b,y_s)dy_s} \\
\geq&\frac{\int_{[\underline{a}_s,v_s]} f_{(st,tr)}(v_b,y_s)dy_s}{\int_{[\underline{a}_s,\bar{a}_s]} p_{(st,tr)}(v_b,y_s)f_{(st,tr)}(v_b,y_s)dy_s} \\
\geq&\frac{\int_{[\underline{a}_s,v_s]} p_{(st,tr)}(v_b,y_s)f_{(st,tr)}(v_b,y_s)dy_s}{\int_{[\underline{a}_s,\bar{a}_s]} p_{(st,tr)}(v_b,y_s)f_{(st,tr)}(v_b,y_s)dy_s} = H_{v_b}(v_s)
\end{aligned}$$

Hence, $\int_{[\underline{a}_s,\bar{a}_s]} v_s dH_{v_b} \geq \int_{[\underline{a}_s,\bar{a}_s]} v_s d\hat{H}_{v_b}$. This implies that for all v_b,

$$\begin{aligned}
&\int_{[\underline{a}_s,\bar{a}_s]} \left(\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right] - v_s\right) p'_{(st,tr)}(v_b,v_s)f'_{tr_s}dv_s \\
=&\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]\int_{[\underline{a}_s,\bar{a}_s]} p_{(st,tr)}(v_b,v_s)f_{tr_s}dv_s - \int_{[\underline{a}_s,\bar{a}_s]} v_s p'_{(st,tr)}(v_b,v_s)f'_{tr_s}dv_s \\
\geq&\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right]\int_{[\underline{a}_s,\bar{a}_s]} p_{(st,tr)}(v_b,v_s)f_{tr_s}dv_s - \int_{[\underline{a}_s,\bar{a}_s]} v_s p_{(st,tr)}(v_b,v_s)f_{tr_s}dv_s \\
=&\int_{[\underline{a}_s,\bar{a}_s]} \left(\left[v_b - \frac{1-F_{st_b}(v_b)}{f_{st_b}(v_b)}\right] - v_s\right) p_{(st,tr)}(v_b,v_s)f_{tr_s}dv_s
\end{aligned}$$

Similarly it can be shown that for all v_s,

$$\int_{[\underline{a}_b,\bar{a}_b]} \left(v_b - \left[v_s + \frac{F_{st_s}(v_s)}{f_{st_s}(v_s)} \right] \right) p'_{(tr,st)}(v_b, v_s) f'_{tr_b} dv_b$$
$$\geq \int_{[\underline{a}_b,\bar{a}_b]} \left(v_b - \left[v_s + \frac{F_{st_s}(v_s)}{f_{st_s}(v_s)} \right] \right) p_{(tr,st)}(v_b, v_s) f_{tr_b} dv_b$$

Hence $(p'_{(d_b,d_s)})_{d_b=st,tr;\, d_s=st,tr}$ must satisfy condition (3). ■

4.2 1/2-Double Auction with Low-Surplus Types more likely to be Trustworthy

The trading mechanism is again a 1/2-double auction with $[\underline{a}_s, \bar{a}_s] = [\underline{a}_b, \bar{a}_b] = [0, 1]$. Also, $F_{st_b} = F_{st_s}$ are uniform on $[0, 1]$. The trustworthy types bid truthfully but now the distributions of their valuation are not uniform. Instead, the valuations of these types are distributed according to $F_{tr_b}(v_b) = 1 - (1 - v_b)^{\frac{1}{\epsilon}+1}$, $v_b \in [0, 1]$ and $F_{tr_s}(v_s) = (v_s)^{\frac{1}{\epsilon}+1}$, $v_s \in [0, 1]$.

This *distribution of trust* has two intuitive assumptions behind it. First, for any positive ϵ, strategic types believe that low valuation types of buyer (high valuation types of seller) have a higher likelihood (i.e. have higher density) among the trustworthy types than high valuation types of buyer (low valuation types of seller); after all, a buyer with low valuation (seller with high valuation) cannot gain much by lying about her valuation. Second, as ϵ falls, the probability that a trustworthy type buyer (seller) has a valuation below (above) a specific value increases. In the limit, as the probability of these trustworthy types goes to zero, the strategic type's belief puts all the weight on the extreme points (0 for buyer and 1 for seller). This is in line with the conjecture that if one is sure that there are no trustworthy types then the most likely answer one will give to the question,"Who will bid equal to their valuation?", is,"The buyer with valuation 0 and seller with valuation 1."

Notice that F_{tr_s} first-order stochastically dominates a uniform distribution on $[0, 1]$ whereas the latter first-order stochastically dominates F_{tr_b}. Hence, if we were considering direct mechanisms, then by proposition 4.1 we can construct a direct mechanism with at least as high probability of trade among strategic types when the *distribution of trust* is uniform compared to when the *distribution of trust* is given by (F_{tr_b}, F_{tr_s}). In section 3.1.1, we saw that if the *distribution of trust* is uniform, then the probability of trade among strategic types in any equilibrium of any k-double auction for any positive *degree of trust* is strictly lower than the C-S equilibrium value. Here, I will show that with the *distribution of trust* given by (F_{tr_b}, F_{tr_s}), it is possible to achieve a higher probability of trade among strategic types than the C-S equilibrium value. Hence proposition 4.1 does not hold for k-double auctions.

Let $\hat{\Gamma}^\epsilon$ denote the game induced by the trading mechanism. Throughout this subsection, $\sigma_i(v_i)$ is a pure strategy.

Lemma 4.2 *In any Bayesian-Nash equilibrium (σ_b, σ_s) of $\hat{\Gamma}^\epsilon$, the bids of the strategic types are non-decreasing in valuation.*

Proof: Since $\bar{p}_{(b,st)}(v_b) = (1-\epsilon)\int_{[\underline{t}_s,t_b]} dG_s + \epsilon\int_{[0,t_b]} dF_{tr_s}$ is weakly-increasing, it must be that $\sigma_b(v_b)$ is weakly-increasing. And since $\bar{p}_{(s,st)}(v_s) = (1-\epsilon)\int_{[t_s,\bar{t}_b]} dG_b + \epsilon\int_{[t_s,1]} dF_{tr_b}$ is weakly-decreasing, it must be that $\sigma_s(v_s)$ is weakly-increasing. ■

The next lemma shows that there is no truthful bidding by the strategic types (except the strategic buyer with valuation 0 and the strategic seller with valuation 1) in any equilibrium of $\hat{\Gamma}^\epsilon$.

Lemma 4.3 *For any equilibrium pair of strategies of* $\hat{\Gamma}^\epsilon$, (σ_b, σ_s), $\sigma_b(v_b) < v_b \,\forall\, v_b > 0$ *and* $\sigma_s(v_s) > v_s \,\forall\, v_s < 1$.

Proof: For any buyer with valuation $v_b > 0$, $Probability(t_s < v_b) > 0$. Hence, we get the result by applying lemma 3.11. ■

Given a pair of strategies for the strategic types for $\hat{\Gamma}^\epsilon$, (σ_b, σ_s), let $\underline{t} = \sigma_s(0)$ and $\bar{t} = \sigma_b(1)$.[10] Since bids of strategic types are non-decreasing in valuations, any strategic type that trades with some strategic type with a positive probability bids in the interval $[\underline{t}, \bar{t}]$. Also, note that for any pair of equilibrium strategies, $\underline{t} > 0$ and $\bar{t} < 1$. Define $\underline{v}_b = \sup\{v_b \mid \sigma_b(v_b) = \underline{t}\}$, $\bar{v}_b = \inf\{v_b \mid \sigma_b(v_b) \geq \bar{t}\}$, $\underline{v}_s = \sup\{v_s \mid \sigma_s(v_s) \leq \underline{t}\}$, and $\bar{v}_s = \inf\{v_s \mid \sigma_s(v_s) = \bar{t}\}$.[11] I am interested in a class D^* of equilibrium strategies σ_b and σ_s for the strategic types that are strictly increasing and C^1 on the intervals $[\underline{v}_b, \bar{v}_b]$ and $[\underline{v}_s, \bar{v}_s]$, respectively.[12] Note that, since σ_b and σ_s are strictly increasing, their inverses, denoted by β and ξ, respectively, exist and are defined on the interval $[\underline{t}, \bar{t}]$.[13]

The next three lemmas provide the necessary conditions that any equilibrium pair of strategies in D^* must satisfy.

Lemma 4.4 *If a strategic buyer bids below* $\underline{t}$*, then she must bid* $\frac{2+2\epsilon}{2+3\epsilon}v_b$ *and if a strategic seller bids above* $\bar{t}$*, then she must bid* $\frac{2+2\epsilon}{2+3\epsilon}v_s + \frac{\epsilon}{2+3\epsilon}$.

Proof: If a strategic buyer bids some t less than $\underline{t}$, then the first order condition is, $(1+\epsilon)(v_b - t - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})t)(t)^{\frac{1}{\epsilon}} = 0$, which implies she must bid $\frac{2+2\epsilon}{2+3\epsilon}v_b$.■

Lemma 4.5 *For any equilibrium in* D^*, $\frac{2+2\epsilon}{2+3\epsilon}\underline{v}_b \leq \underline{t}$ *and* $\frac{2+2\epsilon}{2+3\epsilon}\bar{v}_s + \frac{\epsilon}{2+3\epsilon} \geq \bar{t}$.

Proof: If $\epsilon = 1$, then the unique best response for the strategic buyer is $\frac{2+2\epsilon}{2+3\epsilon}v_b$. Therefore, $\frac{2+2\epsilon}{2+3\epsilon}\underline{v}_b = \underline{t}$.

Suppose $\epsilon < 1$ and that for some equilibrium strategies (σ_b, σ_s) in D^*, we have $\frac{2+2\epsilon}{2+3\epsilon}\underline{v}_b > \underline{t}$.

First I show that then $\underline{v}_s = \xi(\underline{t}) > 0$. If a strategic buyer with valuation v_b bids in the interval $(\underline{t}, \bar{t})$, then the first order condition is,

$$(1-\epsilon)\{(v_b - t)\xi'(t) - \frac{1}{2}\xi(t)\} + (1+\epsilon)\{v_b - t - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})t\}(t)^{\frac{1}{\epsilon}} = 0$$

[10] From now on, I do not include the subscript b or s for the bids.

[11] I restrict attention to those strategies for which $\{v_b \mid \sigma_b(v_b) = \underline{t}\}$ and $\{v_s \mid \sigma_s(v_s) = \bar{t}\}$ are non-empty.

[12] Of course, these intervals must have a non-empty interior. This is true for any strategy pair in D^* *iff* $\underline{t} < \bar{t}$.

[13] With $\beta(\underline{t}) = \underline{v}_b$, $\beta(\bar{t}) = \bar{v}_b$, $\xi(\underline{t}) = \underline{v}_s$ and $\xi(\bar{t}) = \bar{v}_s$.

For any $t \in (\underline{t}, \bar{t})$, $\beta(t)$ satisfies the above condition. Hence,

$$\xi'(t) = \frac{1}{2}(\frac{\xi(t)}{\beta(t) - t}) - (\frac{1+\epsilon}{1-\epsilon})(\frac{\beta(t) - t - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})t}{\beta(t) - t})(t)^{\frac{1}{\epsilon}}$$

Therefore, taking the limit of the last expression as $t \searrow \underline{t}$, we get,

$$\begin{aligned}\lim_{t \searrow \underline{t}} \xi'(t) &= \frac{1}{2}(\frac{\xi(\underline{t})}{\underline{v}_b - \underline{t}}) - (\frac{1+\epsilon}{1-\epsilon})(\frac{\underline{v}_b - \underline{t} - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})\underline{t}}{\underline{v}_b - \underline{t}})(\underline{t})^{\frac{1}{\epsilon}} \\ &= \frac{1}{2}(\frac{\xi(\underline{t})}{\underline{v}_b - \underline{t}}) - (\frac{1+\epsilon}{1-\epsilon})(\frac{\underline{v}_b - (\frac{2+3\epsilon}{2+2\epsilon})\underline{t}}{\underline{v}_b - \underline{t}})(\underline{t})^{\frac{1}{\epsilon}} < \frac{1}{2}(\frac{\xi(\underline{t})}{\underline{v}_b - \underline{t}})\end{aligned}$$

If $\xi(\underline{t}) = 0$, then there will exist $t \in (\underline{t}, \bar{t})$ such that $\xi'(t) < 0$ since ξ is C^1 on $[\underline{t}, \bar{t}]$. Hence, $\xi(\underline{t}) > 0$.

Second, combining the above result, lemma 4.2 and the fact that $\sigma_s(0) = \underline{t}$, we get that all valuation types of strategic seller in $[0, \xi(\underline{t})]$ bid $\underline{t}$.

Third, G_b is discontinuous at $\underline{t}$, that is, the probability that the strategic buyer will bid equal to $\underline{t}$ is positive. By lemma 4.2, all valuation types of strategic buyer with valuations below $\underline{v}_b$ bid less than or equal to $\underline{t}$. By lemma 4.4, if a strategic buyer bids below $\underline{t}$, then she must bid $\frac{2+2\epsilon}{2+3\epsilon}v_b$. But, since $\frac{2+2\epsilon}{2+3\epsilon}\underline{v}_b > \underline{t}$, there exists an interval of valuation types of strategic buyer with valuations below $\underline{v}_b$ that must bid $\underline{t}$ because for such valuation types of strategic buyer $\frac{2+2\epsilon}{2+3\epsilon}v_b > \underline{t}$.

Finally, consider the difference between the payoffs of the seller $v_s > \xi(\underline{t})$ from bidding $\sigma_s(v_s)$ and $\underline{t}$,

$$\begin{aligned}&(1-\epsilon)\int_{[\sigma_s(v_s), \bar{t}]} (\frac{1}{2}(t + \sigma_s(v_s)) - v_s)\, dG_b + \epsilon \int_{[\sigma_s(v_s), 1]} (\frac{1}{2}(t + \sigma_s(v_s)) - v_s)\, dF_{tr_b} \\ &\qquad -(1-\epsilon)\int_{[\underline{t}, \bar{t}]} (\frac{1}{2}(t + \underline{t}) - v_s)\, dG_b - \epsilon \int_{[\underline{t}, 1]} (\frac{1}{2}(t + \underline{t}) - v_s)\, dF_{tr_b}\end{aligned}$$

By the continuity of σ_s and F_{tr_b}, taking the limit of the above expression as $v_s \searrow \xi(\underline{t})$, we get,

$$-(1-\epsilon)(\underline{t} - \xi(\underline{t}))(G_b(\underline{t}) - G_b(\underline{t}^-)) \leq 0,$$

where $G_b(\underline{t}^-) = \lim_{t \nearrow \underline{t}} G_b(t)$. If the above expression is negative, then there exists a strategic seller with valuation $\tilde{v}_s > \xi(\underline{t})$ who prefers to bid $\underline{t}$ instead of $\sigma_s(\tilde{v}_s)$, which is a contradiction. If the above expression is 0, then since $G_b(\underline{t}) - G_b(\underline{t}^-) > 0$, it must be the case that $\xi(\underline{t}) = \underline{t}$. This, however, contradicts lemma 3.11 since there is a positive probability of the buyer bidding above $\underline{t}$. ■

Lemma 4.6 *(β, ξ) are a solution to the following non-autonomous differential equation system in the interval $(\underline{t}, \bar{t})$:*

$$\dot{\beta} = \frac{1}{2}(\frac{1-\beta}{t-\xi}) + (\frac{1+\epsilon}{1-\epsilon})(\frac{\xi - t + \frac{1}{2}(\frac{\epsilon}{1+\epsilon})(1-t)}{t-\xi})(1-t)^{\frac{1}{\epsilon}} \tag{10}$$

$$\dot{\xi} = \frac{1}{2}(\frac{\xi}{\beta - t}) - (\frac{1+\epsilon}{1-\epsilon})(\frac{\beta - t - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})t}{\beta - t})(t)^{\frac{1}{\epsilon}} \tag{11}$$

where $1 \geq \beta \geq t \geq \xi \geq 0$.

Proof: Pick a $t \in (\underline{t}, \bar{t})$. Then, there exists a $v_b \in (\underline{v}_b, \bar{v}_b)$ such that $\sigma_b(v_b) = t$. A strategic buyer's expected payoff if she bids $\tilde{t}$ is

$$(1-\epsilon)\int_{[\underline{t},\tilde{t}]}(v_b - \frac{1}{2}(\tilde{t}+t))dG_s + \epsilon\int_{[0,\tilde{t}]}(v_b - \frac{1}{2}(\tilde{t}+v_s))dF_{tr_s}.$$

Since $v_b \in (\underline{v}_b, \bar{v}_b)$, $\sigma_b(v_b)$ must satisfy the first order condition

$$(1-\epsilon)\{(v_b - \sigma_b(v_b))G_s'(\sigma_b(v_b)) - \frac{1}{2}G_s(\sigma_b(v_b))\} +$$
$$(1+\epsilon)\{v_b - \sigma_b(v_b) - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})\sigma_b(v_b)\}(\sigma_b(v_b))^{\frac{1}{\epsilon}} = 0.$$

This reduces to,

$$G_s'(\sigma_b(v_b)) = \frac{1}{2}(\frac{G_s(\sigma_b(v_b))}{v_b - \sigma_b(v_b)}) - (\frac{1+\epsilon}{1-\epsilon})(\frac{v_b - \sigma_b(v_b) - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})\sigma_b(v_b)}{v_b - \sigma_b(v_b)})(\sigma_b(v_b))^{\frac{1}{\epsilon}}$$

Substituting $v_b = \beta(t)$, $G_s(\sigma_b(v_b)) = \xi(t)$ and $G_s'(\sigma_b(v_b)) = \dot{\xi}(t)$ in above equation, we get (11). A similar proof works for the strategic seller. ■

The next proposition gives a list of conditions that are sufficient to construct a pair of strategies that is indeed in D^*. I will use this proposition to generate a numerical solution.

Proposition 4.7 *Suppose* $(\hat{\beta}, \hat{\xi})$ *are strictly increasing functions that solve the differential equation system in lemma 4.6 in some interval* $I \subset \Re$. *If there exist* $\underline{\tau}$ *and* $\bar{\tau}$ *such that:*

1. $0 < \underline{\tau} < \bar{\tau} < 1$ *and* $[\underline{\tau}, \bar{\tau}] \subset I$.
2. $\hat{\beta}(t) > t$ *and* $\hat{\xi}(t) < t \; \forall t \in [\underline{\tau}, \bar{\tau}]$.
3. $\hat{\beta}(\underline{\tau}) \leq (\frac{2+3\epsilon}{2+2\epsilon})\underline{\tau}$, $\hat{\beta}(\bar{\tau}) \leq 1$, $\hat{\xi}(\underline{\tau}) \geq 0$ *and* $\hat{\xi}(\bar{\tau}) \geq (\frac{2+3\epsilon}{2+2\epsilon})(\bar{\tau} - \frac{\epsilon}{2+3\epsilon})$.
4. $(1-\epsilon)(\frac{1}{2}(1-\hat{\beta}(\underline{\tau})) - (\frac{2+3\epsilon}{2+2\epsilon})t) + (1+\epsilon)(\frac{\epsilon}{2+2\epsilon} - (\frac{2+3\epsilon}{2+2\epsilon})t)(1-t)^{\frac{1}{\epsilon}} \geq 0, \; \forall t \leq \underline{\tau}$.
5. $(1-\epsilon)((1-t)(\frac{2+3\epsilon}{2+2\epsilon}) - \frac{1}{2}\hat{\xi}(\bar{\tau})) + (1+\epsilon)(1 - (\frac{2+3\epsilon}{2+2\epsilon})t)t^{\frac{1}{\epsilon}} \leq 0, \; \forall t \geq \bar{\tau}$.
6. $(1-\epsilon)(\hat{\beta}(\underline{\tau}) - \underline{\tau})\hat{\xi}(\underline{\tau}) = \epsilon\{(\frac{\epsilon}{1+2\epsilon})(\frac{2+2\epsilon}{2+3\epsilon})^{\frac{1}{\epsilon}+1}(\hat{\beta}(\underline{\tau}))^{\frac{1}{\epsilon}+2} - (\hat{\beta}(\underline{\tau}) - (\frac{2+3\epsilon}{2+4\epsilon})\underline{\tau})\underline{\tau}^{\frac{1}{\epsilon}+1}\}$
7. $(1-\epsilon)(\bar{\tau} - \hat{\xi}(\bar{\tau}))(1-\hat{\beta}(\bar{\tau})) = \epsilon\{(\frac{\epsilon}{1+2\epsilon})(\frac{2+2\epsilon}{2+3\epsilon})^{\frac{1}{\epsilon}+1}(1-\hat{\xi}(\bar{\tau}))^{\frac{1}{\epsilon}+2} - (1-\hat{\xi}(\bar{\tau}) - (\frac{2+3\epsilon}{2+4\epsilon})(1-\bar{\tau}))(1-\bar{\tau})^{\frac{1}{\epsilon}+1}\}$

then there exist a pair of strategies $(\hat{\sigma}_b, \hat{\sigma}_s) \in D^*$ *such that* $\hat{\sigma}_b^{-1} = \hat{\beta}$ *and* $\hat{\sigma}_s^{-1} = \hat{\xi}$ *in the interval* $[\underline{\tau}, \bar{\tau}]$.

Proof: Let $\underline{v}_b = \hat{\beta}(\underline{\tau})$, $\bar{v}_b = \hat{\beta}(\bar{\tau})$, $\underline{v}_s = \hat{\xi}(\underline{\tau})$ and $\bar{v}_s = \hat{\xi}(\bar{\tau})$. Then, $\hat{\beta}^{-1}$ and $\hat{\xi}^{-1}$ are well-defined strictly increasing and differentiable functions on the intervals $[\underline{v}_b, 1]$ and $[0, \bar{v}_s]$, respectively. Now, define $(\hat{\sigma}_b, \hat{\sigma}_s)$ as follows:

$$\hat{\sigma}_b(v_b) = \begin{cases} \bar{\tau} & \text{if } v_b \geq \bar{v}_b \\ \hat{\beta}^{-1}(v_b) & \text{if } v_b \in [\underline{v}_b, \bar{v}_b] \\ \frac{2+2\epsilon}{2+3\epsilon} v_b & \text{if } v_b < \underline{v}_b \end{cases}$$

$$\hat{\sigma}_s(v_s) = \begin{cases} \underline{\tau} & \text{if } v_s \leq \underline{v}_s \\ \hat{\xi}^{-1}(v_s) & \text{if } v_s \in [\underline{v}_s, \bar{v}_s] \\ \frac{2+2\epsilon}{2+3\epsilon} v_s + \frac{\epsilon}{2+3\epsilon} & \text{if } v_s > \bar{v}_s \end{cases}$$

I prove that $(\hat{\sigma}_b, \hat{\sigma}_s) \in D^*$. Given $\hat{\sigma}_s$, the strategic buyer's belief about the distribution of seller's bids is:

- With probability $1 - \epsilon$ the seller is strategic and therefore,

$$G_s(t) = \begin{cases} 0 & \text{if } t < \underline{\tau} \\ \underline{v}_s & \text{if } t = \underline{\tau} \\ \hat{\xi}(t) & \text{if } \underline{\tau} \leq t \leq \bar{\tau} \\ \hat{\xi}(\bar{\tau}) & \text{if } \bar{\tau} \leq t \leq (\frac{2+2\epsilon}{2+3\epsilon})\hat{\xi}(\bar{\tau}) + \frac{\epsilon}{2+3\epsilon} \\ (\frac{2+3\epsilon}{2+2\epsilon})(t - \frac{\epsilon}{2+3\epsilon}) & \text{if } (\frac{2+2\epsilon}{2+3\epsilon})\hat{\xi}(\bar{\tau}) + \frac{\epsilon}{2+3\epsilon} \leq t \leq 1 \end{cases}$$

- With probability ϵ the seller is trustworthy type and therefore the bids are distributed as $F_{tr_s}(t)$ for $t \in [0, 1]$.

Suppose a strategic buyer bids above $(\frac{2+2\epsilon}{2+3\epsilon})\hat{\xi}(\bar{\tau}) + \frac{\epsilon}{2+3\epsilon}$, then the derivative of the strategic buyer's expected payoff is,

$$\begin{aligned} \frac{dU_b}{dt} &= (1-\epsilon)((v_b - t)(\frac{2+3\epsilon}{2+2\epsilon}) - \frac{1}{2}G_s(t)) + (1+\epsilon)(v_b - (\frac{2+3\epsilon}{2+2\epsilon})t)t^{\frac{1}{\epsilon}} \\ &\leq (1-\epsilon)((1 - t)(\frac{2+3\epsilon}{2+2\epsilon}) - \frac{1}{2}G_s(t)) + (1+\epsilon)(1 - (\frac{2+3\epsilon}{2+2\epsilon})t)t^{\frac{1}{\epsilon}} \\ &\leq (1-\epsilon)((1 - t)(\frac{2+3\epsilon}{2+2\epsilon}) - \frac{1}{2}G_s(\bar{\tau})) + (1+\epsilon)(1 - (\frac{2+3\epsilon}{2+2\epsilon})t)t^{\frac{1}{\epsilon}} \\ &= (1-\epsilon)((1 - t)(\frac{2+3\epsilon}{2+2\epsilon}) - \frac{1}{2}\hat{\xi}(\bar{\tau})) + (1+\epsilon)(1 - (\frac{2+3\epsilon}{2+2\epsilon})t)t^{\frac{1}{\epsilon}} \leq 0 \end{aligned}$$

The third step follows from the fact that $G_s(\bar{\tau}) \leq G_s(t)$ since $t \geq (\frac{2+2\epsilon}{2+3\epsilon})\hat{\xi}(\bar{\tau}) + \frac{\epsilon}{2+3\epsilon} \geq \bar{\tau}$. The last step uses condition 5. Therefore, $\frac{dU_b}{dt} \leq 0$ for all v_b and all $t \geq (\frac{2+2\epsilon}{2+3\epsilon})\hat{\xi}(\bar{\tau}) + \frac{\epsilon}{2+3\epsilon}$. Hence, no strategic buyer will bid higher than $(\frac{2+2\epsilon}{2+3\epsilon})\hat{\xi}(\bar{\tau}) + \frac{\epsilon}{2+3\epsilon}$.

Suppose a strategic buyer bids in the interval $(\bar{\tau}, (\frac{2+2\epsilon}{2+3\epsilon})\hat{\xi}(\bar{\tau}) + \frac{\epsilon}{2+3\epsilon})$, then the derivative of her expected payoff is,

$$\begin{aligned}\frac{dU_b}{dt} &= -\frac{1}{2}(1-\epsilon)\hat{\xi}(\bar{\tau}) + (1+\epsilon)(v_b - (\frac{2+3\epsilon}{2+2\epsilon})t)t^{\frac{1}{\epsilon}} \\ &\leq -\frac{1}{2}(1-\epsilon)\hat{\xi}(\bar{\tau}) + (1+\epsilon)(1 - (\frac{2+3\epsilon}{2+2\epsilon})t)t^{\frac{1}{\epsilon}} \\ &\leq (1-\epsilon)((1-t)(\frac{2+3\epsilon}{2+2\epsilon}) - \frac{1}{2}\hat{\xi}(\bar{\tau})) + (1+\epsilon)(1 - (\frac{2+3\epsilon}{2+2\epsilon})t)t^{\frac{1}{\epsilon}} \leq 0\end{aligned}$$

Hence, no strategic buyer will bid in this interval as well; therefore, all valuation types of strategic buyer bid less than or equal to $\bar{\tau}$.

If a strategic buyer bids less than $\underline{\tau}$, then, from lemma 4.4, she must bid $\frac{2+2\epsilon}{2+3\epsilon}v_b$. Hence, all valuation types of strategic buyer with valuations $v_b > \frac{2+2\epsilon}{2+3\epsilon}\underline{\tau}$ must bid greater than or equal to $\underline{\tau}$. Consider the function, $f(v_b)$, where $v_b \leq \frac{2+2\epsilon}{2+3\epsilon}\underline{\tau}$, that is the difference between bidding $\underline{\tau}$ and $\tilde{t} = \frac{2+2\epsilon}{2+3\epsilon}v_b$,

$$\begin{aligned}f(v_b) &= (1-\epsilon)(v_b - \underline{\tau})\underline{v}_s + (1+\epsilon)(\int_{[0,\underline{\tau}]}(v_b - \frac{1}{2}(\underline{\tau}+t))t^{\frac{1}{\epsilon}}dt - \int_{[0,\tilde{t}]}(v_b - \frac{1}{2}(\tilde{t}+t))t^{\frac{1}{\epsilon}}dt) \\ &= (1-\epsilon)(v_b - \underline{\tau})\underline{v}_s - \epsilon\{(\frac{\epsilon}{1+2\epsilon})(\frac{2+2\epsilon}{2+3\epsilon})^{\frac{1}{\epsilon}+1}(v_b)^{\frac{1}{\epsilon}+2} - (v_b - (\frac{2+3\epsilon}{2+4\epsilon})\underline{\tau})\underline{\tau}^{\frac{1}{\epsilon}+1}\}\end{aligned}$$

Condition 6 in the proposition guarantees that $f(\underline{v}_b) = 0$. It is easy to show that $f'(v_b) > 0$ for $v_b < \frac{2+2\epsilon}{2+3\epsilon}\underline{\tau}$. Hence, all valuation types of strategic buyer with valuations above $\underline{v}_b$ bid greater than or equal to $\underline{\tau}$.

If a strategic buyer bids in the interval $(\underline{\tau}, \bar{\tau})$, then the first order condition is,

$$(1-\epsilon)\{(v_b - t)\hat{\xi}'(t) - \frac{1}{2}\hat{\xi}(t)\} + (1+\epsilon)\{v_b - t - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})t\}(t)^{\frac{1}{\epsilon}} = 0 \tag{12}$$

$$\implies \hat{\xi}'(t) = \frac{1}{2}(\frac{\hat{\xi(t)}}{v_b - t}) - (\frac{1+\epsilon}{1-\epsilon})(\frac{v_b - t - \frac{1}{2}(\frac{\epsilon}{1+\epsilon})t}{v_b - t})(t)^{\frac{1}{\epsilon}}.$$

$t = \hat{\beta}^{-1}(v_b)$ satisfies the last equation since $(\hat{\beta}, \hat{\xi})$ solve the differential equation system in lemma 4.6. Since $\hat{B}(\bar{v}_b) = \hat{\beta}^{-1}(\bar{v}_b) = \bar{\tau}$, all valuation types of strategic buyer with valuations above $\bar{v}_b$ will bid $\bar{\tau}$. Also, if a strategic buyer with valuation v_b bids $\underline{\tau}$, then the left-hand side of (12) should be non-positive when evaluated at $(v_b, \underline{\tau})$. But, since the left-hand side of (12) is equal to 0 at $(\underline{v}_b, \underline{\tau})$, all valuation types of strategic buyer with valuation in the interval $(\underline{v}_b, \bar{v}_b)$ will prefer to bid $\hat{\beta}^{-1}(v_b)$. Also, since $f(v_b) < 0$ for all valuation types of strategic buyer with valuations less than $\underline{v}_b$, they will prefer to bid less than $\underline{\tau}$. ■

4.2.1 Numerical Solution

The differential equation system in the lemma 4.6 cannot be solved analytically. I numerically solve the system for different values of ϵ with the initial conditions $\beta(0.5) = 5/8$ and $\xi(0.5) = 3/8$ using *Mathematica 5.0.* Then, I take the following steps:

1. I find $\underline{t}$ and $\bar{t}$ as the solution to conditions 6 and 7, respectively, in proposition 4.7. Using the *FindRoot* operator, I look for a solution to condition 6 starting at 0.26 and for condition 7 starting at 0.74.[14]

2. Once I get the particular estimates for $\underline{t}$ and $\bar{t}$, I confirm that the numerical solutions $\beta(t)$ and $\xi(t)$ are strictly increasing in the interval $[\underline{t}, \bar{t}]$. I check that the other conditions listed in proposition 4.7 are also satisfied.

3. I calculate the probability of trade in the event that both players are strategic using three related calculations. Denote by P_j^{ϵ}, $j = 1, 2, 3$, the probability of trade in the event that both players are strategic for a particular value of ϵ which are calculated using the three different methods 1, 2 and 3 given below.

   (a) Start with a strategic buyer with valuation $v_b \geq \underline{v}_b$. The probability she trades with a strategic seller in the event that both players are strategic is

$$\begin{aligned} Probability(v_s \mid \sigma_s(v_s) \leq \sigma_b(v_b)) &= \int_{[0,\sigma_s^{-1}(\sigma_b(v_b))]} dv_s \\ &= \xi(\sigma_b(v_b)) \end{aligned}$$

   Hence,

$$\begin{aligned} P_1^{\epsilon} &= \int_{[\underline{v}_b,1]} \xi(\sigma_b(v_b))dv_b \\ &= \int_{[\underline{t},\bar{t}]} \xi(t)\beta'(t)dt + (1-\beta(\bar{t}))\xi(\bar{t}), \end{aligned}$$

   where I changed the variable $v_b = \beta(t)$ and used the fact that all valuation types of strategic buyer with valuations above $\bar{v}_b = \beta(\bar{t})$ bid equal to $\bar{t}$ and hence trade with all valuation types of strategic seller with valuations below $\xi(\bar{t})$.

   (b) Now instead, begin with a strategic seller with valuation $v_s \leq \bar{v}_s$. Using similar steps as above we can calculate,

$$P_2^{\epsilon} = \int_{[\underline{t},\bar{t}]} (1-\beta(t))\xi'(t)dt + (1-\beta(\underline{t}))\xi(\underline{t})$$

   (c) Finally, I combine the two methods above and calculate,

$$\begin{aligned} P_3^{\epsilon} = &\{\int_{[\underline{t},0.5]} (1-\beta(t))\xi'(t)dt + (1-\beta(\underline{t}))\xi(\underline{t})\} \\ &+\{\int_{[0.5,\bar{t}]} \xi(t)\beta'(t)dt + (1-\beta(\bar{t}))\xi(\bar{t})\} - \frac{9}{64} \end{aligned}$$

   I subtract 9/64 because in adding the first two terms we count the square with side 3/8 twice.

[14]These starting values are a conservative guess about the interval of bids since higher (lower) values of $\underline{t}$ ($\bar{t}$) should in general give a lower probability of trade in the event that both players are strategic.

The results of the numerical solution for different values of ϵ are listed in tables 1, 2 and 3 in the appendix. In the first row of each table, I give the results for $\epsilon = 0$, which pertains to the C-S equilibrium. Define $D_j^\epsilon = P_j^\epsilon - 9/32,\ j = 1, 2, 3$. That is, D_j^ϵ is the the difference between the probability of trade in the event that both players are strategic for a particular ϵ and the probability of trade in the C-S equilibrium, where the former is calculated using method j. Table 3 reports D_1^ϵ, D_2^ϵ, and D_3^ϵ.

Remark 4.8 *A few remarks regarding the tables before I analyze the results:*

1. *Notice that for some small values of ϵ, the value of $\xi(\underline{t})$ is negative. This affects the calculation of P_1^ϵ and, hence, D_1^ϵ. Not surprisingly, D_1^ϵ is in fact negative for most of such values of ϵ. But since this is true even when we numerically solve the C-S equilibrium with $\epsilon = 0$, I attribute it to a numerical approximation error.*

   Also, in the calculation of P_2 and P_3, the term $(1 - \beta(\underline{t}))\xi(\underline{t})$ is negative whenever $\xi(\underline{t})$ is negative. In such a situation, I ignore this term in the calculations since the logic of including this term is that whenever $\xi(\underline{t})$ is positive, all valuation types of strategic buyer who bid above $\underline{t}$ trade with all valuation types of strategic seller with valuations below $\xi(\underline{t})$.

2. *From lemma 4.3, we know that in any equilibrium we must have $\sigma_b(v_b) < v_b$ if $v_b \neq 0$ and $\sigma_s(v_s) > v_s$ if $v_s \neq 1$. However, in table 1 one can see that this is no true for values of $\epsilon \leq 0.07$, since $\beta(\underline{t}) = \underline{t}$ and $\xi(\bar{t}) = \bar{t}$. this again could be a numerical error, but, in any case, this condition is not satisfied only at these boundary points and hence it will not affect the calculation of the probabilities.*

3. *Since these are numerical solutions I cannot report their functional form, but for small values of ϵ one can see that the graphs of the solutions approximately coincide with the graphs of the C-S strategies. This is the reason why in table 1, $\underline{t}$, $\bar{t}$, $\beta(\underline{t})$, $\beta(\bar{t})$, $\xi(\underline{t})$ and $\xi(\bar{t})$ are same as in the C-S equilibrium for small values of ϵ.*

Table 2 shows that the probability of trade in the event that both players are strategic is approximately at least as high as the ex-ante probability of trade in the C-S equilibrium. We can see that the this probability is approximately strictly higher than 9/32 for ϵ above 3%. This increase can be attributed in part to the increase in the highest bid of the strategic buyer and decrease in the lowest bid of the strategic seller (see Table 1). However, even for very high values of ϵ, the gain in probability of trade is not so significant. The highest probability of trade reported in Table 2 is 0.296874 when $\epsilon = 0.5$, which is an increase of approximately just 5% over 9/32.

Similar conclusions can be drawn from Table 3. The reason for reporting D_j^ϵ is that one cannot make out any difference between the probability of trade among strategic types and C-S linear equilibrium for ϵ as large as 3%. Since for small values of ϵ the numerical solution is very close to the C-S equilibrium (see remark 4.8.3 above), the difference in probabilities of trade is so small that *Mathemtica5.0* numerically approximates it to the same number up to 5 decimal places. Hence, to discern such small differences in magnitudes, I subtract 9/32 from P_j^ϵ. In analyzing the results in this table, I compare D_j^ϵ with D_j^0. Note that D_j^0 is the numerical error in calculating the ex-ante probability of trade in the C-S equilibrium using a numerical solution. So I take D_j^0 to be the "tolerance" level of the difference between P_j^ϵ and 9/32 in the sense that if $D_j^\epsilon \geq D_j^0$, I conclude that the equilibrium for that value of ϵ does at least as well as the C-S equilibrium. Those entries for which the "tolerance" limit is breached, that is, $D_j^\epsilon < D_j^\epsilon$ are marked by a star, $\star$.

Looking at Table 3, one can be sure that the numerical solution does better for ϵ greater than or equal to 3%. The numerical value of D_j^ϵ is equal to D_j^0 for all values of ϵ up to 0.007. Thereafter, in general, one can see a increase in D_j^ϵ as ϵ increases, except for ϵ values close to 2%. But, as we concluded from Table 2, the probability of trade among strategic types is not notably higher than that in the C-S equilibrium.

5 Conclusion

This paper introduces uncertainty regarding the presence of trustworthy types of traders in a bilateral trading problem to study the consequence of trust on the behavior of strategic traders. It specifically focuses on two factors, *degree* and *distribution* of trust. A positive result of this study, in contrast to the previously known Meyrson and Satterthwaite (1983) impossibility theorem, is that if there is high enough *degree of trust* in at least one side of the market, then irrespective of the *distribution of trust*, it is possible to construct mechanisms that are ex-post efficient. The particular mechanism that this paper constructs is a direct mechanism in which trustworthy types truthfully report their type. However, I also prove that, unlike these direct mechanisms, real world mechanisms like k-double auctions with or without pre-play communication are always inefficient.

This study also proves that with respect to direct mechanisms, an increase in the *degree of trust* or a change in *distribution of trust* so that now high-surplus types are more likely among trustworthy types leads to higher probability of trade among strategic types. Both these results are not true for k-double auctions. Specifically, I show that when the valuations of all types are uniformly and independently distributed on $[0, 1]$, any equilibrium outcome of any k-double auction for any positive *degree of trust* attains a lower probability of trade among strategic types than when there is no trust among players. Thus a higher *degree of trust* is not necessarily better. The example of a 1/2-double auction in section 4.2 shows that a change in *distribution of trust* that makes high-surplus types more likely among trustworthy types can reduce the probability of trade among strategic types in k-double auctions.

The notably different effects of trust in direct mechanisms vis-à-vis k-double auctions caution against generalizations regarding the consequence of trust in various institutions. The structure and rules of different institutions provide different avenues for trustworthy disposition, which in turn implies different opportunities for strategic behavior.

Appendix

ϵ	$\underline{t}$	$\bar{t}$	$\beta(\underline{t})$	$\beta(\bar{t})$	$\xi(\underline{t})$	$\xi(\bar{t})$
0	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.0001	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.001	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.002	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.003	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.004	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.005	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.006	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.007	0.25	0.75	0.25	1	-1.62204×10^{-10}	0.75
0.008	0.25	0.75	0.25	1	-1.6225×10^{-10}	0.75
0.009	0.25	0.75	0.25	1	-1.6331×10^{-10}	0.75
0.01	0.25	0.75	0.25	1	-2.66222×10^{-10}	0.75
0.012	0.25	0.75	0.25	1	-2.56039×10^{-9}	0.75
0.014	0.25	0.75	0.25	1	-1.15907×10^{-10}	0.75
0.016	0.25	0.75	0.25	1	1.0747×10^{-9}	0.75
0.018	0.25	0.75	0.25	1	3.27144×10^{-9}	0.75
0.02	0.25	0.75	0.25	1	1.70891×10^{-8}	0.75
0.022	0.25	0.75	0.25	1	9.8367×10^{-8}	0.75
0.024	0.25	0.75	0.25	1	3.05627×10^{-7}	0.75
0.026	0.249999	0.750001	0.249999	1	-6.81248×10^{-7}	0.750001
0.028	0.249999	0.750001	0.249999	1	3.83519×10^{-7}	0.750001
0.03	0.249997	0.750003	0.249997	1	-5.60115×10^{-7}	0.750003
0.04	0.249964	0.750036	0.249964	0.999999	5.66771×10^{-7}	0.750036
0.05	0.249817	0.750183	0.249817	1	-4.22282×10^{-7}	0.750183
0.06	0.249452	0.750548	0.249452	1	2.10302×10^{-7}	0.750548
0.07	0.248779	0.751221	0.248779	1	4.12166×10^{-7}	0.751221
0.08	0.247747	0.752253	0.247748	0.999998	1.8364×10^{-6}	0.752252
0.09	0.246339	0.753661	0.246341	0.999996	4.39005×10^{-6}	0.753659
0.1	0.244568	0.755432	0.244573	0.99999	1.00596×10^{-5}	0.755427
0.2	0.217517	0.782483	0.217751	0.999593	4.06889×10^{-4}	0.782249
0.3	0.200606	0.799394	0.201521	0.998096	1.90404×10^{-3}	0.798479
0.4	0.197769	0.802232	0.200245	0.995669	4.33248×10^{-3}	0.799757
0.5	0.198301	0.8017	0.203929	0.993763	6.23858×10^{-3}	0.796073

Table 1: 1/2-*double auction with low-surplus more likely among trustworthy types*: Strategic seller's lowest bid ($\underline{t}$) and strategic buyer's highest bid ($\bar{t}$) and the valuation types of players who bid equal those bids for different values of ϵ.

ϵ	P_1^ϵ	P_2^ϵ	P_3^ϵ
0	0.28125	0.28125	0.28125
0.0001	0.28125	0.28125	0.28125
0.001	0.28125	0.28125	0.28125
0.002	0.28125	0.28125	0.28125
0.003	0.28125	0.28125	0.28125
0.004	0.28125	0.28125	0.28125
0.005	0.28125	0.28125	0.28125
0.006	0.28125	0.28125	0.28125
0.007	0.28125	0.28125	0.28125
0.008	0.28125	0.28125	0.28125
0.009	0.28125	0.28125	0.28125
0.01	0.28125	0.28125	0.28125
0.012	0.28125	0.28125	0.28125
0.014	0.28125	0.28125	0.28125
0.016	0.28125	0.28125	0.28125
0.018	0.28125	0.28125	0.28125
0.02	0.28125	0.28125	0.28125
0.022	0.28125	0.28125	0.28125
0.024	0.28125	0.28125	0.28125
0.026	0.281251	0.281251	0.281251
0.028	0.28125	0.28125	0.28125
0.03	0.28125	0.281251	0.281251
0.04	0.281251	0.281251	0.281251
0.05	0.281256	0.281256	0.281256
0.06	0.281268	0.281269	0.281268
0.07	0.281297	0.281298	0.281297
0.08	0.28135	0.28135	0.28135
0.09	0.281431	0.281431	0.281431
0.1	0.281545	0.281545	0.281545
0.2	0.284604	0.284604	0.284604
0.3	0.289498	0.289498	0.289498
0.4	0.293984	0.293984	0.293984
0.5	0.296874	0.296874	0.296874

Table 2: 1/2-*double auction with low-surplus more likely among trustworthy types*: Ex-ante probability of trade in the event that both players are strategic calculated using three different methods for different values of ϵ.

ϵ	D_1^ϵ	D_2^ϵ	D_3^ϵ
0	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.0001	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.001	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.002	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.003	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.004	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.005	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.006	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.007	-1.55832×10^{-11}	1.15322×10^{-10}	1.06541×10^{-10}
0.008	$-1.55848 \times 10^{-11}\star$	1.15355×10^{-10}	1.06574×10^{-10}
0.009	-1.51101×10^{-11}	1.16083×10^{-10}	1.07858×10^{-10}
0.01	2.42227×10^{-11}	1.86999×10^{-10}	2.25607×10^{-10}
0.012	9.32011×10^{-10}	2.16798×10^{-9}	2.11899×10^{-9}
0.014	1.48061×10^{-10}	$1.07479 \times 10^{-10}\star$	$8.33685 \times 10^{-11}\star$
0.016	7.57626×10^{-10}	2.14957×10^{-9}	1.63589×10^{-9}
0.018	$-2.78251 \times 10^{-9}\star$	$-2.00944 \times 10^{-10}\star$	$-3.65787 \times 10^{-9}\star$
0.02	$-1.59569 \times 10^{-8}\star$	$-1.22766 \times 10^{-9}\star$	$-1.8686 \times 10^{-8}\star$
0.022	$-6.2859 \times 10^{-8}\star$	9.66714×10^{-9}	$-6.61631 \times 10^{-8}\star$
0.024	$-2.02906 \times 10^{-7}\star$	2.83341×10^{-8}	$-2.13202 \times 10^{-7}\star$
0.026	5.90439×10^{-7}	5.71808×10^{-7}	1.07787×10^{-6}
0.028	$-2.52797 \times 10^{-7}\star$	7.02741×10^{-8}	$-2.45648 \times 10^{-7}\star$
0.03	4.95889×10^{-7}	5.06425×10^{-7}	9.4196×10^{-7}
0.04	8.46245×10^{-7}	8.6044×10^{-7}	8.54937×10^{-7}
0.05	5.53745×10^{-6}	5.52162×10^{-6}	5.85117×10^{-6}
0.06	1.83591×10^{-5}	1.85186×10^{-5}	1.83815×10^{-5}
0.07	4.72248×10^{-5}	4.75322×10^{-5}	4.73592×10^{-5}
0.08	9.98131×10^{-5}	9.978×10^{-5}	9.97872×10^{-5}
0.09	1.8066×10^{-4}	1.8056×10^{-4}	1.80601×10^{-4}
0.1	2.94661×10^{-4}	2.94583×10^{-4}	2.94542×10^{-4}
0.2	3.35405×10^{-3}	3.35399×10^{-3}	3.35404×10^{-3}
0.3	8.24839×10^{-3}	8.24838×10^{-3}	8.24839×10^{-3}
0.4	1.27341×10^{-2}	1.27339×10^{-2}	1.2734×10^{-2}
0.5	1.56241×10^{-2}	1.5624×10^{-2}	1.56241×10^{-2}

Table 3: 1/2-*double auction with low-surplus more likely among trustworthy types*: The difference between the ex-ante probability of trade in the C-S equilibrium and the ex-ante probability of trade in the event that both players are strategic for different values of ϵ. $D_j^\epsilon = P_j^\epsilon - \frac{9}{32}$ with $j = 1, 2, 3$. Starred ($\star$) values are those for which $D_j^\epsilon < D_j^0$.

References

[1] H. Aga and R. Malley, Camp David: The Tragedy of Errors, *The New York Review of Books* **48**, Number 13 (August 9, 2001). Available on http://www.nybooks.com/articles/14380.

[2] N. I. Al-Najjar and R. Casadesus-Masanell, Discretion in Agency Contracts, mimeo, Kellogg School of Management, Northwestern University, (2005).

[3] K. Chatterjee and W. Samuelson, Bargaining under Incomplete Information, *Operations Research* **31** (1983), 835-851.

[4] P. Dasgupta, Trust as a Commodity, *in* "Trust: Making and Breaking Cooperative Relations," (D. Gambetta ed.), electronic edition, Department of Sociology, University of Oxford, (2000), 49-72.

[5] J. Farrell and R. Gibbons, Cheap Talk can Matter in Bargaining, *Journal of Economic Theory* **48** (1989), 221-237.

[6] D. Gambetta, Can we Trust Trust?, *in* "Trust: Making and Breaking Cooperative Relations," (D. Gambetta ed.), electronic edition, Department of Sociology, University of Oxford, (2000), 213-237.

[7] D. Good, Individuals, Interpersonal Relations, and Trust, *in* "Trust: Making and Breaking Cooperative Relations," (D. Gambetta ed.), electronic edition, Department of Sociology, University of Oxford, (2000), 31-48.

[8] R. Hardin, "Trust and Trustworthiness," Russell Sage Foundation, New York, (2002).

[9] D.M. Kreps, P. Milgrom, J. Roberts and R. Wilson, Rational Cooperation in the Finitely Repeated Prisoner's Dilemma, *Journal of Economic Theory* **27** (1982), 245-252.

[10] D.M. Kreps and R. Wilson, Reputation and Imperfect Information, *Journal of Economic Theory* **27** (1982), 253-279.

[11] W. Leininger, P.B. Linhart and R. Radner, Equilibria of the Sealed-Bid Mechanism for Bargaining with Incomplete Information, *Journal of Economic Theory* **48** (1989), 63-106.

[12] S.A. Mathews and A. Postlewaite, Pre-play Communication in Two-Person Sealed-Bid Double Auctions, *Journal of Economic Theory* **48** (1989), 238-263.

[13] K.L. McGinn, L. Thompson and M.H. Bazerman, Dyadic Processes of Disclosure and Reciprocity in Bargaining with Communication, *Journal of Behavioral Decision Making* **16** (2003), 17-34.

[14] P. Milgrom and J. Roberts, Predation, Reputation and Entry Deterrence, *Journal of Economic Theory* **27** (1982), 280-312.

[15] R.B. Myerson and M.A. Satterthwaite, Efficient Mechanisms for Bilateral Trading, *Journal of Economic Theory* **29** (1983), 265-281.

[16] R. Radner and A. Schotter, The Sealed-Bid Mechanism: An Experimental Study, *Journal of Economic Theory* **48** (1989), 179-220.

[17] R. Saran, Bargaining with Behavioral Players: Strategic Deception and More Trade, mimeo, Brown University (2006).

[18] M.A. Satterthwaite and S.R. Williams, Bilateral Trade with the Sealed Bid k-Double Auction: Existence and Efficiency, *Journal of Economic Theory* **48** (1989), 107-133.

[19] J. Sobel, A Theory of Credibility, *Review of Economic Studies* **LII** (1985), 557-573.

[20] K. Valley, L. Thompson, R. Gibbons and M.H. Bazerman, How Communication Improves Efficiency in Bargaining Games, *Games and Economic Behavior* **38** (2002), 127-155.