Bid and Guess: A Nested Mechanism for King Solomon's Dilemma*

Cheng-Zhong Qin† Chun-Lei Yang‡

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Abstract

In this paper we propose a mechanism to resolve King Solomon's dilemma about allocating an indivisible good at no cost to the participating agents. A distinctive feature of our mechanism is the design of a two-part contest that makes the agents guess each other's bids in a second-price auction. The accuracy of an agent's guess of the other agent's bid endogenously determines how much she pays for participating in the contest. The truthfully bidding Bayesian-Nash equilibrium of the contesting game results in a reduced game, which has a *unique* and *strict* Bayesian-Nash equilibrium that implements the efficient outcome.

KEYWORDS: King Solomon's dilemma, Nash equilibrium, Bayesian-Nash equilibrium, subgame-perfect equilibrium. (JEL C72, C79, D82)

1 Introduction

King Solomon's dilemma refers to a story of the wisdom of King Solomon (I Kings 3: 16-28). In this story, two women appear before King Solomon seeking for judgement. Having a baby with them, they each claim to be its mother. King Solomon wishes to give the child to the true mother at *no cost* to either woman. The difficulty is,

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†Department of Economics, University of California, Santa Barbara, CA 93106

‡Research Center for Humanity and Social Sciences, Academia Sinica, Taipei, Taiwan

however, that while the women know whose baby the child is King Solomon does not. King Solomon's solution, which consisted in threatening to cut the baby in two, is not foolproof: What would he have done if the fake mother had had the presence of mind to scream like a real mother?

Glazer and Ma (1989) provide a simple and elegant multi-stage mechanism, which implements the efficient outcome of King Solomon's dilemma in a unique subgame-perfect equilibrium without causing any of the women to pay. The information assumption in their paper is that the women's valuations of the baby are common knowledge among the women and King Solomon, with the only missing information being that King Solomon does not know which woman has the higher valuation.[1] They also provide another mechanism in an appendix to cover the case, in which it is commonly known that only the women are aware of each other's valuations of the child, while King Solomon knows only that the true mother values the child more than the fake mother does. Moore (1992) provides a simpler mechanism for resolving the dilemma in this general case. Glazer and Ma (1989) also consider several examples of economics relevance that resemble the features King Solomon dilemma.

The King Solomon's dilemma is generally about allocating an indivisible good to the agent with the highest valuation at no cost to any of the participating agents. In this paper, we propose a mechanism for resolving the dilemma under general information assumptions. We only assume that the agents are informed of whose valuation of the good is higher besides their own valuations of the good.

Because bidding one's true valuation is weakly dominant, it is natural to consider the second-price auction as the mechanism to resolve the allocation problem. The second-price auction alone, however, cannot succeed as it involves payment from the winner to the auctioneer. Furthermore, it is not at all harmful to the less deserving agent to participate in the auction, and hence, leaves her with no good reason not to participate. The next natural step is then to add an endogenous fee for each participant to participate in a second-price auction, such that (i) it maintains weak dominance to truthfully bid; (ii) it is always positive for the less deserving agent so as to deter participation by her; and (iii) it is small enough to guarantee participation by the more deserving agent.

In this paper we show by an explicit design that such endogenous participation fees exist. In a simple form, each agent simultaneously bids and guesses how much the other will bid. She pays as a participation fee a fraction of the difference between

[1] It is assumed that the true mother has a higher valuation of the child than the fake mother does.

her guessed amount and the latter's actual bid. To the extent that the agent is uncertain about the other's exact valuation, this endogenous fee always takes a positive value. In an extended form, at random one agent's participation fee is modified from the simple form, with her guess about the opponent's bid in excess of the latter's guess of her bid as an additional component of the participation fee. Since the more deserving agent's bid is greater than her guess of the less deserving agent's bid, the latter's extended-form participation fee is always positive, even when she has complete information about the more deserving agent's valuation.

Our mechanism works as follows. First, the agents decide individually whether to demand the good. When only one of them does, the good is allocated to whoever demands. If neither does, the good is randomly allocated among the two in the usual way. When both agents demand, however, they will subsequently participate in a second-price auction with endogenous participation fees to determine the allocation.

The truthfully bidding Bayesian-Nash equilibrium for the corresponding second-price auction with endogenous participation fees turns out to imply that given agents's valuations, it is *strictly dominant* for the more deserving agent to demand, to which the *unique* best response for the less deserving agent is not to demand the good. It follows that our mechanism yields the desired implementation in a strict and unique Bayesian-Nash equilibrium in a reduced game, which results from the truthfully bidding Bayesian-Nash equilibrium for the modified second-price auction. It is worth mentioning that since the more deserving agent has a strictly dominant strategy, all the results remain valid if the agents' choices are made sequentially. With the extended-form participation fees, both the strictness and the uniqueness of Bayesian-Nash equilibrium for the reduced game are robust with respect to information settings, in the sense that they remain valid with or without complete information about each other's valuations. Moreover, the results generalize easily to the n-person case.

There have been several papers in the literature on King Solomon's dilemma under general information assumptions. Most of them involve nontrivial refinements of Nash equilibrium. For example, the mechanisms in Yang (1991) and Perry and Reny (1999) implement the efficient outcome in iteratively undominated strategies. The mechanism in Yang (1997) implements the efficient outcome in trembling-hand perfect equilibrium. Olszewski (2003) proposes a mechanism with endogenous side transfers that implements the efficient outcome in iteratively undominated strategies. His mechanism requires the designer to withhold the good in certain situations and to subsidize the agents when they both claim the good. Withholding the baby is arguably a non-credible option for King Solomon. The subsidies, on the other hand, make collusion beneficial to the agents at the designer's cost. The magnitude of the cost to the designer as a result of agents' collusion is bounded above only by the maximum bid the mechanism allows.[2]

In contrast, we assume the same general information structure as in Perry and Reny (1999). Our implementation, however, involves an equilibrium concept which results from backward induction together with weak dominance in the second stage, and is simpler and more natural than those applied in the literature for resolving the dilemma under general information conditions, such as iteratively undominated strategies in Perry and Reny (1999).

The rest of the paper is organized as follows. The next section introduces our nested mechanism in both extended and simple forms. Section 3 briefly extends the mechanism to more than 2-person cases. Section 4 concludes the paper.

2 The Nested Mechanism

In this section we introduce our nested mechanism and we show that the mechanism resolves King Solomon's dilemma under general information assumptions.

2.1 The Nested Mechanism in Extended Form

Consider the problem of efficient allocation of an indivisible good among two agents. Agents' valuations of the good are randomly drawn by nature from [0, 1]. (Normalize the agents' valuations so that the maximum valuation for the good is 1.) Nature informs each agent i of her own valuation v_i but not the valuation of her opponent. As characteristic to King Solomon's Dilemma, nature also informs each agent i of whose valuation is higher as well as a posterior $F_j(\cdot|v_i)$ about agent j's valuation, which is consistent with the information on the ranking of the agents' valuations.

Our mechanism involves the following messages. The agents simultaneously choose between demanding the good and not demanding it. We denote the first choice by "Mine" and the second by "Hers". Without any cost to either agent, the good will be allocated to agent 1 at choice combination (Mine, Hers), to agent 2 at (Hers, Mine), and to each with equal probability at (Hers, Hers). At (Mine, Mine), however, the agents' choices are in real conflict. To decide who gets the good (the baby) at this combination, a contest is applied in which each agent simultaneously announces a bid and a guess. The bids are used to determine the winner and the

[2] More precisely, let $\bar{b}$ be the maximum bid the mechanism allows and let a be the true mother's valuation. Then, in the 2-person case the highest total payoff that the agents can get by colluding is $a + \bar{b}$.

winning price as in second-price auction. The guesses, on the other hand, are used together with the bids to determine how much each agent pays for participating in the contest, which we specify in the following paragraph.

At the beginning of the contest, a lottery with two equally probable states, $s = 1, 2$, is drawn. The states of the lottery are made known to the agents before they bid and guess. The agents thus can condition their bids and guesses on the states of the lottery. The determination of participation fees for the agents depends on the states of the lottery. Since the incentive structure of second-price auction is independent of the lottery states which will become clear in the next paragraph, we can write agent i's bid-guess choice as $(\beta_i, \gamma_{i1}, \gamma_{i2})$, where β_i represents agent i's bid irrespective of the states of the lottery and γ_{is} represents her guess in state $s = 1, 2$. Set $\gamma_i = (\gamma_{i1}, \gamma_{i2})$. We restrict β_i, γ_{i1}, and γ_{i2} to be all in [0, 1].

Given the agents' bid-guess combinations (β_i, γ_i) and (β_j, γ_j), their participation fees in state $s = i$ are $\Delta_{ii}(\gamma_{ii}; \beta_j, \gamma_{ji})$ for agent i and $\Delta_{ji}(\gamma_{ji}; \beta_i)$ for agent j, where [3]

$$\Delta_{ii}(\gamma_{ii}; \beta_j, \gamma_{ji}) = \frac{\delta}{2}\{|\gamma_{ii} - \beta_j| + (\gamma_{ii} - \gamma_{ji})^+\}, \quad \Delta_{ji}(\gamma_{ji}; \beta_i) = \delta|\gamma_{ji} - \beta_i|, \ i \neq j \quad (1)$$

for some constant proportion $\delta \in (0, 1)$. Our analysis does not depend on the choice of proportion δ. In state $s = i$, agent j pays a *simple-form* fee that depends only on the deviation of her guessed amount of i's bid from i's actual bid. Agent i, on the other hand, pays an *extended-form* fee which depends not only on the deviation of her guessed amount of j's bid from j's actual bid, but also on the deviation of this guessed amount from j's guess of her bid. Evidently, the agents' bidding incentives remain unaffected by the guesses and the states of the lottery.

The expected participation fees for the agents over the states of the lottery are:

$$\Delta_i(\gamma_i; \beta_j, \gamma_{ji}) = \frac{1}{2}\Delta_{ii}(\gamma_{ii}; \beta_j, \gamma_{ji}) + \frac{1}{2}\Delta_{ij}(\gamma_{ij}; \beta_j), \ j \neq i. \quad (2)$$

It follows that given actual valuation v_i, agent i's payoff at bid-guess pairs (β_i, γ_i) and (β_j, γ_j) is:

$$U_i(\beta, \gamma, v_i) = \kappa_i(\beta)[v_i - \beta_j] - \Delta_i(\gamma_i; \beta_j, \gamma_{ji}), \ j \neq i, \quad (3)$$

where $\kappa_i(\beta)$ is i's winning probability at bid profile $\beta = (\beta_i, \beta_j)$; $\kappa_i(\beta) = 1$ if $\beta_i > \beta_j$, $\kappa_i(\beta) = 1/2$ if $\beta_i = \beta_j$, and $\kappa_i(\beta) = 0$ if $\beta_i < \beta_j$.

A (pure) strategy for an agent in the contesting game is a mapping that maps her information into a bid-guess pair. Since the agent's information depends on

[3] Given a real number x, $x^+ = x$ when $x \geq 0$ and $x^+ = 0$ when $x < 0$.

her valuation, as usual we write a strategy for the agent simply as a function of her valuation only. Specifically, a strategy for agent i in the contesting game is a pair (b_i, g_i), where $b_i : [0,1] \longrightarrow [0,1]$ specifies a bid $b_i(v_i) \in [0,1]$ for each valuation $v_i \in [0,1]$ and $g_i : [0,1] \longrightarrow [0,1] \times [0,1]$ specifies a pair of guesses $g_i(v_i) = (g_{i1}(v_i), g_{i2}(v_i))$ for each valuation $v_i \in [0,1]$.

Given valuation v_i, agent i's expected payoff, $U_i(b, g, v_i)$, at strategy profile $((b_i, g_i), (b_j, g_j))$ in the contesting game is the expected value of $U_i(b(v_i, v_i'), g(v_i, v_j'), v_i)$ in (3) over agent j's valuations v_j' with respect to posterior $F_j(v_j'|v_i)$, where $b(v_i, v_j') = (b_i(v_i), b_j(v_j'))$ and $g(v_i, v_j') = (g_i(v_i), g_j(v_j'))$.

By (1), (2), and (3), agent i's guesses only affect her participation fee. Given valuation v_i and given agent j's bid-guess strategy (b_j, g_j), to maximize expected payoff in the contesting game, agent i's optimal guesses must minimize the expected participation fee

$$\int \Delta_i(\gamma_i; b_j(v_j'), g_{ji}(v_j'))dF_j(v_j'|v_i).$$

By the Dominated Convergence Theorem (see, e.g., Rudin, 1964, p. 246), the integration $\int \Delta_i(\gamma_i; b_j(v_j'), g_{ji}(v_j'))dF_j(v_j'|v_i)$ is continuous in agent i's guesses γ_i. The minimum is thus well-defined. Denote the minimum expected participation fee for agent i by $\Delta_i(b_j, g_{ji}, v_i)$.[4] When the agents bid truthfully (i.e., when $b_i(v_i') = v_i'$ for $v_i' \in [0,1]$ and $b_j(v_j') = v_j'$ for $v_j' \in [0,1]$), we suppress bidding strategies from the minimum expected participation fees, so as to shorten the notation to $\Delta_i(g_{ji}, v_i)$ for $i \neq j$.

2.11 *A Characterization of Expected Participation Fees when Agents Bid Truthfully*

For ease of exposition, we assume without loss of generality that it is commonly known to the agents that nature draws a higher valuation for agent 1. That is, we assume

$$\text{supp}F_1(\cdot|v_2) \subseteq (v_2, 1] \text{ and } \text{supp}F_2(\cdot|v_1) \subseteq [0, v_1),\ v_1, v_2 \in [0,1] \text{ with } v_2 < v_1 \quad (4)$$

where supp$F_1(\cdot|v_2)$ and supp$F_2(\cdot|v_1)$denotes the supports of $F_1(\cdot|v_2)$ and $F_1(\cdot|v_2)$, respectively.

It turns out that under information assumption (4), the truthfully bidding Bayesian-Nash equilibrium for the contesting game results in a positive expected

[4] Given agent i's own valuation v_i and given that agent i anticipates that agent j bids according to b_j and guesses in state i according to g_{ji}, $\Delta_i(b_j, g_{ji}, v_i)$ is the expected participation fee agent i's optimal guesses in both states generate to her.

participation fee for agent 2 and a positive expected participation fee for agent 1 smaller than her expected value net the expected payment from winning the auction, given valuations of the agents.[5]

THEOREM 1 *Assume posteriors $F_1(\cdot|\cdot)$ and $F_2(\cdot|\cdot)$ satisfies (4). Then, bidding one's true valuation is consistent with Bayesian-Nash equilibrium for the contesting game. Furthermore, in the truthfully bidding Bayesian-Nash equilibrium $((b_1^*, g_1^*), (b_2^*, g_2^*))$:*

$$(i)\quad \Delta_1(g_{21}^*, v_1) < \delta \int (v_1 - v_2')dF_2(v_2'|v_1)$$

and

$$(ii)\quad \Delta_2(g_{12}^*, v_2) > 0$$

for $v_1, v_2 \in [0,1]$ such that $v_2 < v_1$.

PROOF: For each agent, her participation fee does not depend on her bid. On the other hand, neither her winning probability nor her winning price depends on her guess in either state of the lottery. It follows from the properties of second-price auction that truthfully bidding is weakly dominant for each agent. Consequently, bidding one's true valuation is consistent with Bayesian-Nash equilibrium for the contesting game.

Let (b_1^*, g_1^*) and (b_2^*, g_2^*) be the Bayesian-Nash equilibrium for the contesting game with $b_1^*(v_1') = v_1'$ for all $v_1' \in [0,1]$ and $b_2^*(v_2') = v_2'$ for all $v_2' \in [0,1]$. By (1) and (2), for $v_1, v_2 \in [0,1]$ such that $v_2 < v_1$,

$$\begin{aligned}\Delta_1(g_{21}^*, v_1) \;&= \tfrac{\delta}{2} \textstyle\int |g_{12}^*(v_1) - v_2'| dF_2(v_2'|v_1) \\ &+ \tfrac{\delta}{4} \textstyle\int \{|g_{11}^*(v_1) - v_2'| + (g_{11}^*(v_1) - g_{21}^*(v_2'))^+\} dF_2(v_2'|v_1)\end{aligned} \tag{5}$$

and

$$\begin{aligned}\Delta_2(g_{12}^*, v_2) \;&= \tfrac{\delta}{2} \textstyle\int |g_{21}^*(v_2) - v_1'| dF_1(v_1'|v_2) \\ &+ \tfrac{\delta}{4} \textstyle\int \{|g_{22}^*(v_2) - v_1'| + (g_{22}^*(v_2) - g_{12}^*(v_1'))^+\} dF_1(v_1'|v_2).\end{aligned} \tag{6}$$

Given that agent 2 truthfully bids, it follows easily from information assumption (4) and the first term in (5) that, in state $s = 2$ of the lottery, agent 1's optimal guessing strategy satisfies $g_{12}^*(v_1') < v_1'$ for all $v_1' \in (0,1]$. This shows

$$\int |g_{12}^*(v_1) - v_2'| dF_2(v_2'|v_1) < \int (v_1 - v_2') dF_2(v_2'|v_1).$$

[5] Though subsequently we tacitly assume that off-equilibrium beliefs are given by the posteriors, it is evident from the proofs that this assumption is not essential for our results. In fact, the only restriction needed for the off-equilibrium beliefs is the same as (4) for the posteriors.

By definition, $(g_{11}^*(v_1) - g_{21}^*(v_2'))^+$ is nondecreasing in $g_{11}^*(v_1)$. Consequently, from information assumption (4) and the last two terms in (5), it also follows $g_{11}^*(v_1) < v_1$. Thus,

$$\int \{|g_{11}^*(v_1) - v_2'| + (g_{11}^*(v_1) - g_{21}^*(v_2'))^+\} dF_2(v_2'|v_1) < \int (v_1 - v_2') dF_2(v_2'|v_1).$$

Putting the above inequalities together, we have [6]

$$\Delta_1(g_{21}^*, v_1) < \delta \int (v_1 - v_2') dF_2(v_2'|v_1).$$

To show (ii), notice that by (6),

$$\Delta_2(g_{12}^*, v_2) \geq \frac{\delta}{2} \int |g_{21}^*(v_2) - v_1'| dF_1(v_1'|v_2). \tag{7}$$

Consequently, if $F_1(\cdot|v_2)$ is such that agent 2 cannot correctly guess agent 1's bid (i.e. valuation) with probability 1, then (ii) follows directly from (7). Suppose that agent 2 can correctly guess in which case $F_1(\cdot|v_2)$ has v_1 as an atom that has a probability equal to 1. Then, minimizing the participation fee implies $g_{21}^*(v_2) = v_1 = b_1^*(v_1)$. Hence, in this case, (6) reduces to

$$\Delta_2(g_{12}^*, v_2) = \frac{\delta}{4} \{|g_{22}^*(v_2) - v_1| + (g_{22}^*(v_2) - g_{12}^*(v_1))^+\}. \tag{8}$$

Since $g_{12}^*(v_1) < v_1$ as shown above, at least one of the terms in (8) is strictly positive. Consequently, $\Delta_2(g_{12}^*, v_2) > 0$. ■

Consider the case with complete information about valuations v_1 and v_2 as in Glazer and Ma (1989). In this case, the agents under our mechanism can correctly guess each other's bids. With $v_1 > v_2$, it is not hard to see that in truthfully bidding Nash equilibrium for the contesting game, agent 1's guessing strategy satisfies $g_{11}^* = g_{12}^* = v_2$ while agent 2's satisfies $g_{21}^* = v_1$ and $v_2 \leq g_{22}^* \leq v_1$. The participation fees are 0 for agent 1 and $\delta(v_1 - v_2)/4 > 0$ for agent 2. It follows that even with complete information, agent 1 is not discouraged from entering the contest, while the positive participation fee is the deterrence against agent 2's participation.

Let $((b_1^*, g_1^*), (b_2^*, g_2^*))$ be the truthfully bidding Bayesian-Nash equilibrium for the contesting game. We shorten the notation for the agents' expected payoffs at $((b_1^*, g_1^*), (b_2^*, g_2^*))$ to $U_1(g^*, v_1)$ and $U_2(g^*, v_2)$ at valuations v_1 and v_2.

[6] When agent 2 bids truthfully, agent 1 gains $\int (v_1 - v_2') dF_2(v_2'|v_1)$ from the auction. It follows that the suboptimal guesses $g_{11}(v_1) = g_{12}(v_1) = v_1$ already ensures agent 1 a strictly positive payoff in the contesting game.

2.12 *Implementing the Efficient Outcome in a Unique and Strict Bayesian-Nash Equilibrium*

When information assumption (4) is satisfied, Theorem 1 shows that with the truthfully bidding Bayesian-Nash equilibrium $((b_1^*, g_1^*), (b_2^*, g_2^*))$ for the contesting game:

$$\begin{aligned} U_1(g^*, v_1) &= \int [v_1 - v_2'] dF_2(v_2'|v_1) - \Delta_1(g_{21}^*, v_1) \\ &> (1-\delta) \int (v_1 - v_2') dF_2(v_2'|v_1) \\ &> 0 \end{aligned} \tag{9}$$

and

$$U_2(g^*, v_2) \quad = -\Delta_2(g_{12}^*, v_2) < 0 \tag{10}$$

for $v_1, v_2 \in [0,1]$ with $v_1 > v_2$. From (9) and (10) it follows that in the reduced game in Figure 1 that results from the truthfully bidding Bayesian-Nash equilibrium for the contesting game, Mine is strictly dominant for agent 1 and Hers is the unique optimal choice for agent 2 given that agent 1 chooses Mine. In particular, the choice pair (Mine, Hers) is the unique and strict Bayesian-Nash equilibrium outcome for the reduced game.

		Agent 2	
		Mine	Hers
Agent 1	Mine	w_1, w_2	v_1, 0
	Hers	0, v_2	$\frac{v_1}{2}$, $\frac{v_2}{2}$

Figure 1: The Reduced Game Resulting from the Truthfully Bidding Bayesian-Nash Equilibrium $((b_1^*, g_1^*), (b_2^*, g_2^*))$ with $w_1 = U_1(g^*, v_1)$ and $w_2 = U_2(g^*, v_2)$.

In summary, we have shown that the nested mechanism with the extended-form participation fees implements the efficient outcome in a unique and strict Bayesian-Nash equilibrium of the reduced game under information assumption (4).

2.2 The Nested Mechanism in Simple Form

From the proof of Theorem 1 it is clear that the only reason for ever using the extended-form fee is to avoid zero participation fee for the fake mother in case of degenerate posteriors. If the posteriors are non-degenerate, a much simpler mechanism can resolve the King Solomon's dilemma equally successfully. We now simplify the mechanism by replacing the extended-form fees with fees in a simple form.

As before, agent i bids some amount $\beta_i \in [0,1]$ in the contesting game. However, unlike before, her participation fee depends on the accuracy of her guess $\gamma_i \in [0,1]$ of agent j's bid only. Specifically, given agents' bid-guess pairs (β_i, γ_i) and (β_j, γ_j), agent i's participation fee is given by

$$\Delta_i(\gamma_i; \beta_j) = \delta|\gamma_i - \beta_j|, j \neq i, \tag{11}$$

where $\delta \in (0,1]$ is a constant. Since agent i's participation fee in (11) does not depend on agent j's guess, we now let $\Delta_i(v_i)$ denote the minimum expected participation fee for agent i when the agents bid truthfully.

When agent 1 is more deserving and $F_1(\cdot|v_2)$ is non degenerate, the expected participation fees $\Delta_1(v_1)$ and $\Delta_2(v_2)$ satisfy the same properties in Theorem 1. We now summarize this result in Theorem 2 below. Proof of this theorem is omitted as it follows straightforwardly from the proof of Theorem 1.

THEOREM 2 *Assume $F_1(\cdot|\cdot)$ and $F_2(\cdot|\cdot)$ satisfies (4). Assume further $F_1(\cdot|v_2)$ is non degenerate for all v_2. Then, bidding one's true valuation is consistent with Bayesian-Nash equilibrium for the contesting game. Furthermore, in the truthfully bidding Bayesian-Nash equilibrium:*

$$\text{(i)} \quad \Delta_1(v_1) < \delta \int (v_1 - v_2')dF_2(v_2'|v_1)$$

and

$$\text{(ii)} \quad \Delta_2(v_2) > 0$$

for $v = (v_1, v_2)$ such that $v_1 > v_2$.

When the agents bid truthfully, Theorem 2 implies that under information assumption (4) and the non degeneracy of $F_1(\cdot|v_2)$, the nested mechanism with the simple-form participation fees for both agents also makes it strictly dominant for the more deserving agent to choose Mine, against which the unique best response for the less deserving agent is to choose Hers, given the agents' valuations $v_2 < v_1$. It follows that the simplified mechanism also yields the desired implementation in a unique and strict Bayesian-Nash equilibrium for the reduced game in Figure 1 under these information and the non degeneracy assumptions.

3 A Generalization to the n-Person Case

The mechanism for the 2-person case generalizes naturally to the case with n agents as follows. Each agent simultaneously chooses between demanding the good ("Mine") and not demanding it ("Others"). When all the agents choose Others, the good is randomly allocated among them in equal probability. When all but only one agent choose Others, the good is given to the agent who chooses Mine. When two or more agents choose Mine, those who did not make that choice end their participation with a zero payoff.

Let k be the number of the remaining agents. A lottery with k equally probable states is drawn before the agents decide on their bid-guess combinations, so they can condition their choices on the states of the lottery. Let (β_i, γ_i) with $\gamma_i = (\gamma_{ii}, \gamma_{io})$ denote agent i's bid-guess pair, where β_i is her bid, γ_{ii} is her guess in state $s = i$ of the highest bid by the agents other than herself, and γ_{io} is her guess of the highest bid in states $s \neq i$.

Given the other agents' bid-guess pairs (β_j, γ_j), let $j_i \neq i$ be the agent whose bid $\beta_{j_i} = \max_{j \neq i} \beta_j$. When state $s = i$ occurs, agent i's participation fee is given by

$$\Delta_{ii}(\gamma_{ii}; \beta_{j_i}, \gamma_{j_i o}) = \frac{\delta}{2}\{|\gamma_{ii} - \beta_{j_i}| + (\gamma_{ii} - \gamma_{j_i o})^+\},$$

where $\gamma_{j_i o}$ is the guess by agent j_i in state $s \neq j_i$. When state $s \neq i$ occurs, agent i's participation fee is given by

$$\Delta_{io}(\gamma_{i0}; \beta_{j_i}) = \delta|\gamma_{io} - \beta_{j_i}|.$$

It follows that the expected participation fee for agent i over the states of the lottery is:

$$\Delta_i(\beta_i, \gamma_i; \beta_{-i}, \gamma_{-i}, k) = \frac{k-1}{k}\Delta_{io}(\gamma_{io}; \beta_{j_i}) + \frac{1}{k}\Delta_{ii}(\gamma_{ii}; \beta_{j_i}, \gamma_{j_i o}),$$

where β_{-i} and γ_{-i} denote respectively the collections of bids and guesses by all participating agents other than agent i. These participation fees together with a parallel extension of information assumption (4) can carry through the results to the n-person case.

4 Conclusion

This paper proposes a mechanism for resolving King Solomon dilemma under general information assumptions. The mechanism involves a second-price auction with endogenous participation fees. In essence, the success of the mechanism is guaranteed by the properties that these endogenous participation fees maintain the incentives for truthfully bidding in second-price auction, they yield a positive participation fee for the less deserving agent so as to deter participation by her, and they yield a small enough participation fee for the more deserving agent to guarantee her participation.

All in all, the mechanism enables the more deserving agent to upstage the less deserving agent by making sufficient use of the fact that the agents know how deserving they themselves are. This is done by making them guess about each other's bids to determine their fees as well as participate in a second-price auction to determine the winner and the price of the good should a conflict arise.

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