Dynamics of R&D investment strategies in duopoly competitions

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Abstract — *We present new deterministic and stochastic dynamics on the production costs of Cournot competitions, based on Nash and Bayesian Nash equilibriums of nonlinear R&D investment strategies to reduce the production costs of the firms at every period of the game. We study some behaviours of the firms in the case of similar firms and in the case of non-identical firms with different R&D programs. In the deterministic case, we study the transients and the asymptotic dynamics on the production costs of the duopoly competition and their profound implications on the profit and persistence of the firms in the market. In the stochastic case, we analyse the importance of the uncertainty to reverse the initial advantage of one firm with respect to the other.*

1 Introduction

We present new deterministic and stochastic dynamics on the production costs of Cournot competitions, based on R&D investment strategies of the firms with and without uncertainty at every period of the game. At every period of time, the firms involved in a Cournot competition invest in R&D projects to reduce their production costs. By deciding, at every period, to use the Nash and Bayesian Nash equilibriums of the R&D strategies, the firms give rise to deterministic and stochastic dynamics on the production costs characterizing the duopoly competition. Hence, the deterministic dynamics are not obtained by using the well-known adjustment dynamics. In this paper, we present the game theoretical models on the Cournot competition with R&D investment programs that give rise to the deterministic and stochastic dynamical models and we will also study the effect of the dynamics in the profits and persistence of the firms in the market. We study some behaviours of the firms in the case of similar firms and in the case of non-identical firms with different R&D programs.

The Cournot competition with R&D investment programs consists of two subgames in one period of time. The first subgame is an R&D investment program, where both firms have initial production costs and choose, simultaneously, the R&D investment strategies to obtain new production costs. We consider the results of the R&D investment strategies with and without uncertainty in the value of the new production costs. In the presence of uncertainty, the R&D investment strategies determine the expected value of the reduced cost. The second subgame, for simplicity of the model, is a Cournot competition with production costs equal to the reduced cost determined by the R&D investment program. The second subgame has unique Nash and Bayesian Nash equilibriums. In some parameter region of our model, the game presents a unique Nash and Bayesian Nash equilibriums, except for initial costs far away of the minimum attainable reduced production cost where the uniqueness of the equilibrium is broken. This is true in cases of identical and non-identical firms (firms with different R&D programs). We also analyse the loss in the profits of one firm when this firm decides not to invest in R&D projects and the other firm uses the best response strategy (see Figure 1).

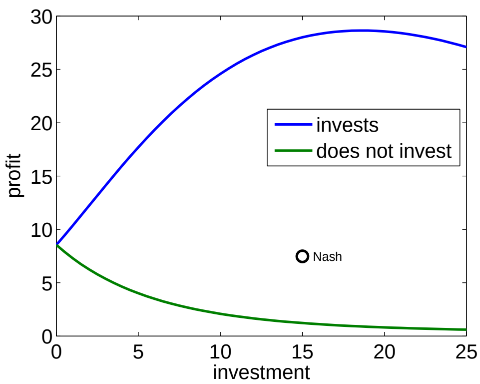

Figure 1: Profits of the firms for different investments of the firm F_2 when the firm F_1 does not invest, in the case of identical firms producing homogeneous goods, with initial production costs $c_{i_1} = 9$ and parameters $\alpha_i = 10$, $\beta_i = 0.013$, $c_{i_L} = 1$, $\epsilon_i = 0.2$, $\lambda_i = 0.1$ (see §2).

The deterministic dynamics on the production costs of the duopoly competition appear from the firms deciding to play the Nash equilibrium in the Cournot competition with R&D investment programs, period after period. For some parameter region of our model, and for identical firms except with respect to their initial production costs, the Figure 2 illustrates the transients and the asymptotic limits of the deterministic dynamics on the production costs of the duopoly competition.

We see that if the production costs of the firm F_1 are smaller than the production costs of the firm F_2, they will keep like that during the all game, although the same conclusion

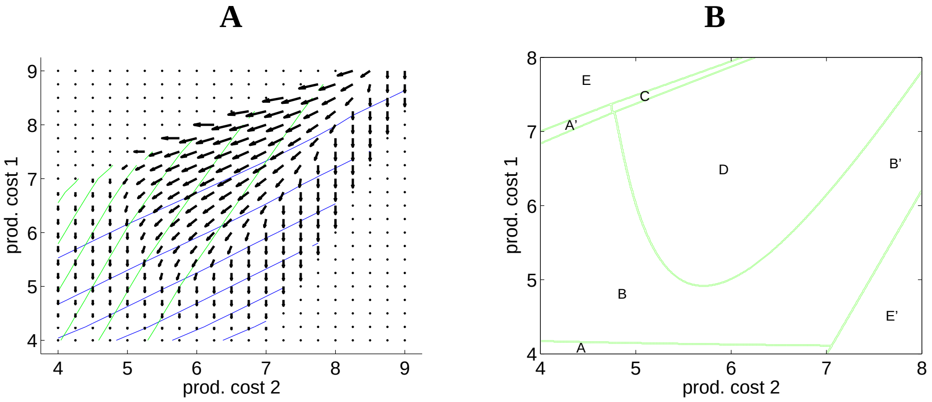

Figure 2: (**A**) The arrows show approximately the directions of the evolution of the production costs determined by the deterministic dynamics. (**B**) Regions with different qualitative dynamic behavior. We consider non-identical firms (firms with different R&D programs) producing homogeneous goods with parameters $\alpha_i = 10$, $\beta_i = 0.013$, $c_{i_L} = 4$, $\epsilon_i = 0.2$, $\lambda_1 = 0.3$, $\lambda_2 = 0.05$.

is not true if the production costs of the firm F_2 is smaller than the production costs of the firm F_1. We observe that if the initial production costs of both firms are high and the production costs of the firm F_1 are much higher than the firm F_2 (region C in Figure 2), then just the firm F_2 invests and after some time the firm F_1 is out of the market (region E in Figure 2). On the other hand, if the initial production costs of both firms are low and the production costs of the firm F_1 are higher than the firm F_2 (region B in Figure 2), then just the firm F_1 invests and the asymptotic production costs will be at the boundary between regions A and B, in Figure 2. If the initial production costs of both firms are high and the production costs of the firm F_1 are higher than the firm F_2 (region B' in Figure 2), then just the firm F_1 invests and after some time the firm F_2 is out of the market (region E' in Figure 2).

An interesting result from the fact that the firms decide their R&D investment strategies just having in mind one period of time, and not all the periods involved in the competitions is the following: for some parameter region of the model both firms have a higher profit, at every period, if they do not invest than if they choose to invest accordingly to the Nash equilibrium, but, surprisingly, the global profit is higher for both firms in the second case (see Figure 3).

The stochastic dynamics on the production costs of the duopoly competition appear from the firms deciding to play the Cournot competition with uncertainty in the results of their R&D investment programs, period after period. For some parameter region of our model, we observe that, in the presence of uncertainty in the model, if the production costs of the firm F_1 is smaller than the production costs of the firm F_2, but sufficiently close, then there is a proportion of outcomes that reverse the initial advantage of the firm F_1 (see Figure 4).

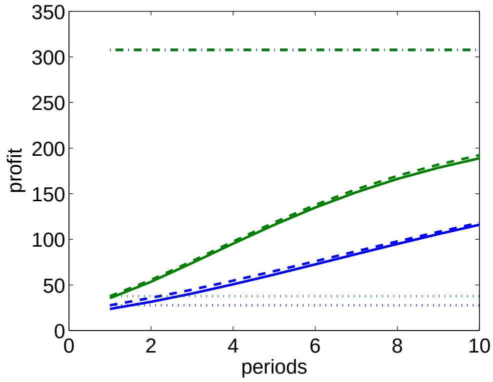

Figure 3: Evolution of the profits along the periods: The "line" corresponds to the evolution of the profit determined by the deterministic dynamics; The "dashes" determines the profit of the firms if they would not invest in that period; The "dash-dot" is the profit if both firms are at the Nash equilibrium of the Cournot game with the minimum attainable production cost. Identical firms producing homogeneous goods, with initial production costs $c_{1_1} = 8.1$, $c_{2_1} = 8.0$ and parameters $\alpha_i = 10$, $\beta_i = 0.013$, $c_{i_L} = 4$, $\epsilon_i = 0.2$, $\lambda_i = 0.06$.

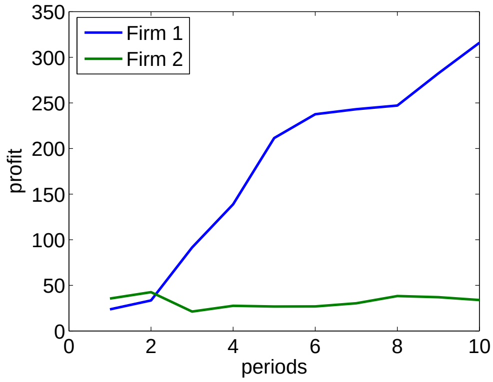

Figure 4: Evolution of the profits along the periods determined by the stochastic dynamics, in the case of identical firms producing homogeneous goods, with initial production costs $c_{1_1} = 8.1$, $c_{2_1} = 8.0$ and parameters $\alpha_i = 10$, $\beta_i = 0.013$, $c_{i_L} = 4$, $\epsilon_i = 0.08$, $\lambda_i = 0.06$.

2 R&D investments on costs

We are going to present the Cournot competition with R&D investment programs. First, we discuss the the Cournot competition and then the R&D investment strategies. We note that the Nash and Bayesian Nash equilibriums are computed in similar ways.

We consider an economy with a monopolistic sector with two firms, F_1 and F_2, each one producing a differentiated good. As considered by N. Singh and X. Vives [4], the representative consumer preferences are described by the following utility function

$$U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \left(\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2\right)/2,$$

where q_i is the amount of good produced by the firm F_i, and $\alpha_i, \beta_i > 0$, for $i \in \{1, 2\}$. The inverse demands are linear and, letting p_i be the price of the good produced by the firm F_i, they are given by

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2,$$

$$p_2 = \alpha_2 - \gamma q_1 - \beta_2 q_2,$$

in the region of quantity space where prices are positive. The goods are substitutes, independent, or complements according to whether $\gamma > 0$, $\gamma = 0$, or $\gamma < 0$, respectively. Demand for good i is always downward sloping in its own price and increases (decreases) with increases in the price of the competitor if the goods are substitutes (complements). When $\alpha_1 = \alpha_2$, the ratio $\gamma^2/\beta_1\beta_2$ expresses the degree of product differentiation ranging from zero when the goods are independent to one when the goods are perfect substitutes. When $\gamma > 0$ and $\gamma^2/\beta_1\beta_2$ approaches one, we are close to a homogeneous market.

The firm F_i invests an amount v_i in a R&D program that reduces the production costs to

$$a_i = c_i - \epsilon_i(c_i - c_{L_i})\left(1 - e^{-\lambda_i v_i}\right),$$

where the parameter $c_{i_L} > 0$ is the minimum attainable production cost for the firm F_i and c_i is the firm F_i's unitary production cost at the beginning of the period satisfying $c_{i_L} \le c_i < \alpha_i$. The parameter $\lambda_i > 0$ can be seen as a "measure" of the quality of the R&D program of the firm F_i, since a bigger λ_i will result in a bigger reduction of the production costs for the same investment. The maximum reduction of the production cost is a percentage $0 < \epsilon_i < 1$ of the difference between the current cost and the lowest possible production cost. The profit $\pi_i(q_i, q_j)$ of the firm F_i is given by

$$\pi_i(q_i, q_j) = q_i\left(\alpha_i - \beta_i q_i - \gamma q_j - a_i\right) - v_i, \tag{1}$$

for $i, j \in \{1, 2\}$ and $i \ne j$. The optimal output level $q_i^* = q_i^*(q_j)$ of the firm F_i in response to the output level q_j of the firm F_j is given by

$$q_i^* = \arg \max_{0 \le q_i \le \alpha_i/\beta_i} q_i\left(\alpha_i - \beta_i q_i - \gamma q_j - a_i\right) - v_i.$$

Hence,

$$q_1^* = \frac{\alpha_1 - a_1 - \gamma q_2}{2\beta_1} \quad \text{and} \quad q_2^* = \frac{\alpha_2 - a_2 - \gamma q_1}{2\beta_2}.$$

Thus, there is a unique Nash equilibrium (q_1^*, q_2^*) for the second subgame of the Cournot competition with R&D investment programs given by

$$q_1^* = \frac{2\beta_2\alpha_1 - \gamma\alpha_2 - 2\beta_2 a_1 + \gamma a_2}{4\beta_1\beta_2 - \gamma^2}, \tag{2}$$

and

$$q_2^* = \frac{2\beta_1\alpha_2 - \gamma\alpha_1 + \gamma a_1 - 2\beta_1 a_2}{4\beta_1\beta_2 - \gamma^2}. \tag{3}$$

Using (2) and (3) in (1), we obtain the following profits for firms F_1 and F_2

$$\pi_1(q_1^*, q_2^*) = \frac{\beta_1 \left(24\beta_2\alpha_1 - 12\gamma\alpha_2 - 2\beta_2 a_1 + \gamma a_2\right)^2}{144(4\beta_1\beta_2 - \gamma^2)^2} - v_1,$$

$$\pi_2(q_1^*, q_2^*) = \frac{\beta_2 \left(24\beta_1\alpha_2 - 12\gamma\alpha_1 + \gamma a_1 - 2\beta_1 a_2\right)^2}{144(4\beta_1\beta_2 - \gamma^2)^2} - v_2.$$

The Nash equilibrium(s) of the first subgame (v_1^*, v_2^*) is (or are) the implicit solution(s) of

$$\max_{v_i \geq 0} \pi_i, \tag{4}$$

for $i \in \{1, 2\}$. In Figure 5, we present the Nash equilibrium investments (v_1^*, v_2^*) for non-identical firms with different R&D programs ($\lambda_1 > \lambda_2$) and with initial production costs (c_1, c_2) in a neighbourhood of the minimum costs (c_{L_1}, c_{L_2}).

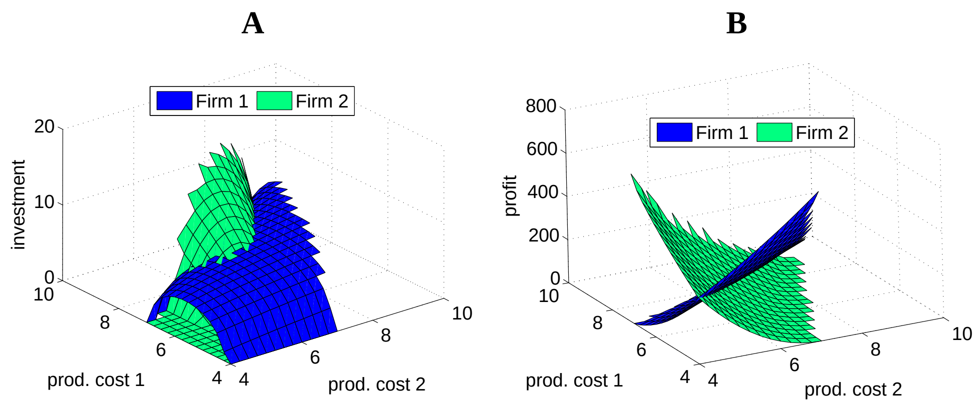

Figure 5: The effect of the production costs: (**A**) on the Nash equilibrium investments, and (**B**) on the corresponding profits. Identical firms producing homogeneous goods with uniform production along the three stages of the period and parameters $\alpha_i = 10$, $\beta_i = 0.013$, $c_{i_L} = 4$, $\epsilon_i = 0.2$, $\lambda_1 = 0.3$, $\lambda_2 = 0.05$.

3 Conclusions

We presented new deterministic and stochastic dynamics on the production costs of Cournot competitions, based on Nash and Bayesian Nash equilibriums of R&D investment strategies of the firms at every period of the game. The following conclusions are valid for identical and non-identical firms (firms with different R&D programs), in some parameter regions of our model of Cournot competition with R&D investment programs. The model presented unique Nash and Bayesian Nash equilibriums, except for initial costs far away of the minimum attainable reduced production cost where the uniqueness of the equilibrium is broken. We characterized the effect of the production costs on the Nash equilibrium investments and on the corresponding profits. We analysed the loss in the profits of one firm when this firm decides not to invest in R&D projects and the other firm uses the best response strategy. We illustrated the transients and the asymptotic limits of the deterministic dynamics on the production costs of the duopoly competition. It can happen that both firms had a higher profit, at every period, if they did not invest than if they have chosen to invest accordingly to the Nash equilibrium, but the global profit was higher for both firms in the second case. We observed that, in the presence of uncertainty in the model, if the production costs of the firm F_1 are smaller than the production costs of the firm F_2, but sufficiently close, then there is a proportion of outcomes that reverse the initial advantage of the firm F_1.

Acknowledgments

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