The effect of a Prisoner's Dilemma in an Edgeworthian Economy

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Abstract

We present a model of an Edgeworthian exchange economy where two goods are traded in a market place. The novelty of our model is that we associate a greediness factor to each participant which brings up a game alike the prisoner's dilemma into the usual Edgeworth exchange economy. Along the time, random pairs of participants are chosen, and they trade or not according to their greediness. If the two participants trade then their new allocations are in the core determined by their Cobb-Douglas utility functions. The exact location in the core is decided by their greediness with an advantage to the greedier participant. However, if both participants are too greedy, they are penalized by not trading. We analyze the effect of the greediness factors in the variations of the individual amount of goods and in the increase of the value of their utilities. We show that it is better to be in minority. For instance, if there are more greedy participants, the increase of the value of their utilities is smaller than the increase of the value of the utilities of the non greedy participants.

1 Introduction

In most economies three basic activities occur: production, exchange and consumption. We analyze the case of a pure exchange economy where individuals trade their goods in the market place for mutual advantage. Two different point of views of modelling the nature of those economic activities are presented below:

- The *Walrasian general equilibrium model* assumes that consumers are passive price takers. They regard a given set of prices as parameters in determining their optimal net demands and supplies. The equilibrium price is such that the market clears. Then the consumers change their endowments by the allocations determined by the equilibrium price. A mechanism that leads to the equilibrium price can be achieved, for instance, through an auctioneer who collects all the offers and demands for each good and adjusts the price vector to clear the market.

- The *Edgeworthian concept* considers consumers as active market participants trading with each other in an attempt to reach a higher level of utility. According to this point of view, an equilibrium is achieved when no person participating in the market can become better off without another person becoming worse off. We will look at the models in this perspective.

Under the appropriate hypothesis, when the number of participants increases to infinity the core shrinks to a point corresponding to the allocation determined by the Walrasian equilibrium price (see Edgeworth [4], Debreu-Scarf [2] and Aumann [1]). In the Edgeworthian exchange economy (*Edgeworth model*) the individuals are no passive price takers but instead they interact in a random meeting market (see Durlauf [3], Gale [8] and [9], Rubinstein and Wolinsky [11] and Wolinsky [12]). In the *Edgeworth model* random pairs of participants are chosen and trade by choosing the allocations determined by the Walrasian equilibrium price of the two participants, also called bilateral equilibrium price. We assume that the utility functions of the participants are of the Cobb-Douglas type. We consider for simplicity that the time is proportional to the number of meetings of the participants. Along the time, the allocations of each participant converge to a limit value, and the bilateral equilibrium prices converge to the market Walrasian price corresponding to the limit allocations (see [5]). We present an Edgeworthian model with greediness (*greed model*) in which to each participant we associate a greediness factor that affects their trade. For instance two non greedy participants will split the benefits by choosing the bilateral competitive equilibrium, a greedy and a non greedy participant will split the benefits with an advantage for the greedy participant, and two greedy participants are penalized by not trading. Hence, the participants are playing a game in the core alike the prisoner's dilemma, where the greediness factor determines their strategy. The greedy participants correspond to the non cooperative players and the non greedy ones correspond to the cooperative players in the prisoner's dilemma. We analyze the effect of the greediness factors in the variation between the limit allocation and the initial endowment of each individual. We also study the increase in the value of the utilities of the participants according to their greediness. In particular, we verify that it is better to be in minority. Since if there are less greedy participants, the increase of the value of their utilities is bigger than the increase of the value of the utilities of the non greedy participants. On the other hand, when the greedy participants are in majority, the increase in the values of their utilities is smaller than the increase of the non greedy participants.

The paper is organized as follows: in section 2, we present the *Edgeworth model* and we show the evolution of the allocations along the time. In section 3, we present the *greed model* and we observe the relation between the increase in the utilities and the greediness. In section 4 we present the conclusions.

2 Edgeworth Model

We look at a pure exchange economy $(\Im, X_i, \succeq_i, w_i)$ where $\Im$ is the population of agents, each of them characterized by a consumption set $X_i \in \mathbb{R}^2_+$ and $\succeq_i$ the individual preferences. So, an exchange economy in which some given amounts of goods X and Y are distributed among n individuals (individual i owns an initial endowment $\bar{x}_i$, $\bar{y}_i$ of good X and Y respectively) is considered. Note that the initial endowments $(\bar{x}_i, \bar{y}_i) \in int(X_i)$.

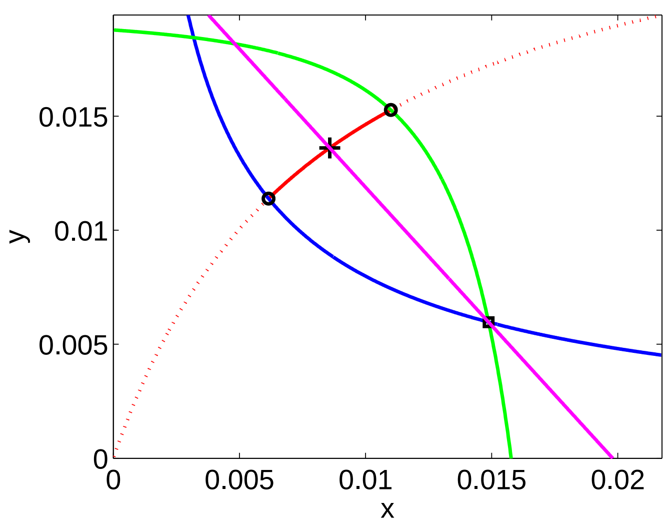

Figure 1: Edgeworth Box with the indifference curves for participant i (blue convex curve) and j (green concave curve). The red curve is the core and the red dots represent the contract curve. The slope of the pink segment line is the bilateral equilibrium price. The interception point (+) of the core with the pink segment line determines the new allocations and the square marks the initial endowments.

We assume individual i obtains utility from the quantities x_i and y_i according to the Cobb-Douglas utility function

$$U_i(x_i, y_i) = x_i^{\alpha_i} y_i^{1-\alpha_i}, 0 < \alpha_i < 1 \tag{1}$$

which represents strictly convex, continuous and nondecreasing preferences where α_i defines the preferences of the goods X and Y for participant i. We consider that preferences, which differ from participant to participant, are drawn from an uniform distribution on the interval $(0, 1)$. Pairs of participants (i, j) are chosen randomly, with equal probabilities, to meet in the market place to trade.

The bilateral equilibrium price of the pair of participants (i, j) is the Walrasian equilibrium price of the economy consisting only of this pair of participants (i, j), and so, not taking in account the other participants of the economy. Considering the good X to be the *numéraire*, it is well known that the bilateral equilibrium price p is given by

$$p = \frac{\alpha_i \bar{y}_i + \alpha_j \bar{y}_j}{(1 - \alpha_i)\bar{x}_i + (1 - \alpha_j)\bar{x}_j} \tag{2}$$

where p is the price of the good Y. The pair of participants (i, j) trade to maximize the values of their utility subject to the constraint that the value of the consumption bundle is equal to the value of the initial endowments, i. e. $p_x x_i + p_y y_i = p_x \bar{x}_i + p_y \bar{y}_i$.

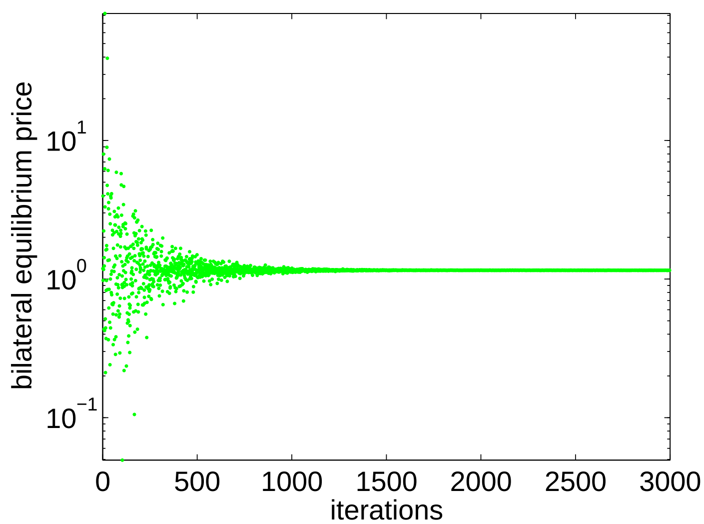

Figure 2: Variation in time of the bilateral equilibrium price. The green dots represent the bilateral equilibrium price at each iteration. The trades between 100 participants were simulated for 3000 iterations.

The bilateral trade is the well known scenario analyzed in the Edgeworth box diagram (see Figure 1). The horizontal axis represents the amount of good X and the vertical represents the amount of good Y of participant i. The point $(\bar{x}_i + \bar{x}_j, \bar{y}_i + \bar{y}_j)$ is the vertex opposite to the origin. The horizontal and vertical lines starting at the opposite vertex are the axes representing the amounts of good X and Y, respectively, of participant j. We represent in the Edgeworth box the indifference curves for both participants passing through the point corresponding to the initial endowments of both participants. The core is the curve where the indifference curves of both participants are tangent and such that the utilities of both participants are greater or equal to the initial ones. The bilateral price determines a segment of allocations that pass through the point corresponding to the initial endowments. The interception of this segment with the core determines the new allocations of the two participants.

In Figure 2, we show the stabilization of the bilateral equilibrium price along the time in a random meeting market place (see [5]).

3 Greed model

The *greed model* is similar to the Edgeworth model in which we introduce a new parameter consisting on the greediness of the participants. If two non greedy participants meet they will trade in the point of the core determined by their bilateral equilibrium price, as in the Edgeworth model.

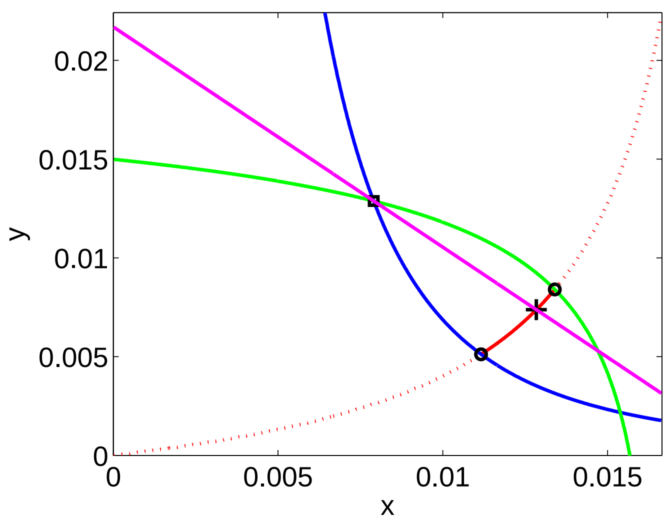

Figure 3: Edgeworth Box with the indifference curves for the greedy participant i (blue convex curve) and non greedy participant j (green concave curve). The red curve is the core and the red dots represent the contract curve. The slope of the pink segment line is a price that gives advantage to the greedy participant. The interception point (+) of the core with the pink segment line determines the new allocations and the square marks the initial endowments.

However, if a greedy participant meets a non greedy participant, they will trade in a point of the core between the point determined by their bilateral equilibrium price and the interception of the core with the indifference curve of the non greedy participant, as can be seen in Figure 3. Finally, if both participants are greedy they are penalized by not being able to trade. This is similar to the prisoner's dilemma, where two non cooperative players are penalized, a non cooperative player has a better payoff than a cooperative player, and two cooperative players have a better payoff than when they meet a non cooperative player but still worse than the payoff of the non cooperative player.

Let the variation of the utility function of a participant be the difference between the limit value of the utility function and the initial value of the utility function. We present, in Figure 4, two cumulative distribution functions of the variation of the utility functions one corresponding to non greedy participants (black) and the other corresponding to the greedy participants (red). This function indicates the proportion of participants that have variations of the utility function less than or equal to its argument. In this figure there are 20% of greedy participants. We observe that the median of the variation of the utility function is higher for the greedy participants.

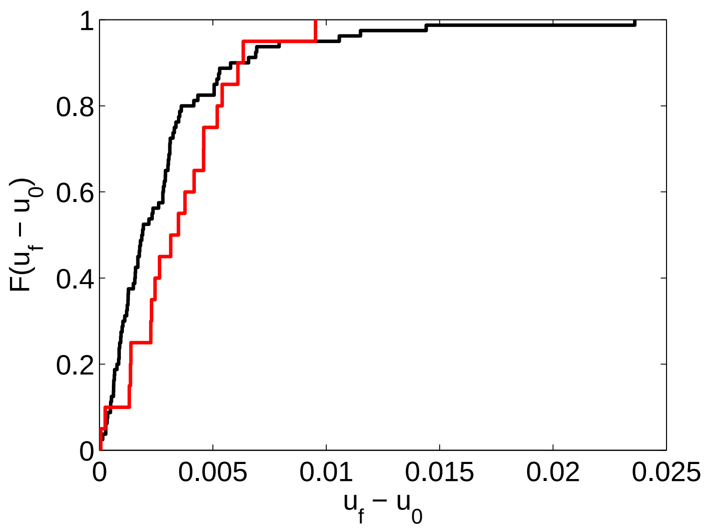

Figure 4: Cumulative distribution function of the variation of the utility ($u_f - u_0$) for the non greedy participants (black) and for the greedy participants (red). Simulation with 20 greedy participants and 80 non greedy participants.

On the other hand, in Figure 5 there are 80% of greedy participants, and we observe that the median of the variation of the utility function is lower for the greedy participants. We show that the strategy corresponding to the minority is the one that provides a higher median variation in the utility function.

4 Conclusion

We presented a model of an Edgeworthian exchange economy where two goods are traded in a market place. The novelty of our model is that we associate a greediness factor to each participant which brings up a game alike the prisoner's dilemma into the usual Edgeworth exchange economy. We analyzed the effect of

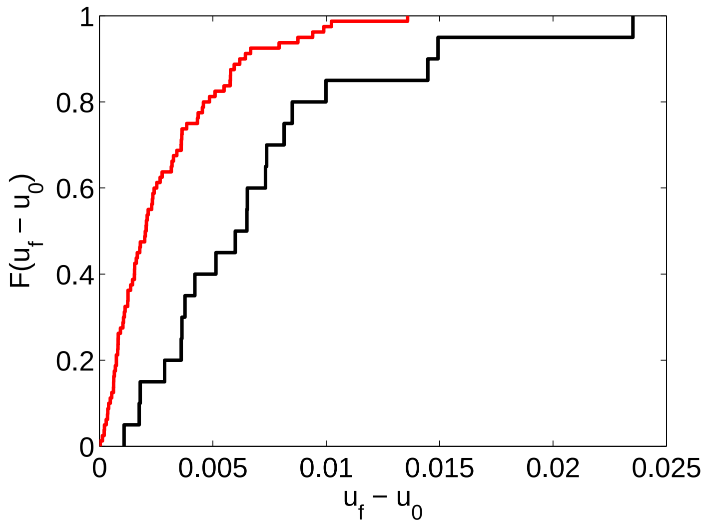

Figure 5: Cumulative distribution function of the variation of the utility ($u_f - u_0$) for the non greedy participants (black) and for the greedy participants (red). Simulation with 80 greedy participants and 20 non greedy participants.

the greediness factors in the variations of the individual amount of goods and in the increase of the value of their utilities. We have shown that it is better to be in minority. For instance, if there are more greedy participants, the increase of the value of their utilities is smaller than the increase of the value of the utilities of the non greedy participants. In the above sense, the optimal proportion of greedy participants is 50%.

Acknowledgments

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