Sequentially Nash Credible Joint Plans in Strategic Networks

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Abstract

I define Sequentially Nash Credible Joint Plans (SN), an *extension* of neologism proofness and a *refinement* of subgameperfect publicly correlated equilibrium. It applies to *three*-player *network* games with *cheap talk* where pairs in a finite rule of order select communication links and actions, have a preexisting common language and *bargain* so that unexpected, "non Nash", *simultaneous* messages' literal meaning are clear as they signal bilateral cooperation. *Multiplicity* is instead obtained in standard networks with bargaining. SN are an *alternative* to evolutionary equilibrium selection in strategic network games. Uniqueness or existence of the related Ferreira's "non-bargained" communication-proof-equilibrium is *not* often the case. As pairs can threat credibly with the unique Harsanyi-Selten (HS) payoff and form a different link, the *smoothed* Nash demand game is SN's *novel* non-cooperative *foundation*. In contrast to HS, a companion paper finds SN in a variation of the Aumann-Myerson game with *infinite* action sets; in the simple majority game, the *nucleolus* is predicted. A version of SN "should" exist for n-person games.

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1 Introduction

A first contribution of this paper is to the literature on *Networks*, represented by sets of links among players. Networks play an important role in the outcome of many social interactions (See Jackson [29] for a survey). In particular, its effect on how payoffs are allocated within a network is important not only in terms of fairness considerations but because it determines players' incentives to form networks. A theory is needed that explains not only how these form but how *payoffs* are allocated and how this depends on relevant *circumstances.*

The present paper focuses on payoff division in three-player *communication* "strategic" networks formation by studying the role played by bilateral sequential *cooperation*, aimed at coordinating actions, *bargaining*, aimed at influencing cooperation, and *focal simultaneous negotiation*, simultaneous message exchange to influence equilibrium outcomes, according to a finite rule of order. The theory of equilibrium selection that is proposed for such *strategic network games,* as link choice and *actions* are strategic variables, is a unified solution concept as it predicts unique payoffs also in the standard case where actions are trivial.

The second contribution is to the literature on strategic transmission that studies cheap talk as a "cooperative" equilibrium selection device for it is assumed that players understand a common language.

I define "Sequentially Nash Credible Joint Plans (SN)" that can be seen as an extension of "neologism proofness" and a refinement of subgameperfect publicly correlated equilibrium. It applies to these sequential bilateral strategic network bargaining environments with cheap talk, the *friendships' environments*, where players have a pre-existing common language so that unexpected, "non Nash", simultaneous messages' literal meaning are clear as they signal bilateral cooperation.

The third contribution concerns the bargaining literature that looks for noncooperative foundations for "reasonable cooperative solutions" to cooperative games as communication network games can be seen as an extension of cooperative games where players cooperate provided they have communication links: SN are an almost non cooperative (ANC) solution for the friendships' environment with a "novel" suggested non cooperative foundation. A particular application yields the nucleolus in coalition structure. Also, SN can be applied to friendships' environments with finite *and* infinite action sets. Finally, a version of SN should exist for n-player games.

With respect to the first contribution, for the most, the network literature has relied on solution concepts where players evaluate prospective networks according to analytical payoff allocation rules that are static in the sense that they depend only on the fixed network structure. As bargaining and transfers in the process of network formation are observed (See Bloch and Jackson [8]), network payoffs should depend on the possibilities of players forming other networks. The network bargaining literature has addressed the problem by allowing link formation and payoff division to happen simultaneously. However, the emphasis on these studies has been on network formation and the tension between stability and efficiency rather than on payoff division. The reason for this emphasis may be found in the well known difficulty of bargaining games yielding unique equilibrium outcomes whenever players can cooperate.

On the other hand the *bargaining environment* induced by the communication technology and contract signing possibilities besides of what is implied from the characteristic function (that gives what any coalition of players can achieve if they cooperate)−or the *value function* in the network literature−are crucial determinants of equilibrium outcomes. As examples, same characteristic functions can give rise to a *coalitional bargaining game* with network effects−a network bargaining game−or without them; same network value functions can be studied in a sequential or simultaneous bargaining network game. Based on Greenberg [23], one could say that this "sensitivity to the details in the specification of the game" is not exclusive of extensive form games[1].

Unless one relies on evolutionary arguments for equilibrium selection (See Goyal [22]), some of the strategic network literature has consequently also such emphasis and sensitivity even though bargaining is not allowed[2]. Moreover, in extensions of equilibrium concepts like coalition-proof-equilibrium to extensive form games with endogenous communication networks, existence and uniqueness (See Ferreira's [18] communication-proof-equilibrium) are not guaranteed.

The present paper in contrast emphasizes on payoff division by providing SN, a single valued solution concept for a specific circumstance, the bargaining environment induced by the communication technology in the friendship's environment; also, SN always exist.

With respect to the second contribution, Myerson [36], based on Farrel's [14] *neologism-proofness* helps address the multiplicity problem in bargaining games by adding focal negotiation in models with sole sequential negotiators with all the bargaining ability. When the sole negotiator uses statements that can be credibly used−based on Myerson's credibility criteria−according to their literal meanings, a reduced set of Nash equilibria is obtained. It is useful to ask if a similar result extends to models with simultaneous pairs of negotiators as each player may have the same bargaining ability to begin with and because models with sequential negotiators may be considered to be equivalent whenever one assumes that mutual consent is needed to yield agreements on literal meanings (See Rabin [45]). SN extends those results.

With respect to the third contribution, the multiplicity and the details of the en-

[1] The quoted phrase is from Ray and Vohra [46]. See also Jackson [29, pp. 29]. The authors seem to imply this exclusivity

[2] The coalitional bargaining game of Ray and Vohra [47] has an underlying strategic game; recall, however, networks effects are not allowed.

vironment in bargaining games documented earlier on[3] makes the necessary mapping from all imaginable coalitional bargaining games to all old and potentially new cooperative solution concepts difficult. Recent previously "unimaginable" improvements in the communication technology reinforce the need of this wider agenda that is neither bounded by existing cooperative solutions nor the aim for realistic bargaining environments.

The present paper contributes to that mapping that is almost inexistent for network games. The model has maybe an even realistic communication technology that together with the focal attribute of words yields naturally a bargaining environment where SN is a "new"[4] ANC solution concept with unique predictions and no restriction to stationary strategies. As pairs can threat credibly with a unique disagreement payoff, in the finite action set case, the unique Harsanyi-Selten prediction [25]−associated to the unique Nash equilibrium of "the disagreement concatenated (See note 8) strategic form game without communication"−and form a different link, the "smoothed Nash demand game" is SN's non-cooperative foundation and differs with most of the literature that uses different versions of the Rubinstein Alternating Offers bargaining model that for the most yield multiplicity of equilibria or use stationary strategies. Finally, note that in the bargaining games where Harsanyi and Selten [25] apply their solution concept strategy sets are finite. A companion paper (Nieva [41]) finds SN for a coalitional network bargaining game, an ANC modification of all three-player Aumann and Myerson [3] network game with infinite action sets. Moreover, in the simple majority game case, the nucleolus is predicted and hence a version of the SN that assumes an "oldest-friend focal effect" may be "the ANC" solution in the friendships' environment. As for the direct proof provided in this paper for the three-player finite action sets case, another version of SN with instead a last-mover-advantage "should" exist for n-person games!

Next, I explain the problems in extending Myerson [36] results and show how my communication environment will yield natural credibility criteria by inducing bargaining possibilities. The model is then presented emphasizing the causal relationship between the communication environment and bargaining possibilities and hence the whole model's justification in terms of the communication technology is clarified.

Negotiations, modelled as a communication game, can influence selection among different Nash equilibria provided one assumes that players understand the negotiator's statements and negotiators are <u>committed</u> to follow through−focal negotiation. For example, in the battle of the sexes game with complete information and communication, there is a Nash equilibrium in which players ignore the male's suggestion to both go shopping and both players choose to go to the Football game. There

[3] See Serrano [53] for an implementation or normative perspective, Hart and Mas-Collel [26] for a general approach and Jackson [29] for the network case.

[4] Actually, this idea has been around in the wage bargaining literature without any non cooperative foundation (See for example Mortensen [31]).

is also an equilibrium where players don't ignore suggestions. Schelling [50] would argue that players would *focus* on the equilibrium that has both players following the male's suggestion to attend the football game if the male is committed to his literal words.

When commitments are not guaranteed, Farrel [14] and Myerson [36] develop criteria to evaluate the credibility of different literal meanings in order to narrow down the number of Nash equilibria in games with sole negotiators. Players will play a Nash equilibrium strategy profile suggested, provided the suggestion passes a credibility test. Credible literal meanings will not be understood and then ignored, but understood, believed and hence followed through.

Whenever the male negotiates and suggests both going to the football game, his suggestion is *tenable,* because it is optimal for the female to go there if she believes he will go there. His suggestion is *reliable* because it is best for him to go if he expects her to follow his suggestion. His suggestion is *credible* or *coherent,* informally, "he means what he says", because it is the best for him out of all tenable and reliable suggestions. In particular, it is better for him than suggesting both going to the ballet concert.

If both players are allowed to formulate negotiation statements simultaneously, then in the associated communication game, there is a Nash equilibrium, where the most preferred suggestion by the male is followed and the female's most preferred one is ignored and vice versa. Even if they mean what they say, when statements conflict, neither Nash equilibrium can be focal because both players would not know what to focus on. Statements with similar suggestions that *happen* to coincide are *not* the exception because there are Nash equilibria with conflicting suggestions! As in Rabin [45], one needs to specify combinations of messages that produce non-binding agreements. In other words, such combinations must suggest or mean for both players—i.e., there is commonknowledge of messages' meanings—playing some equilibrium. It can be shown that if payoff-relevant bargaining that singles out a unique outcome occurs before playing a payoff-relevant game, then players would focus in the unique Nash equilibrium of the communication game associated with two similar tenable and reliable statements that suggest play that yields the unique solution to the bargaining game. Such suggestions will be credible.

I argue that the following communication environment yields naturally such bargaining possibilities and so criteria can be given for the credibility of simultaneous statements in a more general context. I consider pairs of negotiators out of a total of three that can engage at each stage in *preliminary negotiations* with a temporary communication technology according to a <u>finite</u> rule of order; each pair can formulate simultaneous negotiation statements that are suggestions about how to play a *payoff-relevant game* to follow. Statements are represented by correlated strategies, *promise-requests*, in a *simultaneous link choice formation game*, in *a current simultaneous action game* that follows the link formation game and in analogous games for future pairs of negotiators, a *future-request* in the *future game.* A *correlated strategy* is a randomization over action profiles. The formation of communication links is irreversible.

The first assumption studied is that past negotiation statements by other players have no influence. Suppose that pairs of negotiators face a well defined *tenability correspondence* given a negotiation statement. This correspondence represents the set of all correlated strategies that could be rationally implemented by the players in the future game if they believe the negotiation statement by a player in the pair who is the sole negotiator. The associated negotiation statement is defined as *future tenable*.

An individual's negotiation statement is *current reliable and tenable* if given that the sole negotiator believes that players will obey her future-request, the promise-requests in the current games that follow any outcome of the link formation game are reliable and tenable. In this more general context, each promise-request is required to be a "publicly correlated equilibrium" of an associated concatenated strategic form game. An individual statement is *link reliable and tenable* if the promise-request in the link formation game is reliable and tenable in the latter sense. An individual negotiation statement is reliable and tenable iff it is future tenable, link and current reliable and tenable.

It is assumed that during preliminary negotiations pairs have the equivalent of "direct unmediated communication possibilities". This together with the finite rule of order and the natural requirement that in case of disagreement one has a unique self enforcing occurrence, implies that the following *joint plan bargaining problem* can be formulated and solved with the standard Nash Bargaining solution. Moreover, because the finite rule of order implies that the bargaining time within pairs is finite and the disagreement points are self-enforcing by assumption, an adequate smoothed Nash demand game should be the appropriate non cooperative foundation (See Binmore [5, Chapter 1, in particular, pp. 135] for a discussion).

Let players formulate a *joint plan*: two statements with identical promise-requests and future-requests. A joint plan is reliable and tenable if any of the associated individual similar statements is reliable and tenable. The current pair of negotiators bargain *cooperatively* over payoffs that would result if players play according to reliable and tenable joint plans. The payoffs obtained in case of disagreement, the *outside option*s, are in the finite action set case the ones induced by the "unsuccessful Harsanyi and Selten [25] tenable and reliable disagreement joint plan " that is the only tenable and reliable one that suggests link rejection with probability one. Joint plans are *Nash Coherent* or credible if they have associated payoffs consistent with the Nash Bargaining Solution. There is *endogenous cooperation* in the sense that both "successful" and unsuccessful joint plans are possible bargaining outcomes; in particular, there is indeterminacy if the disagreement point is identical to any "individually rational feasible payoff" associated to any *fully successful* tenable and reliable joint plan, one that puts probability one on link formation. Then a *last-mover advantage* is assumed in the sense that the solution involves a fully successful tenable and reliable joint plan and hence the link forms and cooperation occurs.

It is in the latter sense that the communication environment has added to the underlying payoff-relevant game *endogenous effective Nash cooperative negotiation.*

Existence of Nash Coherent Joint Plans is proved whenever one has finite strategy sets as then the current plan bargaining problem is well defined. This the case if future bargaining games in each contingency that follows the pair's negotiation statements and, thus, the tenability correspondence is well defined. Then, recursively, Nash Coherent joint plans at the beginning of the game, *Sequentially Nash Credible Joint Plans (SN)*, exist.

A second assumption studied is based on the idea that past negotiation statements by successful negotiators may be influential. It can happen that one or both players in the pair of negotiators is indifferent to suggesting any joint plan with *individually rational feasible* (IRF) *payoffs,* the ones that are at least as good as the outside options, as individual payoffs for one or both of them may be the same as the ones obtained if agreement is not reached.[5] A credible joint plan is then one with IRF payoffs and future-requested in the negotiation statement by the oldest pair of successful negotiators that formed a link among the past pairs of preliminary negotiators that included one of the indifferent players$-$*Oldest Friend (O-F) Focal Effect.*[6] This may mirror reality as one often observes this subtle loyalty to oldest friends. All such credible joint plans are defined instead as *O-F Joint Plans*.

I show by means of an example that SN under the O-F focal effect exist if action sets are infinite. That this assumption is necessary can be seen in the example; that an extra mild assumption is needed for a generalization is explained in a companion paper in Nieva [41].

In section two, the precise differences with the related literature are given. In section three, I define joint plan bargaining problems, Nash Coherent Joint Plans and O-F Joint Plans. In section four, I derive the tenability correspondence in an underlying payoff-relevant multistage game. In section five, existence of SN is proved assuming a last-mover advantage whenever action sets are finite; that the result could be extended to the n-player case seems to be clear. In section six, I solve a three-player simple majority game with the Aumann-Myerson [3] game and then I illustrate how the O-F focal effect is necessary for existence of SN and implements the nucleolus in coalition structure in a modification of the same game that yields infinite action sets.

2 Related Literature

The model in this paper can be situated within the network formation and bargaining literature. Some papers address the problem of fixed network payoffs by disregarding

[5] For related problems in the "demand commitment model", see Bennett and Van Damme [7]

[6] The relevance of focal effects in experimental bargaining was originally raised in Roth [48].

fixed payoff allocation rules while allowing non cooperative bargaining over the total payoffs a network can achieve (See Jackson [29]for a review). Bargaining in the form of payoff and link proposals, occur multilaterally and simultaneously in Slikker and Van de Noweland [54]. Currarini and Morelli [10] have instead a sequential model and still multilateral model. Bloch and Jackson[8] have a multilateral simultaneous bargaining model with different types of binding transfers at the time of link formation. Navarro and Perea (Bargaining in networks and the Myerson value, mimeo: Universidad Carlos III de Madrid (2001)) use a bilateral sequential model, however, the latter authors' goal objective is to implement the Myerson [32] value. In the present paper, bargaining occurs instead sequentially and bilaterally and is far more general as it assumes that there is an explicit underlying extensive form game. In that latter respect, the present paper is closer to the literature on networks with learning. However, the game played between pairs (see survey in Goyal [22]) is the same and evolutionary equilibrium selection is used. More importantly and for the most, in neither of the positive models above payoff predictions are unique and the emphasis is in the tension between stability and efficiency of Networks.

SN are related to solution concepts proposed to games in strategic form and extensive form game where <u>implicit</u> communication or contract signing are possible and where in general externalities among coalitions cannot be dispensed with and hence the standard characteristic function or value function for networks that allows for externalities lose appeal because of the richness of coalitional interactions[7]. Whenever only communication is possible, the leading solution concepts are that of coalition-proof Nash equilibrium (Bernheim, Peleg and Whinston [4]) for strategic form games, its extension to extensive form games, communication-proof-equilibria (Ferreira [17]) and its extension to extensive form games with endogenous communication networks (Ferreira [18]). The related concept for strategic form games when contract-signing options are implicitly allowed is the notion of equilibrium binding agreements (Ray and Vohra [46]). The key difference is that in the latter notion the behavior of the complement of the deviating coalition is not fixed. All these notions are attractive as they allow for externalities and enjoy the property of consistency (See Greenberg [23] for a general treatment on consistency); however the problem is one of existence or again one of multiple predictions. Within this literature the closest solution to SN is that of Ferreira's [18] communication-proof-equilibria. The most important and attractive similarity is that of consistency. Among the differences, my solution is associated to subgameperfect publicly correlated equilibria rather than only to subgame perfect equilibria in the payoff relevant game. My paper has links and actions be played at the same stage−in some sense, simultaneously−<u>only</u> by the same

[7]This can happen even if one uses the partition function approach that improves on the standard characteristic function without externalities (See Ray and Vohra [46, 47]); in any case, the coalitional bargaining game of Ray and Vohra [47] for this type of environment has still multiple equilibria. In network games with externalities, the network bargaining models with externalities play an identical role (See for example Bloch and Jackson [8]).

pair and cheap talk is model explicitly. This together with the argument that the smoothed Nash demand game would be the appropriate non cooperative foundation makes my solution concept "more non cooperative". More importantly my focus is on equilibrium selection and this is achieved by allowing cooperative "bargaining over strategies" whenever links are proposed. Finally, and in principle, there is no restriction on only finer coalitions "being able to block" as it is the case in coalition-proof Nash equilibria like concepts.

This paper can be seen as an extension of Myerson's [36] *coherent plans* for sole sequential negotiators to the case of pairs of sequential simultaneous negotiators whenever cooperative negotiation possibilities are endogenous as defined previously (in Myerson [33, 33], bilateral cooperation is not endogenous) and, however, there is complete information.

Loosely, one could say that in contrast to Rabin's [45] complete-information model which studies bilateral simultaneous repeated pre-play communication before a simultaneous two-player payoff-relevant game, I assume either endogenous effective Nash or O-F Nash cooperative negotiation possibilities (See 3.2.3 and 3.2.4 for the precise definition) as pairs Nash bargain over payoffs associated to tenable and reliable joint plans. This assumption deals with the problem of multiple equilibria that would result if one would use instead Rabin's model applied to my payoff-relevant games. Alternatively and applying Rabin's model to my payoff relevant-games, one can assume that pairs focus in the Rabin's [45] *negotiated equilibrium* that yields the Nash bargaining solution. Without assuming such cooperative negotiation, only lengthy-negotiation environments with pure coordination payoff relevant games yield unique payoff predictions (See Rabin [45, pp. 373]).

From a broader perspective, in contrast to Aumann and Hart [2] and the literature reviewed in their paper that studies strategic information transmission as expanding the set of outcomes, my work emphasizes its study as *restricting* the set of outcomes. In particular, I focus on *long bounded* cheap talk whereas the authors focus on *long cheap talk*.

With respect to the bargaining literature that looks for noncooperative foundations for reasonable cooperative solutions to cooperative games and based on the technologically induced rules of the game in this paper and the idea of consistency of the prekernel, it would not be a surprise that a different or more non cooperative foundation of the Nucleolus−that belongs to the prekernel−in TU than the one in Serrano [51] and possibly in NTU may be obtained in the sense of Serrano [52].

With respect to the extensive literature on pairwise bargaining in many person bargaining games, including the ones that are in addition coalitional bargaining games, it is argued that in my game the smoothed Nash demand game should be appropriate. In contrast, the standard model used in pairwise meetings in such models is some version of the Rubinstein alternating offers game (See Serrano [52], Gale [20] Gul [24] Rubinstein and Wolinsky [49] among numerous ones). Papers in the bargaining literature that have dealt in addition with stage payoff-relevant games are only two-player infinitely repeated games (Okada [43] [42]), Bush and Wen [9] and Houba [27]). There, bargaining is over long term contracts, thus binding agreements are assumed. The complexity of the analysis associated to endogenous outside options (as in case of disagreement the stage game is played non cooperatively) is avoided in my paper as the feasible bargaining set is composed by payoffs associated to tenable and reliable plans, thus outside options are already self enforcing and unique as for the Harsanyi and Selten [25] prediction in the finite action set case. Besides, as the rest of the bargaining literature, their model is a modification of the Rubinstein alternating offers model. In other models (See Dekel [11] for the two-player and-two stage case), the Nash demand game is used in a first stage and if disagreement occurs a second stage the demand game is played again. They find of course multiplicity of equilibria and even inefficient ones. Recall that in my paper an "appropriate" smoothed Nash demand game should be used.

3 Simultaneous Negotiation Problems

3.1 A Two-Player Negotiation Problem

I consider the problem of two players i and j, the *negotiators*, when they have the opportunity to make simultaneous negotiation statements to players i, j and l in *preliminary negotiations,* say with a temporary communication technology*,* about a *payoff-relevant game* with finite horizon to follow. Suppose first that past statements that negotiators i and j may know about at the time they negotiate are not influential.

The payoff-relevant game begins with a simultaneous communication *link formation game* where the *choice* sets that negotiators i and j have available are denoted by the sets $A_i = A_j = \{y, n\}$. The communication link is assumed to be permanent. Such a set A_l for player l has a trivial unique payoff-irrelevant choice, "move nothing". Denote by $A = A_i \times A_j \times A_l$ the associated *choice profile set* in the payoff-relevant game Also, the two-player choice profile set for i and j is denoted by A_{ij}. A bilateral link ij between players i and j forms if and only if both players play y. Hence n is considered a *unilateral rejection.*

Play of a choice $a = (a_i, a_j, a_l) \in A$ can be identified with an *immediate contingency* a, the one that occurs right after a is played, or alternatively a *current contingency* a in the payoff-relevant game, at which a *current simultaneous game* a takes place where the *action* sets that negotiators i and j have available are denoted by $B_{i,a}$ and $B_{j,a}$. Such a set $B_{l,a}$ for player l has as trivial unique payoff-irrelevant action. Denote by $B_a = B_{i,a} \times B_{j,a} \times B_{l,a}$ the associated *action profile set.* Individual sets are assumed to be the same regardless of a, i.e., $B_a = B$ for all a. The set of *current joint strategies* $\times B$ in the payoff-relevant game is the Cartesian product of B_a for all a, that is,

$$\times B = \prod_a B_a = B^4,$$

An element of $\times B$ is denoted by $\times b$ and b_a, is the *a-th component* of $\times b$.

In any immediate contingency (a, b), the one that can be identified with play of choice a followed by action profile $b \in B$, a *future* game (a, b) takes place. The set of joint strategies in this game are denoted by $\times Z_{p/(a,b)}$, a Cartesian product of $Z_{p(a,b)}$ sets, that is,

$$\times Z_{p/(a,b)} = \prod_{p(a,b)} Z_{p(a,b)} \tag{1}$$

. Each $Z_{p(a,b)}$ stands for the choice or action profile set in any contingency of the payoff-relevant game that may follow the (a, b) occurrence including immediate contingency (a, b). Any such a contingency is denoted by $p(a, b)$. It is assumed that $\times Z_{p/(a,b)}$ only depends on the link ij forming or not. The set of *future joint strategies* in the payoff-relevant game is

$$\times Z = \prod_{(a,b)} \prod_{p(a,b)} \times Z_{p(a,b)} \tag{2}$$

, or, using Eq. (1),

$$\times Z = \prod_{(a,b)} \times Z_{p/(a,b)} \tag{3}$$

, the Cartesian product of sets of joint strategies in all possible future games (a, b). Any contingency in any future game (a, b) is defined as a *future contingency* p of the payoff-relevant game.

For any $(a, \times b, \times z)$, where $a \in A$, $\times b \in \times B$ and $\times z \in \times Z$, $U_m (a, \times b, \times z)$ denotes the expected utility payoff outcome for player $m = i, j, l$ if a, $\times b$ and $\times z$ are played in the payoff-relevant game.

For simplicity (and wlg. for the model in Nieva [41]), I define a *correlated strategy* on a strategy profile set T as a function τ from T to the Real interval $[0, 1]$ such that $(\tau(t))_{t \in T^{\subset}} \in \Delta T^{\subset}$ is a probability distribution over some finite strategy profile subset $T^{\subset}$ of T, and $\tau(t) = 0$ if $t \notin T^{\subset}$. The set of correlated strategies on T is denoted by $\mho^T$. A given correlated strategy τ may be implemented with a mediator that randomly chooses a profile t of pure strategies in $T^{\subset}$ with probability $\tau(t)$. Then the mediator would *recommend* each player, say i, j and l, *publicly* to implement the strategy t_i, t_j and t_l respectively. If such mediation is possible one has the equivalent of *direct unmediated communication possibilities.*

A vector of correlated strategies ϑ on a Cartesian product of action profile sets that depend on events $e \in E$ and denoted by $\times T = \prod_e T_e$ is defined as:

$$\vartheta = \prod_e \vartheta_e \tag{4}$$

, where ϑ_e is a correlated strategy on T_e. The interpretation is that if event $e \in E$ occurs, the mediator would implement correlated strategy ϑ_e, the e-th component of ϑ. The set of all vectors of correlated strategies on $\times T$ is denoted by $\mho^{\times T}$.

A *negotiation statement* for player i μ_i *suggests* play in the communication game *corresponding* to the payoff-relevant game. Any contingency in the communication game corresponds to a given contingency in the payoff-relevant game in the sense that besides past negotiation statements and recommendations by different negotiators and mediators respectively any such corresponding contingency includes the same sequence of choices and actions that led to the given contingency in the payoff-relevant game.

A negotiation statement μ_i is represented, abusing notation, by three components. The first component is a correlated strategy on action profile set A in the simultaneous link formation game, a *link promise-request* $\alpha_i \in \mho^A$. The second component is a correlated strategy on action profile sets in current contingencies a, a current *promise-request* $\beta_i \in \mho^{\times B}$. The third component is a correlated strategy on choice or action profile sets in future contingencies p, a *future-request* $\zeta_i \in \mho^{\times Z}$, using Eqs. (2) and (4). It is implicit that these suggestions are, wlg., the *same* regardless of specific recommendations that may have occurred as in the theory proposed in this paper credibility of negotiation statements will depend on past sequences of choices and actions and later on even on past negotiation statements but not on specific recommendations. For simplicity in the notation, we thus abstract from recommendations (See also 4.2.2).

Using Eqs. (3) and (4), it will be useful to express ζ_i as follows:

$$\zeta_i = \prod_{(a,b)} \zeta_{i,p/(a,b)} \tag{5}$$

, where $\zeta_{i,p/(a,b)} \in \mho^{\times Z_{p/(a,b)}}$.

In particular, if the negotiator announces $\beta_{i,a}$ $\left(\zeta_{i,p}\right)$,for the corresponding current (future) contingency in the communication game to the current (future) contingency a (p) of the payoff-relevant game, she is *requesting* player j to obey player i's mediator according to $\beta_{i,a}$ $\left(\zeta_{i,p}\right)$. She is *promising* to obey her own mediator according to $\beta_{i,a}$ $\left(\zeta_{i,p}\right)$. She is requesting player j to obey player i's mediator according to $\beta_{i,a}$ $\left(\zeta_{i,p}\right)$. The request to player l is trivial in this particular case.

A negotiation statement for player i is thus an element of $\mho = \mho^A \times \mho^{\times B} \times \mho^{\times Z}$ and it is denoted by $\mu_i = (\alpha_i, \beta_i, \zeta_i) \in \mho$.

A negotiation statement for player j is defined analogously and her negotiation statement $\mu_j \in \mho$.

To formalize the credibility, reliability and tenability of a negotiation statement whenever there are two simultaneous negotiators, one needs to deal first with the problem of conflicting simultaneous negotiation statements. To set up this problem precisely, I will define first a tenable and reliable statement for a player when she is the sole negotiator.

Let player i be the *sole* negotiator with negotiation statement $\mu_i = (\alpha_i, \beta_i, \zeta_i)$ given player $j's$ statement $\mu_j = \left(\alpha_j, \beta_j, \zeta_j\right)$, where the latter is to be regarded as noise. I assume in this section that there exists a well defined non empty *tenability* *correspondence* $Q : \mho \rightarrow\rightarrow \mho^{\times Z}$, where $Q(\mu_i)$ represents the set of all vectors of correlated strategies that could be rationally implemented by the players in future contingencies of the communication game p_c—that correspond to future contingencies p—following the negotiator's statement if they would believe negotiation statement μ_i. A negotiation statement μ_i is *future tenable* iff $\zeta_i \in Q(\mu_i)$. One writes then $\mu_i \in \underline{\mho} \subset \mho$.

Let $\mu_i = (\alpha_i, \beta_i, \zeta_i) \in \mho$ and, wlg., noise $\mu_j = (\alpha_j, \beta_j, \zeta_j)$ be given. If $a' \in A$ was played following μ_i in the communication game, consider the following a'-*concatenated*[8] *strategic form game* $\left(B_i \times B_j, \pi_{ij}^{\mu_i/a'}\right)$, where payoffs are given by

$$\pi_{ij}^{\mu_i/a'}(b_i, b_j) = \left[\sum_z U_i(.) \Pr[U_i(.)], \sum_z U_j(.) \Pr[U_j(.)]\right] \tag{6}$$

if (b_i, b_j) is played, $U_m(.) = U_i(a', \times b, \times z)$, the a'-th component of $\times b$ is such that $b_{a'} = (b_i, b_j, b_l)$ and $\Pr[U_m(.)]$ is the probability that $U_m(.)$ results given that contingency $(a', b_{i,a'})$ occurred and play is consistent with ζ_i thereafter, for $m = i, j$.

Note that $\pi_l^{\mu_i/a'}(b_i, b_j)$, the associated payoff to player l can be computed analogously and $\pi^{\mu_i/a'}(b_i, b_j)$ would then refer to a payoff triplet for all players. Recall, b_l is trivial.

Let μ_i and the a'-concatenated game be given and hence players are expected to obey future-request ζ_i. A request in $\beta''_{i,a'}$ by player i is tenable if it is optimal for player j to obey player i's mediator given that player i is believed to fulfill his promise to obey the mediator. A promise in $\beta''_{i,a'}$ by player i is reliable if it is optimal for player i to obey the mediator given that player j is expected to obey the mediator. Equivalently, I will say that a promise-request $\beta''_{ia'}$ by player i is reliable and tenable given μ_i if $\beta''_{ia'}$ is a *publicly correlated equilibrium* of $\left(B_i \times B_j, \pi_{ij}^{\mu_i/a'}\right)$. A statement $\mu_i = (\alpha_i, \beta_i, \zeta_i) \in \mho$ is *current reliable and tenable* iff any promise-request $\beta_{i,a'}$ is reliable and tenable for all $a' \in A$ given μ_i.

Given $\mu_i = (\alpha_i, \beta_i, \zeta_i) \in \mho$, α_i is tenable and reliable iff α_i is a publicly correlated equilibrium of $\left(A_i \times A_j, \pi_{ij}^{\mu_i}\right)$, where payoffs for $\left(a'_i, a'_j\right) \in A_i \times A_j$ are

$$\pi_{ij}^{\mu_i}\left(a'_i, a'_j\right) = \sum_b \beta_{i,a'}(b)\, \pi_{ij}^{\mu_i/a'}(b_i, b_j) \tag{7}$$

, the expected payoffs for players i and j if current contingency a' occurs and play is consistent with β_i and ζ_i thereafter. A statement $\mu_i = (\alpha_i, \beta_i, \zeta_i) \in \mho$ is *link reliable and tenable* iff α_i is reliable and tenable.

A statement μ_i is reliable and tenable iff it is future tenable, link and current reliable and tenable. Such a statement will be said to belong to $\widetilde{\mho}$.

[8]The term concatenated is taken from Gibbons [21, pp. 85-86] that uses the Nash equilibria of a one shot concatenated strategic form game (see figure 2.3.4) to find the subgame perfect equilibria of the two period repeated game in figure 2.3.3.

As for the Aumann [1] critique, one could consider only $\mu_i = (\alpha_i, \beta_i, \zeta_i) \in \widetilde{\mho}$ where any $\beta_{i,a'}$ implies putting positive probability only on *self-signaling* Nash equilibria—that is player i wants to suggest any Nash equilibrium if and only if it is true (See Farrel and Rabin [16] for a detailed explanation of the term)—of $\left(B_i \times B_j, \pi_{ij}^{\mu_i/a'}\right)$ for all $a' \in A$. Note that if one does not have self-signalling Nash equilibria nothing could be necessarily achieved with pre-play communication. For simplicity, as in Rabin [45], I don't discriminate among Nash equilibria.

Analogously, one defines reliability and tenability of μ_j for player j whenever she is the sole negotiator and has her own mediator. Note that $\mu_i, \mu_j \in \mho$, so μ_i is tenable and reliable whenever player i is the sole negotiator if and only if μ_j is tenable and reliable whenever player j is the sole negotiator.

In case neither of the negotiation statements by players i and j are noise, the tenability of one player's statement—link, current and future tenability—depends on the statement of the other one. If one has conflicting requests, who would players obey if they are willing to obey either of the negotiators, or equivalently, if both negotiators' statements are tenable whenever they are the sole negotiators? The subsections that follow address this problem.

A simultaneous negotiation problem for players i and j as just described is denoted by $\Phi_{ij} = (A, B, \times Z, U, Q)_{ij}$.

3.2 O-F and Nash Coherent Joint Plans

3.2.1 Preliminary Definitions

We define for any two vectors x and y in $\mathbb{R}^2$

$x \geq y$ (x is *as least as good as* y) iff $x_i \geq y_i$ and $x_j \geq y_j$, and

$x > y$ (x is *strictly better* than y) iff $x_i > y_i$ and $x_j > y_j$, $i \neq j$.

A *bargaining problem* for agents i and j consists of a pair (F, ψ), where F is a closed convex subset of $\mathbb{R}^2$, $\psi = \left(\psi_i, \psi_j\right)$ is a vector in $\mathbb{R}^2$ and the set of *individually rational feasible allocations (IRF set)*

$F \cap \left\{(x_i, x_j) \,|x_i \geq \psi_i \text{ and } x_j \geq \psi_j \text{ or } x_{ij} \geq \psi_{ij}\right\}$

is non-empty and bounded. Here F represents the set of feasible payoff allocations or the *feasible set*, and ψ represents the disagreement payoff allocation or the *outside options.*

A bargaining game (F, ψ) is *essential* iff there exists at least one allocation x in F that is strictly better for agents than the disagreement allocation ψ, i.e., $x > \psi$.

A point x in F is *strongly (Pareto) efficient* iff there is no other point y in F such that $y \geq x$ and $x_w > y_w$ for at least one player $w \in \{i, j\}$. A point x in F is *weakly (Pareto) efficient* iff there is no other point y in F such that $y > x$. The *feasible frontier* is the set of feasible payoffs allocations that are strongly Pareto efficient in F. The IRF *frontier* is the set of points in F that are strongly Pareto efficient in the IRF set.

3.2.2 A Joint Plan Bargaining Problem

Before I develop a notion of credibility whenever negotiation statements are simultaneous by adding "Nash or O-F Nash effective cooperative negotiation", necessary conditions for simultaneous statements to be reliable and tenable *in this context* have to be given for these eventually to be credible.

Negotiation statements for both players are *similar* if $\mu_i = \mu_j$. A *joint plan* is a negotiation statement $\mu \in \mho$ such that there exists similar statements for players 1 and 2 and $\mu_1 = \mu_2 = \mu$. Abusing notation, μ will also refer to (μ_1, μ_2), where it may seldom be the case that $\mu_1 \neq \mu_2$, in which case there will be no confusion as the term joint plan will not be implicit! Such a joint plan μ is tenable and reliable iff μ is tenable and reliable for player i or j whenever any of them is the sole negotiator. Finally, only joint plans can be tenable and reliable *in this context*.

If one would not restrict players to enunciate only tenable and reliable joint plans then tenable and reliable statements that happen to coincide could not be focal because it can be shown (at the reader's request) that there are both Nash equilibria of the communication game associated, say, to a payoff-relevant simultaneous game where tenable and reliable individual statements conflict and ones where such statements don't conflict. Outside players or the two players themselves would ask, does the pair mean what it says? Is the pair really agreeing? In a different way, how could the pair be agreeing if there is the possibility that the pair could enunciate such conflicting statements. For a pair of tenable and reliable statements to be focal this restriction is necessary.

Next, a *joint plan bargaining problem* (F, ψ, Φ_{ij}) for players i and j derived from a simultaneous negotiation problem $\Phi_{ij} = (A, B, \times Z, U, Q)$ is a bargaining problem (F, ψ) with two characteristics:

1. For each payoff pair $(x_i, x_j) \in F$, there exists an associated tenable and reliable joint plan

   $\mu = (\alpha, \beta, \zeta) \in \widetilde{\mho}$ such that $(x_i, x_j) = \sum_a \alpha(a)\, \pi_{ij}^{\mu}(a_i, a_j)$, where $\pi_{ij}^{\mu}(a_i, a_j)$ is as defined in Eq. (7).

2. The disagreement payoff allocation is $\psi = (x_i, x_j) = \pi_{ij}^{\widehat{\mu}}(\widehat{a}_i, \widehat{a}_j)$ the payoff associated to the *disagreement joint plan* $\widehat{\mu} = \left(\widehat{\alpha}_{\widehat{a}}, \widehat{\beta}, \widehat{\zeta}\right) \in \widetilde{\mho}$ that is tenable and reliable and derived as follows: Let the *disagreement future-request* for now be given by $\widehat{\zeta}$ (See 4.2.2 for derivation). The disagreement current promise-request $\widehat{\beta}$ is such that $\widehat{\beta}_{a'}$ for all $a'_{ij} \neq (y, y)$, corresponds to a unique Nash equilibrium strategy profile of $\left(B_i \times B_j, \pi_{ij}^{\widehat{\mu}/a'}\right)$; in the case individual strategy sets B_m, $m = i, j, l$, are finite, I choose it to be the one based on the theory of equilibrium selection of Harsanyi and Selten [25]; wlg., $\widehat{\beta}_{a'}$ if $a'_{ij} = (y, y)$ is arbitrary fixed to a given value. Finally, wlg., $\widehat{\alpha}_{\widehat{a}}$ is a degenerate correlated

strategy that puts probability one on unilateral rejections, that is $\widehat{\alpha}_{\widehat{a}}(\widehat{a}) = 1$, where $\widehat{a}_{ij} = (n, n)$[9].

Denote the $\widehat{a}$-*disagreement concatenated strategic form game without communication*—in the sense that players think that the communication link won't form—by $\left(B_i \times B_j, \pi_{ij}^{\widehat{\mu}/\widehat{a}}\right)$. As argued in the introduction, play suggested by this joint plan $\widehat{\mu}$ is then self-enforcing. If the individual action sets are infinite another unique self-enforcing disagreement point would be needed (See example and the associated general case in Nieva [41]).

Definition 1 *A tenable and reliable joint plan $\mu = (\alpha, \beta, \zeta)$ such that its link promise-request puts positive probability on link formation, that is, $\alpha(a) > 0$ where $a_{ij} = (y, y)$, is called* ***successful*** *otherwise it is* ***unsuccessful*** *and one says preliminary negotiations or negotiators are successful or otherwise* ***unsuccessful****. If $\alpha(a) = 1$, a tenable and reliable joint plan is* ***fully successful****.*

Note that the associated communication link forms form only if $a_{ij} = (y, y)$ is played. Also $\widehat{\mu}$ is unsuccessful.

Definition 2 *The technology of communication implicit in definition 1 is characterized as* ***apparent and contingent*** *in a sense explained in 3.2.4, remark 3.*

Any such plan bargaining game will be denoted by (F, ψ, Φ_{ij}).

3.2.3 Nash Coherent Joint Plans

Define a *solution* of the joint plan bargaining problem (F, ψ, Φ_{ij}) to be a payoff pair $(x_i, x_j) \in F$ and an associated tenable and reliable joint plan $\mu \in \widetilde{\mho}$.

Players i and j can carry out negotiations *endogenously, Nash effectively* and *cooperatively* if given the simultaneous negotiation problem Φ_{ij}, they can construct and solve (F, ψ, Φ_{ij}) as follows:

1. The solution is derived from the *non transferable utility* (NTU) *Nash Bargaining Rule* (NBR) applied to the associated (F, ψ). The NTU NBR solution solves: $\arg\max_{x \in F(h),\, x \geq \psi} (x_i - \psi_i)\left(x_j - \psi_j\right)$.
2. If the IRF set is a singleton, i.e., $\nexists (x_i, x_j) \in IRF$ s.t. $x > \psi$ and the disagreement point is identical to any individually rational feasible payoff associated to any fully successful tenable and reliable joint plan, a *last-mover advantage* is assumed in the sense that the solution is required to consist of a fully successful tenable and reliable joint plan $\mu \in \widetilde{\mho}$ and hence the link would form if μ is followed through.

[9] Note that if one would require in addition that the Nash equilibria in concatenated games to be self-signalling then it would be natural not to require $\widehat{\beta}_{a'}$ to be self-signalling in $\left(B_i \times B_j, \pi_{ij}^{\widehat{\mu}/a'}\right)$ for $a' \in A$ and $a'_{ij} \neq (y, y)$.

There is *endogenous cooperation* in the sense that failed cooperation is possible and *meaningful* as both successful and unsuccessful preliminary negotiations are possible Nash bargaining outcomes.[10]

A joint plan μ is *Nash Coherent* and hence a *credible joint plan* if it is the solution component of a joint plan bargaining problem (F, Φ_{ij}, ψ) where players can negotiate Nash effectively and cooperatively[11]. Whenever I want to refer to players i and j's set of Nash Coherent Joint Plans in Φ_{ij} given ψ, I write $\eta(\Phi_{ij}, \psi) \subset \widetilde{\mho}$.

3.2.4 Oldest-Friends Joint Plans

I will be interested in developing credibility criteria for simultaneous statements assuming instead that past joint plans by successful negotiators may influence a negotiation problem in a future contingency of the communication game p_c corresponding to p.

So let pairs of players, out of a total of three, take turns to conduct preliminary negotiations and then play a respective payoff-relevant game according to a finite rule of order in stages k of a corresponding multistage game with communication, where $k = 1, ...K + 1$. Also, let statements enunciated by different past pairs that were involved in preliminary negotiations be denoted by μ^{k^-}, i.e., $\mu^{k^-} = (\mu^1, ..., \mu^{k-1})$, where $\mu^t = \left(\mu_j^t, \mu_l^t\right)$ with $\mu_j^t = \mu_l^t$ as for restriction on 3.2.2, for $t = 1, ...k - 1$, $k > 1$ and $i \neq j \neq l$. To allow for such influence, it will be useful to think of contingencies in the communication game p_c having not just a past sequence of choices $a^{k^-}$and actions b^{k^-}but, in addition, a sequence of joint plans μ^{k^-}. The current negotiation problem at contingency p_c at stage $k-$history (defined in 4.2.2) of the corresponding multistage game with communication is then denoted by $\Phi_{ij,\mu^{k-}}$ and the tenability correspondence by $Q_{\mu^{k-}}$.

To formulate these criteria, I make the following assumption:

Oldest-Friends Focal Effect: Let one or both players in the pair of negotiators be indifferent between joint plans with payoffs in the IRF set of a joint plan bargaining problem $\left(F^k, \psi^k, \Phi_{il,\mu^{k-}}\right)$. If $k > 1$, the solution to $\left(F^k, \psi^k, \Phi_{il,\mu^{k-}}\right)$ involves the payoff in the IRF that is future-requested in the tenable and reliable joint plan by

[10] In standard bargaining problems disagreement or failed cooperation is not meaningful in the sense that in general it does not occur and if it "would occur" only the disagreement payoffs pair is obtained. In this paper disagreement is in contrast meaningful as different payoffs for the third player may occur after disagreement and an opportunity to form a permanent link has been not used.

[11] One can think of pairs having possibilities to set up a smooth Nash demand game that yields in the limint as unique equilibrium outcome the NTU NBR payoff. Then the unique Rabin's [45] negotiated equilibrium in a game with preplay communication where there is such a payoff-relevant bargaining game would be associated to the tenable and reliable joint plan that yields the NTU NBR. Alternatively and without any *cooperative transformation* (See Myerson [37]) of the original payoff-relevant game, one could assume that pairs focus in the Rabin's negotiated equilibrium equilibrium that yields the NBR prediction. Without any of these assumptions, unique payoff predictions are only possible if negotiations are lengthy and one has pure coordination payoff relevant games.

the *oldest pair* of successful negotiators that formed their link–according to the rule of order–among the past preliminary negotiators that included one of the indifferent players. Otherwise $\eta\left(\Phi_{il,\mu^{k-}},\psi^k\right)$ is used.

The credible joint plans that are predicted under this assumption will be defined as *Oldest-Friends Joint Plans (O-F Joint Plans)* and its set is denoted by $\eta^f\left(\Phi_{ij,\mu^{k-}},\psi^k\right)$. One then says that pairs can carry out negotiations *endogenously, O-F Nash effectively* and *cooperatively.* For existence purposes, specially when action sets are infinite, note that the outside options depend on past sequences of choices and actions in the multistage game with communication.

Remark 3 *It is said that there is an apparent and contingent technology of communication because even though a link forms iff a reliable and tenable joint plan is enunciated and no player chooses a unilateral rejection*[12]*, link formation does not occur in equilibrium of the communication game unless such joint plan is in addition successful and Nash Coherent or O-F and no player chooses a unilateral rejection.*

Remark 4 *Disagreement joint plans enunciated by older pairs of negotiators are trivially followed as any* $\widehat{\mu}=\left(\widehat{\alpha}_{\widehat{a}},\widehat{\beta},\widehat{\zeta}\right)$ *future-requests in* $\widehat{\zeta}$ *optimal future play in case of disagreement (See Remark 6).*

Remark 5 *If a pair can carry out negotiations endogenously, either Nash or O-F Nash effectively and cooperatively, contingencies of the communication game,* p_c*, include only tenable and reliable joint plans, that is,* μ^{k^-} *consists of* $\mu^t=\left(\mu_j^t,\mu_l^t\right)$*, such that* $\mu_j^t=\mu_l^t\in\widetilde{\mho}^t$ *for all* $t=1,...k-1$*,* $k>1$*. Hence, abusing notation,* μ^t *could be regarded as a tenable and reliable joint plan with no risk of confusion whatsoever from now on, i.e.,* $\mu^t=\mu_j^t=\mu_l^t$

Remark 6 *It is implicit in Remark 4 that if any tenable and reliable joint plan that is not part of a solution to an older joint plan bargaining problem is enunciated, this joint plan still may*[13] *influence future play in the relevant subgames of the communication game*[14]*.*

The situation in last Remark 6 is analogous to the case where say only the male enunciates a negotiation statement a day before the battle of the sexes game is played.

[12] Even under a disagreement joint plan, players by mistake may chose (y,y) and hence the link forms!

[13] For that to happen the plan has to be succesful, the link has to form and the indiference cases have to occur.

[14] These are "credible neologisms" in the sense of Farrel[14, pp. 515]. That is they would be understood and would signal bilateral cooperation and believed if the pair of negotiators would enunciate such tenable and reliable joint plans. However, these are not "the best joint plan" the pair can enunciate because these are not associated to the NBR.

Suggesting both going to the Ballet concert influences play in the subgame that follows this statement where the battle of the sexes game is played (because it suggests a Nash equilibrium in that subgame) in the second day. However, it is not even a Nash equilibrium in the whole game that begins the day before. when the male statement is enunciated, because "its not the best the male can say" (See Myerson [37, pp. 110-111]

O-F Joint Plans Formal Definition Let $i, j, l \in \{1, 2, 3\}$, and $i \neq j \neq l$. Suppose players i and j had successful preliminary negotiations, link ij formed and have enunciated, as part of their future-request, the tenable and reliable joint plan $\gamma \in \widetilde{\mho}_{il}$ and only then j and l successfully negotiated, formed link jl and future-requested $\delta \in \widetilde{\mho}_{il}$ where it maybe that $\gamma \neq \delta$. Schematically, as the bargaining problem for i and l follows, one has the following physical apparent and contingent order of link formation:

$(i, j) \qquad (j, l) \qquad (i, l)$.

For all essential bargaining problems for i and l, set

$\eta^f \left(\Phi_{il,\mu^{k-}}, \psi^k \right) = \eta \left(\Phi_{il,\mu^{k-}}, \psi^k \right)$.

Otherwise[15]:

<u>Case 1.</u> If $\nexists (x_i, x_l) \in IRF^k$ s.t. $x_i^k > \psi_i^k$, however $\exists (x_i, x_l) \in IRF^k$ s.t. $x_l^k > \psi_l$, set $\eta^f \left(\Phi_{il,\mu^{k-}}, \psi^k \right) = \gamma$;

<u>Case 2.</u> If $\nexists (x_i, x_l) \in IRF^k$ s.t. $x_l^k > \psi_l^k$, however $\exists (x_i, x_l) \in IRF^k$ s.t. $x_i^k > \psi_i^k$ set $\eta^f \left(\Phi_{il,\mu^{k-}}, \psi^k \right) = \delta$;

graphically, in the plane (x_i, x_l), the IRF^k set for $\left(F^k, \psi^k, \Phi_{il,\mu^{k-}} \right)$ is a straight closed vertical and horizontal closed segment respectively.

<u>Case 3.</u> If $\nexists (x_i, x_l) \in IRF^k$ s.t. $x^k > \psi^k$ set $\eta^f \left(\Phi_{il,\mu^{k-}}, \psi^k \right) = \gamma$.

In words, there are three cases in which the assumption turns out to imply a not essential $\left(F^k, \psi^k, \Phi_{il,\mu^{k-}} \right)$ to be "effectively" a singleton. As oldest friends' tenable and reliable statements are the only ones that are credibly understood by their literal meanings, the only possible payoff $(x_i, x_l) \in IRF^k$ and associated joint plan to be bargained about by players i and l is the one that confirms the joint plan by the oldest successful pair of friends that has one of its member, i or l, indifferent between any payoff in IRF^k.

In addition, if one only has pair (i, j) enunciating as part of its future-request $\gamma \in \widetilde{\mho}_{il}$ and thus one has schematically,

$(i, j) \qquad (i, l)$,

[15] Note that if one would require the plans different than the disagreement joint plan associated to payoffs in the feasible set "to be self-signalling" and none of such plans would exist then disagreement should be the unique bargaining outcome. See end of 3.1 and note 9.

For all essential bargaining problems for i and l, set
$\eta^f\left(\Phi_{il,\mu^{k-}}, \psi^k\right) = \eta\left(\Phi_{il,\mu^{k-}}, \psi^k\right).$
Otherwise
<u>Case 1.</u> $\nexists\left(x_i^k, x_l^k\right) \in IRF^k$ s.t. $x_i^k > \psi_i^k$, however $\exists\left(x_i, x_l\right) \in IRF^k$ s.t. . $x_l^k > \psi_l^k$
set $\eta^f\left(\Phi_{il,\mu^{k-}}, \psi^k\right) = \gamma;$
<u>Case 2.</u> $\nexists\left(x_i, x_l\right) \in IRF^k$ s.t. $x^k > \psi^k$
set $\eta^f\left(\Phi_{il,\mu^{k-}}, \psi^k\right) = \gamma.$

If there are no past successful negotiators, that is, no apparent and contingent links have formed,
$\eta^f\left(\Phi_{il,\mu^{k-}}, \psi^k\right) = \eta\left(\Phi_{il,\mu^{k-}}, \psi^k\right).$

In contrast to the case with a last-mover advantage, in *all* non essential bargaining games in this bilateral sequential negotiation and bargaining environment an O-F Joint Plan may be the disagreement one, in which case, unsuccessful preliminary negotiations occurs and the link does not form.

It is clear that if the tenability correspondence is non-empty and the joint plan bargaining game is well defined then, depending on the assumption, either Nash Coherent or O-F Joint Plans exist in any corresponding contingency p_c (corresponding to p should be understood) of the communication game. In what follows, I develop a theory of rational behavior in corresponding future contingencies in the latter game and formalize the idea of a history. This theory will be relevant for the construction of the tenability correspondence that has been assumed so far as given, in particular, the disagreement future-request $\widehat{\zeta}$. More importantly, it will make it possible to define Sequentially Nash Credible Joint Plans.

4 The Tenability Correspondence

4.1 The Payoff Relevant Multistage Game

To derive the tenability correspondence, I will consider a $K+1$-*multistage game with payoff-relevant observed actions* M based in Fudemberg and Tirole [19], however with substages.

4.1.1 Choice and Actions Sets and Histories

At the beginning of the first stage 1, all players $m = 1, 2, 3$ select simultaneously from choice sets A_{m,h^1}, where A_{m,h^1} for the player m that is not associated to the link being proposed has a trivial unique payoff-irrelevant action "move nothing". For the other two players $A_{m,h^1} = \{y, n\}$. A permanent communication link forms if and only if both latter players play y. I let the initial history be $h^1 = \varnothing$ at the start of play. At the end of the first substage, all players observe the substage 1's choice profile. Let $a^1 = (a_1^1, a_2^1, a_3^1)$ be the first substage's choice profile. At the beginning of the second substage players know history $h^{1.2}$ that can be identified with a^1 given that h^1 is trivial. In the second substage, regardless of a^1, or equivalently, of the link forming or not, all players $m = 1, 2, 3$ choose simultaneously from the same action sets, that is, $B_{m,h^{1.2}} = B_{m,h^1}$, $m = 1, 2, 3$, where B_{m,h^1} for the player m that is not associated to the link being proposed has a trivial unique payoff-irrelevant action. At the end of the second substage, all players observe the second substage's action. Let $b^1 = (b_1^1, b_2^1, b_3^1)$ be the second substage action profile. At the beginning of stage 2 players know history h^2 that can be identified with (a^1, b^1) or, equivalently, $(h^{1.2}, b^1)$.

In general, choices and actions for player m will depend on previous choices and actions, so I let A_{m,h^2} denote the action set for player m at history h^2 and B_{m,h^2} denote the action set for player m at history $h^{2.2}$. By iteration, histories in general are

$h^k = \left(a^1, b^1, a^2, b^2, ..., a^{k-1}, b^{k-1}\right)$

and

$h^{k.2} = \left(a^1, b^1, a^2, b^2, ..., a^{k-1}, b^{k-1}, a^k\right)$

and B_{m,h^k} is the action set for player m at stage k when the history is $h^{k.2}$ and A_{m,h^k} is the action set for player m at stage k when the history is h^k. I let $K + 1$ be the total number of stages in the game. By definition each h^{K+1} describes an entire sequence of choices and actions from the start of the game on. I denote H^{K+1} as the set of all terminal histories that can be identified with the set of possible *outcomes* when the game is played.

Note that this is a model where pairs of players have non trivial stage action sets whenever they follow−depending on a link being formed or not and according to the rule of order−to propose a link. The third player has a trivial unique payoff-irrelevant choice or action. If the last pair in the rule of order played the associated stage games all players move nothing there after, that is choice and action profile sets are singletons there after. Let $K + 1$ be the total number of stages in the game. By definition each h^{K+1} describes an entire sequence of actions from the start of the game on. I denote H^{K+1} as the set of all terminal histories that can be identified with the set of possible *outcomes* when the game is played.

4.1.2 Pure Strategies and Payoff Outcomes

A pure strategy for player i is a contingent plan on how to play in the first and second substage at stage k of the game for respective possible histories h^k and $h^{k.2}$. I let H^k or $H^{k.2}$ denote the set of all substage k-histories, and

$A_{i,H^k} = \cup_{h^k \in H^k} A_{i,h^k}$ and

$B_{i,H^{k.2}} = \cup_{h^{k.2} \in H^{k.2}} B_{i,h^{k.2}}$.

A pure strategy for player i is a sequence of maps $\{s_i^k\}_{k=1}^K$, where each s_i^k maps $H^k \cup H^{k.2}$ to the set of player i's feasible choices A_{i,H^k} and actions $B_{i,H^{k.2}}$ (i.e., satisfies $s_i^k(h^k) \in A_{i,h^k}$ and $s_i^k(h^{k.2}) \in B_{i,h^{k.2}}$ for all $h^k \in H^k$ and $h^{k.2} \in H^{k.2}$). The set of all pure strategies for player i in the payoff-relevant multistage game is denoted by S_i.

A sequence of choices and actions for a profile for such strategies $s \in S$ is called the *path* of the strategy profile, where S is the set of all strategy profiles: the first substage choices are $a^1 = s^1(h^1)$. Second substage actions are $b^1 = s^1(a^1)$. The first substage choices in stage 2 are $a^2 = s^2(a^1, b^1)$. The second substage actions in stage 2 are $b^2 = s^2(a^1, b^1, a^2)$ and so on. Since the terminal histories represent an entire sequence of play or path associated with a given strategy profile, one can represent each players' corresponding *overall's* payoff as a function $u_i : H^{K+1} \to \mathbb{R}$. Abusing notation, I denote the payoff vector to profile $s \in S$ as $u(s) = u\left(h^{K+1}\right)$, as one can assign an outcome in H^{K+1} to each strategy profile $s \in S$.

4.1.3 Nash Equilibrium

A pure-strategy *Nash equilibrium* in this context is a strategy profile s such that no player i can do better with a different strategy or, using standard Fudemberg and Tirole's [19] notation, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_i' \in S_i$.

4.1.4 Subgameperfect Equilibrium

Since all players know the history h^k or $h^{k.2}$, one can view respectively the game from stage k on with history h^k or $h^{k.2}$ as an extensive form game in its own and denote it by $M\left(h^k\right)$ or $M\left(h^{k.2}\right)$. To define the payoff functions in this game, note that if the sequence of choices and actions or path in stages k through K are a^k or b^k through b^K, the final history will be $h^{K+1} = (h^k, a^k, b^k, ..., b^K)$ or $h^{K+1} = (h^{k.2}, b^k, ..., b^K)$. The payoffs for player i will be $u_i(h^{K+1})$.

Strategies in $M\left(h^k\right)$ or $M\left(h^{k.2}\right)$ respectively are defined in a way where the only histories one needs consider are those consistent with h^k or $h^{k.2}$. Precisely, any strategy profile s of the whole game induces a strategy profile $s|h^k$ on any $M\left(h^k\right)$ or $s|h^{k.2}$ on any $M\left(h^{k.2}\right)$. For each i, $s_i|h^k$ or $s_i|h^{k.2}$ is the *restriction* of s_i to the histories consistent with h^k or $h^{k.2}$. One denotes the restriction profile set by $S|h^k$ or $S|h^{k.2}$.

Let histories h^{K+1} be such that $h^{K+1} = (h^k, a^k, b^k, ..., b^K)$ or $h^{K+1} = (h^{k.2}, b^k, ..., b^K)$ and the associated subset of H^{K+1} be denoted by $H^{K+1}(h^k)$ or $H^{K+1}(h^{k.2})$. As one can assign respectively an outcome in $H^{K+1}(h^k)$ or $H^{K+1}(h^{k.2})$ to each restriction profile $s|h^k$ or $s|h^{k.2}$ where $s \in S$, the overall payoff vector to the restriction $s|h^k$or $s|h^{k.2}$, will be denoted abusing notation by $u(s|h^k)$ or $u(s|h^{k.2})$. Thus, one can speak of Nash equilibria of $M\left(h^k\right)$ or $M\left(h^{k.2}\right)$.

It will be useful to express $S|h^k$ as a set of points rather than a set of mappings. Hence, consider the following Cartesian product derived recursively for all h^k:

$$S|h^k = A_{h^k} \times B^4_{h^k} \times \prod_{\left(a^k, b^k\right)} S|\left[h^k, a^k, b^k\right] \tag{8}$$

. A strategy profile s of a multi-stage payoff-relevant game with observed actions M is a *subgame-perfect equilibrium* if, for every h^k and $h^{k.2}$, the restriction $s|h^k$ and $s|h^{k.2}$ to $M\left(h^k\right)$ and $M\left(h^{k.2}\right)$ respectively is a Nash equilibrium of $M\left(h^k\right)$ and $M\left(h^{k.2}\right)$.

4.1.5 Vectors of Correlated Strategies

I will be interested in representing negotiation statements about a payoff-relevant game to follow$-$as defined in section 3$-$in the multistage payoff-relevant game as a vector of correlated strategies .

A vector of correlated strategies is a sequence of maps $\{\omega^k\}_{k=1}^K$, where each ω^k maps H^k and $H^{k.2}$ to the set of correlated strategies on elements of A_{H^k} and $B_{H^{k.2}}$ (i.e., $\omega^k(h^k)$ is a correlated strategy on A_{h^k} for all $h^k \in H^k$ and $\omega^k(h^{k.2})$ is a correlated strategy on B_{h^k} for all $h^{k.2} \in H^{k.2}$). I denote by $W|h^1$ the set of all vectors of correlated strategies in history h^1.

Given $\omega|h^1 \in W|h^1$, I am interested in the probability of the path $\left(a^1, b^1, a^2, ..., a^K, b^K\right)$ corresponding to strategy profile $s \in S$ and denoted by $\Pr\left[s/\omega|h^1\right]$. This will be given by the expression

$\Pr\left[s/\omega|h^1\right] = \omega^1_{h^1}\left(a^1\right) * \omega^1_{(a^1)}\left(b^1\right) * \omega^2_{(a^1,b^1)}\left(a^2\right) *, ..., *\omega^K_{(a^1,b^1,a^2,...,a^K)}\left(b^K\right).$

Let $\omega|h^k \in W|h^k$ be a given vector of correlated strategies in the subgame that begins in history h^k. It will be also of interest to know the probability of the path $\left(h^k, a^k, b^k, ..., a^K, b^K\right)$ corresponding to the restriction $s|h^k$ of $s \in S$ on $M\left(h^k\right)$ for any $h^k \in H^k$ for all k and denoted by $\Pr\left[\left(s|h^k\right)/\omega|h^1\right]$. This will be given by

$$\Pr\left[\left(s|h^k\right)/\omega|h^k\right] = \omega^k_{h^k}\left(a^k\right) * \omega^k_{\left(h^k,a^k\right)}\left(b^k\right) *, ..., *\omega^K_{\left(h^k,a^k,b^k,...,a^K\right)}\left(b^K\right) \tag{9}$$

.

4.2 Credibility in the Communication Game Histories

I want to add both endogenous, Nash and O-F Nash effective cooperative negotiation, as defined in 3.2.3 and 3.2.4 respectively, to the multistage payoff-relevant game. In order to do so, at every relevant "history" of the associated multistage game with communication game a player $m = i, j, l$ that moves non trivially can engage in preliminary negotiations. To set up negotiation problems as in section 3, future joint strategies in the associated payoff-relevant games to follow are defined in this context. Note that different vectors of correlated strategies enunciated at different stages of the communication game by the same player should be implemented by having respectively different mediators that, at each stage, make a public announcement or recommendation observed by all players. For simplicity, the associated notation in the multistage game with communication will be abstracted from for the most!

4.2.1 Future Joint Strategies

I denote the set of future joint payoff relevant strategies at stage $k \leq K$ as $\times Z_{h^k}$. Using Eq. (3), one can recursively derive $\times Z_{h^k}$ for all h^k as a Cartesian product of joint strategies in future games $\left[h^k, a^k, b^k\right]$:

$$\times Z_{h^k} = \prod_{\left(a^k, b^k\right)} \times Z_{h'^k_{(.)}/\left[h^k, a^k, b^k\right]} \tag{10}$$

, where $h^{k'}_{(.)}$ should be interpreted as a future contingency in the payoff-relevant game to follow at h^k. For any restriction of strategy profiles S can be expressed as a Cartesian product from Eq. (8) , at each recursion, when obtaining $\times Z_{h^k}$,

$$\times Z_{h'^k_{(.)}/\left[h^k, a^k, b^k\right]} = S| \left[h^k, a^k, b^k\right] \tag{11}$$

Hence, at each recursion

$$\times Z_{h^k} = \prod_{\left(a^k, b^k\right)} S| \left[h^k, a^k, b^k\right] \tag{12}$$

.

4.2.2 Negotiation Problems and The Tenability Correspondence

Now one can define utility functions at histories $h^k_{(.)}$ where the arguments are link choices, current actions and future joint strategies by using:

$U_{h^k_{(.)}}\left(a^k, \times b^k, \times z^k\right) = u(s|h^k)$,

where $s|h^k = \left(a^k, \times b^k, \times z^k\right)$ after using Eq. (8) and (12) . This expression refers to the expected utilities for the three players if $a^k \in A_{h^k}$, $\times b^k \in \times B^4_{h^k}$ and $\times z^k \in \times Z_{h^k_{(.)}}$ are played following $h^k_{(.)}$.

To formulate negotiation problems and joint plan bargaining problems in the notation of section 3, I assume that a *history of the multistage game with communication* corresponding to the multistage payoff-relevant game, $\mathring{h}^k_{(.)} \neq h^k_{(.)}$, includes in the subscript (.), in addition to a sequence of past choices $a^{k^-}$and actions, b^{k^-}, a sequence of past tenable and reliable joint plans $\left(\mu^1, ..., \mu^{k-1}\right) = \mu^{k^-}$ (See Remark 5) and past recommendations by different mediators. Abstracting from recommendations, for each negotiation problem in $\mathring{h}^k_{\left(\mu^{k^-}, a^{k^-}, b^{k^-}\right)}$, a *corresponding history* to the unique $h^k_{\left(a^{k^-}, \beta^{k^-}\right)}$, one sets $B = B_{h^k_{(.)}}$ and $\times Z = \times Z_{h^k_{(.)}}$ and $U = U_{h^k_{(.)}}$.

The negotiation problem is trivial in histories where players move nothing. So assume a history is reached where after any tenable and reliable joint plan is enunciated, choices and then actions are taken, players move nothing thereafter. Either Nash Coherent or O-F Joint Plans are defined in such histories $\mathring{h}^k_{(.)}$, where say players i and j move non trivially, as follows:

The set of future joint strategies $\times Z_{\mathring{h}^k_{(.)}} = \times Z_{h^k_{(.)}}$, or $\times Z^k$, if no confusion arises, is the Cartesian product of singleton action profile sets. So the tenability correspondence in $\mathring{h}^k_{(.)}$ is trivially defined as

$Q_{\mathring{h}^k_{(.)}}\left(\mu^k\right) = \mho^{\times Z_{\mathring{h}^k_{(.)}}}$.

If no confusion arises, I will write only $\mho^{\times Z^k}$, also a singleton, a Cartesian product of functions that put probability one on the unique element of the trivial action set profiles at each future history of the payoff-relevant game to follow $\mathring{h}^k_{(.)}$. Set the tenability correspondence in section 3, $Q_{\mu^{k-}}(\mu) = Q_{\mathring{h}^k_{\left(\mu^{k-}\right)}}\left(\mu^k\right)$. If there is a last-mover advantage indexing by μ^{k^-} is not necessary!

For any trivial (as a trivial future game follows) a'^k-concatenated strategic form game $\left(B_i^k \times B_j^k, \pi^{\mu_m^k/a'^k}\right)$, where $\mu_m^k = \left(\alpha_m^k, \beta_m^k, \zeta_m^k\right) \in \mho^k$, to be well defined, one sets for any $b^k \in B_{h^k_{(.)}}$ and the unique trivial $z^k \in \times Z^k$

$$\Pr\left[U_{m,h^k_{(.)}}(.)\right] = 1 \tag{13}$$

, where as for Eq.(6), $U_{m,h^k_{(.)}}(.) = U_{m,h^k_{(.)}}\left(a'^k, \times b^k, \times z^k\right)$, the a'^k-th component of $\times b^k$ $b^k_{a'^k} = \left(b_i^k, b_j^k, b_l^k\right)$ and $\Pr\left[U_{m,h^k_{(.)}}(.)\right]$ is the probability that $U_{m,h^k_{(.)}}(.)$ results given that $\left(a'^k, b^k_{i,a'^k}\right)$ occurred and play is consistent with ζ_m^k thereafter, for $m = i, j$.

The outside options in the joint plan bargaining problem are $\psi^k = \left(x_i^k, x_j^k\right)$ with disagreement plan $\widehat{\mu}^k = \left(\widehat{\alpha}^k_{\widehat{a}^k}, \widehat{\beta}^k, \widehat{\zeta}^k\right)$ where the disagreement future-request $\widehat{\zeta}^k$ is trivial and thus can be obtained!

Finally, if the sequence of past joint plans is given by μ^{k^-} one sets $\Phi_{ij,\mu^{k-}} = \Phi_{\mathring{h}^k_{\left(\mu^{k-}\right)}}$, or simply Φ_{ij}^k.

Recall that to each such history in the communication game $\mathring{h}^k_{(.)}$, there are associated future-requests by pairs il or jl that may have formed in some order. Suppose that $\left(F, \Phi_{ij}^k, \psi\right)_{\mathring{h}^k_{(.)}}$ is well defined. Depending on the assumptions either $\eta_{\mathring{h}^k_{(.)}}\left(\Phi_{ij}^k, \psi^k\right)$ or $\eta^f_{\mathring{h}^k_{(.)}}\left(\Phi_{ij}^k, \psi^k\right)$, the credible joint plan set, exists.

In general, suppose that one has inductively defined a non empty credible joint plan set in any $\mathring{h}^k_{(.)}$; either one has

$$\eta_{\mathring{h}^k_{(.)}}\left(\Phi_{ij}^k, \psi^k\right) \neq \varnothing \text{ or } \eta^f_{\mathring{h}^k_{(.)}}\left(\Phi_{ij}^k, \psi^k\right) \neq \varnothing \tag{14}$$

, where $i \neq j$ and $i, j = 1, 2, 3$.

As for Eqs. (5) and (12) the vector of correlated strategies ζ^k of section 3 in future histories of the future game can be expressed in terms of a vector of correlated strategies of the payoff-relevant game to follow history h^k in the multistage payoff-relevant game of subsection 4.1 for

$$\zeta^k = \prod_{\left(a^k,b^k\right)} \zeta_{h_{(.)}^{k\prime}/\left[h^k,a^k,b^k\right]} \in \prod_{\left(a^k,b^k\right)} W|\left[h^k,a^k,b^k\right] \tag{15}$$

. Let $\mu^k = \left(\alpha^k, \beta^k, \zeta^k\right)$ have $\zeta^k \in Q_{\mathring{h}_{(.)}^k}\left(\mu^k\right)$, that is, μ^k is future tenable. The future-request ζ^k should be such that for any $\left(a^k, b^k\right) \in A_{h_{(.)}^k} \times B_{h_{(.)}^k}$, depending on the assumption, either

$\zeta_{h_{(.)}^{\prime k}/\left[h^k,a^k,b^k\right]} \in \eta_{\mathring{h}_{(.)}^{k+1}}\left(\Phi^{k+1}, \psi^{k+1}\right)$ or $\zeta_{h_{(.)}^{\prime k}/\left[h^k,a^k,b^k\right]} \in \eta^f_{\mathring{h}_{(.)}^{k+1}}\left(\Phi^{k+1}, \psi^{k+1}\right)$,

where $\mathring{h}_{(.)}^{k+1} = \left[\mathring{h}_{(.)}^k, r_a^k, a^k, r_b^k, b^k, \mu^k\right]$, for all link choice recommendations $r_a^k \in A_{h_{(.)}^k}$ and action recommendations $r_b^k \in B_{h_{(.)}^k}$. That is, any $\zeta_{h_{(.)}^{\prime k}/\left[h^k,a^k,b^k\right]}$, should equal the identical Credible Joint Plans in the histories that follow $\mathring{h}_{(.)}^k$ after players i and j enunciated μ^k, $\left(a^k, b^k\right)$ was played and any pair of recommendations occurred; for all possible recommendations belong to the support of α^k and β^k in the given $\mu^k = \left(\alpha^k, \beta^k, \zeta^k\right)$. Recall from section 3, that credibility of joint plans depend only on past choices and actions in the last-mover advantage case and, under the O-F focal effect, credibility depends in addition on past successful joint plans and not on its specific recommendations. Hence, for simplicity, I will ignore recommendations and write instead $\left[\mathring{h}_{(.)}^k, \beta^k, \mu^k\right]$, provided indexing by μ^{k^-} is not relevant.

It is implicit that if $\mu^k = \left(\alpha^k, \beta^k, \zeta^k\right)$ is such that histories $\mathring{h}_{(.)}^{k+1}$ have players move nothing, $\eta^f_{\mathring{h}_{(.)}^{k+1}}\left(\Phi^{k+1}, \psi^{k+1}\right)$ is a trivial joint plan, as actions profile sets there and thereafter are singletons.

Remark 7 *If one assumes the O-F focal effect, tenable future-requests ζ^k in $\mu^k = \left(\alpha^k, \beta^k, \zeta^k\right)$, i.e., $\zeta^k \in Q_{\mathring{h}_{(.)}^k}\left(\mu^k\right)$, may be different depending on the μ^{k^-} associated to $\mathring{h}_{\left(\mu^{k^-}\right)}^k$, as different past successful joint plans may influence play in each history in a different way.*

By the inductive assumption in Eq. (14), $Q_{\mathring{h}_{(.)}^k}\left(\mu^k\right) \neq \varnothing$. Next, for any a'^k-concatenated strategic form game $\left(B_i^k \times B_j^k, \pi_{ij}^{\mu_m^k/a'^k}\right)$, where $\mu_m^k = \left(\alpha_m^k, \beta_m^k, \zeta_m^k\right) \in \mho^k$, to be well defined, one sets for any $b^k \in B_{h_{(.)}^k}$ and $\times z^k \in \times Z^k$

$\Pr\left[U_{m,h_{(.)}^k}(.)\right] = \Pr\left[\left(s|h^k\right)/\omega|h^k\right]$,

where $U_{m,h_{(.)}^k}(.)$ is defined as in Eq.(13) and $\Pr\left[U_{m,h_{(.)}^k}(.)\right]$ equals to $\Pr\left[\left(s|h^k\right)/\omega|h^k\right]$ in Eq. (9); the latter is the probability of the path corresponding to the restriction $s|h^k = \left(a'^k, \times b^k, \times z^k\right)$, from Eqs. (8) and (12), given the vector of correlated strategies $\omega|h^k$ that can be expressed as $\ddot{\mu}^k = \left(\ddot{\alpha}_{a'^k}^k, \ddot{\beta}^k, \zeta^k\right)$ for Eq. (15); $\ddot{\mu}^k$ puts probability

1 on both a'^k and on b^k after a'^k occurred, i.e., $\ddot{\beta}^k_{a'^k}\left(b^k\right) = 1$ and is consistent with ζ^k_m thereafter, for $m = i, j$.

The outside options in the associated joint plan bargaining problem are $\psi^k = \left(x^k_i, x^k_j\right)$ with tenable and reliable $\widehat{\mu}^k = \left(\widehat{\alpha}^k_{\widehat{a}^k}, \widehat{\beta}^k, \widehat{\zeta}^k\right)$, i.e., $\widehat{\mu}^k \in \widetilde{\mho}$. Note that the disagreement future-request $\widehat{\zeta}^k$ <u>can be derived</u> given the finite rule of order and has the peculiar feature that it depends only on statements of pairs that have had or will have successful preliminary negotiations; of course, it "depends" on the current pair's disagreement future-request trivially (See Remark 4). Also, recall, it was fixed when outside options were defined in 3.2.2.

In general, history $\mathring{h}^k_{(.)}$ has future-requests by pairs that have successfully negotiated before in a given order. The ones of pairs that were unsuccessful are "basically" ignored. Assume that $(F, \Phi_{ij}, \psi)_{\mathring{h}^k_{(.)}}$ is well defined. Either $\eta_{\mathring{h}^k_{(.)}}\left(\Phi_{ij}, \psi\right)$ or $\eta^f_{\mathring{h}^k_{(.)}}\left(\Phi_{ij}, \psi\right)$ exist for any possible history.

5 Sequentially Nash Credible Joint Plans

Suppose either that O-F or Nash Coherent Joint Plans exist for all histories, only then the inductive assumption in Eq. (14) is justified. Then, the credible joint plans at the beginning of play are defined as *Sequentially Nash Credible Joint Plans (SN)*. Formally,

$$SN = \eta\left(\Phi_{ij}, \psi\right) \text{ or } SN = \eta^f\left(\Phi_{ij}, \psi\right) \tag{16}$$

, where $\eta\left(\Phi_{ij}, \psi\right) = \eta_{\mathring{h}^1}\left(\Phi_{ij}, \psi\right)$ and $\eta^f\left(\Phi_{ij}, \psi\right) = \eta^f_{\mathring{h}^1}\left(\Phi_{ij}, \psi\right)$ were defined in 3.2.3 and 3.2.4 respectively.

Clearly SN suggest subgameperfect publicly correlated equilibria in the multistage game with communication.

Next a theorem is given for *non degenerate* finite concatenated strategic form games[16], where a last-mover advantage is assumed.

Theorem 8 *Let B_{m,h^k} be a finite set for $m = i, j, l$ for any possible history h^k and each concatenated strategic form game be nondegenerate. Assume a last-mover advantage. SN exist.*

Proof. If the sets B_{m,h^k} for $m = i, j, l$ are finite then the joint plan bargaining problem $(F, \Phi_{ij}, \psi)_{\mathring{h}^k_{(.)}}$ in any history $\mathring{h}^k_{(.)}$is well defined. In particular, feasible sets are closed as its elements are convex combinations of the unique outside options and a finite number T of payoff pairs $\left\{(x_i, x_j)_t\right\}_{t=1,\ldots,T}$, where each $(x_i, x_j)_t$is

[16] Wilson [55, Theorem 1, pp. 85] has shown that, excluding certain degenerate cases, in any finite game the number of Nash equilibria is finite and odd.

associated to an "almost degenerate" publicly correlated equilibrium $\beta^k_{m,a^k,t}$ corresponding to an a^k-concatenated strategic form game $\left(B^k_i \times B^k_j, \pi^{\mu_{m,t}/a^k}_{ij}\right)$ given $\mu^k_{m,t} = \left(\alpha^k_{m,a^k}, \beta^k_{m,t}, \zeta^k_m\right) \in \widetilde{\mho}^k$ and $a^k_{ij} = (y, y)$, for all $t = 1, ..., T$; that is, $\mu^k_{m,t} \neq \mu^k_{m,t'}$ if and only if $\beta^k_{m,a^k,t} \neq \beta^k_{m,a^k,t'}$ where $t \neq t'$ and $t, t' = 1, ..., T$; also $\beta^k_{m,a^k,t}$ is almost degenerate in the sense that it implies putting probability 1 on a mixed strategy Nash equilibrium of such concatenated game for all t, for $m = i, j$ ■

As the reader may have anticipated, the feasible sets are not necessarily closed whenever the B_{m,h^k} sets are infinite or, if finite, oldest friends can suggest randomizing between the NBR rule payoff and the outside options whenever the *IRF* sets are closed segments. In the latter case, the associated tenable and reliable joint plans belong to an infinite choice set even though action sets are finite. See Nieva [41] where joint plan bargaining games are nevertheless well defined for an almost non cooperative modification of the three-player Aumann-Myerson [3] (A-M) network formation game where an extra mild assumption is needed. In the modified game pairs of players choose proposals from infinite action sets. The continuity of the Nash Bargaining solution with respect to outside options is key in showing general existence. The example in the next section is one parameter case extracted from that paper and does not need the extra mild assumption.

The following theorem is work in progress.

Theorem 9 *Let B_{m,h^k} be a finite set for $m = i, j, l$ for any possible history h^k. Assume the O-F focal effect. SN exist.*

Proof. The proof is analogous to the one in Nieva [41]. The key argument uses the result that the Nash equilibrium correspondence has a closed graph and that the Nash bargaining solution is continuous in the Haussdorff space whenever using the Haussdorff distance ■

6 A Modified Simple Majority Game

In what follows the A-M model is modified by adding bargaining or transfer possibilities and in addition endogenous O-F Nash effective cooperative negotiation. The latter almost non cooperative (ANC) model yields unique payoff predictions instead of the multiple prediction obtained in the modified game without negotiation possibilities or the original A-M model to begin with.

Consider the three-player simple majority cooperative game with characteristic function:

$v(1) = 0,$	$v(2) = 0,$	$v(3) = 0,$
$v(13) = 1$	$v(23) = 1$	$v(12) = 1,$
$v(123) = 1.$		

where, for example, $v(13)$ is the total wealth players 1 and 3 can assure if they collude and cooperate.

Suppose payoffs accrued to each player in such a cooperative game depends on the communication structure, that is, it depends on who is at least indirectly communicated or *linked* with other players. Formally, payoffs depend on the communication link structure represented by a *graph*, a set of bilateral links.

Graph g^{ij} is the one that only has a link between player i and j, ij. Graph g^{ij+jl} is the one that would result if links jl is added to graph g^{ij} for $i \neq j \neq l$, where $i, j, l \in \{1, 2, 3\}$. Graph g^N denotes the complete graph where all players are linked. Also, if I write that some values for player i and j are (x, y), the first (second) value component refers to player i (j). *Myerson* [32] *values* for each player were derived axiomatically and are given below for different graphs (the first, second third component in the triplet corresponds to player 1,2, and 3 respectively):

One-link	Values	Two-Link	Values	Complete	Values
g^{13}	$(\frac{3}{6}, 0, \frac{3}{6})$	g^{13+32}	$(\frac{1}{6}, \frac{1}{6}, \frac{4}{6})$	g^N	$(\frac{2}{6}, \frac{2}{6}, \frac{2}{6})$
g^{23}	$(0, \frac{3}{6}, \frac{3}{6})$	g^{12+23}	$(\frac{1}{6}, \frac{4}{6}, \frac{1}{6})$		
g^{12}	$(\frac{3}{6}, \frac{3}{6}, 0)$	g^{21+13}	$(\frac{4}{6}, \frac{1}{6}, \frac{1}{6})$		

Note how the player who has relatively more links or friends gets more.

In the A-M sequential network formation model, pairs of players propose indestructible bilateral communication links following a bridge-like rule order and evaluate induced communication structures, graphs, using the Myerson values. Links are formed if the pair agrees. As in bridge, after the last link has been formed, each of the pairs must have a last chance to form an additional link. If then every pair rejects, the game *ends*. This game is of perfect information. Hence, it has subgame perfect equilibria in pure strategies. Each equilibrium has a unique graph formed at the end of play.

Assume that links 12, 23 and 13 are proposed in that order.

Claim 10 *The A-M solution has three subgame perfect equilibrium outcomes in which either of the one link graph is the last to form.*

Proof. From any two link graph the complete graph follows as the players not linked get more if they link, $\frac{2}{6}$ instead of $\frac{1}{6}$. A one link graph is last to form as any player in that link would reject a second link as the complete graph would follow next in which case her payoff would go down from $\frac{3}{6}$ to$\frac{2}{6}$.

Suppose links 12 and 23 have been rejected. Link 13 would form as players 1 and 3 would expect to get half instead of zero payoffs in case the game would end after rejection. One stage backwards, player 3 is indifferent between linking or not with player 2. One more stage backwards, player 2 is indifferent between linking or not with player 1 if players expect link 23 to form. On the other hand, player 1 is indifferent between linking or not with 2 if players expect link 23 not to form and instead link 13 to form. Thus, depending on the decision of the indifferent player, there are several subgame perfect equilibria outcomes in which either of the one link graph forms. ■

Consider the following modification, at each stage of the A-M game, a link, say ij, may form if both players i and j play *choice* y in the *simultaneous link choice "formation" game*. If at least one of them plays a *unilateral rejection*, n, the link does not form.

Following any outcome of the link "formation" game a *current simultaneous action game* takes place. Actions are interpreted as *proposals pairs.* Each player proposes a non negative payoff for player i and for player j. Proposals pairs are *feasible* if they add to the sum of the pair's Myerson values in the *immediate prospective graph*, the one that would form if the link ij forms. Proposals pairs *match* if they are feasible and coincide. If the compete graph is the immediate prospective graph then only the Shapley values that coincide with the associated Myerson values are feasible.

A link forms and a given transfer scheme is binding for players i and j, if and only if both choose y and match proposals pairs. Otherwise, the link does not form. With respect to payoff outcomes, if the immediate prospective graph does not form and the game ends, payoffs in the *last proposal match*−the one that led to the formation of the last graph−are realized. The third player gets her Myerson value in such last graph. Otherwise stage payoffs are zero unless the complete graph forms, in which case the Shapley values are realized. Note that whenever a pair of players did not form its link, the underlying two-player strategic form game has the same action profile set but play of any action profile is payoff-irrelevant. This is needed for my modification of the A-M model to fit the model in this paper. Note, however, the link forms provided the pair matches proposals.

To formulate joint plan bargaining problems, I assume that each pair can engage in preliminary negotiations with a temporary communication technology and can enunciate negotiation statements in the corresponding communication game represented by a correlated strategy in the link formation game, correlated strategies in the current simultaneous games and correlated strategies in future contingencies of the payoff-relevant game to follow, a future-request. The disagreement joint plan suggests unilateral link rejections, that is, $\widehat{\mu}^k$ is such that $\widehat{\alpha}^k_{\widehat{a}^k}$ has $\widehat{a}^k_{ij} = (n, n)$ and $\widehat{\beta}^k_{a'^k}$ for any a'^k such that $a'^k_{ij} \neq (y, y)$ is the same arbitrarily given payoff-irrelevant correlated strategy. Wlg., $\widehat{\beta}_{a'^k}$ if $a'^k = (y, y)$ is arbitrary fixed to any two given *unilateral proposals pair rejection*, (unfeasible proposals pairs) as any of these are a Nash equilibrium of any a^k−concatenated game.

Note, any proposal match associated to a given μ^k that has induced payoffs $\pi^{\mu^k/a^k}_{ij}(b_i, b_j)$ that are greater or equal than the disagreement payoffs are a Nash equilibium of the a^k-*concatenated game,* where $a^k_{ij} = (y, y)$. If not, a player could get his outside option by proposing something unfeasible. Hence, the given μ^k is current reliable and tenable. By identical reasons it is link reliable and tenable. If μ^k is in addition future tenable, this type of payoffs belong to the feasible set in the joint plan bargaining problem for players i and j. The IRF set consists of convex combinations of all such payoffs and the outside options. The latter will become clear in the proof.

Proposition 11 *In the modified three-player simple majority game, a SN has the first pair suggesting "half-each" payoffs and future-requesting joint plans that suggest consecutive rejection of the next two links in the order and the nucleolus in coalition structure is implemented.*

Proof. Let the first two links in the rule of order 12 and 23 be rejected in stage 1 and 2 of the game respectively. Next to propose in stage 3 is pair $(1,3)$.[17]

Part 1

I. Suppose that players 1 and 3 have a fully successful joint plan (that is, it is reliable, tenable, suggests link forming, that is, $\alpha^3_{a^3}$ puts probability one on a^3 where $a^3_{13} = (y, y)$) that suggests[18] a half-each payoff proposal match, that is, it recommends each one to propose $(\frac{3}{6}, \frac{3}{6})$, a payoff for player 1 and another one for player 3. Suppose after link 13 forms, link 12 is rejected in stage 4 and link 32 is being discussed in stage 5. I want to find out, to begin with, what are all the tenable future-requests for players 1 and 3 on players 3 and 2 in this contingency.

First, let's see what players 3 and 2 can achieve by enunciating a future tenable joint plan that suggests a proposal match (Note this joint plan is not necessarily reliable and tenable) such that player 2 is offered (out of the sum of their Myerson values in the immediate prospective graph g^{13+32}, $\frac{4}{6} + \frac{1}{6}$) less than what she would get in the complete graph, $\frac{2}{6}$. After link 32 forms, as a future tenable joint plan, it would have to future-request players 1 and 2 to enunciate their unique O-F Joint Plan that suggests link formation and a proposal match (both propose their Shapley values) and thus form the third link 12. This is the case as the latter players' joint plan bargaining problem would be "essential", both gain by linking. The expected payoffs for player 3 and 2 associated to their joint plan (Plan a) would be $(\frac{2}{6}, \frac{2}{6})$, their Myerson values in the complete graph.

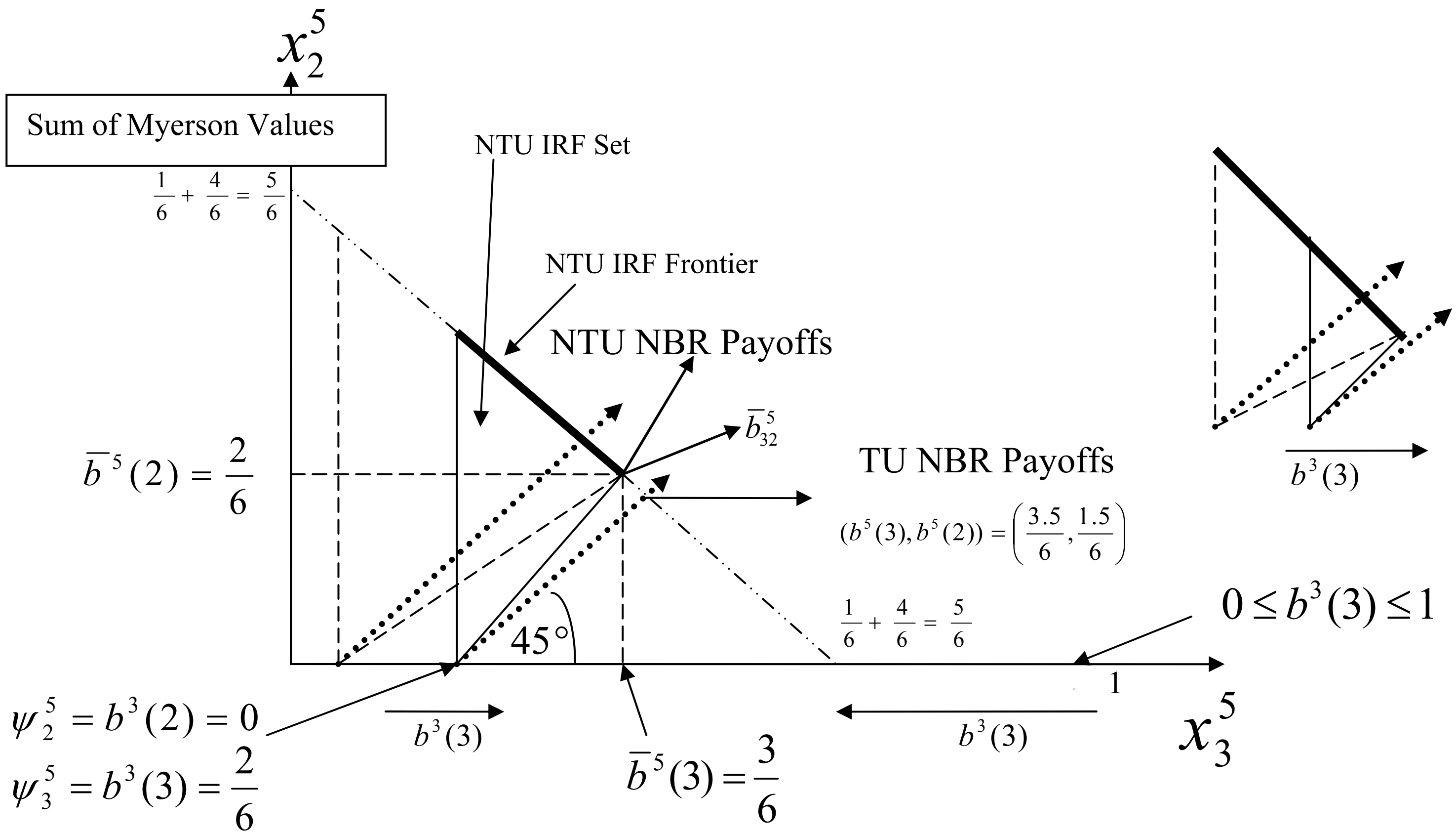
Second, if instead players 3 and 2 can enunciate a future tenable joint plan that suggests a proposal match such that player 2 is offered strictly more than $\frac{2}{6}$, this joint plan has to future-request players 2 and 1 to enunciate the unique O-F Joint Plan that suggests both unilaterally rejecting the third link. Link 32 would be the last to form (Plan of type b). The associated expected payoffs pair (x^5_3, x^5_2) for players 3 and 2 would lie on the diagonal in figure 1 to the northwest of $\overline{b}^5_{32} = (\frac{3}{6}, \frac{2}{6})$.

Third, if instead players 3 and 2's future tenable joint plan suggests a proposal match that offers exactly $\frac{2}{6}$ to player 2, proposal match b^5 such that $b^5_{32} = (\frac{3}{6}, \frac{2}{6}) = \overline{b}^5_{32}$ in figure 1, player 2 would be indifferent between forming or not the third link.

[17] Note that if pair $(1,3)$ rejects the game ends with zero payoffs. If it accepts, pair $(1,2)$ follows; in turn, if $(1,2)$ rejects, pair $(2,3)$ is next; because every not linked pair must have a last opportunity to propose (as in bridge). If link 23 does not form the game ends, and so on.

[18] In the language of section 3, this plan has a promise-request, a degenerated correlated strategy, that puts probability 1 on both proposing $(\frac{3}{6}, \frac{3}{6})$.

Players 3 and 2's Bargaining Game-Figure 1



Link 12 and 23 were rejected in stage 1 and 2 respectively. In stage 3, link 13 formed with payoff proposal match b^3 with $\left(b^3(1), b^3(3)\right) = \left(\frac{4}{6}, \frac{2}{6}\right)$. Link 12 was rejected. In stage 5, link 32 is proposed with induced outside options (ψ_3^5, ψ_2^5). Link 32 is last to form with payoffs (x_3^5, x_2^5) equal to an associated proposal match with $\left(b^5(3), b^5(2)\right) = \left(\frac{3}{6}, \frac{2}{6}\right) = \bar{b}_{32}^5$.

As players 3 and 2 could be the only relevant oldest pair of successful negotiators according to the O-F focal effect, there are three types of future tenable joint plans (provided these are tenable and reliable, there are three more types of histories in the communication game corresponding to the payoff-relevant contingency that follows $b_{32}^5 = (\frac{3}{6}, \frac{2}{6})$) if $b_{32}^5 = \overline{b}_{32}^5$. One type of joint plan would future-request an O-F Joint Plan that suggests link 12 to be formed (Plan d1). The other one would future-request an O-F Joint Plan that suggests link 12 to be rejected (Plan d2). The third one consist of mixes (Plan**s** d3). The associated expected payoffs (x_3^5, x_2^5) for players 3 and 2 would be respectively $(\frac{2}{6}, \frac{2}{6})$, $\overline{b}_{32}^5 = (\frac{3}{6}, \frac{2}{6})$ and convex combinations of the latter pairs of payoffs. Note how the O-F focal effect prevents the IRF^5 set to be open at $\overline{b}_{32}^5 = (\frac{3}{6}, \frac{2}{6})$!

As outside options for players 3 and 2 are $(\frac{3}{6}, 0)$, joint plan d2 with payoffs $(\frac{3}{6}, \frac{2}{6})$ is the only reliable, tenable that has strong Pareto efficient payoffs (Note that the joint plan that suggests link 32 rejection is also tenable and reliable;d1 and d2 are only future tenable). Thus, d2 is the unique <u>Nash Coherent</u> Joint Plan for players 3 and 2. Moreover it is fully successful. Player 1 would get in the latter case her Myerson value in graph g^{13+32}, $\frac{1}{6}$. See figure 1, however, set the outside options for players 3 and 2 $\left(\psi_3^5, \psi_2^5\right) = (\frac{3}{6}, 0)$.

Back to players' 1 and 3's discussion, as player 3 gets the same independently of link 32 forming or not, the O-F focal effect implies that <u>O-F</u> Joint Plans whenever link 32 is being discussed are up to the oldest fully successful friends 1 and 3. Fully successful joint plans for players 1 and 3 vary if the O-F Joint Plan they future-request either suggest link 32 rejection (type 1 plan**s**), link formation with proposal match $(\frac{3}{6}, \frac{2}{6})-$and thereafter link 12 rejection$-$(type 2 joint plan) or mixes (type 3 joint plans). Associated expected payoffs for players 1, 2 and 3 would be respectively $(\frac{3}{6}, 0, \frac{3}{6})$, $(\frac{1}{6}, \frac{2}{6}, \frac{3}{6})$ and convex combinations between $(\frac{3}{6}, 0, \frac{3}{6})$ and $(\frac{1}{6}, \frac{2}{6}, \frac{3}{6})$. As it will become clear soon, the O-F focal effect ensures payoffs will be $(\frac{3}{6}, 0, \frac{3}{6})$ and hence the nucleolus will be implemented in coalition structure!

One stage backwards, as of link 12 discussions in stage 4, one can now characterize all possible type 1 fully successful joint plan**s** for players 1 and 3. As the outside option pair for players 1 and 2 is $(\frac{3}{6}, 0)$, using analogous reasons as in bargaining among players 3 and 2 above, a fully successful joint plan for players 1 and 3 would have to future-request an O-F Joint Plan that suggests either unilaterally rejecting link 12 (type 1.1 Joint Plan) or link formation with a proposal match $(\frac{3}{6}, \frac{2}{6})$ (type 1.2 Joint Plan) or a mix.(type 1.3 Joint Plan**s**). Expected payoffs pairs for players 1 and 3 would be respectively $(\frac{3}{6}, \frac{3}{6})$, $(\frac{3}{6}, \frac{1}{6})$ and convex combinations between $(\frac{3}{6}, \frac{3}{6})$ and $(\frac{3}{6}, \frac{1}{6})$. On the other hand, one can characterize the unique type 2 joint plan for players 1 and 3. As the outside options pair for players 1 and 2 is $(\frac{1}{6}, \frac{2}{6})$, their joint plan bargaining game is essential and such a fully successful joint plan for players 1 and 3 would have to future-request an O-F Joint Plan for players 1 and 2 that suggests link formation and a proposal match. Also, analogously as before, an O-F Joint Plan that suggests link 23 rejection after link 12 forms would be future-requested. The

NTU NBR yields payoffs of $(\frac{1}{6}+\frac{1}{6}, \frac{2}{6}+\frac{1}{6})$ for players 1 and 2. Player 3 would get her Myerson value in g^{13+12}, $\frac{1}{6}$. Under any joint plan of type 3, the bargaining game for players 1 and 2 is also essential, thus player 3 would get also $\frac{1}{6}$ and player 1 could not get more than $\frac{3}{6}$!

II. Suppose that players 1 and 3 have a fully successful joint plan that suggests proposal matches where player 3 is offered less than half.

If link 12 is rejected then in any O-F Joint Plan for players 3 and 2, they would suggest link formation and a proposal match as the joint plan bargaining game is essential (See figure 1 where player 3 is offered $b^3(3) = \frac{2}{6}$ and hence outside options are $\left(\psi_3^5, \psi_2^5\right) = (\frac{2}{6}, 0))$. Based on the analysis in I, link 23 would be the last link to form. In particular, if player 3's outside option is zero (Note that player 2's outside option is, as in I, again zero) the NTU NBR would give player 2 half of the sum of their Myerson values, that is, $\frac{2.5}{6}$. That is the most she would get. The least she may get is, following I, $\frac{2}{6}$ (See figure 1 where she gets exactly that).

One stage backwards, as player 1's outside option is $\frac{1}{6}$ and that of 2's is at most $\frac{2.5}{6}$, the joint plan bargaining game as of link 12 discussions is essential (as $\frac{1}{6}+\frac{2.5}{6} < \frac{5}{6}$, the sum of players 1 and 2's Myerson values) whenever player 3 is offered less than half. Analogously as in the case of type 2 joint plan in I, it can be shown that under any fully successful joint plan by players 1 and 3 with future-requests consistent with the previous analysis, link 12 would form right after link 13 forms and then the third link 23 would be rejected.

III. Now suppose player 3 is offered more than half.

If link 12 is rejected then in any O-F Joint Plan for players 3 and 2, they suggest unilateral rejections. Note that as link 23 does not form, player 2 gets zero in g^{13}, and player 3 would get more than $\frac{3}{6}$.

One stage backwards as of link 12 discussions, as the outside option pair for players 1 and 2 is $(\psi_1^4, 0)$, where $\psi_1^4 < \frac{3}{6}$, as in II, a fully successful joint plan for players 1 and 3 consistent with the previous analysis would have to future-request on players 1 and 2 an O-F Joint Plan that suggests link formation and a proposal match. Again, link 12 would be the last link to form.

Fully successful joint plans in cases II, III and I, where in the latter case one does not include the fully successful joint plan for players 1 and 3 that future-requests unilateral rejections of links 12 and 32−in that order−after link 13 forms (type 1.1 plan), have expected payoffs for players 1 and 3 that would give at least one player (either 1 or 3) less than a half and the other one at most $\frac{3}{6}$.

Part 2 . Because the outside options are zero as of link 13 discussions, from Part 1, out of any fully successful tenable and reliable joint plan, type plan 1.1 is the only one that yields strong Pareto efficient payoffs, $(\frac{3}{6}, \frac{3}{6})$, if obeyed. Thus, it is the unique O-F Joint Plan as of link 13 preliminary negotiations. Note that as of link 13 preliminary negotiations no link has formed−as past preliminary negotiations have been unsuccessful−so the unique tenable and reliable joint plan by not linked pairs, the disagreement joint plan is basically ignored or trivially followed.

Part 3. One stage backwards, fully successful joint plans for players 2 and 3 are analogous to the one in the bargaining problem for players 1 and 3. In contrast, outside options are zero for player 2 and a half for player 3. As players 2 and 3 have no preceding oldest successful negotiators, the unique O-F Joint Plan suggests link formation and a half-half proposal match and future-requests consecutive rejection of the next two links in the order (it is a plan analogous to type 1.1 plan). At the beginning of the game, a similar argument can be applied as of link 12 discussions and the claim follows ■

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