

Negative value of information in an informed principal problem with independent private values.

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Abstract

This note demonstrates that in a principal-agent environment with independent private values and generic payoffs, the mechanism implemented by an informed principal is *not* ex-ante optimal. This result implies that in (generic) settings where the principal can covertly acquire private information before selecting a mechanism, she will fail to select an ex-ante optimal mechanism. Furthermore, the principal is indifferent whether to become informed before or after selecting a mechanism.

Keywords: ex-ante optimal allocation, informed principal, value of information, information acquisition/gathering.

1 Introduction

This note studies the principal-agent environment with independent private values, in which the principal may have private information at the moment of mechanism selection. It consists of two parts. In the first part, I demonstrate that for generic payoff functions, reservation utilities and prior beliefs, the equilibrium allocation implemented by an informed principal is not ex-ante optimal.¹ Because an uninformed principal can implement an ex-ante optimal allocation as an equilibrium in a corresponding (optimal) direct mechanism, the arrival of the principal's private information before mechanism design has negative impact on her ex-ante payoff.

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¹An ex-ante optimal allocation maximizes the ex-ante payoff of the principal among all interim incentive-compatible and individually rational allocations and can be implemented by an uninformed principal as an equilibrium in an optimal mechanism. A property is generic if it holds on a dense open set of the payoff functions satisfying the assumptions of the model. This genericity concept is the same as in Mas-Colell [11] and Maskin and Tirole [14]. The quasilinear preferences are non-generic in our model and our results will not hold for these preferences.

The tension between what the informed principal can implement and what the principal would like to implement ex-ante has implications for her behavior: at the ex-ante stage the principal may be willing to delay gathering her private information, restrict the set of available mechanisms, or even give up the principal's rights.² The second part of this note analyzes one such implication. I consider a setting in which a principal covertly chooses between acquiring private information either before selecting a mechanism or after selecting a mechanism but before executing it. I demonstrate (1) that in equilibrium the principal is indifferent about when to become informed and (2) that the equilibrium allocation is equivalent to the allocation implemented by the informed principal and thus generically is not ex-ante optimal. In particular, there is a surprising equilibrium in which the principal stays uninformed until after selecting a mechanism and yet selects a mechanism which would be implemented if she were informed. Importantly, these results imply that the principal will not select an ex-ante optimal mechanism in (generic) settings where she can covertly acquire private information before mechanism design.

In our model, the principal always observes her private information by the time of execution of a mechanism. Therefore, early realization of information may only decrease the principal's ex-ante payoff. One channel through which this can happen is the information leakage through mechanism design: if the agent infers something about the principal's private information from the choice of mechanism, his incentives constraints would be tightened. However, the Inscrutability Principle by Myerson [16] states that the mechanism selected by an informed principal need not signal any information: any allocation which can be achieved by an informed principal can also be achieved without revealing any private information until after the mechanism is played. Moreover, Maskin and Tirole [14] (henceforth, MT) show that generically in equilibrium no principal's private information is revealed to the agent until after mechanism design.

Yet, neither the Inscrutability Principle nor the results in MT imply that an informed principal can achieve an arbitrary allocation and, in particular, an ex-ante optimal allocation. What the principal can achieve in equilibrium depends on the deviations available to her different types. In an ex-ante optimal allocation the principal gets insured by increasing payoffs of some types at the expense of other types. This is done to such an extent that these latter types can always construct profitable deviations guaranteed to be accepted by the agent. Thus, the principal is unable to implement an ex-ante optimal allocation not because of information leakage to the agent but rather because of the conflict among her own types.

The crucial step in proving this result is to establish a restriction on an ex-ante optimal allocation that must be satisfied if this allocation is implemented by an informed principal. Following MT, I analyze the problem in terms of *indirect* payoff functions of the principal's types defined on the space of slacks in incentive compatibility and individual rationality constraints of the agent. In an ex-ante optimal allocation, the marginal values of these payoff functions are equal across the types of

²Mylovanov [17] presents an example in which the mechanism selected by the informed principal delivers a lower ex-ante payoff than the mechanism selected by her opponent.

the principal. The allocation selected by the informed principal violates this condition for generic *indirect* payoff functions that are *differentiably strictly concave*. This note complements the work of MT by proving the genericity result explicitly through invoking the Transversality and the Implicit Function Theorems.

Under additional assumptions on the primitives of the model, which guarantee that the allocation selected by the informed principal is deterministic, this result implies the equivalent result in the space of *direct* payoff functions defined over allocations. Because of technical difficulties I have been unable to establish this equivalence for stochastic allocations. However, the result for deterministic allocations is still a valuable observation since due to similar technical reasons the literature almost exclusively restricts attention to deterministic allocations.

Our results do not imply that an informed principal implements an allocation that is worse than an ex-ante optimal allocation for *all* types of the principal. Both allocations are interim incentive-efficient in the sense of Holmström and Myerson [9]: there are no other incentive-compatible and individually rational allocations that generate higher payoffs for all types of the principal. However, (generically) the allocation selected by the informed principal does not generate the highest feasible ex-ante payoff and hence allowing the principal to observe her private information before selecting a mechanism has negative value from the ex-ante perspective. At the same time, as MT has demonstrated (generically) the allocation selected by the informed principal achieves for each type of the principal a payoff strictly higher than would be possible if the principal's information were common knowledge. This reveals that allowing the agent to observe the principal's private information before selecting a mechanism has negative value from both the ex-ante and the interim perspectives.

The consequence of our genericity result is that in (generic) environments where the principal can covertly acquire information before mechanism design she will be unable to implement an ex-ante optimal allocation. In equilibrium, the principal can always deviate and learn her information early. Because the agent anticipates this the only equilibrium outcome is an allocation which would be implemented by the informed principal, regardless of whether the principal actually becomes informed. In fact, in equilibrium the principal is indifferent about when to acquire information.

The question of information acquisition in mechanism design has been recently studied by Bergemann and Välimäki [3]. Their focus is on the existence of mechanisms that simultaneously create efficient incentives for the information acquisition by agents and achieve an ex-post efficient allocation. In contrast, in our model the issue is the timing of the information acquisition by the principal; information acquisition is costless and the principal always observes her information before execution of the mechanism. Our model is also related to Cremer and Khalil [5], who consider a setting in which the agent has a choice about whether to become informed before or after signing a contract.

The rest of the note is organized as follows. Section 2 presents the environment. Section 3 shows the genericity result. Section 4 considers a setting in which the principal has a choice over when to become informed.

2 Model

Environment. Our environment is identical to the one in MT. There is a principal (she) and an agent (he), who can contract on an observable and verifiable allocation $(y, t) \in \Omega$, where $\Omega = \Omega_y \times \Omega_t$ is a convex and compact subset of $\mathbb{R} \times \mathbb{R}$.³ (As an example, one might think of the principal and the agent as a buyer and a seller, in which case y is the amount of good exchanged and t is the price paid.)

The agent has a twice continuously differentiable von Neumann - Morgenstern payoff function $U_j(y, t)$, where $j \in \{1, 2\}$ is the agent's type.⁴ The payoff function $U_{(\cdot)}(\cdot)$ is decreasing in y , increasing in t , and strictly concave in (y, t) .⁵ It decreases in j ,

$$U_1(y, t) > U_2(y, t) \text{ for all } (y, t) \in \Omega,$$

and also satisfies the single-crossing condition,

$$-\frac{\partial U_1/\partial y}{\partial U_1/\partial t} < -\frac{\partial U_2/\partial y}{\partial U_2/\partial t} \text{ for all } (y, t) \in \Omega.$$

The principal has a twice continuously differentiable von Neumann - Morgenstern payoff function $V_i(y, t)$, where $i \in \{1, \dots, N\}$, $N \geq 2$, is the principal's type. The payoff function $V_{(\cdot)}(\cdot)$ is increasing in y , decreasing in t , and strictly concave in (y, t) .

The agent has interim individual rationality constraint: in any contract *each* type of the agent has to obtain at least his reservation utility \bar{u} . There are no individual rationality constraints for the principal.

Let \underline{t} be the minimal transfer in Ω_t . It is assumed that payoff functions are such that (1) given this transfer the agent is necessarily worse off than without a contract, regardless of his type or the action y :

$$U_j(y, \underline{t}) < \bar{u} \text{ for all } (y, \underline{t}) \in \Omega \text{ and } j \in \{1, 2\}.$$

and (2) regardless of the types, there exists an allocation such that both parties prefer it to no contract.

The types of the parties i and j are their private information, independently realized with strictly positive probabilities ϕ_i and p_j .⁶ The structure of preferences and rules of the games are common knowledge.

Allocations and mechanisms. Let \mathcal{P} be the set of probability measures over the set of allocations Ω . An allocation rule is a function which maps the types of the parties into a distribution over allocations $\nu : \{1, \dots, N\} \times \{1, 2\} \rightarrow \mathcal{P}$. Let Ψ be the

³The convexity and compactness of Ω are required for existence of equilibrium in the informed principal game.

⁴MT require the payoff functions to be continuously differentiable. However, twice continuous differentiability is needed for the genericity result.

⁵MT require strict concavity of the parties' payoffs only in y . The genericity results in MT and this note will not hold if payoffs are linear in t . See Proposition 11 in MT, Yilankaya [23] and Mylovanov [18] for the corresponding results.

⁶MT allow for degenerate probability distributions.

set of allocation rules. For the principal denote the expected payoff of type i , who obtains the allocation of type i' , by

$$V_i(i', \nu) = \sum_j p_j \int_{\Omega} V_i(y, t) d\nu(i', j).$$

Define similarly for the agent

$$U_j(j', \nu) = \sum_i \phi_i \int_{\Omega} U_j(y, t) d\nu(i, j').$$

An allocation rule ν is *incentive-compatible* if for $i, i' \in \{1, \dots, N\}$

$$V_i(i, \nu) \geq V_i(i', \nu) \quad (PIC)$$

and for $j, j' \in \{1, 2\}$

$$U_j(j, \nu) \geq U_j(j', \nu). \quad (AIC)$$

An allocation rule is *individually rational* if for $j \in \{1, 2\}$

$$U_j(j, \nu) \geq \bar{u}. \quad (AIR)$$

A *mechanism* is a game form whose outcome is a probability distribution over allocations.⁷ The solution concept for a mechanism is Perfect Bayesian equilibrium. Following MT, the set of allowed mechanisms is determined by two properties: for any mechanism (a) there exists equilibrium regardless of the agent's beliefs about the principal, and (b) the equilibrium payoff correspondence is upper-hemicontinuous and convex-valued in beliefs.⁸

Uninformed and informed principal games. Figure 1 depicts the timing of the game in which the principal selects a mechanism before learning her type. The highest expected payoff for the principal in a Perfect Bayesian equilibrium of this game is obtained by an *ex-ante optimal allocation*, which is a solution of the following program:

$$\max_{\nu \in \Psi} \sum_i \phi_i V_i(i, \nu) \quad (EA)$$

⁷In our model, the continuation equilibrium in a mechanism offered off the equilibrium path depends on the out-of-equilibrium beliefs of the agent. The Revelation Principle (e.g., Myerson [15] or Dasgupta et al. [6]) does not apply off the equilibrium path and hence we cannot restrict attention to direct mechanisms. However, the Revelation Principle applies on the equilibrium path: any equilibrium outcome can be described by an incentive-compatible and individually rational allocation rule, which is a truth-telling equilibrium outcome of some direct mechanism.

⁸These properties are needed for existence of equilibrium; standard existence results do not apply because the action space includes a choice of a mechanism. Any set of one-stage mechanisms with finite strategy spaces satisfies these properties, given the parties can use a public correlation device (needed for convexity of equilibrium correspondence). The requirement (b) could be somewhat weakened. See p. 398 in MT for details.

subject to (PIC), (AIC), and (AIR). An ex-ante optimal allocation exists, since the set of allocations rules satisfying the constraints is compact in the weak topology and the objective function is continuous.

Figure 2 depicts the timing of the informed principal game, in which the principal selects a mechanism after she learns her type. The equilibrium allocation in the informed principal problem is characterized in MT. Following MT define an indirect payoff function $V_i(c_i, r_i)$ to be the value function of the program

$$\max_{\nu \in \Psi} V_i(i, \nu) \quad (IU_i)$$

subject to

$$\int_{\Omega} U_1(y, t) d\nu(i, 1) - \int_{\Omega} U_1(y, t) d\nu(i, 2) = -c_i,$$

$$\int_{\Omega} U_2(y, t) d\nu(i, 2) - \bar{u} = -r_i.$$



Figure 1: Uninformed Principal: Ex-ante selection of a mechanism.

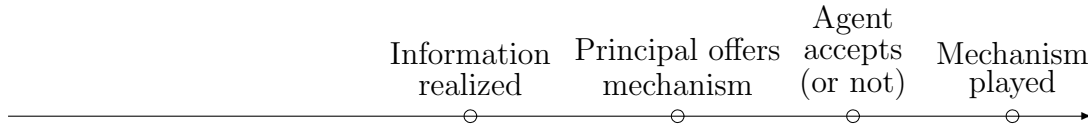


Figure 2: Informed Principal: Interim selection of a mechanism.

Propositions 2 – 4, 6, and 7 in MT prove existence of Perfect Bayesian equilibrium and show that an allocation ν is an equilibrium outcome if and only if there exist strictly positive real numbers μ_r and μ_c such that

1. for $i = 1, \dots, N$ the allocation $\nu(i, \cdot)$ solves program IU_i given the slacks r_i and c_i that are a solution of the program

$$\max_{(r_i, c_i) \in Z_i} V_i(c_i, r_i) \quad (WE_i)$$

$$\text{s.t. } \mu_r r_i + \mu_c c_i = 0, \quad (BC_i)$$

where Z_i is the set of (r_i, c_i) for which there exists a solution of program IU_i for i with $\nu(i, \cdot)$ that has support on Ω (see Maskin and Tirole [12], the proof of Proposition 2); and

2. ν satisfies *AIR* for $j = 2$ and *AIC* for $j = 1$,

$$\sum_i \phi_i r_i = 0, \quad (IIR)$$

$$\sum_i \phi_i c_i = 0. \quad (IIC)$$

MT call these allocations *strong unconstrained Pareto optimum* (henceforth, SUPO).⁹

Interior Solution. I will further assume that the set of payoff functions and the allocations Ω is such that the first order conditions of program WE_i are satisfied with equality, i.e., its solution is interior. Without this restriction the results in this note would not be valid for two reasons. First, Proposition 4 in MT, which is further used for the equilibrium characterization in Proposition 6 in MT, makes use of first order conditions that hold with equality. Second, the first order conditions which hold with equality are a crucial building block in the proof of Lemma 3 in this note.

Properties of indirect payoff function. I also make

Assumption 1. The payoff functions $U_i(y, t)$ and $V_i(y, t)$ are such that the indirect payoff functions $V_i(r_i, c_i)$ are strictly concave and twice continuously differentiable.

These properties are required to prove the regularity of SUPO, i.e., that the Jacobian of the system of first-order optimality conditions describing SUPO allocations has non-zero determinant. Unfortunately, the general conditions under which these properties hold have not been established in the literature and I have also been unable to do so.

On the other hand, these properties are straightforward to obtain if the solution of program IU_i is a deterministic allocation. In the setting with two types of the principal, Quesada [20] imposes the following additional assumptions on preferences:

Assumption 2. 1. The principal has two types.¹⁰

2. $V_i(y, t) = V(S_i(y) - t)$ and $U_j(y, t) = U(t - \psi_j(y))$, where V and U are increasing and strictly concave, S_i is increasing and concave and $S_2(y) > S_1(y)$, and ψ_j is increasing and convex and $\psi_2(y) > \psi_1(y)$;

3. The function $\Phi(x, y) = U(U^{-1}(x) + \psi_2(y) - \psi_1(y))$ is strictly convex.

⁹More precisely, MT (1) define Walrasian equilibrium to be the triple (μ_r, μ_c, ν) that solves programs WE_i and satisfies *IIC*, *IIR* and (2) establish equivalence between SUPO and ν of the Walrasian equilibrium for strictly positive beliefs. As MT show SUPO exists and generically there are at most finitely many of them. Under additional assumptions on preferences, Quesada [20] establishes some further properties of SUPO. However, in environments other than with independent private values SUPO may not exist. See Maskin and Tirole [13] and Tisljar [22] for analysis of common values environments and Cella [4] and Kosenok and Severinov [10] for analysis of environments with correlated types.

¹⁰Although MT allow for $n \geq 2$ types of the principal, the proof of the uniqueness of the equilibrium (Proposition 7) in MT is carried out for the case of $n = 2$. MT state that their results are straightforward to generalize for the environments with more than two types.

Proposition 1 in Quesada states that under Assumption 2 any SUPO allocation is deterministic or, more precisely, that any SUPO allocation has support consisting of one allocation (y, t) for each combination of types of the principal and the agent.¹¹ However, the proof of Proposition 1 in Quesada does much more: it shows that any solution of program IU_i has support consisting of one allocation (y, t) for each type of the agent. This allows us to obtain

Lemma 1. *If Assumption 2 holds, then the solution of program IU_i is unique and the indirect payoff function $V_i(r_i, c_i)$ is strictly concave and twice continuously differentiable.*

Proof. “Uniqueness of solution”. Imagine there are two distinct allocations ν_1 and ν_2 that solve program IU_i . Then, the stochastic allocation ν_S which mixes with equal probabilities between ν_1 and ν_2 is also a solution of program IU_i : it satisfies constraints IIR , IIC and achieves the same value of the objective function as ν_1 and ν_2 . However, this allocation has support consisting of two points (y, t) for each agent’s type and therefore, as shown in the proof of Proposition 1 in Quesada, cannot be a solution of program IU_i .

“Properties of $V_i(r_i, c_i)$ ”. Strict concavity of $V_i(r_i, c_i)$ follows directly from the uniqueness of the solution of program IU_i over the set of deterministic allocations. An application of the Implicit Function Theorem to the first-order conditions describing the solution of program IU_i over the set of deterministic allocations gives continuous differentiability of the Lagrange multipliers and thus twice continuous differentiability of the value function. \square

3 Negative Value of Information

An SUPO allocation is computed subject only to IIR and IIC and ignoring other incentive compatibility and individual rationality constraints. To facilitate the comparison between the ex-ante optimal allocation and SUPO, define an *unconstrained ex-ante optimum* (UEAO) to be the allocation which maximizes the ex-ante expected payoff of the principal $\sum \phi_i V_i(c_i, r_i)$ subject to constraints IIR and IIC . The following lemma establishes restrictions on allocations which are simultaneously SUPO and UEAO. It states (equation (6)) that marginal values of indirect utilities should be equal across the types of the principal.

Lemma 2. *If SUPO is UEAO, it solves*

¹¹Quesada shows existence of payoff functions satisfying her assumptions. It can be further shown that the set of payoff functions satisfying Assumption 1 is open in the set of payoff functions satisfying assumptions of our model. The topology for the space of payoff functions on compact Ω is given by C^2 uniform convergence: a sequence of functions f_n converges to a payoff function f , $f_n \rightarrow f$, if and only if every derivative of $f_n \rightarrow f$ from 0-th to 2-nd order uniformly converges to zero.

$$\frac{\partial V_i(r_i, c_i)}{\partial r_i} - \gamma_i \mu_r = 0 \quad (1)$$

$$\frac{\partial V_i(r_i, c_i)}{\partial c_i} - \gamma_i \mu_c = 0 \quad (2)$$

$$\mu_r r_i + \mu_c c_i = 0 \quad (3)$$

$$\sum_i \phi_i r_i = 0 \quad (4)$$

$$\mu_r = 1 \quad (5)$$

$$\gamma_1 - \gamma_2 = 0 \quad (6)$$

where $\gamma_i, \mu_c > 0$.

Proof. Equations (1)–(5) are satisfied by any SUPO. Let γ_i be the Lagrange multiplier corresponding to constraint BC_i in program WE_i . Equations (1) and (2) are the first order conditions with respect to r_i and c_i , and (3) is constraint (BC_i) of programs WE_i . Equation (4) is constraint (IIR). Equation (5) is normalization. The last equation holds if SUPO is UEAO. The first order conditions of the program determining UEAO with respect to r_i and c_i are

$$\begin{aligned} \frac{\partial V_i}{\partial r_i}(r_i, c_i) - \lambda_r &= 0 \quad (\text{for } i = 1, 2), \\ \frac{\partial V_i}{\partial c_i}(r_i, c_i) - \lambda_c &= 0 \quad (\text{for } i = 1, 2). \end{aligned}$$

Together with (1) and (2) it implies $\mu_r = \lambda_r/\gamma_i$, $\mu_c = \lambda_c/\gamma_i$ and $\gamma_1 = \gamma_2$. \square

The next Lemma indicates that the conditions under which SUPO is UEAO are limited. We say that a property is generic if it holds on a dense open set of the payoff functions satisfying the assumptions of the model.¹²

Lemma 3. *SUPO is not UEAO generically on the set of the principal's payoff functions and the agent's prior beliefs about the principal.*

Proof. We will prove that the set the principal's payoff functions and the agent's prior beliefs about the principal for which SUPO is not UEAO is dense in the set of principal's payoffs and the agent's prior beliefs satisfying the assumptions of the model by demonstrating that

(*) if for some principal's payoff functions, $V_i(y, t)$, and the agent's prior beliefs about the principal, ϕ_i , SUPO is UEAO, then in any open set containing these payoffs and beliefs there exist $V_i(y, t)$ and ϕ_i for which SUPO and UEAO differ.

¹²I do not use a more recent concept of genericity in Anderson and Zame [2] because my results rely on some genericity results proven in MT using the older concept. See footnote 11 for topology on the space of payoff functions.

Proposition 1 in MT implies that $r_i \neq 0$ at SUPO holds generically on the set of payoff functions. Hence, by the Baire property we only need to prove (*) for $V_i(y, t)$ and ϕ_i for which $r_i \neq 0$.

Let SUPO be UEAO for some such $V_i(y, t)$ and ϕ_i . Claim (*) holds for these payoffs and beliefs if there is no open set containing ϕ_i for which SUPO is UEAO.

Assume otherwise and denote by B_ϕ an (arbitrary) open set of ϕ_i for which SUPO is UEAO for the payoff functions $V_i(y, t)$. Now let us consider an open set B_V of principal's payoff functions containing $V_i(y, t)$ such that for all payoff functions in B_V and all beliefs in B_ϕ , $r_i \neq 0$ at SUPO. Then, there exists $\varepsilon > 0$ close to zero such that for all $\epsilon \in (-\varepsilon, \varepsilon)$ the payoff functions,

$$\tilde{V}_i(y, t) = (1 + a_i\epsilon)V_i(y, t), \quad \tilde{a}_i = \begin{cases} 1, & i = 1; \\ -1, & i = 2; \\ 0, & \text{otherwise.} \end{cases}$$

belong to B_V .

Because $\tilde{V}_i(y, t)$ are monotone transformations of $V_i(y, t)$, the solution of program IU_i is the same for both functions. Hence,

$$\tilde{V}_i(r_i, c_i) = (1 + a_i\epsilon)V_i(r_i, c_i).$$

Since $r_i \neq 0$, there exists $r_{i'}, i' \neq i$, such that $r_1 - r_{i'} \neq 0$. Because SUPO is UEAO, Lemma 2 implies that the following equations have to be satisfied,

$$(1 + a_i\epsilon)\frac{\partial V_i(r_i, c_i)}{\partial r_i} - \gamma = 0 \quad (R_i)$$

$$(1 + a_i\epsilon)\frac{\partial V_i(r_i, c_i)}{\partial c_i} - \gamma\mu_c = 0 \quad (C_i)$$

$$r_i + \mu_c c_i = 0, \quad (BE_i)$$

$$\sum_{i \neq i'} \phi_i(r_i - r_{i'}) = 0. \quad (IIR')$$

where IIR' is obtained from (4) using $\phi_{i'} = 1 - \sum_{i \neq i'} \phi_i$. The Jacobian of this system with respect to $(r_1, c_1, \dots, r_N, c_N, \gamma, \epsilon, \phi_1, \mu_c)$ evaluated at $\epsilon = 1$ is

$$J = \begin{bmatrix} \frac{\partial^2 V_1}{\partial r_1^2} & \frac{\partial^2 V_1}{\partial r_1 \partial c_1} & 0 & 0 & \dots & -1 & 1 \cdot \gamma & 0 & 0 \\ \frac{\partial^2 V_1}{\partial r_1 \partial c_1} & \frac{\partial^2 V_1}{\partial c_1^2} & 0 & 0 & \dots & -\mu_c & \mu_c \cdot \gamma & 0 & -\gamma \\ 0 & 0 & \frac{\partial^2 V_2}{\partial r_2^2} & \frac{\partial^2 V_2}{\partial r_2 \partial c_2} & \dots & -1 & -1 \cdot \gamma & 0 & 0 \\ 0 & 0 & \frac{\partial^2 V_2}{\partial r_2 \partial c_2} & \frac{\partial^2 V_2}{\partial c_2^2} & \dots & -\mu_c & -\mu_c \cdot \gamma & 0 & -\gamma \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & \mu_c & 0 & 0 & \dots & 0 & 0 & 0 & c_1 \\ 0 & 0 & 1 & \mu_c & \dots & 0 & 0 & 0 & c_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \phi_1 & 0 & \phi_2 & 0 & \dots & 0 & 0 & r_1 - r_{i'} & 0 \end{bmatrix}$$

where the order of equations is $(R_1), (C_1), \dots, (R_N), (C_N), (BE_1), \dots, (BE_N), (IIR')$ and it is assumed that $i' \neq 2$. Its dimensionality is $2(N+1)+1$ by $2(N+2)$.

Observe that the second last column of Jacobian contain one non-zero element and $2(N+1)$ zeros. This implies that $\text{rank}(J) = 1 + \text{rank}(J_1)$, where is the submatrix of J obtained from J by removing the $2(N+1)+1$ -th row and the $2(N+1)+1$ -th column. Now, form a matrix by taking first $2(N+1)$ rows with first $2(N+1)$ elements of J . Clearly, $\text{rank}(J_1) \geq \text{rank}(J_2)$. The matrix J_2 is square and has a determinant, which is equal to

$$2\gamma \cdot \prod_{i=1}^2 \left(\frac{\partial^2 V_i}{\partial r_i^2} \mu_c^2 - 2 \frac{\partial^2 V_i}{\partial r_i \partial c_i} \mu_c + \frac{\partial^2 V_i}{\partial c_i^2} \right) \prod_{i=3}^N \left(\frac{\partial^2 V_i}{\partial r_i^2} \frac{\partial^2 V_i}{\partial c_i^2} - \left(\frac{\partial^2 V_i}{\partial r_i \partial c_i} \right)^2 \right). \quad (7)$$

The last product is strictly positive because its each term is a Hessian of V_i . The first product is non-zero because its each term is a quadratic polynomial in μ_c , whose discriminant,

$$D = 4 \left[\left(\frac{\partial^2 V_i}{\partial r_i \partial c_i} \right)^2 - \frac{\partial^2 V_i}{\partial r_i^2} \frac{\partial^2 V_i}{\partial c_i^2} \right],$$

is proportional to the negative of the determinant of the Hessian of $V_i(r_i, c_i)$ and is less than zero. Hence, there is no real μ_c setting any of the terms to zero. This gives $\text{rank}(J_2) = 2(N+1)$ and $\text{rank}(J) = 2(N+1)+1$.

Because J has full rank, the Transversality Theorem implies that the system $(R_1), (C_1), \dots, (R_N), (C_N), (BE_1), (BE_2), (IIR')$ in variables $(r_1, c_1, \dots, r_N, c_N, \gamma, \epsilon, \mu_c)$ has a regular solution on some open set $B'_\phi \subseteq B_\phi$.¹³ By the Implicit Function Theorem $(r_1, c_1, \dots, r_N, c_N, \gamma, \epsilon, \mu_c)$ can be locally solved as a function of ϕ_1 for $\phi_i \in B'_\phi$.¹⁴ Therefore, for $\phi_i \in B'_\phi$ system $(R_1), (C_1), \dots, (R_N), (C_N), (BE_1), (BE_2), (IIR')$ has a solution only on a set of ϵ of Lebesgue measure zero. This implies that for any $\phi_i \in B'_\phi$ we can always find an ϵ arbitrarily close to 1 such that the system $(R_1), (C_1), \dots, (R_N), (C_N), (BE_1), (BE_2), (IIR')$ does not have a solution and hence SUPO is not UEAO. This establishes claim (*).

The fact that the set of $V_i(y, t)$ and ϕ_i on which SUPO is not UEAO is open follows from the upperhemicontinuity of the solutions of programs IU_i and the program corresponding to UEAO in the payoff functions and prior beliefs. \square

I now present the main result that generically the informed principal will fail to select an ex-ante optimal allocation.

Proposition 1. *Strong unconstrained Pareto optimum is not ex-ante optimal allocation generically on the set on the set of the principal's payoff functions and the agent's prior beliefs about the principal.*

Proof. Lemma 1 in MT, Proposition 1 in MT, and the proof of Corollary to Proposition 4 in MT imply that the subset of the set of payoff functions and beliefs satisfying

¹³See, for instance, Abraham and Robin [1], or Mas-Colell [11], Theorem I.2.2, p. 45, and, in particular, Proposition 8.3.1, p. 320.

¹⁴See Hirsch, [7] or Mas-Colell [11], Theorem H.2.2, p. 38.

the assumptions of our model for which in SUPO the only binding constraints are IIR (AIR for $j = 2$) and IIC (AIC for $j = 1$) is dense and open. By Lemma 3 the subset on which SUPO is not UEAO is dense and open. By the Baire property, the intersection of these two subsets is also dense and open. \square

Hirshleifer [8] was first to point out that premature acquisition or release of information may destroy beneficial trading opportunities. Proposition 1 in MT shows that generically SUPO Pareto dominates the allocation implemented under common knowledge of the principal's type, i.e., the principal would be strictly hurt if her information were released to the agent. The result that generically SUPO is not ex-ante optimal allocation complements these findings by showing that the principal's ex-ante payoff is decreased even if the information is released only to the principal and not to the agent.

The results obtained in this section will not hold if the players have quasilinear preferences. Proposition 11 in MT and Proposition 1, and Corollary 2 in Mylovanov [18] imply that in this environment the sets of SUPO, UEAO, and ex-ante optimal allocations coincide.

4 Information Acquisition

The results in the previous section could be used to study the incentives of the principal to acquire information before selecting a mechanism. Consider a mechanism selection game with covert information acquisition, in which the principal can privately decide whether to acquire private information before selecting a mechanism or after selecting but before executing a mechanism (Figure 3). As Neyman [19] points out, in equilibrium a player cannot be made strictly worse off by a unilateral improvement in her information, which implies that the principal will always (weakly) prefer to learn her type early. Moreover, as MT show in the proof of Proposition 7, the principal who knows her type can always implement SUPO. We have,

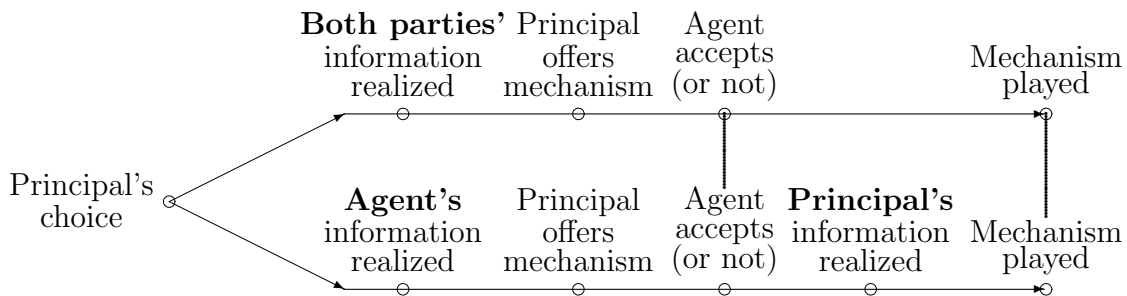


Figure 3: Principal secretly chooses between acquiring information before selecting a mechanism or after selecting but before executing a mechanism. Vertical lines indicate that the agent does not observe when the principal acquires information.

Proposition 2. *In the mechanism selection game with covert information acquisition, an allocation rule ν is an outcome of a Perfect Bayesian equilibrium if and only if it is SUPO.*

Proof. Necessity. If in equilibrium, the principal gets informed before selecting a mechanism with probability one, then Proposition 7 in MT implies that she will implement SUPO. Suppose now that in equilibrium with probability $p < 1$ the principal stays uninformed at the moment of mechanism selection. The proof of Proposition 7 in MT shows that the principal can always deviate, get informed with probability 1 and, regardless of the agents' beliefs, implement an allocation that yields payoffs that are arbitrary close to those of SUPO. Hence, her equilibrium payoff should weakly dominate her payoff from SUPO. By definition of SUPO, there is no allocation that strictly dominates SUPO and hence SUPO is the equilibrium allocation.

Sufficiency. Fix any $p \in [0, 1]$. Now, consider an equilibrium candidate in which with probability p the principal stays uninformed and with probability $1 - p$ she gets informed. In both cases she offers a mechanism whose equilibrium outcome given prior beliefs is the same SUPO. The proof of Proposition 6 in MT establishes that for any SUPO there exist beliefs for any alternative (out-of-equilibrium) offer of a mechanism such that in the continuation equilibrium the principal's payoff is weakly lower than in this SUPO for all types and hence the principal does not want to deviate and offer a different mechanism. \square

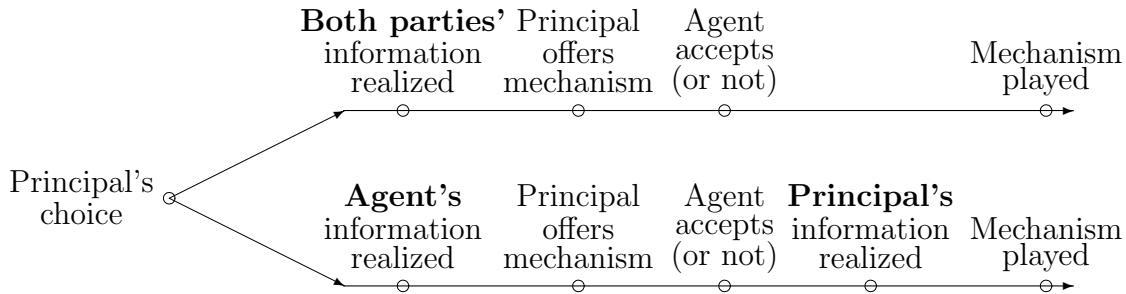


Figure 4: Principal publicly chooses between acquiring information before selecting a mechanism or after selecting but before executing a mechanism.

This result together with Proposition 1 implies that generically (on the set of the principal's payoff functions and the agent's prior beliefs about the principal) the principal who can covertly acquire information before designing a mechanism will fail to implement an ex-ante optimal allocation.

Corollary 1. *In the mechanism selection game with covert information acquisition the equilibrium allocation is not ex-ante optimal generically.*

As can be seen from the sufficiency part of the proof of Proposition 2, there are multiple equilibria that differ in the probability with which the principal gets informed before selecting a mechanism. It follows that generically there is an equilibrium in which the *uninformed* principal fails to implement an *ex-ante optimal* allocation.

Corollary 2. *In the mechanism selection game with covert information acquisition there generically exists an equilibrium in which the principal is uninformed at the moment of mechanism design and the equilibrium allocation is not ex-ante optimal.*

It is interesting to contrast these results with the principal’s behavior in the environments with other information acquisition structures. In the setting in which the principal’s decision about when to become informed is observed by the agent and is common knowledge (Figure 4), the principal cannot increase her ex-ante payoff by acquiring private information early and (generically) the only equilibrium for the principal is to stay uninformed and implement an ex-ante optimal allocation.¹⁵ The observation that common knowledge of information acquisition may benefit the principal by substituting for the principal’s inability to commit not to acquire information early is similar to the result in Shavell [21] that mandatory disclosure is often socially desirable. In Shavell, however, mandatory disclosure limits the cost of information acquisition exorted by the seller and ensures that socially desirable information is revealed (Shavell, [21], p. 28), whereas in our model it prevents the principal from acquiring information early and does not allow the conflict among different types of the principal to decrease the principal’s ex-ante payoff.

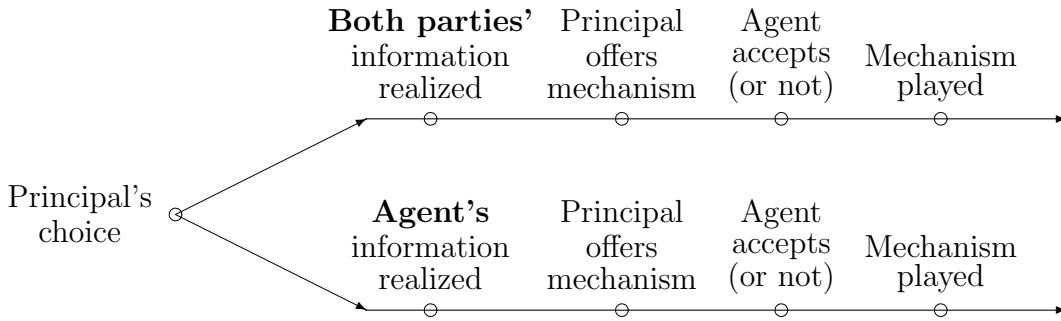


Figure 5: Principal has a choice between acquiring information before selecting a mechanism or never acquiring it.

On the other hand, in the environments in which the principal faces a choice between acquiring information before selecting a mechanism or never acquiring it (Figure 5), the principal will never benefit from postponing information acquisition, regardless of whether the agent observes her decision. The uninformed principal has to restrict herself to allocations which do not condition on her type. These allocations are Pareto dominated by the allocation that would be implemented if the principal’s type were common knowledge, which in turn are generically strictly Pareto dominated by the allocation implemented by the informed principal.

¹⁵In the (non-generic) settings where SUPO is an ex-ante optimal allocation, there also exist equilibria in which the principal becomes informed.

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