A common agency model with pre-contractual information gathering*

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Abstract

Most advances in the common agency literature assume that the agent always decides to acquire her private information before the contract is offered by the principals. We find this assumption unrealistic, since acquiring this information long time before the contract is even offered is normally very costly for the agent.

We propose a common agency model where the agent can strategically decide *whether or not* to gather information before receiving the contract offers from two principals. Our results show some differences with the common agency model resulting from allowing the agent to gather such information, and relevant differences with respect to the information gathering models, due to the introduction of a second principal.

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1 Introduction

Many economic problems can be modelled as a game with a unique (common) agent and various principals who want to influence the agent's decisions. For example, when different retailers buy their products from a common wholesaler, each retailer offers a payment scheme that tries to induce the wholesaler to produce a combination of output and prices that maximizes the retailer's profits. Similarly, when various manufacturers use the same marketing company to sell their products in the market, they try to control the marketing company's actions. In the same way, this structure is suitable for modelling the situation of different lobbyists who want to control a common politician by persuading (or even bribing) him. Finally, in the theory of optimal taxation, the Federal and State revenue departments may want to maximize the taxes collected from their common agent –the taxpayer– but minimize the distortion that these taxes introduce.

All of these models, which involve different principals trying to control a common agent, are denoted by the literature as *common agency* games. This class of games has received much attention since the seminal paper of Bernheim and Winston (1986), but especially since Stole (1991) and Martimort (1992) introduced the possibility of a privately-informed agent.

All of this literature assumes that the agent privately observes some piece of information which cannot be observed by the principals. For instance, the wholesaler privately learns his cost structure, but neither retailers can observe it. It is assumed, however, that the wholesaler *always* acquires this pre-contractual information about her cost structure, even if it is costly for her to do so. Instead, a more realistic approach would allow the wholesaler to decide whether to acquire such information only when this strategy is optimal for her.

Therefore, we will assume that information acquisition before receiving the contract offers cannot be taken for granted, because it may be costly for the agent to gather. On the contrary, we will allow the agent to optimally decide when it is worthwhile to gather such information. Hence, this paper belongs to a line of research that studies the strategic information gathering decisions of the agent[1].

Specifically, we want to modify the above example in the following way: a wholesaler doesn't know her cost structure before its conversations with the retailers, and gathering information about it implies incurring some costs (such as the reports from consulting and engineering firms, comparing similar experiences of other companies, etc.). We allow the wholesaler to decide whether incurring this cost is worth it. If the wholesaler decides that it is not worth it, then she decides whether or not to accept the retailers' contracts. In this case, she will learn her costs when the contract is close to being implemented, e.g. when all the new machinery is installed, an additional plant is built, etc. and

[1] See Cremer, Khalil and Rochet (1998a) and (1998b).

she is almost at the point of delivering the amounts of output ordered by the retailers in the contract.

Hence, we will assume that information acquisition is costly when gathered at a pre-contractual stage, while it is available at no cost afterwards (but only observable for the agent). Thus, this kind of information acquisition is considered *socially wasteful* (in terms of Cremer, Khalil and Rochet (1998a)) because it is conducted only for the purpose of rent seeking. That is, it improves the bargaining position of the agent (the wholesaler), since by gathering information she can obtain a higher utility than if she didn't have this opportunity. Moreover, we will assume that the agent can gather information at some cost c while it is prohibitively expensive for any principal (retailer) to become informed. For instance, when two retailers try to control the production of a wholesaler (common agent), they can't observe her cost structure, and trying to gather accurate information about it is extremely expensive for them (or even illegal if industrial spying is banned by law).

Thus, we will construct a common agency model with strategic information gathering before the signing of the contract. In particular, we will build on Cremer, Khalil and Rochet's (1998a) information-gathering model by extending it to the case of two principals (retailers). In this way we can address two questions:

1. What are the effects of introducing a new retailer in a single principal-agent model with information gathering? Does it lead to higher production from the wholesaler?

2. What are the effects of allowing the wholesaler to decide if she wants to observe her cost structure in a common agency model? That is, what are the effects of endogenizing the information acquisition decision in a common agency game?

Regarding the first question, our results are consistent with Cremer *et. al.*'s (1998a) model. However, given the existence of competition among the retailers we can specify instances where the agent's production schedule for different cost structures is dramatically above or below her optimal production schedule in the case of a single retailer. Hence, we will specify cases for which the introduction of a second principal (retailer) leads to greater (or lower) productive efficiency than their model with a single retailer

With respect to second question, as we will show, we find that the agent will only decide to always gather information under very particular circumstances. In other contexts, she will decide to never gather information, or to acquire it only sometimes. This shows that the common agency results, where the agent is supposed to always gather such pre-contractual information, is just one of the equilibria we describe.

Therefore, we will structure the paper as follows: firstly, we present a description of the common agency model, without specifying the agent's decision about whether or not to acquire information. Afterwards, we describe the information gathering game within the common agency model described in the previous section, and we will define the conditions for a contract in this setting to be commonly feasible. In section four, we find the pure strategy equilibria of the information gathering game, one in which the wholesaler always acquires pre-contractual information about her costs and another one in which he never does so. In section five we focus on mixed strategy equilibria of this common agency game with information gathering. Finally, in the last section we present our conclusions.

2 Model

We analyze a common agency model with two principals (retailers) and one agent (wholesaler), who produces a level of output q_j for each principal $j = \{1, 2\}$. For ease of exposition we will refer to each principal as "he" whereas we will deal with the agent as "she".

Retailers (principals)

Retailer j's profit function is given by his profit function $\pi^j(q_j, t_j)$, which is twice continuously differentiable, decreasing in the transfer he pays to the wholesaler t_j, $\pi^j_{t_j}(\cdot) < 0$ and $\pi^j(\cdot)$ have partial derivatives up to third order which are uniformly bounded. Additionally, note that the wholesaler's type (θ) doesn't influence the retailer's welfare.

We will assume that retailer's preferences are quasilinear:

$$\pi^j(q_j, t_j) = \mathcal{V}^j(q_j) - t_j$$

That is, transfers are considered a "bad" for the retailer who pays them, and $\mathcal{V}^j(q_j)$ represents the benefit that retailer j obtains from selling the output produced by the wholesaler. Additionally, we assume that $\pi^j(\cdot)$ is strictly concave in q_j, i.e. it increases for higher values of q_j but at a decreasing rate.

Note that we assume that retailers don't need to incur any cost of stocking or packaging the products that they buy to the wholesaler before selling them to their final customers. So, we make the simplifying assumption that the only costs of a retailer are related with buying goods from the wholesaler. It is also important to clarify that we don't consider the possibility of what Martimort and Stole (2003) call *contractual externalities*. Indeed, the fact that retailer 1 increases the amount of q_1 sold in the market doesn't affect retailer 2's profits via a reduction in the market price. Thus, we will assume that retailers 1 and 2 don't compete in the retailing market. For example, they are local monopolies that sell in very distant areas. More specifically, in the above quasilinear utility function, $\mathcal{V}^j_{q_1 q_2}(\cdot) = 0$.

Wholesaler (agent)

Wholesaler's utility function is given by $\mathcal{U}(q_1, q_2, t_1 + t_2, \theta)$ which is twice continuously differentiable and strictly increasing in the aggregate transfer, $t_1 + t_2$. Additionally, $\mathcal{U}(\cdot)$ has partial derivatives up to the third order which are uniformly bounded and there exist no fixed costs for the wholesaler. That is, producing no output, $q_1 = q_2 = 0$, implies no costs at all and no payoff for the wholesaler because no transfer will be received, $\mathcal{U}(0,\ 0,\ 0,\ \theta) = 0$.

The wholesaler's cost structure is represented by the parameter $\theta \in \Theta$ where $\Theta \equiv [\underline{\theta}, \overline{\theta}]$, with cumulative distribution function $F(\theta)$, with strictly positive density in its support, $f(\theta) > 0$ for all $\theta \in \Theta$, which is assumed to be common knowledge among all the players. We will additionally suppose that the distribution of types satisfies the monotone hazard rate property, i.e. $\frac{f(\theta)}{1-F(\theta)}$, is nondecreasing in θ.

The wholesaler's utility function is increasing in θ,

$$\mathcal{U}_\theta(q_1, q_2, t_1, t_2, \theta) > 0 \text{ for all } q_1, q_2, t_1, t_2, \theta.$$

That is, for a given contract that pays a transfer t_1 and t_2 in exchange of output q_1 and q_2 to retailers 1 and 2 respectively, the wholesaler is made better-off as her cost structure parameter, θ, increases. This assumption will become clearer when we introduce the Spence-Mirlees single-crossing condition, since the latter implies that an increase in θ makes the agent ask for smaller increases in the transfer $t_j(\cdot)$ for any marginal increase in the delivered output q_j.

The wholesaler's outside option is normalized to zero, i.e. $\overline{\mathcal{U}}$=0.

Additionally, we will assume that wholesaler's payoff function is quasilinear:

$$\mathcal{U}(q_1, q_2, t_1, t_2, \theta) = t_1 + t_2 - \mathcal{C}(q_1, q_2, \theta)$$

where the wholesaler's indifference curves are strictly convex in q_j.

Note that by definition, the indifference curves of a quasilinear utility function are parallel displacements to each other, i.e. for a given pair of output $\{q_1, q_2\}$ the slope of $\mathcal{U}(\cdot)$ is always the same for any level of $t_1 + t_2$. Furthermore, the agent with type θ incurs a cost $\mathcal{C}(q_1, q_2, \theta)$ from producing the combination of output $\{q_1, q_2\}$. We will not assume that the production processes of q_1 and q_2 are totally independent, i.e. $\mathcal{C}_{q_1 q_2}(\cdot) \neq 0$, since, as we will observe, the presence of complementarities or substituibilities between them enriches the analysis about when it is optimal for the wholesaler to gather pre-contractual information.

As discussed above, every retailer submits his contract offers, which specify the transfer he will pay for every possible amount of output delivered by the wholesaler, $t_j\,(q_j)$. Given these offers by every retailer, the wholesaler needs to optimally choose the production that she will deliver to each retailer, q_1 and q_2. The indirect utility function generated from this optimal choice of the wholesaler will be denoted by,

$$\mathrm{U}(\theta) = \max_{q_1, q_2 \geq 0} \mathcal{U}(q_1, q_2, t_1(q_1), t_2(q_2), \theta)$$

Furthermore, the wholesaler's payoff function also satisfies the Spence-Mirlees single-crossing property:

$$\frac{\partial}{\partial\theta}\left(\frac{\mathcal{U}_{q_j}(q_1, q_2, t, \theta)}{\mathcal{U}_t(q_1, q_2, t, \theta)}\right) < 0 \text{ that is, } \frac{\partial \mathrm{MRS}^{Agent}_{q_j, t}}{\partial\theta} < 0$$

In the case we analyze, this condition can be expressed simply by $\mathcal{C}_{q_i,\theta}(q_1, q_2, \theta) < 0$. Note that this derivative is well defined, since $\mathcal{U}(q_1, q_2, t, \theta)$ is twice continuously differentiable and $\mathcal{U}_t(q_1, q_2, t, \theta) > 0$ by assumption.

The economic intuition behind this assumption is that the marginal cost of producing additional units decreases for larger values of θ, i.e. a wholesaler with a higher value of θ is more efficient in the production of marginal units of output. In other words, for a given point like $\left(q_j^o, t^o\right)$ and for a given value of q_l for $l \neq j$, the slope of the wholesaler's utility function decreases with the wholesaler's cost structure, θ.

All of these assumptions will be assumed to hold throughout the paper, except where noted otherwise. Additional assumptions will be added when needed.

Finally, the wholesaler must either accept both contracts, or reject both and obtain $\overline{\mathcal{U}}$=0. That is, we are considering a model of *intrinsic* common agency, in which the wholesaler either accepts both contracts or none. The wholesaler's only choice is to leave the market, i.e. don't participate in the mechanism. The distinction between an intrinsic and a *delegated* common agency model is that the agent can choose whether to contract with one, both or none of the principals. This difference is less important when the contracting activities of the two principals are complementary in terms of the common agent utility, i.e. in any situation where the agent finds profitable to contract exclusively with one principal, she will find it also attractive to contract with the other one.

3 Information gathering

We now introduce into our analysis the possibility that the agent doesn't know the value of θ at the beginning of the game, and has to decide whether to incur a cost c in order to learn it before she receives any contract offers. Otherwise, she will learn her costs structure, θ, when the contract is accepted and the production process is ready to be started. That is, after installing new machinery, adjusting the size of the plant, etc.

Then, the time structure of the model is the following:

1. Nature draws the exact realization of θ, which is not revealed to any player.
2. The wholesaler decides whether to learn or not learn her type.
3. θ is learned by the wholesaler (or nothing is observed), and it is not learned by anyone else. Additionaly, retailers can't even observe the wholesaler's decision on whether to gather information or remain uninformed.
4. Each retailer j (non-cooperatively) offers a menu of contracts to the wholesaler.
5. The wholesaler accepts or rejects the contract
6. The wholesaler observes her cost structure θ if he decided not to gather information about it during stage 2.
7. The contract is implemented.

It is important to notice that we assume that any allocation of output and transfer resulting from the contract is both observable and verifiable ex-post by a third party such as a court of law. Hence, no party is able to deviate from the output and transfer specified in equilibrium, since such a breach of the contract could be immediately proved in a court of law. That is, retailers can check whether the information revealed by the agent was truthful or not. For example, the court can have access to the original report of an independent consulting company which conducted an accurate study about the wholesaler's cost structure, θ, during stage 2.

Furthermore, we focus on *deterministic* mechanisms in the sense that the wholesaler chooses a output level but not a probability distribution over them. As Martimort and Stole (2003) point out, these kind of randomizations are rarely observed and would make the contract extremely difficult to enforce by a court of law, since the breach of the contract could only be detected after acquiring sufficient statistical observations of that randomization.

3.1 Some comments on the Revelation Principle

One could be tempted to use the Revelation Principle in this common agency model with information gathering. However, as we explain below, this principle doesn't generally hold in this context. Let's start by the single principal-agent model. In the proof of the Revelation Principle in that case, we focus on eliminating all those possible messages $m(\theta)$ from the indirect revelation mechanism that are never chosen on the equilibrium path. Later, we take only those remaining messages (i.e. equilibrium messages, $m(\theta)$), and construct a mapping from the equilibrium messages sent by the agent to her corresponding type, constructing a direct revelation mechanism[2]. In this mechanism the agent only needs to report his type and the principal chooses in his behalf the equilibrium message associated with this reported type.

As Martimort and Stole (2002) show, however, the standard proof of the Revelation Principle *fails* for common agency games. That is, for any Bayesian Nash equilibrium (BNE) of the common agency game we cannot find a BNE in the corresponding direct revelation mechanism, in which the agent truthfully reports his type to both principals. The problem is that off-the-equilibrium path messages which had no use in the single principal-agent model (by definition), may be used to sustain equilibria of the common agency model. The reason for this problem is that the agent's strategy is no longer only a mapping from the agent's type space to the agent's messages. On the contrary, now the agent's choice from the contract offered by principal 1 depends upon principal 2's offer.

There exist different approaches in order to solve this problem, see Martimort and Stole (2002). One of them is to use the Revelation Principle but assuming that the underlying preferences of all individuals are sufficiently concave (in particular, globally strictly concave). This assumption guarantees that both retailers' maximization problem will be concave in q_j^i, and will involve $q_j^i(\theta) > 0$ for all θ. As a consequence, all those messages that can be used to sustain equilibria in common agency games, become economically irrelevant. This is due to the fact that these messages now are sent off-the-equilibrium path, as they do in the principal-agent model with a single principal. Hence, we will use the Revelation Principle with this additional condition on concavity. Specifically, in order to get a clearer picture of the time structure of the direct revelation mechanism, we can describe the sequence of events in the following way (taken from Cremer *et. al.* (1998a)):

> At stage 5, when the wholesaler has to decide whether to accept or refuse the retailers' contracts, the retailers can ask the agent "tell me whether you are informed or not. If you are, tell me your information. If you are not, you will learn θ during stage 6, and you will tell me θ then". To each of these announcements are attached production levels and transfer payments.

[2] Note that this is only the case when the message function $m(\theta)$ is onto (or surjective). That is, when for every message $m \in \mathcal{M}$ sent by the agent there exists an element in her type space $\theta \in \Theta$, such that $m = m(\theta)$.

3.2 Wholesaler's optimal reports

In the single principal-agent model, the principal can induce the agent to truthfully report her type by offering an appropiately designed menu of contracts. However, as soon as we allow the principals non-cooperatively to make their contract offers to the agent, the agent's reports are influenced by both principals' offers. Indeed, contrary to the single principal-agent model, note that in a common agency setting each principal will try to induce the agent falsely to report to her rival in order to extract a larger share from the agent's information rents. Hence, the above influence of contract offers on reports has to be considered.

In particular, if we want to use a direct revelation mechanism, we must introduce some additional notation to account for this kind of effect on the agent's reports. So let's denote

$$\hat{\theta}_2\left[\hat{\theta}_1|q_1(\theta),q_2(\theta),t_1(\theta),t_2(\theta)\right] = \arg\max_{\theta'} \ t_1^i\left(\hat{\theta}_1\right)+t_2^i\left(\theta'\right)-\mathcal{C}\left(q_1^i\left(\hat{\theta}_1\right),q_2^i\left(\theta'\right),\theta\right)$$

to be the optimal announcement of agent θ to principal 2, $\hat{\theta}_2$, given the announcement made to principal 1, $\hat{\theta}_1$, and given the contracts offered by both principals. Note that, holding principal 2's contract fixed, a change in principal 1's contract will affect the report of the agent to principal 2. For the sake of notation, we will simply denote the above optimal announcement as $\hat{\theta}_2\left[\hat{\theta}_1|q_1(\cdot)\right]$. In a direct-revelation equilibrium, each principal chooses her contract offer taking the offer of the other principal as given.

Note that we have assumed that the agent announces his type to each principal separately, denoting this report by $\hat{\theta}_1$ and $\hat{\theta}_2$ respectively. Various reasons can be used to justify why a principal can only observe the announcement of type meant for him, Stole (1991). For instance, if principals could jointly observe the agent's reports, the possibility of secret side contracts between each principal and the agent before the agent's type is announced may make such joint observations useless. Additionally, antitrust laws might punish collusive activities such as the side contracts resulting from the above joint observation of reports.

3.3 Feasibility under information gathering

In this section we start our construction of a common agency model with information gathering. To our knowledge, there is no previous literature in this area, in spite of the existence of many papers both in the common agency and in the information gathering literature. For the sake of exposition, we will maintain our notation, and we will proceed as in Fudenberg and Tirole (1991): we will define feasibility in this framework (Bayesian incentive compatibility and individual rationality conditions) and we will find the exact conditions that guarantee that a contract is feasible in this context.

Note that all of this characterization of the set of feasible contracts just determines what contracts are optimal for the principal given truthtelling and participation of the agent. Afterwards, we will explicitly find the equilibria of the information gathering game, in which the agent has to decide whether it is optimal for her to incur the costs of gathering pre-contractual information.

Let's firstly define what do we mean by an output function and a transfer function:

Definition 1. *A retailer j's output function $q_j : \Theta \rightarrow X^2$ maps the type $\hat{\theta}_j$ that the wholesaler reported to retailer j into a pair of output $\left\{q_j^i(\hat{\theta}_j), q_j^u(\hat{\theta}_j)\right\}$ to be implemented by the wholesaler.*

Note that the wholesaler can determine $q_j^i(\hat{\theta}_j)$ as soon as the contract is offered when she is informed about her type (when she previously gathered information). On the other hand, $q_j^u(\hat{\theta}_j)$, cannot be known until stage 6 when the uninformed wholesaler is able to finally observe her type θ. Meanwhile, if she accepts the contract, she can only construct an expectation of $q_j^u(\tilde{\theta})$, based on $\tilde{\theta}$ =E$\left[\tilde{\theta}\right]$

Definition 2. *A transfer function $t_j : \Theta \rightarrow X^2$ maps the type $\hat{\theta}_j$ that the agent reported to retailer j into a positive monetary amount $\left\{t_j^i(\cdot), t_j^u(\cdot)\right\}$ that will be received by the wholesaler.*

Using these definitions of transfer and output function, we can define the wholesaler's indirect utility function resulting from optimally choosing the reports $\hat{\theta}_1$ and $\hat{\theta}_2$ to each retailer. In particular, $\mathrm{U}^i(\theta)$ denotes the wholesaler's indirect utility function when she knows the exact realization of θ (because she gathered information) and she chooses a contract meant for the informed wholesaler (superscript i). That is,

$$\mathrm{U}^i(\theta) = \max_{\hat{\theta}_1, \hat{\theta}_2} t_1^i\left(\hat{\theta}_1\right) + t_2^i\left(\hat{\theta}_2\right) - \mathcal{C}\left(q_1^i\left(\hat{\theta}_1\right), q_2^i\left(\hat{\theta}_2\right), \theta\right) \tag{$\mathrm{U}^i(\theta)$}$$

Similarly, $\mathrm{U}^u(\theta)$ expresses the informed wholesaler's indirect utility (she knows the value of θ) when choosing the contract meant for the uninformed wholesaler,

$$\mathrm{U}^u(\theta) = t_1^u\left(\tilde{\theta}\right) + t_2^u\left(\tilde{\theta}\right) - \mathcal{C}\left(q_1^u\left(\tilde{\theta}\right), q_2^u\left(\tilde{\theta}\right), \theta\right) \tag{$\mathrm{U}^u(\theta)$}$$

where $\mathrm{E}[\theta] = \tilde{\theta}$. Note that one if the wholesaler chooses the uninformed wholesaler's contract, then no information about θ is revealed to the retailer, and $\tilde{\theta}$ is used as the expected value of this (supposedly) unknown parameter.

Conversely, for the case of the uninformed wholesaler, she only knows that $\mathrm{E}[\theta] = \tilde{\theta}$. Then, her indirect utility functions for the case of choosing a contract for the informed and the uninformed agent, respectively, are

$$\mathrm{E}\left[\mathrm{U}^i(\theta)\right] = \mathrm{U}^i(\tilde{\theta}) = \max_{\hat{\theta}_1,\hat{\theta}_2} t_1^i\left(\hat{\theta}_1\right) + t_2^i\left(\hat{\theta}_2\right) - \mathcal{C}\left(q_1^i\left(\hat{\theta}_1\right), q_2^i\left(\hat{\theta}_2\right), \tilde{\theta}\right) \tag{$\mathrm{E}\left[\mathrm{U}^i(\theta)\right]$}$$

$$\mathrm{E}\left[\mathrm{U}^u(\theta)\right] = \mathrm{U}^u(\tilde{\theta}) = t_1^u\left(\tilde{\theta}\right) + t_2^u\left(\tilde{\theta}\right) - \mathcal{C}\left(q_1^u\left(\tilde{\theta}\right), q_2^u\left(\tilde{\theta}\right), \tilde{\theta}\right) \tag{$\mathrm{E}[\mathrm{U}^u(\theta)]$}$$

Note that we assume the wholesaler to be risk neutral. Additionally it is important to notice that $\mathrm{E}\left[\mathrm{U}^i(\theta)\right]$ is not simply the expectation of $\mathrm{U}^i(\theta)$ over θ, since the optimal pair of reports $\left(\hat{\theta}_1, \hat{\theta}_2\right)$ for the case of the informed wholesaler may be different to the case of the uninformed wholesaler.

We now define common feasibility of a contract in this setting. A pair of output functions $\left\{q_j^i(\cdot), q_j^u(\cdot)\right\}_{j=\{1,2\}}$ is commonly feasible if there exists a pair of transfer functions $\left\{t_j^i(\cdot), t_j^u(\cdot)\right\}_{j=\{1,2\}}$ such that the contracts offered by each principal satisfy:

1. the pair of output functions $\left\{q_j^i(\cdot), q_j^u(\cdot)\right\}_{j=\{1,2\}}$ are implementable, i.e. the contract satisfies the following two sets of BIC constraints

   a) The agent chooses the contract meant for her:

$$\mathrm{U}^{\mathrm{i}}(\theta) \geq \mathrm{U}^u(\theta) \tag{IC_{iu}}$$

$$\mathrm{E}_\theta\left[\mathrm{U}^u(\theta)\right] \geq \mathrm{E}_\theta\left[\mathrm{U}^{\mathrm{i}}(\theta)\right] = \mathrm{U}^{\mathrm{i}}(\tilde{\theta}) \tag{IC_{ui}}$$

for any combination of reported and true type $(\hat{\theta}_1, \hat{\theta}_2, \theta) \in \Theta^3$.

b) Thuthteling conditions[3]:

$$q_j^k(\cdot) \text{ is nondecreasing in } \theta, \text{ for all } \theta \in \Theta \text{ and for all } k = \{i, u\}$$

$$\dot{\mathrm{U}}^k(\theta) = -\mathcal{C}_\theta^k(q_1^k(s), q_2^k(\hat{\theta}_2\left[s|q_1^k(s)\right]), s) \text{ for all } k = \{i, u\} \text{ and for all } \theta \in \Theta$$

These two conditions imply that $\mathrm{U}^k(\theta)$ is increasing in θ, convex and continuous. Therefore, we obtain the following (usual) truthtelling BIC constraint to be used in the principal j's maximization problem below

[3] These are the usual truthtelling Bayesian incentive compatibility conditions for the common agency model with incomplete information and a continuum of types. For completeness, we include its derivation, based on Laffont and Martimort (2002), in the Appendix.

$$\mathrm{U}^k(\theta) = \mathrm{U}^k(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \mathcal{C}_\theta^k(q_1^k(s), q_2^k(\hat{\theta}_2\left[s|q_1^k(s)\right]), s)ds \text{ for all } k = \{i, u\} \quad \text{(BIC)}$$

2. participation (or IR) constraints

$$\mathrm{U}^i(\theta) \geq 0 \text{ for all } \theta \in \Theta \quad (\mathrm{IR}_i)$$

$$\mathrm{E}_\theta\left[\mathrm{U}^u(\theta)\right] \geq 0 \text{ for all } \theta \in \Theta \quad (\mathrm{IR}_u)$$

Note that IR_i and IC_{iu} can be merged in $\mathrm{U}^i(\theta) = \max\{0, \mathrm{U}^u(\theta)\}$. So, the only remaining incentive compatibility condition among those IC conditions which specify that the agent chooses the contract meant for her is IC_{ui}.

That is, a pair of output functions $\left\{q_j^i(\cdot), q_j^u(\cdot)\right\}_{j=\{1,2\}}$ is feasible if there exists a pair of transfer functions $\left\{t_j^i(\cdot), t_j^u(\cdot)\right\}_{j=\{1,2\}}$ such that the agent always prefers to truthfully reveal his type θ, and whether she has previously gathered information or not. Additionally, by doing so he gets a higher payoff than by rejecting both contracts.

4 Pure Strategy Equilibria

In the previous section, we characterized the conditions that a contract offered by principal j needs to satisfy in order to be feasible. That is, the conditions that it has to satisfy in order to be accepted by the appropiately informed type of agent, to induce truthful report of her type and finally to be accepted by her (individual rationality).

Now we can proceed to find the equilibrium strategies for the agent, i.e. her optimal strategy in terms of gathering information. We will start with the pure strategy equilibria where she always gathers information, where she never gathers, and later on we will focus on the mixed strategy equilibrium.

We will denote retailer j's beliefs about the probability that the wholesaler gathers information by $\omega \in [0, 1]$, where $\omega = 1$ indicates a wholesaler who always acquires information. We start with this case.

4.1 Informed Agent, $\omega = 1$

4.1.1 Best Response of retailer j to $\omega = 1$

In the case of an informed wholesaler, the expected utility of the retailer is given by

$$\mathrm{E}_\theta \left[\mathcal{V}_j(q_j^i(\theta)) - t_j^i(\hat{\theta}_j \left[\theta | q_l^i(\theta) \right]) \right]$$

and substituting $t_j^i(\hat{\theta}_j \left[\theta | q_l^i(\theta) \right])$ from the informed agent's indirect utility function,

$$\mathrm{E}_\theta \left[\mathcal{V}_j(q_1^i(\theta)) - \mathrm{U}^i(\theta) - \mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2 \left[\theta | q_1^i(\theta) \right]), \theta) + t_2^i(\hat{\theta}_2 \left[\theta | q_1^i(\theta) \right]) \right]$$

using BIC and integrating by parts we can reduce the wholesaler's objective function to the following[4],

$$\mathrm{E}_\theta \left[\mathcal{V}_j(\cdot) + \frac{1 - F(\theta)}{f(\theta)} \mathcal{C}_\theta^i(\cdot) - \mathcal{C}^i(\cdot) + t_2^i(\cdot) \right] - \mathrm{U}^i(\underline{\theta})$$

Using the "no rent at the bottom" condition, we have $\mathrm{U}^i(\underline{\theta}) = 0$. Suppose, on the contrary, that $\mathrm{U}^i(\underline{\theta}) = \varepsilon > 0$. Then, retailer j can decrease $\mathrm{U}^i(\underline{\theta})$ (and $\mathrm{U}^i(\theta)$ for all $\theta \in \left(\underline{\theta}, \bar{\theta}\right]$) by ε and gain this amount ε. Therefore, $\mathrm{U}^i(\underline{\theta}) = 0$ is optimal.

Then, the first order condition with respect to q_j^i of the retailer j's maximization problem is given by

$$\frac{\partial \mathcal{V}_j(\cdot)}{\partial q_j^i} + \frac{1 - F(\theta)}{f(\theta)} \left(\frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_j^i} - \frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_l^i} q_l^{i\prime}(\cdot) \frac{\frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_j^i \partial q_l^i}}{\frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_l^i \partial \theta} + \frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_j^i \partial q_l^i(s)} q_l^{i\prime}} \right) - \frac{\partial \mathcal{C}^i(\cdot)}{\partial q_j^i} = 0$$

Hence, principal j's contract offers exactly what he would offer in a common agency game in which the agent is always informed, see Stole (1991).

As a consequence, when the principal believes that the agent will always acquire information, he will simply offer a unique type of contract to the agent, regardless that she acquired information or not. And he will do so by finding a pair of indirect utility functions for the agent $\left(\mathrm{U}^i, \mathrm{U}^u\right)$ that satisfy the above feasibility conditions.

[4] We didn't include the calculations that lead to the result in the case of the informed agent, $\omega = 1$. In spite of this fact, in the appendix we included a detailed solution of the retailer j's maximization problem for any value of ω in the mixed strategy Nash equilibrium of section 5. Hence, the above results are just a corollary of the more general result we find for any value of $\omega \in [0, 1]$.

The fact that retailer j offers a unique type of contract that satisfies the above first order condition doesn't mean that this first order condition has a unique nondecreasing solution. In fact, as Stole (1991) shows, this solution is only unique for goods which are considered contract substitutes in the wholesaler's production process. On the contrary, when these goods are complements, there exists a continuum of symmetric Nash equilibria with nondecreasing solutions. In particular, we adapt his results to the present context in the following Lemma.

Lemma 1
In a symmetric contracting game where $\pi_1(\cdot) = \pi_2(\cdot)$ and $\mathcal{C}_{q_1 q_2 \theta} \geq 0$, $\mathcal{C}_{q_1 \theta\theta} \geq 0$ and $\mathcal{C}_{q_i q_i \theta} + \mathcal{C}_{q_1 q_2 \theta} \geq 0$, a continuum of differentiable Nash equilibria exist in the case of contract complements, while a unique equilibrium contract exists in the case of contract substitutes.

This implies that there exists a multiplicity of equilibrium contract offers when q_1 and q_2 are complementary goods, where each of these contracts generates a different utility level for the informed wholesaler. This fact will become relevant when we analyze when it is optimal for the wholesaler to acquire precontractual information about her cost structure; what we do next.

4.1.2 Wholesaler's Best response

We analyzed what is the best response of retailer j to the wholesaler's strategy of always acquiring information, $\omega = 1$. On the other hand, regarding the wholesaler's equilibrium strategy, she will acquire information when the cost of acquiring it is smaller than its ex-ante expected benefits,

$$c \leq \mathrm{E}\left[\mathrm{U}^i(\theta)\right] - \mathrm{E}\left[\mathrm{U}^u(\theta)\right]$$

and using IC_{ui} we have,

$$c \leq \mathrm{E}\left[\mathrm{U}^i(\theta)\right] - \mathrm{U}^i\left(\tilde{\theta}\right) \leq \mathrm{E}\left[\mathrm{U}^i(\theta)\right] - \mathrm{E}\left[\mathrm{U}^u(\theta)\right]$$

What provides the wholesaler with a cutoff value for her decision on whether or not to acquire information. Therefore we can summarize this equilibrium in the following proposition, similar to the results of Cremer *et. al.* (1998a), but applied to a common agency context.

Proposition 1
There exists a pure strategy Nash equilibrium in which the agent always gathers information, $\omega = 1$, and principal j offers a contract like that in a common agency model, satisfying the above first order condition, with no differences regarding the agent's strategy during the information gathering stage if and only if

$$c \leq c_1 = \mathrm{E}\left[\mathrm{U}^i(\theta)\right] - \mathrm{U}^i\left(\tilde{\theta}\right)$$

where c_1 is the ex-ante value of information for an agent who is offered the common agency contract.

Note that, given the multiplicity of equilibrium contract offers for the informed wholesaler commented above for the case of contract complements, c_1 will be in some interval $\left[c_1^0, c_1'\right]$. Otherwise, when producing contract substitutes, c_1 will be ex-ante uniquely determined.

4.2 Uninformed Agent, $\omega = 0$

When the wholesaler never purchases information in advance about her cost structure, it is optimal for the retailer to "sell the store" to the wholesaler, as if she controlled a vertically integrated firm that produces and sells products to her final customers. That is, by making the wholesaler responsible for the optimal decision on production and sales, the wholesaler bears all the risk in the relationship. The retailer will choose a fixed price for his store that eliminates all his risks, and at the same time leaves the wholesaler with zero expected rents.

We will firstly show the equilibrium strategies for retailer j in this case (i.e. the retailer's best response when $\omega = 0$), and the conditions under which $\omega = 0$ is an optimal response for the wholesaler when the retailer has chosen the above optimal strategy.

4.2.1 Best Response of retailer j to $\omega = 0$

Retailer j must find a menu of contracts $\left\{q_j^i(\theta), t_j^i(\theta), q_j^u(\theta), t_j^u(\theta)\right\}$ that maximizes his expected payoff. Since we are looking for retailer j's equilibrium strategy, we fix the wholesaler's optimal strategy of $\omega = 0$ into the retailer j's expected utility,

$$\mathrm{E}_\theta \left[\mathcal{V}_j(q_j^u(\theta)) - t_j^u(\hat{\theta}_j\left[\theta | q_l^u(\theta)\right])\right]$$

The best response of the retailer is to choose a transfer $t_j^u(\hat{\theta}_j\left[\theta | q_l^u(\theta)\right])$ such that

$$t_j^u(\hat{\theta}_j\left[\theta | q_l^u(\theta)\right]) = \mathcal{V}_j(q_j^u(\theta)) - S_j$$

where S_j is the amount of money that the wholesaler pays for having bought the retailing store to the retailer. The exact value of S_j is determined by,

$$S_j = \mathrm{E}_\theta\left[S_j(\theta)\right], \text{ where } S_j(\theta) = \max_{q_j} \mathcal{V}_j(q_j(\theta)) - \mathcal{C}\left(q_j(\theta), q_l^{BR}(\theta), \theta\right)$$

where q_l^{BR} indicates that the other retailer l is also playing his best response to $\omega = 0$, what we take as fixed.

That is, $S_j(\theta)$ is the maximum utility that the wholesaler with type θ can obtain if she produces and sells $\{q_1, q_2\}$ herself.

4.2.2 Wholesaler's Best Response

The indirect utility function of the uninformed wholesaler, given the equilibrium strategy of the retailers specified above, is then,

$$\mathrm{U}^{u}(\theta) = S_1(\theta) + S_2(\theta) - S_1 - S_2$$

That is, the uninformed wholesaler receives the net benefits from producing and selling the optimal output $\{q_1, q_2\}$ (as if he was the principal who sells them and the agent who produces them) minus the price of the firm (since he "bought the retailing store").

It is clear that if the wholesaler accepts $t_j^u(\hat{\theta}_j[\theta|q_l^u(\theta)])$ he will produce an efficient allocation since deriving her new objective function with respect to q_j^u, we obtain

$$\frac{\partial S_j(\theta)}{\partial q_j^u} = 0 \Longleftrightarrow \frac{\partial \mathcal{V}_j(\cdot)}{\partial q_j^u} = \frac{\partial \mathcal{C}^u(\cdot)}{\partial q_j^u}$$

and the agent will receive zero expected surplus, $\mathrm{E}[\mathrm{U}^{u}(\theta)] = 0$. In this case, the wholesaler internalizes all the effects of varying q_j and the retailer extracts all surplus from the wholesaler, i.e. the retailer obtains a utility level as high as in the principal-agent model with complete information.

In order to induce an uninformed wholesaler to choose $t_j^u(\hat{\theta}_j[\theta|q_l^u(\theta)])$, the transfer $t_j^i(\hat{\theta}_j[\theta|q_l^i(\theta)])$ must be chosen in such a way that the uninformed wholesaler doesn't want to lie about her information acquisition, reporting that she acquired information when she didn't. Hence, we need that the corresponding indirect utility function $\mathrm{U}^{i}(\theta)$ satisfies the incentive compatibility and individual rationality conditions stated above,

$$\mathrm{E}_\theta[\mathrm{U}^{u}(\theta)] \geq \mathrm{U}^{i}(\tilde{\theta}) \qquad (\mathrm{IC}_{ui})$$

$$\mathrm{U}^{i}(\theta) \geq \max\{0, \mathrm{U}^{u}(\theta)\} \qquad (\mathrm{IR}_i + \mathrm{IC}_{iu})$$

Rewriting $(\mathrm{IR}_i + \mathrm{IC}_{iu})$ in this context,

$$\mathrm{U}^{i}(\theta) \geq \max\{0,\ S_1(\theta) + S_2(\theta) - S_1 - S_2\}$$

That is, the smallest function $t_j^i(\cdot)$ that satisfies this condition is the one that implies

$$\mathrm{U}^{i}(\theta) = \max\{0,\ S_1(\theta) + S_2(\theta) - S_1 - S_2\}.$$

So, the constraint $(\mathrm{IR}_i + \mathrm{IC}_{iu})$ holds. Regarding IC_{ui}, we can check that it also holds,

$$\mathrm{E}[S_j(\theta) - S_j] \geq S_j\left(\tilde{\theta}\right) - S_j$$

$$\Longleftrightarrow \mathrm{E}[S_j(\theta)] \geq S_j\left(\tilde{\theta}\right)$$

$$\iff S_j \geq S_j\left(\mathrm{E}\left[\theta\right]\right)$$

Hence, IC_{ui} is also satisfied.

Therefore, $\left(\mathrm{U}^i, \mathrm{U}^u\right)$ is a best response of the principal to the agent choosing $\omega = 0$ in equilibrium. Thus, the principal offers only one contract, namely the contract which is optimal in the uninformed agent case. If the wholesaler (agent) deviates from the equilibrium strategy of $\omega = 0$, then she will accept this unique type of contract if (IR_i + IC_{iu}) holds.

We can summarize these results in the following proposition.

Proposition 2 [based on Cremer *et. al.* (1998a)]

There exists an equilibrium with the wholesaler not gathering information, $\omega = 0$, and retailer j "selling the store" to the wholesaler for a price S_j if and only if the costs of gathering information for the wholesaler are higher than a cutoff level given by

$$c_0 = \mathrm{E}\left[\max\left\{0,\ S_1 + S_2 - S_1\left(\theta\right) - S_2\left(\theta\right)\right\}\right]$$

Proof [in Appendix]

5 Optimal contracts for a more general setting

In the previous two sections, we studied the optimal contracts offered by the retailers when the wholesaler acquires information with probability one or zero, respectively. In this section, we analyze the contracts that the retailers will optimally offer for the case in which the probability that the wholesaler has asquired information, ω, is strictly between zero and one.

We will start by describing what are the optimal contracts that will be offered by retailer j among the set of feasible contracts defined on section 3.3, for a given value of ω. Afterwards, in section 6, we will discuss the wholesaler's best response to the optimal menu of contracts offered by the retailers.

5.1 Retailer j's maximization problem

Let's denote by $\omega \in (0, 1)$ the probability that retailer j assigns to the wholesaler gathering pre-contractual information at stage 2. Then, retailer j maximizes the following expected utility function:

$$\omega \mathrm{E}_\theta\left[\mathcal{V}_j(q_1^i(\theta)) - t_j^i(\hat{\theta}_l\left[\theta|q_1^i(\theta)\right])\right] + (1-\omega)\,\mathrm{E}_\theta\left[\mathcal{V}_j(q_j^u(\theta)) - t_j^u(\hat{\theta}_l\left[\theta|q_j^u(\theta)\right])\right]$$

And by quasilinearity of the wholesaler's utility function, $\mathrm{U}^k(\theta)$,

$$t_1^k(\hat{\theta}_2\left[\theta|q_1^k(\theta)\right]) = \mathrm{U}^k(\theta) + \mathcal{C}^k(q_1^k(\theta), q_2^k(\hat{\theta}_2\left[\theta|q_1^k(\theta)\right]), \theta) - t_2^k(\hat{\theta}_2\left[\theta|q_1^k(\theta)\right])$$

Hence, substituting this expression into retailer j's program, we obtain

$$\begin{aligned}\max_{\left\{q_j^k(\theta), \mathrm{U}^k(\theta)\right\}_{k=\{i,u\}}} \quad & \omega \mathrm{E}_\theta \mathcal{V}_j(q_1^i(\theta)) - \mathrm{U}^i(\theta) - \mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta) \\ & + t_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]) + (1-\omega)\, \mathrm{E}_\theta \mathcal{V}_j(q_1^u(\theta)) - \mathrm{U}^u(\theta) \\ & - \mathcal{C}^u(q_1^u(\theta), q_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]), \theta) + t_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right])\end{aligned}$$

subject to (1)-(4)

$$q_j^k(\cdot) \text{ is nondecreasing in } \theta, \text{ for all } \theta \in \Theta \text{ and for all } k = \{i, u\} \tag{1}$$

$$\mathrm{U}^k(\theta) = \mathrm{U}^k(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \mathcal{C}_\theta^k(q_1^k(s), q_2^k(\hat{\theta}_2\left[s|q_1^k(s)\right]), s)ds \text{ for all } k = \{i, u\} \tag{2}$$

$$\mathrm{U}^k(\theta) \geq 0 \text{ for all } \theta \text{ for all } k = \{i, u\} \tag{3}$$

$$\mathrm{E}_\theta\left[\mathrm{U}^u(\theta)\right] \geq \mathrm{U}^{\mathrm{i}}(\tilde{\theta}) \tag{4}$$

Hence, constraints (1) and (2) would represent BIC for truthful report of θ for the wholesaler. On the other hand, (4) stands for truthtelling about whether or not she gathered pre-contractual information since, as we noted above, IC_{ui} is the only incentive compatibility condition remaining among those which specify that the agent truthfully reveals whether she acquired information or not. Finally, (3) summarizes all of the above individual rationality conditions.

Note that constraint (4) must be binding in equilibrium $\mathrm{E}_\theta\left[\mathrm{U}^u(\theta)\right] = \mathrm{U}^{\mathrm{i}}(\tilde{\theta})$. Otherwise we could lower $\mathrm{U}^u(\theta)$ by a constant, which is possible since $\mathrm{U}^u(\underline{\theta})$ is not constrained below. Additionally, $\mathrm{U}^i(\underline{\theta}) = 0$ for the usual reasons that no rent is left for the agent with the worst type.

Simplifying the objective function of this maximization problem, using integration by parts and rearranging, we can obtain the principal's relaxed maximization problem (a complete construction of this maximization problem is detailed in the Appendix).

$$\max_{\left\{q_j^i(\theta),q_j^u(\theta)\right\}} \quad \begin{aligned} &\omega \mathrm{E}_\theta \mathcal{V}_j(q_1^i(\theta)) + \frac{1-F(\theta)}{f(\theta)}\mathcal{C}_\theta^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]),\theta) \\ &-\mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]),\theta) + t_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]) \\ &+(1-\omega)\mathrm{E}_\theta \mathcal{V}_j(q_1^u(\theta)) - \mathcal{C}^u(q_1^u(\theta), q_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]),\theta) \\ &+t_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]) + (1-\omega)\int_{\underline{\theta}}^{\tilde{\theta}} \mathcal{C}_\theta^i(q_1^i(s), q_2^i(\hat{\theta}_2\left[s|q_1^i(\theta)\right]),s)ds \end{aligned}$$

subject to (1).

Applying first order conditions with respect to q_j^i, we obtain the following Lemma.

Lemma 2

In the information gathering game with two retailers, there exists a mixed strategy Nash equilibrium where the wholesaler gathers pre-contractual information only sometimes, and retailers offer a menu of contracts to the wholesaler. Specifically, retailer j's output function offered to the informed wholesaler must satisfy the following first order condition

$$\omega\left[\frac{\partial \mathcal{V}_j(\cdot)}{\partial q_j^i} + \frac{1-F(\theta)}{f(\theta)}\left(\frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_j^i} - \frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_l^i} q_l^{i\prime}(\cdot)\frac{\frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_j^i \partial q_l^i}}{\frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_l^i \partial \theta} + \frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_j^i \partial q_l^i(s)} q_l^{i\prime}}\right) - \frac{\partial \mathcal{C}^i(\cdot)}{\partial q_j^i}\right] f(\theta)$$
$$+(1-\omega)\frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_j^i} 1_{\theta<\tilde{\theta}} = 0$$

If we denote by CA the informed wholesaler information rents in a common agency model,

$$\mathrm{CA} = \frac{1-F(\theta)}{f(\theta)}\frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_j^i} - \frac{1-F(\theta)}{f(\theta)}\frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_l^i} q_l^{i\prime}(\cdot)\frac{\frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_j^i \partial q_l^i}}{\frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_l^i \partial \theta} + \frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_j^i \partial q_l^i(s)} q_l^{i\prime}}$$

Then, the above first order condition gets reduced to the following expression,

$$\frac{\partial \mathcal{V}_j(\cdot)}{\partial q_j^i} = \frac{\partial \mathcal{C}^i(\cdot)}{\partial q_j^i} - \mathrm{CA} - \frac{1-\omega}{\omega f(\theta)}\frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_j^i} 1_{\theta<\tilde{\theta}} \qquad (\mathrm{FOC}_{q_j^i})$$

Let's build some economic intuition on this result. The left hand side simply represents retailer j's surplus from marginally increasing the production received from the wholesaler: retailer j derives a higher utility $\mathcal{V}_j(\cdot)$ because he can sell a higher amount of product to the market. Conversely, the right hand side measures marginal loss of rents when he increases q_j^i, i.e. the effect of this marginal increase in q_j^i on additional expected rents given to the wholesaler. Firstly, in order to induce the wholesaler to deliver him a higher q_j^i, he needs to compensate her for her increase in expected production costs, $\frac{\partial \mathcal{C}^i(\cdot)}{\partial q_j^i}$.

Secondly, in order to induce the informed wholesaler to truthfully reveal her private information about θ, both retailers compete in order to extract as much rents as possible from her. The effects of this competition on the term CA depend ultimately on whether the wholesaler is producing two goods which are considered complements or substitutes in her production process.

Specifically, in the case in which $\frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_j^i \partial q_l^i} < 0$ the wholesaler produces two complementary goods, and the increased production of one of them decreases the marginal cost of the other. That is, economies of scope must necessarily exist (see Panzar (1989)). As can be checked in $\text{FOC}_{q_j^i}$, this decreases the wholesaler's information rents derived from the common agency nature of the game (an increase in the CA term). This decrease is due to the fact that retailers increase their competition. Thus, in the case of complements, a retailer j will decrease the ordered amount of q_j^i to attempt that the wholesaler decreases the amount of q_l^i contracted with retailer l. This will make truthtelling of θ cheaper for retailer j. In equilibrium both retailers proceed in this way, and this competition implies a reduction in the wholesaler's information rents, and a great distortion in the amount of produced output, since very little output is now contracted by retailers.

In the case of $\frac{\partial^2 \mathcal{C}^i(\cdot)}{\partial q_j^i \partial q_l^i} > 0$ we have that q_j^i and q_l^i are contract substitutes in the retailer's production process, and then increased production of one of them increases the marginal cost of the other. Here we have diseconomies of scope. Note that, algebraically, the term CA becomes smaller. Intuitively, now the retailers' competition works in a different way from above. In particular, retailer j will increase his order of q_j^i with the objective of making the wholesaler decrease her order with retailer l, q_l^i, and making truthtelling cheaper. In equilibrium both retailers behave in this way, what implies an increase in the wholesaler's information rents. Additionally, there exists a small distortion in productive efficiency, since a lot of output is contracted by retailers due to their own competition for the wholesaler's rents.

Hence, the wholesaler's information rents are increased for contract substitutes and decreased for contract complements.

The last term reflecting retailer j's marginal loss of rents when increasing q_j^i depends on the efficiency level of the wholesaler. In particular, if the wholesaler is relatively efficient, $\theta > \tilde{\theta}$, then she doesn't receive any additional rent, and his production schedule would coincide with the one in a common agency game. However, when the informed wholesaler is inefficient, $\theta < \tilde{\theta}$, a marginal increase in q_j^i decreases her information rents. This decrease increases in the marginal costs' responsiveness to changes in θ, $\frac{\partial \mathcal{C}_\theta^i(\cdot)}{\partial q_j^i}$, and in the retailer's subjective probability that the wholesaler is uninformed, $\frac{1-\omega}{\omega}$. For instance, a inefficient wholesaler with very steep cost functions will observe her information rents decreased if, in addition, the retailer believes that she didn't gather pre-contractual information, i.e. $(1-\omega) \to 1$.

Intuitively, when the retailer expects that the wholesaler doesn't acquire information, the output schedule q_j^i is not so important for the case of the informed wholesaler, given that he expects her to not be informed. Instead, q_j^i becomes relevant in this situation because it helps in reducing the expected rent of the uninformed wholesaler. This is achieved by reducing q_j^i for all $\theta < \tilde{\theta}$, which reduces $\mathrm{U}^i\left(\tilde{\theta}\right)$.

Now we describe retailer j's contract offers intended to be chosen by the uninformed wholesaler, resulting from taking first order conditions with respect to q_j^u.

Lemma 3
In the information gathering game with two retailers, there exists a mixed strategy Nash equilibrium where the wholesaler gathers pre-contractual information only sometimes, and retailers offer a menu of contracts to the wholesaler. Specifically, retailer j's output function offered to the uninformed wholesaler must satisfy the following first order condition

$$(1-\omega)\left[\frac{\partial \mathcal{V}_j(\cdot)}{\partial q_j^u} - \frac{\partial \mathcal{C}^u(\cdot)}{\partial q_j^u}\right] = 0$$

That is,

$$\frac{\partial \mathcal{V}_j(\cdot)}{\partial q_j^u} = \frac{\partial \mathcal{C}^u(\cdot)}{\partial q_j^u} \qquad (\mathrm{FOC}_{q_j^u(\cdot)})$$

Hence, the uninformed wholesaler implements an efficient output. That is, the level of output q_j^u is raised until the point where the marginal increase in retailer j's utility from a higher output equals the marginal increase in the uninformed wholesaler's costs. Moreover, the uninformed wholesaler accepts $\mathrm{FOC}_{q_j^i}$ and $\mathrm{FOC}_{q_j^u}$ since by constraint (3) of the retailer's maximization problem $\mathrm{U}^i(\theta) \geq 0$ and $\mathrm{U}^u(\theta) \geq 0$.

We can now summarize our results regarding the retailers' contract offers, which extends Cremer *et. al.* (1998a) results to a common agency game.

Proposition 3

In a common agency model with information gathering, the optimal contract for retailer j among all the set of feasible contracts (those satisfying Bayesian incentive compatibility and individual rationality conditions) satisfies the following properties:

1. *The output function of the uninformed agent $q_j^u(\cdot)$ is efficient,*

2. *The output function of the informed agent $q_j^i(\cdot)$ coincides with the one in the exogenously informed common agency game with incomplete information, for all $\theta \geq \tilde{\theta}$, and it is strictly lower for all $\theta < \tilde{\theta}$.*

5.2 Numerical approximations

The above $\text{FOC}_{q_j^i}$ for the informed wholesaler produces an expression of $q_j^{i\prime}(\theta)$ defined by a first order non-linear differential equation. Its non-linearity means that no explicit solution of $q_j^i(\theta)$ can be found from this differential equation, and only a numerical methods approximation can be used to obtain some additional intuition about the behavior of $q_j^i(\theta)$.

In particular, we assume a parametric expression for wholesaler and retailers' utility function. We will assume that the wholesaler's utility function is

$$\mathcal{U}(q_1, q_2, t_1, t_2, \theta) = t_1 + t_2 - (2 - \theta)\left[q_1^2 + q_2^2 - \alpha q_1 q_2\right]$$

where the parameter α represents the degree of complementarity, when $\alpha > 0$, or substituibility, when $\alpha < 0$, between goods q_1 and q_1 in the wholesaler's production process. Additionally, we will assume that retailer j's utility function is given by $\pi^j(q_j, t_j) = q_j - t_j$. Finally, we will assume that $\theta \sim$U$[0, 1]$.

Therefore, applying them to the above first order condition, we obtain Figure 4, where we compare our results with those of Cremer *et. al.* (1998a) and Stole (1991) in the case of contract complements.

Insert Figure 1, Comparison of $q_j^i(.)$ when Complements

Specifically, each series represents the following results:

1. series EFF represents the efficient outcome, obtained where the marginal utility that the retailer obtains from marginally increasing q_j^i equals the additional cost incurred by the wholesaler in order to produce this extra output.

2. series COOP represents the situation where both retailers offer their contracts cooperatively, and the results coincide with those in the single principal-agent model.

3. series SP_{IG}, represents the information gathering game with a single principal, i.e. Cremer *et. al.* (1998a) results.

4. series CA illustrates the common agency results, where both retailers submit their contract offers non-cooperatively.

5. series CA_{IG} represents common agency with information gathering. We have assumed high values of $\frac{1-\omega}{\omega}$ in order to emphasize the difference with the previous series (when $\frac{1-\omega}{\omega} \to 0$ both series coincide).

The important point of Figure 4 is that CA and CA_{IG} results are both below all the other series. Specifically, they are both below SP_{IG} results, what implies that introducing a new retailer in a model of information gathering leads to *less* efficient results when the wholesaler produces contract complements. On the other hand, comparing series CA and CA_{IG}, we can conclude that allowing the wholesaler to gather information leads to slightly more efficient production schedules than in the common agency model for $\theta < \tilde{\theta}$.

Constructing the same figure for contract substitutes, we obtain the following production schedules for each of the cases analyzed above

Insert Figure 2, Comparison of $q_j^i(.)$ when Substitutes, $\alpha = -3$

That is, for high values of α (the substituibility between q_1 and q_2 in the wholesaler's production process) we obtain that now both CA and CA_{IG} are always above the results for an information gathering game with a single principal, SP_{IG}. In fact they are also above the efficient production schedule. Intuitively, the retailers' competition for the wholesaler's information rents becomes so tough when dealing with contract substitutes that the wholesaler ends up producing more than what maximizes retailers' joint profits if they could coordinate their offers, COOP, and more than what would be efficient for the society, EFF. Thus, introducing a new retailer dramatically increases the wholesaler's production schedule for all values of θ, what constitutes a significant difference with respect to the information gathering game with a single principal in Cremer *et. al.* (1998a).

Finally, comparing CA and CA_{IG}, we can observe that when we allow the wholesaler to acquire pre-contractual information, her production schedule is lower than in the common agency model for all inefficient types, $\theta < \tilde{\theta}$.

We can summarize our results in the following corollary.

Corollary 1

Introducing a new retailer in the information gathering game leads to an informed wholesaler's production schedule $q_j^i(\theta)$ *which is:*

a. *further away from the first best outcome than when a single retailer is considered, if the wholesaler produces contract complements.*

b. *above the efficient production schedule and above the information gathering game with a single retailer, if the wholesaler produces contract substitutes.*

Additionally, allowing the wholesaler to gather pre-contractual information about her cost structure implies that her production schedules for $\theta < \tilde{\theta}$*:*

a. *slightly decreases when she produces contract complements, and*

b. *decreases when she produces contract substitutes.*

In the following table we emphasize the differences between our results and those in the information gathering literature with a single principal, SP_{IG}, and those in the common agency models with a exogenously informed agent, CA, in terms of the optimal production schedule for the wholesaler in each case.

	SP_{IG}	CA
Complements	below	slightly below
Substitutes	above	below

6 Description of the Equilibria

We solved retailer j's maximization problem for a given belief about ω. The solutions to this maximization problem define the retailer j's best-response given his beliefs about the wholesaler's decision of gathering information, ω, and given the incentive compatibility and individual rationality constraints of the wholesaler.

So, given a belief of ω, the retailer j's best-response correspondence defines an entire menu of contracts offered to the wholesaler

$$\left\{t_j^i\left(\hat{\theta}_j\right), q_j^i\left(\hat{\theta}_j\right), t_j^u\left(\hat{\theta}_j\right), q_j^u\left(\hat{\theta}_j\right)\right\}$$

for every $\hat{\theta}_j \in \Theta$ reported by the wholesaler to retailer j

Note that, by making the appropriate change of variables (Mirlees' trick), we can say that for every ω the retailer can maximize his expected utility by choosing a pair of wholesaler's indirect utility functions $\left(\mathrm{U}_\omega^i(\theta), \mathrm{U}_\omega^u(\theta)\right)$. Therefore, since we already analyzed the retailer's best response, we now describe what is the wholesaler's optimal strategy of gathering information. Her ex-ante value of information (given that retailer's beliefs are ω) is given by

$$\mathrm{W}(\omega) = \mathrm{E}\left[\mathrm{U}_\omega^i(\theta)\right] - \mathrm{E}\left[\mathrm{U}_\omega^u(\theta)\right]$$

Note that since the wholesaler constructs the value of acquiring information at the ex-ante stage, she needs to form an expectation about θ.

In order to obtain some intuition about this result, let's work with the extreme cases of this mixed strategy Nash equilibrium. If the retailer believes that the wholesaler will probably acquire information, then the ex-ante value of this information decreases, since its acquisition doesn't improve the wholesaler's bargaining position if ω is close to 1. On the contrary, if the retailer believes that the wholesaler will almost never gather information about θ, then acquiring it becomes more (ex-ante) profitable for the wholesaler, because it can improve her position during the negotiation of the contract with the retailers.

Let's analyze the equilibria that we described for different values of c:

Insert Figure 3a: Wholesaler's optimal strategy for contract substitutes
Insert Figure 3b: Wholesaler's optimal strategy for contract complements

1. $c_1 \geq c$. Then, the ex-ante value of acquiring information for the wholesaler, c_1, is higher than the cost of gathering it, c, for any beliefs that retailer j may sustain about ω. That is,

$$\mathrm{W}(\omega) > c \text{ for any } \omega$$

   and the wholesaler always gathers information, which is the pure strategy Nash equilibrium described in subsection 4.1. Note that, as we described on that section, there exists a continuum of equilibrium contracts that the retailer can offer when dealing with contract complements, what implies a continuum of values for c_1, one for each equilibrium contract. We denote this fact by the interval $c_1 \in \left[c_1^0, c_1'\right]$, as in the graph. For the case of contract substitutes, however, the value of c_1 is uniquely determined.

2. $c \geq c_0$. If the cost of gathering information is higher than the ex-ante value of acquiring such information, W_0, then the wholesaler will never acquire it. In this case, retailer j forms a belief of $\omega = 0$ what implies that his best response to the wholesaler's equilibrium strategy is to offer him the possibility of "selling the store" at a price S_j as was specified in the pure strategy Nash equilibrium described in subsection 4.2.

3. $c \in [c_1, c_0]$. In this case, the cost of gathering information, c, is above the expected benefits for the agent in the case that she always gathers information, c_1. That is, c is not high enough to support a strategy of always gathering information. At the same time, this cost is low enough to induce the wholesaler to never acquire information, i.e. $c < c_0$. Hence, the wholesaler will play mixed strategies, gathering information only part of the time for values of ω which make her indifferent between acquiring and not acquiring information, $\mathrm{W}(\omega) = c$

The next proposition summarizes the above results. We obtain a set of equilibria relatively similar to Cremer *et. al.* (1998a), but extended to a common agency setting with two retailers.

Proposition 4
In the information gathering game in which the wholesaler can acquire pre-contractual information at a cost c there exist the following three equilibria:

1. *if $c \geq c_0$, the wholesaler never acquires information and the retailer "sells the store" to the wholesaler at a price S_j.*
2. *if $c \in [c_1, c_0]$, the wholesaler randomizes with a probability ω such that* $\mathrm{W}(\omega) = c$, *and the retailer offers a menu of contracts that satisfies first order conditions* $\mathrm{FOC}_{q_j^i(\cdot)}$ and $\mathrm{FOC}_{q_j^u(\cdot)}$.
3. *if $c \leq c_1$ the wholesaler always gathers information, and the retailer offers a menu of contracts equivalent to the one offered in a common agency game.*

Graphically, an increase in the costs of gathering information, c, implies that we move from left to right in the description of equilibria of figure 6. That is, for small increases in c the wholesaler will initially stop playing pure strategies (always acquiring information, $\omega = 1$). After that, for greater increases in c, the set of mixed strategy Nash equilibria will shrink, i.e. a reduction in the set of retailer's beliefs ω that can sustain a mixed strategy Nash equilibrium gets reduced from $[0, \omega]$ to $[0, \omega']$ where $\omega > \omega'$. Finally, if the costs of gathering information increase enough, we can reach the pure strategy Nash equilibrium in which the wholesaler never acquires information about his cost structure, θ, before the beginning of her conversations with the retailers.

7 Conclusions

Common agency games are the appropriate way to model many economic situations in which different principals try to influence a common agent who can privately observe some piece of information, such as her cost structure. However, the costs of acquiring such information are *not* negligible, specially when the agent (e.g. the wholesaler) wants to learn this information before her negotiations about the contract with the principals (e.g. retailers).

This suggests that the wholesaler will only acquire private information if by doing so she can improve her bargaining position. We have shown that for high enough costs of acquiring information, the wholesaler may decide to remain uninformed and start her conversations with the principals as poorly informed as they are. In this respect, the standard result in the common agency literature, where the agent always gathers information, is just one of the equilibria we found, while the results in all the other equilibria are different.

Furthermore, we showed that by allowing the wholesaler to decide about her acquisition of pre-contractual information, we obtain production schedules which are clearly below (slightly below) the common agency results when the wholesaler produces contract substitutes (complements). These results confirm the importance of considering an information gathering stage before the common agency game. Indeed, only under very particular circumstances we can expect the agent to decide that gathering information is always optimal for her (which is taken for granted by the common agency models) without considering whether it is optimal for the agent to incur the costs of learning such information.

On the other hand, our contribution in terms of the information gathering models is the introduction of a second principal (retailer) who competes with the first one for the common wholesaler's rents. Thanks to this introduction, we find that the information gathering game with two principals leads to higher production schedules when the wholesaler produces contract substitutes in the wholesaler's production process, compared with the information gathering game with a single retailer. Conversely, when she produces goods which are contract complementaries, her production schedule shifts downwards with respect to the case of a single retailer, leading to lower levels of output, and even greater levels of productive inefficiency than in the single principal-agent model.

These results suggest that, for example, when allowing the introduction of a second retailer into a market, the regulatory authorities shouldn't assume that its introduction will ensure an increase in output. On the contrary, they should consider the existence of complementarities or substitutabilities in the wholesaler's production function. As we proved, each case leads to radically different results.

Extensions:

Many extensions can be naturally applied to this model. For example, we have assumed that the wholesaler acquired all or none of the information about her cost structure, θ. However, if she goes to a consulting firm in order to obtain a report about her unknown costs, the consulting firm can offer her very accurate reports which will normally be more expensive than less accurate ones. That is, the greater her expenditure on information gathering, the more accurate the information about θ that she will get. This increases the wholesaler's strategies at a pre-contractual stage, by allowing him to gather different amounts of information, depending on his preferred degree of accuracy.

Additionally, by only allowing pre-contractual information gathering, we haven't allowed the principals (retailers) to influence the wholesaler's information gathering strategy with their contract offers. By changing the timing of the information gathering decision to *after* the contract offers have been submitted, we might obtain different results from the ones found here.

Last but not least, we assumed that both retailers are monopolists in their corresponding markets, which eliminates any further interaction when each of them sells to his final customers the products that he bought to the wholesaler. Relaxing this assumption, and allowing competition in the customer market, can lead to richer and more realistic results.

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8 Appendix

8.1 Truthtelling Bayesian Incentive Compatibility conditions

Let's use the wholesaler (agent) payofffunction

$$\mathcal{U}(q_1(\theta), q_2(\theta), t_1(\theta), t_2(\theta), \theta) = t_1(\theta) + t_2(\theta) - C(q_1(\theta), q_2(\theta), \theta)$$

that we will denote by $\mathcal{U}(\theta)$.

The local incentive compatibility condition[5] can be written as

$$\begin{aligned} \dot{\mathcal{U}}(\theta) &= \frac{\partial \mathcal{U}(\theta)}{\partial \theta} = \dot{t}_1(\theta) + \dot{t}_2(\theta) - \mathcal{C}_{q_1}(q_1(\theta), q_2(\theta), \theta)\dot{q}_1(\theta) \\ &\quad - \mathcal{C}_{q_2}(q_1(\theta), q_2(\theta), \theta)\dot{q}_2(\theta) - \mathcal{C}_\theta(q_1(\theta), q_2(\theta), \theta) \end{aligned}$$

Rearranging,

$$\begin{aligned} \dot{\mathcal{U}}(\theta) &= -\mathcal{C}_\theta(q_1(\theta), q_2(\theta), \theta) + \dot{t}_1(\theta) + \dot{t}_2(\theta) \\ &\quad - \mathcal{C}_{q_1}(q_1(\theta), q_2(\theta), \theta)\dot{q}_1(\theta) - \mathcal{C}_{q_2}(q_1(\theta), q_2(\theta), \theta)\dot{q}_2(\theta) \end{aligned}$$

But in order to induce a wholesaler of type θ to truthfully reveal his type, we need,

$$\dot{t}_1(\theta) + \dot{t}_2(\theta) - \mathcal{C}_{q_1}(q_1(\theta), q_2(\theta), \theta)\dot{q}_1(\theta) - \mathcal{C}_{q_2}(q_1(\theta), q_2(\theta), \theta)\dot{q}_2(\theta) = 0$$

By using the Envelope Theorem, the above incentive compatibility condition reduces to

$$\dot{\mathrm{U}}(\theta) = -\mathcal{C}_\theta(q_1(\theta), q_2(\theta), \theta)$$

where $\mathrm{U}(\theta)$ is the indirect utility function of a wholesaler with type θ. Applying integrals on both sides of the equality, we obtain

$$\mathrm{U}(\theta) = \mathrm{U}(\underline{\theta}) - \int_{\theta}^{\bar{\theta}} \mathcal{C}_\theta(q_1(s), q_2(s), s)\, ds$$

[5] Note that the single-crossing property defined in our description of the model ensures that this local incentive compatibility condition is also global.

8.2 Proposition 2

This proof is based on Cremer *et. al.* (1998a)

The wholesaler will only choose to not gather information ($\omega = 0$) if and only if her expected utility when uninformed is larger than when informed,

$$\mathrm{E}\left[\mathrm{U}^{u}(\theta)\right] \geq \mathrm{E}\left[\mathrm{U}^{i}(\theta)\right] - c$$

$$\Longleftrightarrow c \geq \mathrm{E}\left[\mathrm{U}^{u}(\theta)\right] - \mathrm{E}\left[\mathrm{U}^{i}(\theta)\right]$$

On one hand, $\mathrm{E}[\mathrm{U}^{u}(\theta)] = 0$ because the uninformed agent was "buying the store" and obtaining no expected rents. On the other hand,

$\mathrm{U}^{i}(\theta) = \max\left\{0,\ S_1 + S_2 - S_1(\theta) - S_2(\theta)\right\}$ that we derived from the above constraints.

Then,

$$\begin{aligned} c &\geq 0 - \mathrm{E}\left[\max\left\{0,\ S_1(\theta) + S_2(\theta) - S_1 - S_2\right\}\right] \\ &\Longleftrightarrow c \geq \mathrm{E}\left[\max\left\{0,\ S_1 + S_2 - S_1(\theta) - S_2(\theta)\right\}\right] \end{aligned}$$

and denoting $c_0 = \mathrm{E}[\max\left\{0,\ S_1 + S_2 - S_1(\theta) - S_2(\theta)\right\}]$ as the cutoff level of the information gathering costs, we obtain the result of Proposition 2 in the text: the agent prefers not to gather information about θ if and only if the costs of acquiring such information are above the cutoff level c_0.

8.3 Retailer j's maximization problem

Let's simplify the objective function of this maximization problem firstly by inserting constraint (2) in the first term of the maximand for $\mathrm{U}^{i}(\theta)$,

$$\begin{aligned} &\omega \mathrm{E}_{\theta}[\mathcal{V}_j(q_1^i(\theta)) - \mathrm{U}^i(\bar{\theta}) + \int_{\underline{\theta}}^{\theta} \mathcal{C}_{\theta}^i(q_1^i(s), q_2^i(\hat{\theta}_2\left[s|q_1^i(s)\right]), s)ds \\ &-\mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta) + t_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right])] \\ &+ (1-\omega)\,\mathrm{E}_{\theta}[\mathcal{V}_j(q_1^u(\theta)) - \mathrm{U}^u(\theta) - \mathcal{C}^u(q_1^u(\theta), q_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]), \theta) \\ &+ t_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right])] \end{aligned}$$

s.t (1), (3) and (4)

Applying integration by parts, noting that $\mathrm{U}^{k}\left(\bar{\theta}\right) = 0$, and inserting our results back into the maximand we obtain,

$$\omega \mathrm{E}_\theta[\mathcal{V}_j(q_1^i(\theta)) + \frac{1-F(\theta)}{f(\theta)}\mathcal{C}_\theta^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta)$$
$$-\mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta) + t_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right])]$$
$$+(1-\omega)\,\mathrm{E}_\theta[\mathcal{V}_j(q_1^u(\theta)) - \mathrm{U}^u(\theta) - \mathcal{C}^u(q_1^u(\theta), q_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]), \theta)$$
$$+t_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right])]$$

Rearranging,

$$\omega \mathrm{E}_\theta[\mathcal{V}_j(q_1^i(\theta)) + \frac{1-F(\theta)}{f(\theta)}\mathcal{C}_\theta^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta)$$
$$-\mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta) + t_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right])]$$
$$+(1-\omega)\,\mathrm{E}_\theta[\mathcal{V}_j(q_1^u(\theta)) - \mathcal{C}^u(q_1^u(\theta), q_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]), \theta)$$
$$+t_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right])] - (1-\omega)\,\mathrm{E}_\theta\left[\mathrm{U}^u(\theta)\right]$$

s.t (1) and (4)

and noting that $\mathrm{U}^i\left(\tilde{\theta}\right) = -\int\limits_{\underline{\theta}}^{\tilde{\theta}} \mathcal{C}_\theta^i(q_1^i(s), q_2^i(\hat{\theta}_2\left[s|q_1^i(\theta)\right]), s)ds$, and adding and substracting in the third term of the above expression,

$$\omega \mathrm{E}_\theta[\mathcal{V}_j(q_1^i(\theta)) + \frac{1-F(\theta)}{f(\theta)}\mathcal{C}_\theta^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta)$$
$$-\mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta)$$
$$+t_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right])] + (1-\omega)\,\mathrm{E}_\theta[\mathcal{V}_j(q_1^u(\theta)) - \mathcal{C}^u(q_1^u(\theta), q_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]), \theta)$$
$$+t_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right])] - (1-\omega)\,\mathrm{E}_\theta \mathrm{U}^u(\theta)$$
$$+\mathrm{U}^i\left(\tilde{\theta}\right) + \int\limits_{\underline{\theta}}^{\tilde{\theta}} \mathcal{C}_\theta^i(q_1^i(s), q_2^i(\hat{\theta}_2\left[s|q_1^i(\theta)\right]), s)ds$$

Rewriting it,

$$\omega \mathrm{E}_\theta \mathcal{V}_j(q_1^i(\theta)) + \frac{1-F(\theta)}{f(\theta)}\mathcal{C}_\theta^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta)$$
$$-\mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]), \theta) + t_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]) + (1-\omega)\,\mathrm{E}_\theta \mathcal{V}_j(q_1^u(\theta))$$
$$-\mathcal{C}^u(q_1^u(\theta), q_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]), \theta) + t_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right])$$
$$-(1-\omega)\left[\mathrm{E}_\theta\left[\mathrm{U}^u(\theta)\right] - \mathrm{U}^i\left(\tilde{\theta}\right)\right]$$
$$+(1-\omega)\int\limits_{\underline{\theta}}^{\tilde{\theta}} \mathcal{C}_\theta^i(q_1^i(s), q_2^i(\hat{\theta}_2\left[s|q_1^i(\theta)\right]), s)ds$$

Given functions $q^i(\cdot)$ and $q_j^u(\cdot)$, the difference $\mathrm{E}_\theta\left[\mathrm{U}^u(\theta)\right] - \mathrm{U}^i\left(\tilde{\theta}\right)$ can be made as small as feasible. Hence, the principal j's relaxed maximization problem becomes,

$$\begin{aligned}\max_{\left\{q_j^i(\theta),q_j^u(\theta)\right\}} \quad & \omega \mathrm{E}_\theta[\mathcal{V}_j(q_1^i(\theta)) + \frac{1-F(\theta)}{f(\theta)}\mathcal{C}_\theta^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]),\theta) \\ & -\mathcal{C}^i(q_1^i(\theta), q_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right]),\theta) + t_2^i(\hat{\theta}_2\left[\theta|q_1^i(\theta)\right])] \\ & + (1-\omega)\,\mathrm{E}_\theta[\mathcal{V}_j(q_1^u(\theta)) - \mathcal{C}^u(q_1^u(\theta), q_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right]),\theta) \\ & + t_2^u(\hat{\theta}_2\left[\theta|q_1^u(\theta)\right])] + (1-\omega)\int_{\underline{\theta}}^{\tilde{\theta}} \mathcal{C}_\theta^i(q_1^i(s), q_2^i(\hat{\theta}_2\left[s|q_1^i(\theta)\right]),s)ds\end{aligned}$$

subject to

$$q_j^k(\cdot) \text{ is nondecreasing in } \theta, \text{ for all } \theta \in \Theta \text{ and for all } k = \{i,u\} \qquad ((1))$$

As usual, we will firstly solve the problem ignoring the monotonicity constraint (1) and later on we will impose some conditions on $F(\theta)$ that guarantee that the solution for $q_j^k(\cdot)$ is indeed monotonic, and safisfies constraint (1).

Omitting the contents in parenthesis for ease of notation, and solving for the first order (necessary) conditions,

$$\omega\left[\frac{\partial\mathcal{V}_j(\cdot)}{\partial q_j^i} + \frac{1-F(\theta)}{f(\theta)}\left(\frac{\partial\mathcal{C}_\theta^i(\cdot)}{\partial q_j^i} + \frac{\partial\mathcal{C}_\theta^i(\cdot)}{\partial q_l^i} q_l^{i\prime}(\cdot)\frac{\partial\hat{\theta}_l\left[s|q_j^i(\theta)\right]}{\partial q_j^i}\right) - \frac{\partial\mathcal{C}^i(\cdot)}{\partial q_j^i}\right]f(\theta)$$

$$+ (1-\omega)\,\frac{\partial\mathcal{C}_\theta^i(\cdot)}{\partial q_j^i}1_{\theta<\tilde{\theta}} = 0$$

Simplifying, and solving[6] for $\frac{\partial\hat{\theta}_l\left[s|q_j^i(\theta)\right]}{\partial q_j^i}$, we obtain the following expression, which coincides with ($\mathrm{FOC}_{q_j^i}$) in the text.

$$\omega\left[\frac{\partial\mathcal{V}_j(\cdot)}{\partial q_j^i} + \frac{1-F(\theta)}{f(\theta)}\left(\frac{\partial\mathcal{C}_\theta^i(\cdot)}{\partial q_j^i} - \frac{\partial\mathcal{C}_\theta^i(\cdot)}{\partial q_l^i} q_l^{i\prime}(\cdot)\frac{\frac{\partial^2\mathcal{C}^i(\cdot)}{\partial q_j^i\partial q_j^i}}{\frac{\partial^2\mathcal{C}^i(\cdot)}{\partial q_l^i\partial\theta} + \frac{\partial^2\mathcal{C}^i(\cdot)}{\partial q_j^i\partial q_j^i(s)} q_l^{i\prime}}\right) - \frac{\partial\mathcal{C}^i(\cdot)}{\partial q_j^i}\right]f(\theta)$$

$$+ (1-\omega)\,\frac{\partial\mathcal{C}_\theta^i(\cdot)}{\partial q_j^i}1_{\theta<\tilde{\theta}} = 0$$

[6] A detailed explanation of how to find $\frac{\partial\hat{\theta}_l\left[s|q_j^i(\theta)\right]}{\partial q_j^i}$ is included for completeness in the last section of this Appendix.

The first order condition when derivating with respect to the output function of the *uniformed* agent, $q_j^u(\cdot)$, is,

$$(1-\omega)\left[\frac{\partial \mathcal{V}_j(\cdot)}{\partial q_j^u}-\frac{\partial \mathcal{C}^u(\cdot)}{\partial q_j^u}\right]=0$$

That is,

$$\frac{\partial \mathcal{V}_j(\cdot)}{\partial q_j^u}=\frac{\partial \mathcal{C}^u(\cdot)}{\partial q_j^u} \qquad (\mathrm{FOC}_{q_j^u(\cdot)})$$

8.4 Derivation of $\frac{\partial \hat{\theta}_l[\theta|q_j(\theta)]}{\partial q_j(\theta)}$, from Stole (1991)

Let's suppose that $\{q_1, q_2, t_1, t_2\}$ is a pure-strategy Nash equilibrium of the common agency game. Then the first order conditions found in the text hold for the contract offered by each retailer. Let's check what are the effects of a change in retailer 2's menu of contracts.

A necessary condition for $\hat{\theta}_1$ to be chosen by the wholesaler when her true type is θ and retailer 2's contract is $q_2(\cdot)$ is that $\hat{\theta}_1$ is the solution to the wholesaler's maximization problem

$$\hat{\theta}_1=\arg\max_{\theta'}\ t_1(\theta')+t_2(\theta)-\mathcal{C}(q_1(\theta'),q_2(\theta),\theta)$$

That is, derivating with respect to θ' and since in equilibrium we must have that $\hat{\theta}_1=\theta'$, then

$$t_1'\left(\hat{\theta}_1\right)=\mathcal{C}_{q_1}\left(q_1\left(\hat{\theta}_1\right),q_2(\theta),\theta\right)\times q_1'(\theta)$$

From implementability in a common agency game we have that

$$t_j'(\theta_j)=\mathcal{C}_{q_j}(q_1,q_2,\theta)\times q_j'(\theta)$$

Then, the above first order condition becomes

$$\mathcal{C}_{q_1}\left(q_1\left(\hat{\theta}_1\right),q_2^e\left(\hat{\theta}_1\right),\hat{\theta}_1\right)\times q_1'(\theta)=\mathcal{C}_{q_1}\left(q_1\left(\hat{\theta}_1\right),q_2(\theta),\theta\right)\times q_1'(\theta)$$

where $q_2^e\left(\hat{\theta}_1\right)$ is what retailer 1 expects of retailer 2 to offer to the wholesaler in equilibrium. Then we obtain,

$$\left[\mathcal{C}_{q_1}\left(q_1\left(\hat{\theta}_1\right),q_2^e\left(\hat{\theta}_1\right),\hat{\theta}_1\right)-\mathcal{C}_{q_1}\left(q_1\left(\hat{\theta}_1\right),q_2(\theta),\theta\right)\right]\times q_1'(\theta)=0$$

And since $q_1(\theta)$ is increasing, then the difference

$$\left[\mathcal{C}_{q_1}\left(q_1\left(\hat{\theta}_1\right), q_2^e\left(\hat{\theta}_1\right), \hat{\theta}_1\right) - \mathcal{C}_{q_1}\left(q_1\left(\hat{\theta}_1\right), q_2(\theta), \theta\right)\right] \text{ must be zero.}$$

Totally differentiating with respect to $q_2(\theta)$ and $\hat{\theta}_1$,

$$\left[0 - \mathcal{C}_{q_1 q_2}\left(q_1\left(\hat{\theta}_1\right), q_2(\theta), \theta\right)\right] dq_2(\theta)$$

$$\left[\mathcal{C}_{q_1 q_1}\left(q_1\left(\hat{\theta}_1\right), q_2^e\left(\hat{\theta}_1\right), \hat{\theta}_1\right) \times q_1'\left(\hat{\theta}_1\right) + \mathcal{C}_{q_1 q_2}\left(q_1\left(\hat{\theta}_1\right), q_2^e\left(\hat{\theta}_1\right), \hat{\theta}_1\right) \times q_2^{e\prime}\left(\hat{\theta}_1\right)\right] d\hat{\theta}_1 +$$

$$+\left[\mathcal{C}_{q_1 \theta}\left(q_1\left(\hat{\theta}_1\right), q_2^e\left(\hat{\theta}_1\right), \hat{\theta}_1\right) - \mathcal{C}_{q_1 q_1}\left(q_1\left(\hat{\theta}_1\right), q_2(\theta), \theta\right) \times q_1'\left(\hat{\theta}_1\right)\right] d\hat{\theta}_1$$

Note that in a pure strategy Nash equilibrium expectations must be correct, and hence $q_2^e(\theta) = q_2(\theta)$. Moreover, in a pure strategy Nash equilibrium the agent must be telling the truth to each principal, that is $\hat{\theta}_1 = \hat{\theta}_2 = \theta$. Inserting these two equalities into the above results,

$$\left[-\mathcal{C}_{q_1 q_2}\left(q_1(\theta), q_2(\theta), \theta\right)\right] dq_2(\theta)$$

$$\left[\mathcal{C}_{q_1 q_1}\left(q_1(\theta), q_2(\theta), \theta\right) \times q_1'(\theta) + \mathcal{C}_{q_1 q_2}\left(q_1(\theta), q_2(\theta), \theta\right) \times q_2'(\theta)\right] d\hat{\theta}_1 +$$

$$+\left[\mathcal{C}_{q_1 \theta}\left(q_1(\theta), q_2(\theta), \theta\right) - \mathcal{C}_{q_1 q_1}\left(q_1(\theta), q_2(\theta), \theta\right) \times q_1'(\theta)\right] d\hat{\theta}_1$$

Which gets simplified to

$$-\mathcal{C}_{q_1 q_2} dq_2(\theta) = \left[\mathcal{C}_{q_1 \theta} + \mathcal{C}_{q_1 q_2} q_2'(\theta)\right] d\hat{\theta}_1$$

Therefore, the marginal effect of a change in the retailer 2's contract into the wholesaler's report to retailer 1 is given by

$$\frac{d\hat{\theta}_1}{dq_2(\theta)} = -\frac{\mathcal{C}_{q_1 q_2}}{\mathcal{C}_{q_1 \theta} + \mathcal{C}_{q_1 q_2} q_2'}$$

and generally, for any two retailers l and j,

$$\frac{d\hat{\theta}_l\left[\theta | q_j(\theta)\right]}{dq_j(\theta)} = -\frac{\frac{\partial^2 \mathcal{C}(\cdot)}{\partial q_j \partial q_l}}{\frac{\partial^2 \mathcal{C}(\cdot)}{\partial q_l \partial \theta} + \frac{\partial^2 \mathcal{C}(\cdot)}{\partial q_j \partial q_l} q_l'}$$