The Efficiency of Decentralized Trading

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Abstract

In a decentralized market traders are matched into pairs and sellers make price offers. Traders have a finite life expectancy, exiting the market with a constant hazard rate δ. With vanishing δ it is shown that an equilibrium exists and that the market converges to the efficient competitive outcome. Additional assumptions that can be found in the literature and that are favorable to the efficient outcome are not needed.

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1 Introduction

It is a common claim that decentralized markets clear and become efficient as "frictions" vanish. Typical decentralized markets are the markets for housing, used cars, and labor. To support the claim of market clearance, economists commonly refer to the following informal story: Suppose prices in a market are constantly too high. Then some sellers must be *rationed*, i.e. trade less than they desire. This gives them an incentive to decrease their prices to increase their trading volume by making their offer acceptable to more buyers. This incentive upsets any equilibrium candidate in which prices are too high. The story relies on two ingredients: rationing of sellers and the existence of additional buyers at lower prices. Here, we will show that one can indeed model the story formally and show that the two ingredients are indeed sufficient for a decentralized market to become efficient once frictions vanish. In particular, additional favorable assumptions for the competitive outcome that are not part of the story but that were made in the existing literature on decentralized trading are not needed for the convergence result. The results of this paper suggest that convergence to efficiency is a robust property of decentralized trading that is largely independent of the exact trading rules.

We use the following dynamic matching and bargaining game following Gale (1987): There is a large pool (a continuum) of traders with a finite expected lifetime. These traders want to sell or buy one unit of an indivisible good. Sellers have zero costs[1] and buyers have valuations $v \in [0, 1]$ and these types are private information. In every period, all sellers and buyers from the pool are matched into pairs. The seller in each pair makes a price offer p to the buyer. If the buyer accepts the price, the pair exits the market. If he declines, the match is broken up and both traders return to the pool and wait to be rematched with new partners in the next period. While waiting, traders exit with a constant hazard rate δ. The hazard rate introduces costs of waiting for better offers. At the end of every period an equal mass of new buyers and sellers flows new into the market.

With vanishing frictions, that is with $\delta \to 0$, the trading outcome converges towards efficiency and all trade happens close to the price $p = 0$. An equilibrium exists in this game if δ is not too large.[2]

The intuition follows the one given in the introductory paragraph: If all sellers to set a price $p' > 0$, then some of them will be rationed. Rationing means that the probability to trade sometimes during their life must be strictly smaller than one. However, setting a slightly lower price p'' would allow them to trade in addition with those buyers with $v \in [p'', p']$ and increases the trading probability for sellers strictly by the share of these buyers. When δ becomes small and sellers can sample buyers more and more often, then in the limit a seller will become almost certain to find a buyer with $v \in [p'', p']$ who accepts his price p'' slightly below p'. This incentive to decrease prices upsets every equilibrium candidate with $p' > 0$.

[1] This assumption can be relaxed, see section 3.4.

[2] Existence is a nontrivial problem in this class of models, see e.g. the discussion in Satterthwaite and Shneyerov (2005, p. 11).

The simplification we reach in our proof relative to the ones found in the literature is mainly due to our findings about the relation between the evolution of the population and the incentive compatibility constraints. We consider the provision of the proof technique to be a major contribution of this paper.

However, the convergence result is not immediate. One has to account for the possibility that sellers do not set a single price but rather mix over a range of prices and that they do it just in the right way to give incentives for buyers to accept high prices (by setting high prices most of the time), while balancing the distribution of buyers to avoid accumulation of low valuation buyers (by setting low prices some of the time). The main part of the proof consists in showing that this is not possible for vanishing δ.

Diamond (1971) shows that even with small trading frictions sellers can have considerable market power, which is reflected in the well known Diamond paradox: Given any common price p' set by sellers and any arbitrarily small friction δ, buyers with $v > p'$ are willing to pay an additional premium of $\delta(v - p')$ to save the waiting costs. This allows all sellers to mark up the price p' and provides incentives against the competitive outcome. Because in Diamond's model valuations are homogeneous, sellers cannot reach additional buyers by decreasing their prices. Therefore, prices are equal to their monopolistic level.

Here this is different because buyers are heterogeneous: When we show existence of an equilibrium, we prove that for each δ there exists a price level $p^*(\delta)$ at which the incentives for sellers to mark up the price by the waiting costs are just balanced by the incentives to decrease the price to reach additional buyers. With decreasing δ, the potential premium $\delta(v - p^*(\delta))$ decreases while the incentives to reach additional buyers remain, therefore $\lim_{\delta\to 0} p^*(\delta) = 0$.

Giving sellers all the bargaining power is our crucial departure from the literature on dynamic matching and our model is standard in most other respects. The basic framework of the steady state model with heterogeneous agents, pairwise matching, and an exogeneous inflow of agents was introduced by Gale (1987). Recent models like Inderst (2001) and Satterthwaite and Shneyerov (2004, 2005) extended this framework to asymmetric information.[3] Following McAffee (1993) and Satterthwaite and Shneyerov (2005) we introduce an exogeneous exit rate. Given the exit rate, we drop time discounting as an additional friction. Also, we assume that all inflowing buyers enter the pool while the existing literature assumes an additional participation decision of inflowing buyers to become active part of the pool. For a discussion, see section 4.1.

We start with a section introducing the model, then a statement and proof of our main theorem, followed by an extensive discussion of our modelling choices and extensions. In particular, we show which additional assumptions the existing literature makes to ensure convergence and how these assumptions translate into forces towards market clearance.

[3] Moreno and Wooders (2001) also analyse convergence with asymmetric information but in a non-stationary market with one-time inflow and only two types.

2 Model

2.1 The Players

Buyers want to buy one unit of the good and their valuation is $v \in [0,1]$. The mass of inflowing buyers with a valuation $v \leq v'$ is $G(v')$; $G(\cdot)$ is convex and twice continuously differentiable with a strictly positive density $g(\cdot)$ on $[0,1]$, so we can define $\min_v g(v) \equiv g_l > 0$.[4] Sellers are endowed with one unit of a good, for which they have no use value, so costs are zero. We index them by $c \in [0,1]$ and let the mass of inflowing sellers with an index $c \leq c'$ be $F(c') = c'$, i.e. the index is uniformly distributed. This allows us to define distributional strategies in c.[5] Valuations v and index c are private information.

The instantaneous payoffs of trading at a price p are $\pi(p,c) = p$ for the seller and $u(v,p) = v - p$ for the buyer. Payoffs are zero when traders exit without trading. Traders maximize their expected payoffs. Every period they exit with probability δ which makes them inpatient, i.e. the "hazard rate" δ acts in a similar way as discounting. Finally, information of traders is restricted to their own trading history.

At the beginning of each period the population in the market consists of a continuum of sellers and buyers, each of equal size. All traders are matched into pairs. In each pair the seller announces a price p and then the buyer announces whether he accepts or rejects the offer. If he accepts, both receive their payoffs π, u. Those players who have traded are removed from the market. Of those who did not trade another share δ is removed (discouraged), before new players flow into the market. The inflow of buyers and sellers has mass one. With the inflow of new traders the period ends and the next starts according to the same rules. The masses of sellers and buyers are identical in each period.[6]

To summarize, we have the following timing within each period:

1. All sellers and buyers are matched into pairs
2. The seller offers price p
3. The buyer accepts or rejects
4. If he accepts, both are removed from the market
5. If he rejects, each is removed with probability δ
6. New sellers and buyers arrive, each with mass one

[4] Convexity is only needed for our constructuve proof of existence, not for the characterization.

[5] The reader is invited to think of c as a cost variable which purifies the equilibrium. We will indeed introduce cost heterogeneity later and show that convergence still holds, see section 3.4.

[6] The assumption of equal inflows of buyers and sellers can be relaxed, see the note on page 12.

The payoff to a seller in a steady state who sets the same price p every period can be derived as follows: Denote by $D(p)$ his probability to trade at p in a given period. Then his expected life-time profit is the infinite sum of the expected profit per period t, $pD(p)$ times the probability to live for t periods, which consist of the probability not to die before t, $(1-\delta)^t$, and of the probability not to trade before t, $(1-D(p))^t$. Together:

$$\begin{aligned}\Pi(p) &= \sum_{t=0}^{\infty} pD(p)(1-D(p))^t(1-\delta)^t \\ &= p\underbrace{\frac{D(p)}{1-(1-D(p))(1-\delta)}}_{\equiv q(p)} \\ &= q(p)\,p\end{aligned}$$

where $q(p)$ is the *ultimate* probability to trade sometimes during the seller's life when setting p every period. Similarly, $(1-q(p))$ is the probability of such a seller to exit before being able to trade. Sellers use price strategies $p(c)$ which are are weakly increasing in the index c (possibly reordering the index appropriately).

Similarly, the expected payoff to a buyer is the infinite sum of his per period payoff, weighted by the probability to reach each period. It is well known that the optimal search strategy of a buyer who samples prices from a known and constant distribution of prices can be described by a threshold, the reserve price r, such that he accepts a prices p if and only if $p \leq r$, see McMillan and Rothchild (1994). The expected per period payoff of a buyer with a reserve price r depends on the expected price offer, $E[p|p \leq r]$ and the probability to receive an acceptable offer $p \leq r$, denoted $S(r)$.[7] His expected overall payoff is:

$$\begin{aligned}U(r,v) &= \sum_{t=0}^{\infty} S(r)(1-S(r))^t(1-\delta)^t(v-E[p|p \leq r]) \\ &= \underbrace{\frac{S(r)}{1-(1-S(r))(1-\delta)}}_{\equiv w(r)}(v-E[p|p \leq r]) \\ &= w(r)(v-E[p|p \leq r])\end{aligned}$$

where $w(r)$ is the *ultimate* probability to trade for a buyer who accepts all prices $p \leq r$ and $(1-w(r))$ is the probability that a buyer with reserve price r exits the market before trading. Let $V(v) = \max_r U(r,v)$ be the maximized expected lifetime payoff. Given the stationarity of the problem, the solution $r(v)$ must satisfy:

$$r(v) = v - (1-\delta)V(v) \qquad (1)$$

that is at the reserve price $r(v)$ buyers must be indifferent between acceptance and rejection, so $v-r(v) = (1-\delta)V(v)$ and buyers accept all prices p such that their payoff from accepting the price is (weakly) larger than their continuation payoff from rejection which is $(1-\delta)V(v)$.

[7] Define $E[p|p \leq r] = r$ if $S(r) = 0$.

Behind the assumption that buyers use such a reserve price in equilibrium is the notion of sequential rationality. First, we assume that buyers do not update their belief about the distribution of prices after observing off-equilibrium prices. Second, given their beliefs buyers' acceptance decisions must be optimal. In particular, buyers' types v who will never trade on the equilibrium path and have zero expected payoffs, accept all (off-equilibrium) prices $p \leq v$ that give them positive payoffs. In a Nash-equilibrium these assumptions do not need to hold.[8] [9]

2.2 Steady State Conditions and Equilibrium

The market is in a steady state if the mass of traders, denoted by T, the distribution of offered prices, denoted $S(\cdot)$, and the distribution of reserve prices $D(\cdot)$ is constant, where $D(p)$ is the share of buyers in the pool with $r(v) \geq p$ and $S(p)$ is the share of buyers offering a price $p' \leq p$. Thus, $D(p)T$ is the mass of buyers accepting a price p. We will give necessary and sufficient conditions for these parameters to describe a steady state.

The evolution of the traders' population in the market is governed by the in- and outflow processes: Suppose that the density of buyers with valuations v was $T_{t-1}\phi^b_{t-1}(v)$ in the previous period. In period t, $T_t\phi^b_t(v)$ will consist of the new inflow $g(v)$ and those buyers who remained. The share of buyers v who remain in the population consists of those who did not trade and who did not become discouraged: $(1 - S_{t-1}(r(v)))(1-\delta)$, that is[10]

$$T_t\phi^b_t(v) = g(v) + T_{t-1}\phi^b_{t-1}(v)(1 - S_{t-1}(r(v)))(1-\delta) \tag{2}$$

The pool is in a steady state if $\phi^b_t = \phi^b_{t-1}$, which is the case if the outflow and the inflow matches. Reformulating (2) gives:

$$\phi^b(v) = \frac{g(v)}{T(S(r(v)) + (1 - S(r(v)))\delta)} \tag{3}$$

Integrating over $\phi^b(v)$ yields $D(p)$: [11]

$$D(p) = \int_{\inf\{v|r(v)\geq p\}}^{1} \frac{g(v)}{T(S(r(v)) + (1 - S(r(v)))\delta)}dv \tag{4}$$

For sellers' densities ϕ^s we have the following flow conditions:

$$T_t\phi^s_t(c) = T_{t-1}\phi^s_{t-1}(c)(1 - D_{t-1}(p(c)))(1-\delta) + 1 \tag{5}$$

[8] In a Nash equilibrium we could sustain an equilibrium in which all sellers offer some arbitrary price p^n: Because without the first assumption, we could postulate that whenever a seller deviates from p^n to offer a lower price, buyers belief that from now on all sellers offer prices 0, so that they reject the offer. Without the second assumption we could assume that buyers with valuations $v < p^n$ always reject all prices below p^n. Then again sellers would have no incentives to decrease their prices.

[9] Serrano (2002) assumes that traders use a double auction in which both traders submit their bids simultaneously and trade happens at the midpoint of the bids, provided the seller bids below the buyer. As Serrano notes, the simultaneity of bidding precludes the use of sequential rationality and non-competitive equilibria may exist.

[10] In accordance with the literature on dynamic matching games with a continuum of players we take these evolution conditions of the population as fundamentals of the model instead of deriving it from individual players' matching and trading probabilities (the "law of large number convention").

[11] The integral is well defined because the denominator is monotone.

from where we can derive a similar condition for $\phi^s(c)$ in the steady state and derive $S(p)$ by integrating over $\phi^s(c)$:

$$\phi^s(c) = \frac{1}{T\left(D\left(p\left(c\right)\right)+\left(1-D\left(p\left(c\right)\right)\right)\delta\right)} \tag{6}$$

$$\Rightarrow S(p) = \int_0^{\sup\{c|p(c)\leq p\}} \frac{1}{T\left(D\left(p\left(c\right)\right)+\left(1-D\left(p\left(c\right)\right)\right)\delta\right)}dc \tag{7}$$

In addition, for $S(\cdot)$ and $D(\cdot)$ to be proper cumulative distribution functions it must be that

$$S(1) = 1 \tag{8}$$

$$D(0) = 1 \tag{9}$$

Evaluating (7) at $p=1$ this implies for T:

$$T = \int_0^1 \frac{1}{\left(D\left(p\left(c\right)\right)+\left(1-D\left(p\left(c\right)\right)\right)\delta\right)}dc \tag{10}$$

The conditions for $S(\cdot)$, $D(\cdot)$, and T given in this section are necessary and sufficient for the market to be in a steady state. They are necessary, because we derived them as direct implication of the steady state. And they are sufficient: Given $S(\cdot)$, $D(\cdot)$, and T we can derive $\phi^b(\cdot)$ and $\phi^s(\cdot)$ using (3) and (6). By their definitions, these $\phi^b(\cdot)$ and $\phi^s(\cdot)$ lead to a steady state. And if conditions (4), (7), (8), and (9) hold, $D(\cdot)$ and $S(\cdot)$ are indeed the shares derived from $\phi^b(\cdot)$ and $\phi^s(\cdot)$.

Given the steady state conditions we are ready to define an equilibrium:

Definition 1 *A steady state equilibrium is described by the pair $p(\cdot)$ and $r(\cdot)$, the pair of steady state distributions $S(\cdot)$ and $D(\cdot)$, and the mass T of traders, such that*

- $p(c) \in \arg\max \Pi(p) \quad \forall c$
- $r(v) = v - (1-\delta)V(v) \quad \forall v$
- *the steady state conditions (4), (7),(8) and (9) hold.*

We can show that such an equilibrium exists for δ not too large:

Theorem 1 *There exists a δ_g such that for every $\delta \leq \delta_g$ there exists a steady state equilibrium.*

Proof: See Appendix

3 Main Results

We want to characterize the set of equilibria with $\delta \to 0$. For this, we will look at a strictly decreasing sequence of exit rates $\{\delta_k\}_{k=1}^{\infty}$ with $\lim_{k\to\infty} \delta_k = 0$ and $\delta_1 \leq \delta_g$. We know that for each δ_k there exists at least one equilibrium and we select one equilibrium for each k, which gives us a sequence $\{(p_k(\cdot), r_k(\cdot), S_k(\cdot), D_k(\cdot), T_k)\}_{k=1}^{\infty}$.

We will show that for every such sequence the support of prices at which trade happens shrinks to a singleton, i.e. a "law of one price" holds. This is the first theorem. Given that this "law" holds, we then show that the "one price" must be the Walrasian price of zero, which is stated in the second theorem. And finally, given that all sellers are willing to trade at a price of zero, it follows that the equilibria become efficient, i.e. all buyers and sellers are actually able to trade, which is stated in the last theorem.

Before we prove the theorems, we introduce some notation and make some general observations. To define the support of trading prices, we define h_k to be the highest accepted price, $h_k = r_k(1)$ and l_k to be the lowest offered price, $l_k = p_k(0)$. Then, for every k and in every equilibrium sellers must make strictly positive profits: Because even if all other sellers were to offer a price of zero, most buyers are willing to pay some $p' > 0$ to avoid delay given $\delta_k > 0$. This means that all sellers offer prices $p_k(c) \leq h_k$. By their definitions, $w_k(\cdot)$ is monotone increasing and $q_k(\cdot)$ is decreasing. In addition, reserve prices $r_k(\cdot)$ are increasing in buyers valuations (by increasing differences of $U(r, v)$).

3.1 The Law of One Price

Theorem 2 *For every sequence of sequential steady state equilibria with* $h_k \equiv r_k(1)$ *and* $l_k \equiv p_k(0)$:

$$\lim_{k\to\infty} (h_k - l_k) = 0$$

We prove the theorem by contradiction, i.e. we want to show that it cannot be that along a (sub)sequence indexed by k', $(h_{k'} - l_{k'}) \geq C$ for some $C > 0$. Without loss of generality, the subsequence k' is k itself. We will introduce two lemmas that state the implication of this hypothesis and then show that these implications lead to a contradiction.

Before giving the lemmas in detail, we provide some intuition. With $\delta_k \to 0$, no buyer would accept $h_k \geq l_k + C$ if all sellers would offer a common price l_k. Therefore prices must be dispersed. Now, take some $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$. As said in the introduction, prices can be dispersed when $k \to \infty$ only if two opposing conditions hold: $r_k(1) = h_k$ requires that low prices $p \leq h_k - \varepsilon_1$ are offered rarely enough to make waiting for good prices unprofitable for the buyer. At the same time, $p_k(0) = l_k$ requires that buyers who have a reservation price above $l_k + \varepsilon_2$ trade frequently enough so that they do not accumulate in the pool and the seller would not want to deviate from l_k to $l_k + \varepsilon_2$. The following two lemmas correspond to these conditions: In the first lemma we conclude from the scarcity of low prices, that there are some buyers who accept prices above $l_k + \varepsilon_2$, while they do not find such prices frequently. Given that these buyers trade rarely, the second lemma shows that these buyers accumulate and have a strictly positive share in the pool. Finally, we conclude that sellers would indeed deviate from $p_k(0) = l_k$ to $l_k + \varepsilon_2$. So the two conditions for price dispersion cannot be fulfilled simultaneously and we have $(h_k - l_k) \to 0$.

The first lemma states that for some $\varepsilon_1 > 0$ the ultimate trading probability for the type $v = h_k - \varepsilon_1 < h_k$ is bounded away from one for k large enough. In addition, the type $v = h_k - 2\varepsilon_1$ accepts a price above the very best offer, i.e. $r_k(h_k - 2\varepsilon_1) > l_k$:

Lemma 1 *Given $(h_k - l_k) \geq C\ \forall k$, there exist some $\varepsilon_1 > 0$,$\varepsilon_2 > 0$, $\bar{w} < 1$, and $\bar{k}$ such that for all $k \leq \bar{k}$:*

$$\begin{aligned} w_k(r_k(h_k - \varepsilon_1)) &\leq \bar{w} < 1 \\ r_k(h_k - 2\varepsilon_1) &\geq l_k + \varepsilon_2 \end{aligned}$$

Proof: As said, for some buyer to accept $h_k = r_k(1)$ all lower prices must be rare. Otherwise, the buyer would just wait for a better offer. Buyers care only about the ultimate trading probability $w_k(r)$, so scarcity means that the ultimate trading probability with reservation prices $r < h_k$ must be small and in particular bounded away from one.

The two statements in the lemma follow as implications of this scarcity: First, the ultimate trading probability of buyers with valuations $v \in (l_k, h_k)$ is bounded away from one, because their reservation price must be below their valuation, $r_k(v) \leq v$ and so $r_k(v) < h_k$. From before, we know that $r < h_k$ implies that $w_k(r)$ is bounded away from one. Formally, take any $\varepsilon_1 \in \left(0, \frac{1}{3}C\right)$ and suppose the first statement would not hold. Then $\limsup_{k\to\infty} w_k(r_k(h_k - \varepsilon_1)) = 1$ and by $r_k(h_k - \varepsilon_1) < h_k$, $U_k(r_k(h_k - \varepsilon_1), 1) \geq w_k(r_k(h_k - \varepsilon_1))(1 - (h_k - \varepsilon_1))$, $\limsup_{k\to\infty} U_k(r_k(h_k - \varepsilon_1), 1) \geq (1 - h_k + \varepsilon_1)$, so $\limsup V_k(1) \geq (1 - h_k + \varepsilon_1)$. However, $r_k(1) = h_k$ requires $V_k(1) \leq \frac{1-h_k}{1-\delta_k}$ by definition and therefore $\limsup_{k\to\infty} V_k(1) \leq (1 - h_k)$, leaving us with a contradiction.

Second, taking the type $v_k = h_k - 2\varepsilon_1 > l_k$, we can find a bound on his payoffs. This bound on payoffs is also a bound on the prices he accepts. We can bound $V_k(v) = \sup_r U(r, v) = w_k(r)(v - E_k[p|p \leq r])$ by using that l_k is the lowest offered price, i.e. $E[p|p \leq r] \geq l_k$ and by using the bound from before on $w_k(\cdot)$: Because the reservation price of $v_k = h_k - 2\varepsilon_1$ is lower than the reservation price of $v = h_k - \varepsilon$, the trading probability of v_k is lower as well and therefore $\bar{w}$ is also a bound on $w_k(r_k(v_k))$. Together, type v_k can trade with probability no more than $\bar{w}$ at a price not better than l_k, i.e. for all $k \geq \bar{k}$:

$$V_k(v_k) = \sup_r U(r, v_k) \leq \bar{w}(v_k - l_k) < v_k - l_k$$

So the payoff for type v_k is strictly smaller than the payoff from accepting a price l_k. Because of the strict inequality, acceptance of some offer even slightly above l_k will still make v_k better of, so for some ε_2, $r_k(v_k) \geq l_k + \varepsilon_2$. Formally, substituting the bound on V_k into the definition of $r_k(v)$ and rewriting terms yields $r_k(v_k) \geq l_k + (1 - \bar{w})(v_k - l_k) + \delta_k V_k(v_k)$ and defining $\varepsilon_2 \equiv (1 - \bar{w})(v_k - l_k)$ gives $r_k(v_k) \geq l_k + \varepsilon_2$ for all $k \geq \bar{k}$■

The next lemma states that the mass of buyers of a certain type in the pool is connected to their ultimate probability of trading. Intuitively, the less likely some buyers are to trade, the larger is their share because they stay longer in the pool. Specifically, if the trading probability $q_k(v)$ for a set of buyers is bounded away from one, we can find a lower bound on their mass in the pool.

The total mass of all buyers is maximal if no buyer ever trades. Then every period only a share δ_k exits while a mass 1 enters and the total mass is $\frac{1}{\delta_k}$. Now, suppose there is some interval $[x_k, y_k]$ of types which has some mass M in the inflow and every $v \in [x_k, y_k]$ has a trading probability of at most $\bar{w}$. Then every period at least a mass $(1-\bar{w})M$ enters the pool who will stay until they become discouraged with rate δ_k. Therefore the mass of types $v \in [x_k, y_k]$ is at least $\frac{(1-\bar{w})M}{\delta_k}$. From before, the total mass of buyers is $\frac{1}{\delta_k}$. So the share of types $v \in [x_k, y_k]$ is at least $\frac{(1-\bar{w})M}{\delta_k}\left[\frac{1}{\delta_k}\right]^{-1} = (1-\bar{w})M$ which is independent of δ_k and strictly positive.

And finally: If the reserve price of the lowest type $v = x_k$ is $r_k(x)$, then also all higher $v \in [x_k, y_k]$ accept $r_k(x)$. Therefore, the share of buyers accepting $p_k = r_k(x)$ is at least $(1-\bar{w})M$, i.e. $D_k(r_k(x)) \geq (1-\bar{w})M$. Stated formally:

Lemma 2 *If the ultimate probability of trading is bounded for some interval of buyers, $w_k(v) \leq \bar{w} < 1\ \forall v \in [x_k, y_k]$, then for $p_k = r_k(x)$:*

$$D_k(r_k(x)) \geq g_l(y-x)(1-\bar{w}) \tag{11}$$

Proof: Immediate from substituting $w(\cdot)$ into the definition of $D(\cdot)$ and using the bound $\bar{w}$■

Proof of Theorem 2: Now we connect the two lemmas to show that sellers would indeed want to deviate if $(h_k - l_k) \geq C\ \forall k$. By the first lemma, for all $v \in [h_k - 2\varepsilon_1, h_k - \varepsilon_1]$, $w_k(v) \leq \bar{w}$ so the interval $[h_k - 2\varepsilon_1, h_k - \varepsilon_1]$ meets the requirement of the second lemma, so for all $k \geq \bar{k}$:

$$D_k(r_k(h_k - 2\varepsilon_1)) \geq g_l\varepsilon_1(1-\bar{w}) > 0$$

Demand $D_k(p)$ is higher for lower prices, so by $r_k(h_k - 2\varepsilon_1) \geq l_k + \varepsilon_2$ from the first lemma, we have $D_k(l_k + \varepsilon_2) \geq D_k(r(h_k - 2\varepsilon))$ and therefore:

$$D(l_k + \varepsilon_2) \geq \varepsilon_1 g_l(1-\bar{w})$$

So a seller who offers a price $p'_k = l_k + \varepsilon_2$ has a strictly positive probability to be matched with a buyer who accepts his offer. This is true for all k and with $k \to \infty$, the seller can be sure to trade because he can wait indefinitely long:

$$\begin{aligned}\lim_{k\to\infty} q_k(l_k + \varepsilon_2) &= \lim_{k\to\infty} \frac{D_k(p'_k)}{1-(1-D_k(p'_k))(1-\delta_k)} \\ &= 1 \end{aligned} \tag{12}$$

But if a seller can be sure to trade at some $p' > l_k$, offering $l_k = p_k(0)$ cannot be optimal. So we have found a contradiction when $(h_k - l_k)$ does not vanish, proving the "law of one price".■

3.2 Convergence to the Walrasian Price

Theorem 3 *For every sequence of sequential steady state equilibria, prices converge to the Walrasian Price:*

$$\lim_{k\to\infty} p(c) = 0 \ \forall c$$

For the second theorem it is sufficient to prove $\limsup_{k\to\infty} h_k \to 0$, given $h_k \geq p_k(c) \geq 0$ Again, we prove the theorem by contradiction and assume that $\limsup_{k\to\infty} h_k = p^c > 0$. Under this assumption, there exists a convergent subsequence indexed by k' with $\lim_{k'} h_{k'} = \lim_{k'} l_{k'} = p^c$ using theorem (2) and without loss of generality k' is equal to k. We construct the contradiction through two lemmas, proven in the appendix.

For the first lemma, observe that along this sequence, for any $p' < p^c$, buyers with $v \in (p', l_k)$ do not trade but accumulate in the market. Hence, they accept a price $p' \leq v$ and have a strictly positive share in the pool. Therefore, with $\delta_k \to 0$, a seller offering any p' slightly below p^c can be sure to trade in the limit, i.e. $q_k(p') \to 1$. Therefore, every seller can guarantee himself a trade at a price close to his equilibrium price $p \in [l_k, h_k]$ and hence, sellers must be sure to trade in equilibrium:

Lemma 3 *If* $\lim_{k\to\infty} l_k = \lim_k h_k = p^c > 0$ *along some (sub-)sequence* k *it must be that:*

$$\lim_{k\to\infty} q_k(p_k(c)) = 1 \ \forall c \in [0,1]$$

On the other hand, if $p^c > 0$ then p^c is above the "market clearing level", i.e. the mass of buyers who are willing to trade is strictly smaller than the mass of sellers, i.e. $(1 - G(p^c)) < 1$ by $g(v) \geq g_l$. But the mass of buyers and sellers who trade in equilibrium must be equal. Therefore some of the sellers must be rationed, implying the second lemma:

Lemma 4 *If* $\lim_{k\to\infty} l_k = \lim_k h_k = p^c > 0$ *along some (sub-)sequence* k *it must be that:*

$$\lim_{k\to\infty} q(p_k(c)) \neq 1 \text{ for some } c$$

So $\limsup_{k\to\infty} l_k = 0$. Otherwise, the two lemmas give us a contradiction■

3.3 Efficiency

Theorem 4 *For every sequence of sequential steady state equilibria, the equilibrium outcome becomes efficient for* $\delta \to 0$*:*

$$\begin{aligned} \lim_{k\to\infty} w_k(v) &= 1 \ \forall v \in (0,1] \\ \lim_{k\to\infty} q_k(c) &= 1 \ a.e. \end{aligned}$$

Proof: For the last theorem, just note that $h_k = r_k(1) \to 0$ implies that

$$\lim_{k\to\infty} r_k(v) = 0 \ \forall v$$

and by definition of $r_k(v) = v - (1-\delta) V_k(v)$:

$$\lim_{k\to\infty} V_k(v) = v$$

and this implies that all types $v > 0$ trade with probability converging to one, $w_k(v) \to 1$. And if the mass of trading buyers in the inflow becomes one, the mass of trading sellers must become one as well, so $q_k(c) \to 1$■

3.4 Heterogeneous Cost

We can show that our characterization result holds also with cost heterogeneity. For this we do not need to alter the proof. Let costs $c \in [0,1]$ be distributed according to some cdf $F(\cdot)$ with continuous density $f(c)$.[12] Sellers' profits are given by

$$\Pi(p,c) = q(p)(p-c)$$

Denote by p^w the market clearing price such that the masses of buyers and sellers who would be willing to trade at p^w are equal:

$$F(p^w) = 1 - G(p^w)$$

Assume that this price is interior, $p^w \in (0,1)$. Now we can proof that all equilibria must be market clearing as well: The reader might want to check that we did not use the sellers' costs in the first part of the proof. We only used that $q_k(p') \to 1$ for some $p' > l_k$ implies that l_k cannot be a profit maximizing price, independent of the the cost of the seller. So the "law of one price" holds, i.e. $(r_k(1) - p_k(0)) \to 0$ even with heterogeneous costs.

For the second part, we can state an equivalent to lemma 3. Let $p^c = \limsup p_k(0) = \limsup r_k(1)$. We reason similar to before that it must be that $q_k(p_k(c)) \to 1$ for all $c > p^c$ and $w_k(r_k(v)) \to 1$ for all $v < p^c$. Again, by deviating to any lower price $p' < p^c$ sellers could be sure to trade with buyers with $v \in [p', p^c]$ who have a strictly positive share in the pool. For buyers, $l_k \to h_k$ implies that the expected trading price is approximately h_k for all $v > p^c$. If at this price the ultimate trading probability $w_k(h_k)$ is strictly smaller than one, the payoff of type $v = 1$ will be strictly smaller than $(1 - h_k)$ and $v = 1$ would accept prices $p' > h_k$, contradicting the definition of h_k.

Taken together, the requirement that all buyers with $v > p^c$ and all sellers with $c < p^c$ must be able to trade implies immediately that p^c must be the market clearing price p^w. Otherwise, one of the trading sides would have to be rationed. The proof of convergence is thus a straightforward extension, building upon the same intuition as the proof with homogeneous sellers.

[12] Our model can capture unbalanced inflows as follows: Suppose that the mass of one side is a and the mass of the other is b. W.l.o.g., $a \geq b$ and we can normalize the larger side to one and the smaller to $\frac{b}{a}$. Now, if the smaller side is the buyers' side, we can "fill" the inflow with buyers with $v = 0$ until their total mass is also one (and similarly for sellers with $c = 1$). Now our proof holds, observing that p^w being interior implies that we do not need to worry about non-differentiability at the extremes.

Note however, that our constructive proof of existence does not work for heterogeneous sellers. While we needed to construct only one single price with homogeneous sellers, heterogeneity would require us to construct a full price schedule, one price $p(c)$ for each c. For this reason, we restricted our analysis to the homogeneous seller case.

4 Discussion

4.1 Existing Literature and Other Market Clearing Forces

We show that the incentives to reach out for additional buyers is all that is needed to guarantee the efficient outcome. In the existing literature, one can find two assumption that give sellers additional incentives to decrease their prices. In the main strand of the literature[13] within each pair *both* sides of the market have a chance to make an offer. In recent models only sellers can make the offer but in addition, buyers have the chance to simultaneously receive several offers from *competing sellers*.[14]

For illustration, take the basic model with homogeneous sellers where the market clearing price is zero. Suppose that sellers would set a common price $p' > 0$ even for small δ. At this price, not all sellers will be able to trade so their ultimate trading probability is bounded away from one. This implies that their profits are strictly smaller than p'. Therefore, a seller would accept price offers p'' from buyers which are considerably less than p'. Given δ close enough to zero, buyers now have the chance to trade at a price p'' in the future when it is their turn to propose. This makes them unwilling to accept p' and gives strong incentives to seller to propose price below p' themselves.

In a model with a positive chance that a seller competes directly with the offer of another seller, we have another pressure on prices: given the common price level p', an incremental decrease below p' increases the trading probability strictly whenever p' is strictly positive. This is the additional incentive for sellers to decrease prices when directly competing with other sellers.

Of course, in both kinds of models it has to be proven that there is no price dispersion. But our proof of the law of one price can be applied to both situations to yield this conclusion.

So we can distinguish three forces towards the competitive price level, the incentive to reach additional buyers analyzed here, the incentive to undercut the competitors, and the better outside option for buyers if they have some bargaining power. Rationing on the sellers' side is their common starting point. But there is an important qualitative difference between the three forces: While the existence of additional buyers at lower prices is a basic feature shared by most markets, the possibility of directly competing offers or the distribution of bargaining power between traders depends on the fine details of the situation and the model. By showing that the convergence results do not depend on

[13] Mortensen (1982), Rubinstein&Wolinsky (1985) and Gale (1986, 1987) initiated the analysis for complete information. Serrano (2002), Moreno&Wooders (2002) and Inderst (2001) extended it towards incomplete information.

[14] See Satterthwaite and Shneyerov (2004, 2005). Though without convergence results, this structure can be found in the literature on nosiy search, e.g. in Burdett and Judd (1983) and the literature on dynamic labor search, e.g. in Cahuc, Zylberberg (2004).

these latter details, we provide evidence for the robustness of the prediction that decentralized trading is efficient.

Clearly, whenever the first market clearing force is present, convergence is only strengthened when we introduce additional market clearing forces. Therefore, our result extend to a setting where sellers are matched with a random number of buyers and hold optimal auctions. This is a slight generalization of Satterthwaite and Shneyerov (2005) who restrict sellers to set competitive (hidden) reserve prices and also restrict the set of (sequences of) equilibria.

When modelling the evolution of the pool of traders we follow McAfee (1993) and assume that there is some death rate, letting the rate converge to zero. This seems a natural condition when analyzing steady states. The main alternative would be to follow Gale (1987) and later authors who assume that traders are literally infinitely lived. However, these authors need to assume that not all traders enter in order to ensure existence of a steady state with finite populations. Under the assumption of an additional entry decision, we could have sustained equilibria in which only buyers above some threshold $p^c \in (0, 1)$ enter: Given that only such buyers are available in the pool, sellers would have no incentives to decrease their price below such a threshold p^c and all p^c could be sustained. But such an equilibria might be considered unstable because it relies on a literal impossibility of sellers to reach those inactive buyers with valuations $v < p^c$ who accumulate outside the market. If sellers could e.g. advertise their prices at some cost k per ad to buyers and $k \to 0$ we could restore the convergence result. The assumption of exogenous entry in our model might therefore be considered as shorthand for a more complex model that has infinitely lived agents, endogeneous entry, and advertising.

Also, the alternative assumption of infinitely lived agents has direct implications for the set of possible equilibria by introducing a sort of "zero profit condition" for sellers: To ensure a steady state, the inflows of buyers and sellers must be identical with infinitely lived agents. Then either all buyers have a strictly positive probability of trading. In this case, sellers must offer prices close to zero, make zero profits, and the equilibrium outcome is close to efficiency. Otherwise, not all buyers enter. Then some sellers must choose to stay out of the market to balance the inflows of sellers and buyers. But sellers will stay out only if their equilibrium profits are zero.

So already by the seemingly technical assumption of infinitely lived agents we know that sellers must make zero profits, independent of further strategic considerations and even away from the limit. Given this observation, it is then possible to show again that prices are zero and the equilibrium outcome must be efficient with vanishing frictions, see Satterthwaite, Shneyerov (2004). Illustrating these stronger implications for convergence, in Lauermann (2005) we argue that because of this implicit "zero profit condition", decentralized trading can be efficient with literally infinitely lived agents even when it is not so with finitely lived agents.

Finally, our model might be considered as going back to the very roots of the dynamic matching and bargaining literature. Already in the seventies and long before Wolinsky and Rubinstein (1985) published their seminal paper on dynamic matching and bargaining, Butters began working on a model where only sellers make price offers and other market clearing forces are absent. He also allowed for non-stationary inflows, a more general matching technique and two-sided heterogeneity. Thus, relative to later authors building upon his unfinished typescript, his analysis was much more ambitious. But it remained cumbersome and was not published. Although we have not reached the level of generality envisioned by Butters yet, this note might be a step in the direction of the analysis he had in mind.

4.2 Full Information

In this paper we assume that sellers do not observe the buyers' valuation before making an offer. In Lauermann (2005b) we explore a situation where sellers can perfectly observe the types. Surprisingly, we find that with symmetric information the outcome does not converge to the competitive outcome but stays bounded away from it. The reason is that asymmetric information shields the buyers from price discrimination by sellers. However, we also show that whenever sellers receive a imperfect signal about v with an arbitrarily small amount of noise, the convergence result is restored.

Basically, with asymmetric information sellers who want to trade with buyers with low valuations must set low prices for *all* buyers. This is not so with symmetric information. Therefore, rationing at a common high price p^c translates into incentives for price decreases only under asymmetric information. Under symmetric information, sellers can target lower valuation buyers individually.

4.3 Conclusion

In our analysis, we checked the robustness of the market clearing hypothesis and the underlying intuition. We were able to prove asymptotic efficiency of decentralized trading by appealing to the basic economic forces of rationing of traders at non-market clearing prices.

While we were able to prove efficiency in a setting with extreme market power, the existing literature suggests that there are important conditions which are not checked in this paper: Gale (2000) suggests in his book that a complete model of decentralized trading should include the problem of coordination across markets (and time) for different products, e.g. the market for labor and the market for consumption goods. This problem is central to a market economy and it is not even remotely included in the existing models.[15] Second, Wolinsky's (1990) analysis suggests that decentralized markets might not work efficiently with aggregate uncertainty. Again, robustness to aggregate uncertainty is central to the efficiency of market economies and it is not included in the analysis given here.

A Existence and Characterization of a Pure Strategy Equilibrium

Here we will derive and characterize an equilibrium to prove:

[15] However, in recent auction theory the static version of the coordination problem receives much attention in the discussion of demand for complementary goods.

Proposition 5 *There exists a* δ_g *such that for every* $\delta \leq \delta_g$ *there exists a sequential steady state equilibrium in which all sellers set a common price* $p^*_\delta \in (0,1)$ *and this price converges to zero:* $\lim_{k\to\infty} p^*_\delta = 0$.

Proof: To derive the equilibrium, we start with the hypothesis that all sellers offer a common price p^* and use the first order condition to characterize the equilibrium. The profit of a seller offering p^* every period is the sum of the expected profit of the current period, $D(p^*)\,p^*$ plus the expected future profit $q(p^*)\,p^*$weighted by the probability to be active in the next period $(1-\delta)(1-D(p^*))$:

$$\pi(p^*) = D(p^*)\,p^* + (1-\delta)(1-D(p^*))\,q(p^*)\,p^*$$

It is straightforward to derive the ultimate probability of trading $q(p^*)$, by observing that exactly a mass $1-G(p^*)$ of the inflowing buyers trades. Because the mass of trading buyers and sellers must be identical, it follows that a mass $1-G(p^*)$ of the inflowing sellers trade as well, i.e.

$$q(p^*) = 1-G(p^*)$$

Now, we look at the profit of deviating from p^* to p once today, keeping the common price p^* from tomorrow onward:

$$\pi(p|p^*) = D(p)\,p + (1-\delta)(1-D(p))\,q(p^*)\,p^*$$

With $p^b \equiv \arg\max_p \pi(p|p^*)$, we want to show that there exist a p^* such that $p^b = p^*$. For p^b to be a solution to $\arg\max_p \pi(p|p^*)$ it is sufficient that the first order condition holds:

$$\frac{d}{dp}\pi\left(p^b|p^*\right) = 0$$

and that $\pi(p|p^*)$ is globally concave given p^*:

$$\frac{d}{dp^2}\pi(p|p^*) < 0 \text{ for } p \leq r(1)$$

Because a seller can make strictly positive profits for every p^*, we cannot have boundary solutions at $p=0$ or $p \geq r(1)$ where profits would be zero.

To derive the profit function we need to derive the "demand function" $D(\cdot|p^*)$. In the following we drop its dependency on p^* for better readability. The first important observation regards the buyers' reserve price $r(v)$. From (1) we have $r^+(v) = v-(1-\delta)(v-p^*)$ for $v \geq p^*$ and $r^-(v) = v$ for $v \leq p^*$. We can define its inverse $v^+(p) = \frac{p-p^*(1-\delta)}{\delta}$ as the type for which $r^+(v) = p$. Note the regime change at p^* which we capture by the superscripts.

Using the steady state conditions we can now derive $D(\cdot)$ and its derivatives:

$$\begin{aligned}
D^+(p) &= \frac{1}{T}\int_{v^+(p)}^{1} g(v)\,dv & d^+(p) &= -g\left(v^+(p)\right)\frac{1}{\delta T} & \text{for } p \geq p^* \\
D^-(p) &= 1-\frac{G(p)}{T\delta} & d^-(p) &= -g(p)\frac{1}{\delta T} & \text{for } p \leq p^* \\
T &= \frac{G(p^*)}{\delta} + (1-G(p^*)) & & &
\end{aligned}$$

These conditions are derived as follows: For the first line, we use the identity of the outflow of buyers with $v \geq p$ and their inflow. Because all buyers with $v \geq p$ trade immediately, the mass of their outflow is $TD^+(p)$ which has to match the inflow, which is $\int_{v(p)}^1 g(v)\,dv$. So we have

$$TD^+(p) = \int_{v(p)}^1 g(v)\,dv$$

and reformulating gives $D^+(\cdot)$. For the derivative note that $\frac{d}{dp}v^+(p) = \frac{1}{\delta}$. Therefore $\frac{d}{dp}D^+ = -g(v^+(p))\frac{1}{\delta T}$. For prices $p < p^*$ note that buyers with $v < p^*$ never trade, so the mass of their outflow comes only through exit: $T(1-D^-(p))\delta$ which must be equalized with their inflow $G(p)$: $T(1-D^-(p))\delta = G(p)$, reformulating gives the second line and its derivative. For T, note that the total mass T is equal to the accumulating mass of buyers with $v \leq p^*$, $\frac{G(p^*)}{\delta}$ plus the mass of the inflow of new buyers with $v \geq p^*$, $(1-G(p^*))$.

Because $d^+(p^*)$ is equal to $d^-(p^*)$ we have continuity of the full marginal "demand" function $\frac{d}{dp}D(p)$. The first derivative of $\pi(p|p^*)$ writes now as:

$$\frac{d}{dp}\pi(p|p^*) = d(p)\,p + D(p) - d(p)(1-\delta)\,q(p^*)\,p^*$$

Substituting D and d and solving for p^* shows that the first order condition $\frac{d}{dp}\pi(p|p^*)|_{p=p^*} = 0$ implies:

$$\frac{(1-G(p^*))\,\delta}{(G(p^*)+\delta(1-G(p^*)))\,g(p^*)} = p^* \qquad (13)$$

From continuity of $K(p^*,\delta) \equiv \frac{(1-G(p^*))\delta}{(G(p^*)+\delta(1-G(p^*)))g(p^*)}$ in p^* and the fact that $K(0,\delta) > 0$ and $K(1,\delta) = 0$ it follows that a solution exists.

Now fix a solution p^*_δ for every δ and look at its behavior. We want to show that p^*_δ converges to zero, $\lim_{\delta\to\infty} p^*_\delta = 0$. This can be derived from inspection of $K(p^*,\delta)$: Suppose p^* would not converge and $p^*_{\delta'} \geq c > 0$ for a subsequence δ'. Then $\lim_{\delta'\to\infty} K(p^*,\delta') = 0$ for $p^* \geq c$:

$$\begin{aligned}\lim_{\delta'\to\infty}\frac{(1-G_{\delta'}(p^*))\,\delta'}{\left(G_{\delta'}(p^*)+\delta'(1-G_{\delta'}(p^*))\right)g_{\delta'}(p^*)} &= \frac{(1-G_{\delta'}(p^*))\,0}{(G_{\delta'}(p^*)+0)\,g_{\delta'}(p^*)}\\ &= 0 < p^*_{\delta'}\end{aligned}$$

which yields a contradiction to (13).

To check concavity on $p \in [0, r_k(1|p^*)]$, we derive $\frac{d^2}{dp^2}\pi(p|p^*)$ as follows:

$$\begin{aligned}\frac{d^2}{dp^2}\pi(p|p^*)^+ &= -g'\left(v^+(p)\right)\frac{1}{\delta}\left(p-(1-\delta)\,q(p^*)\,p^*\right) - 2g\left(v^+(p)\right) \quad for \quad p \in [p^*, r(1|p^*)]\\ \frac{d^2}{dp^2}\pi(p|p^*)^- &= -g'(p)\left(p-(1-\delta)(1-G(p^*))\,p^*\right) - 2g(p) \quad \text{for } p \in (0,p^*)\end{aligned}$$

For $p \in [p^*, r(1|p^*)]$, the first term is negative because $(p-(1-\delta)\,q(p^*)\,p^*) \geq 0$ and $-g'(v^+(p)) \geq 0$ by assumption. Therefore $\frac{d^2}{dp^2}\pi(p|p^*)^+ < 0$. For

$p \in (0, p^*)$, $\lim_\delta p^*(\delta) \to 0$, implies that $\lim_\delta r_\delta(1|p^*) = 0$, so $p \le r(1|p^*)$ and therefore $(p - (1-\delta)(1 - G(p^*))p^*)$ is close to zero for δ small enough. Because of $G(\cdot)$ being C^2, $g'(\cdot)$ is continuous on $[0,1]$ and so $-g'(p)$ is bounded. Therefore $\lim_{\delta \to 0} -g'(p)(p - (1-\delta)(1 - G(p^*))p^*) = 0$ and we can find some δ_g such that for all $\delta \le \delta_g$, $-g'(p)(p - (1-\delta)(1 - G(p^*))p^*) < 2g_L \le 2g(p)$. Therefore, $\frac{d^2}{dp^2}\pi(p|p^*)^- < 0$ for $\delta \le \delta_g$ ■[16]

B Algebra

B.1 Proof of Lemma 3

For given $\lambda > 0$, if $l_k \to p^c > 0$, there is some $\bar{k}_2(\lambda)$ such that $p' \equiv p^c - \frac{\lambda}{2} < l_k$ for all $k \ge \bar{k}_2(\lambda)$. Therefore $w_k(v) = 0$ for all $v \in \left(l_k - \lambda, l_k - \frac{\lambda}{2}\right)$ and $k \ge \bar{k}_2$. Using lemma 2:

$$D_k(p^c - \lambda) \ge \frac{\lambda}{2} g_l > 0$$

Similar to the derivation of (12), the inequality implies $q_k(p^c - \lambda) \to 1$. By profit maximization, $\Pi_k(p_k(1)) \ge \Pi_k(p^c - \lambda)$ so

$$q_k(p_k(1))\, p_k(1) \ge q_k(p^c - \lambda)(p^c - \lambda) \ \ \forall \lambda$$

and by $\lim_{k\to\infty} p_k(1) = p^c$:

$$\begin{aligned} \lim_{k\to\infty} q_k(p_k(1))\, p_k(1) &\ge \lim_{k\to\infty} \underbrace{q_k(p^c - \lambda)}_{\to 1}(p^c - \lambda) \\ \Rightarrow \lim_{k\to\infty} q_k(p_k(1)) &\ge 1 - \frac{\lambda}{p^c} \ \ \forall \lambda \in (0,1) \end{aligned}$$

implies that $q_k(p_k(1)) \to 1$ by choosing λ small enough. By monotonicity, $q_k(p_k(c)) \ge q_k(p_k(1))$ for $c \in [0,1]$, extending this to $q_k(p_k(c)) \to 1$ ■

B.2 Proof of Lemma 4

Lets define the masses of inflowing sellers and buyers who will ultimately trade by $T_k^B = \int_{l_k}^1 w_k(r_k(v))\, g(v)\, dv$ and $T_k^S = \int_0^1 q_k(p_k(c))\, dc$. Suppose the lemma did not hold, then $\lim_{k\to\infty} q(p_k(c)) = 1$ for all c and

$$\begin{aligned} \lim_{k\to\infty} T_k^S &= \lim_{k\to\infty} \int_0^1 q_k(p_k(c))\, dc \\ &= 1 \end{aligned}$$

But $T_k^B = \int_{l_k}^1 w_k(r_k(v))\, g(v)\, dv \le (1 - G(l_k))$ and with $l_k \to p^c > 0$, for every $\lambda \in (0,1)$ there is a $\bar{k}_3$ such that for all $k \ge \bar{k}_3$: $T_k^B \le 1 - G(p^c) + \lambda$ and choosing λ small enough, $T_k^B \le 1 - G(p^c) + \lambda < 1$ for all $k \ge \bar{k}_3(\lambda)$. Hence the identity of T_k^S and T_k^B is violated because

$$T_k^B \nrightarrow 1$$

■

[16] The requirement that $G(\cdot)$ be convex is equivalent to the requirement needed to show that the monopolistic pricing problem $\pi(p) = (1 - G(p))\,p$ is globally concave and the monopolistic price can be derived from the first order condition $\frac{d}{dp}\pi(p) = 0$.

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