Fairness in a Student Placement Mechanism with Restrictions on the Revelation of Preferences*

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1 Extended Abstract

In a school choice problem (Abdulkadiroğlu and Sönmez, 2003) a number of students has to be assigned to a number of schools, each of which has a limited capacity of seats. Students have preferences over schools and remaining unassigned and schools have priority rankings over students. For example, students who live closer to a school or have siblings attending a school have higher priority to be admitted at the school. Another example is the assignment based on one or several entrance exams. Then students who achieve higher test scores in the entrance exam of a school have higher priority for admission at the school than students with lower test scores. We call the collection of strict priority rankings a priority structure.

School choice problems are closely related to two-sided matching problems called college admissions problems (Gale and Shapley, 1962).[1] Similar as in a school choice problem, in a college admissions problem there is a set of students and a set of schools with fixed capacities, and students have again preferences over schools and remaining unassigned. The key difference is that in a college admissions problem schools also have preferences over students. In other words, while in school choice problems schools are mere objects to be "consumed by students," in college admissions problems schools are agents with possible strategic considerations. By formally treating school priorities as school preferences, a school choice problem can be seen as a special college admissions problem. As a consequence, most concepts and results from college admissions can be reinterpreted in school choice.[2]

A central concept in the two-sided matching literature is stability.[3] In the context of college

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[1] See Roth and Sotomayor (1990) for a comprehensive survey on two-sided matching.

[2] See, for instance, Abdulkadiroğlu and Sönmez, 2003; Balinski and Sönmez, 1999; Ehlers and Klaus, 2006a,b; Ergin, 2002; Kesten, 2006a,b.

[3] In many centralized labor markets, clearinghouses are most often successful if they produce stable assignments. Empirical evidence is given in Roth (1984, 1990, 1991) and Roth and Xing (1994).

admissions, an assignment (or matching) of students to schools is stable if there are no school and no student that prefer one another to (one of) their respective matches. Its counterpart in school choice is called fairness. An assignment of students to schools is fair if no student has justified envy. Student i has justified envy if there is a school s and another student j such that j is assigned a seat at s and i has a higher priority for s than j, but i is nevertheless assigned to a less preferred seat. In their seminal paper, Gale and Shapley (1962) showed that for any college admissions problem a stable matching can be found by applying their Deferred Acceptance algorithm (the DA-algorithm for short). They also proved that each of the two versions of their algorithm (either students or schools "proposing") yields an optimal matching for the problem at hand. More precisely, if the students "propose" the resulting stable matching is the best (worst) among all stable matchings for all students (schools) simultaneously. For obvious reasons, the matching obtained from the DA-algorithm with students proposing is called the student-optimal stable matching, and the mechanism that assigns to each college admissions problem its student-optimal stable matching, the student-optimal stable mechanism. (Similarly there is a school-optimal stable matching and mechanism.) In the context of school choice, Balinski and Sönmez (1999) noted that since only the welfare of students matters, the student-optimal stable mechanism Pareto dominates any other fair mechanism. Ergin (2002) showed that the student-optimal stable mechanism is Pareto efficient if and only if the priority structure is acyclic. Roughly speaking, a priority structure is acyclic if it never gives rise to a situation in which a student can block a potential settlement between any other two students without affecting his own position.

The student-optimal stable mechanism is employed in several real-life two-sided matching markets. For instance, the National Resident Matching Program, which assigns medical graduates to hospitals in the US, was redesigned in 1998 and it was decided to switch from the school-optimal to the student-optimal stable mechanism (Roth and Peranson, 1999; Roth, 2002). An additional important property of the student-optimal stable mechanism is that it is strategy-proof for the students (Dubins and Freedman, 1981; Roth, 1982). In terms of school choice, no matter the declared preferences of the other students nor the priority structure of the schools, a student cannot do better than by declaring his/her true preferences. In practice this is very helpful for students as they need not worry about which strategy to adopt.

Yet, in several real-life school choice situations, students are asked to elicit a preference list containing only a limited number of schools.[4] In some regions of Spain, for instance, students can apply to at most 8 out of hundreds of different academic programs. Imposing a curb on the length of the submitted lists, though certainly having the merit of "simplifying" matters, has the perverse effect of forcing participants not to be truthful, and eventually compel them to adopt a strategic behavior when choosing which ordered list to submit. In fact, given the capacity of modern computers there is no technical need to "simplify" or accelerate computations by imposing this restriction. Participants may adopt strategic behavior because the "quantitative" effect (participants cannot reveal their complete preference lists) is likely to have a "qualitative" effect since participants may self-select by not declaring their most preferred options. For instance, if a student fears rejection by the best academic programs, it can be advantageous not to apply to these programs and use instead its allowed application slots for lower ranked academic

[4] A noticeable exception is the assignment of students to secondary schools in Singapore where students have to submit a list that contains *all* schools. Teo *et al.* (2001) show that even for the school-optimal stable mechanism this leaves little room for profitable manipulation.

programs.

In this paper we study the effects of imposing such a *quota* (*i.e.*, a maximal length of submitted preference lists) on the strategic behavior of students in school choice problems. Thereby we revive an issue that was in fact initially discussed by Romero-Medina (1998).[5] By imposing a quota, students may no longer have a weakly dominant strategy. Students are now typically "forced" not to reveal truthfully their preferences. Thus, the Gale-Shapley student-optimal stable mechanism induces a non-trivial preference revelation game where students can only declare up to a fixed number (quota) of schools to be acceptable. Each possible quota, from 1 up to the total number of schools, defines a non-cooperative "quota-game."

We analyze the Nash equilibria of the collection of quota-games. We first establish the existence of Nash equilibria for any quota by invoking Gale and Shapley's (1962) existence result and by showing that any fair assignment can be supported at a Nash equilibrium. Nevertheless, as we will show, for all nonextreme quotas, unfair assignments cannot even be discarded as outcomes of strong Nash equilibria in undominated "truncation" strategies. Moreover, the violation of fairness is not due to the presence of a quota. More precisely, if a strategy profile is a Nash equilibrium for some quota then it is also a Nash equilibrium for all higher (*i.e.*, less stringent) quotas. It follows that if for some quota a Nash equilibrium induces an unfair assignment then this unfair assignment is also supported at (the same) Nash equilibrium when we do not impose any quota on students' lists (*i.e.*, when the quota is equal to the total number of schools). Our main result identifies Ergin's (2002) acyclicity condition as a necessary and sufficient condition on the priorities to guarantee fair Nash equilibrium outcomes. In particular, as a policy implication, our result suggests that fairness in the restrictive procedure is recovered through strategic interaction if the assignment of students is based on a centralized entrance exam. Subsequently, we extend our results by using a refinement of Nash equilibrium, alternative concepts of fairness, and controlled school choice. Finally, we discuss the issue of a quota for other mechanisms. We conclude with some suggestions for (natural) experiments as future research.

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[5] To the best of our knowledge, Romero-Medina (1998) is the only paper that explicitly analyzes restrictions on the length of submitted preference lists. Unfortunately, his paper contains several flaws that make it difficult to draw meaningful conclusions.

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