Technology adoption with forward-looking agents*

Paolo Colla
Università Bocconi - IEP
Via Gobbi, 5
20136 Milan (Italy)

Filomena Garcia
ISEG - Departamento de Economia
Rua Miguel Lupi, 20
1249-078 Lisbon (Portugal)

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Abstract

We investigate the effects of forward looking behavior in technology adoption. Within an overlapping generations model agents choose between two alternative networks taking into consideration both the installed base and the expected base. The latter element is the distinctive feature of our approach, and in general brings in multiple equilibria. We use results from the supermodular games literature to guarantee equilibrium existence and we prove uniqueness. We consider both the cases of incompatible and compatible technologies and show that technologies cannot lock-in, while the adoption path exhibits hysteresis. Network choices are characterized both in terms of their long run properties and expected time of adoption.

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†jeea_ver.rap

1 Introduction

We characterize the unique equilibrium in a game of technology adoption where consumers are forward-looking. Our analysis focuses on technologies exhibiting network externalities, in that they become more valuable for each individual the more consumers adopt them (see Rolfhs (1974) for a seminal contribution). The literature has treated the problem of both static and dynamic decision-making in the presence of network externalities (Katz and Shapiro (1986), Shy (1996)). However, the possibility for agents to consider the behavior of their future counterparts –obviously ruled out within a static setup– has not received much attention within dynamic frameworks. In fact, most of the papers assume that agents realize their payoffs at the time they purchase a given technology. From then onwards they belong to the network, thus increasing future adopters' benefits but not receiving any further payoff. As a consequence, agents regard predecessors' adoptions only, that is they consider the installed base when opting for one technology. In this paper we deal with consumers receiving payoffs in all periods of their permanence in the network, in which case expectations over future agents' choices need to be factored in.

We consider an overlapping generations setup in which consumers live for two periods and opt for one of two technologies. We focus on stochastic technology values, i.e. consumers are not aware of the technology value in successive periods. This feature is in line with the analysis in Adner (2003): the marginal utility of technological improvements for consumers is not constant in every step of the technology evolution. In fact, it assumes the form of an S-curve: agents have a very high willingness to pay for the first improvements, but this willingness decreases after a certain threshold. If we think about two technologies whose values evolve through time, it is possible that at a certain point consumers have higher willingness to pay for one technology, and at another point they prefer the other technology.

Consumers obtain utility from the value of the technology, the so called *stand alone value*, and from interaction with other consumers, the *network value* as in Arthur (1989), Choi (1994), Farrell and Saloner (1992) and Shy (1996). We distinguish between direct and indirect network externalities. Direct effects are related to the increase in the quality of the product due to an increment in the number of contemporaneous users. Indirect effects entail broader benefits stemming from past adoptions. Even though direct interaction with remote consumers is not possible, the fact that a technology has had many previous users increases its value for the consumer: there is higher likelihood that the technology has less flaws, technical assistance might be prompter and more and better components might be available (see Liebowitz and Margolis (1994) for a complete characterization of indirect network externalities).

Before making their choice, consumers observe technology values and predecessors' actions, and form expectations about future behavior. The main result of our paper is that a unique equilibrium exists in which individuals adopt technologies via switching strategies that depend on predecessors' choices. The presence of strategic complementarities due to network externalities induces multiple equilibria in adoption. If individuals expect the adoption of a certain technology, they have an incentive to choose this same technology and enjoy the network benefit. As Levin (2001) points out, 'because the expectation of a certain behavior tends to be self-fulfilling, there is the strong possibility of multiple expectations-driven equilibria' (Levin (2001), p. 1). The main problem is therefore to select among these equilibria the one that will actually be played. Given stochastic technology values and network effects, our game is a (dynamic) Bayesian game of strategic complementarities. By Van Zandt and Vives (2003) , a static Bayesian game of strategic complementarities admits a greatest and a least Bayes-Nash equilibrium in monotone strategies. We show that, in our dynamic framework, this result still holds, and moreover that the greatest and least equilibria coincide. Hence the game is dominance solvable and has a unique equilibrium.

Our uniqueness result relates to the literature on so-called global games (see Morris and Shin (2003) for a general overview).[1] The main idea is that if 1) each player observes a noisy signal of the payoffs and 2) the payoffs' space includes values that make each action strictly dominant, then iterative dominance leads to a unique equilibrium as the noise becomes small. In our setup, stochastic technology values serve the purpose of a noisy signal and strategic complementarities arise due to network externalities.

[1] More recently, Burdzy, Frankel and Pauzner (2001), Frankel and Pauzner (2000) and Frankel (2001) study the conditions under which an equilibrium is selected for dynamic games with binary actions and strategic complementarities. Giannitsarou and Toxvaerd (2003) provide similar results for recursive games and Levin (2001) extends the main findings to overlapping generation games. For an application of Levin's result to the technology adoption game described in the present article see Colla and Garcia (2004).

A second result of our analysis is that technologies cannot *lock in* by historical events. Instead they will alternatively be chosen through time. Arthur (1989) presents a dynamic model in which agents choose between two competing technologies based on their personal preferences (stand alone value) as well as the network benefit, with the latter described by a function of the total number of previous adopters. His main finding is that whenever a given technology achieves a sufficient mass of adopters, it locks in attracting all future consumers. He considers as well the possibility that consumers take into account the future base. However he restricts his analysis to the case of lock-in by one of the technologies, neglecting in the first place how forward-looking behavior interplays with the existence of lock-in. In the present paper we address this issue, allowing agents to obtain payoffs every period of their lives and thus to form expectations about the future base. A similar standpoint is taken in Ochs and Park (2004), that highlight the importance of forward looking behavior in network formation.[2]

The absence of lock in is in line with Liebowitz and Margolis (1994; 1995), that question the empirical relevance of lock-in patterns in technology adoption, arguing that lock-ins are extremely unlikely to occur. We provide a different rationale for the absence of lock-ins. Agents incorporate successors' choices and face technologies with stochastic stand alone values. Given that these values are independent of the number of adopters, network externalities are not the only driving force behind technology adoption. A lead in terms of installed base is not enough to attract all consumers. This is because agents are concerned with the stand alone value granted by the technology upon purchase, as well as with the value provided in subsequent periods.

Even though lock-in does not emerge in our setup, path dependence in technology choices is still present. Once the model parameters are laid out, the technology adopted by the ancestor affects the current user's decision. When this occurs the equilibrium path exhibits *hysteresis*.

One empirical prediction of our model is that competing technologies coexist in the market. Several cases within the computer industry are compatible with our results. In fact, the computer

[2] Their model differs from ours in that individual types (or technology values as in our setup) are uncorrelated, thus the global games equilibrium selection approach cannot be applied. In Ochs and Park (2004) it is possible to identify a unique symmetric perfect bayesian equilibrium since agents choose both when and whether to join a network.

industry does not comply entirely with early leading technologies becoming dominant (see Gandal, Greenstein and Salant (1999) for further details). In the early 80's, CP/M, a highly adopted operating system, was orphaned both by users and developers at the advantage of DOS, an operating system that had been recently developed.[3] In this case the technology that had initially gained some market share lost its relevance over time. One reason for this is the heterogeneity of adopters: early buyers are usually highly skilled, while late buyers are less skilled and might go for a more user friendly platform. Another example that contradicts the idea of lock-in is the persistence in time of the two computer platforms Macintosh and PC (IBM-PC). These two platforms became important at the time the 16-bit microprocessor was invented, and have shared the market since then. Passing through several waves of technological advances none of the two managed to become dominant in the market (see Bresnahan and Greenstein (1999)). An explanation for this evidence is the idea that technologies' relative values oscillate through time. Improvements in microprocessors' speed stand as a clear example of this. In the beginning microprocessors were very slow and any marginal increase in the speed was extremely valued by the consumers. However, as the microprocessors proved faster, speed increases became less important. In the Macintosh/PC example, the permanence in the market of the two standards is justified by the oscillating preferences of the consumers for the technological improvements in the two platforms.

We characterize as well the unique equilibrium in technology adoption when a converting device is available and allows agents from different networks to interact. The conclusion on the absence of lock in (that is a direct consequence of the stochastic pattern of the technology value), does not change when converters are allowed. However converters contribute to mitigate hysteresis: agents have weaker incentives to coordinate their choices, and as a consequence technology adoption depends less on predecessors' decisions.

Since technologies do not lock in, a different measure of dominance should be studied. The first measure we propose is the limiting probability of technology adoption. This describes the likelihood that each technology is chosen in the long run, and provides a rough estimate of its

[3]'Orphaning occurs when late adopters choose a technology incompatible with the technology adopted by early users and suppliers of supporting services' (Gandal, Greenstein and Salant (1999), p. 88)

expected demand schedule over long horizons. We show that technologies are more likely to be adopted in the long run if they 1) provide higher stand alone values, or 2) agents prefer bigger networks, or 3) they embed a converter device. In short run equilibrium a technology will be adopted for a certain number of consecutive periods, and then replaced. From the producer's point of view, it is therefore important to determine what is the expected time of adoption, our second measure for dominance. We find that the expected time of adoption decreases in the presence of converters due to the fact that compatible technologies reduce path dependence and one observes switching between technologies more often.

The remainder of the paper is organized as follows. The sequential move game with incompatible technologies is described in section 2. Section 3 characterizes the unique equilibrium and analyses the impact of the underlying parameters on the equilibrium outcome. Partial converters are introduced in section 4. The long run behavior of our adoption game is described in section 5. Section 6 focuses on the robustness of the equilibrium outcome to alternative specifications. Finally section 7 concludes.

2 Theoretical model

We consider a sequence of users planning to adopt a technology within a discrete time and infinite horizon setting. At each time $n \in N$, player n enters the game and chooses among two competing technologies A and B. Each player lives for two periods. We borrow the terminology from the OLG literature and refer to the young (resp. old) generation at time n as the n-th player (resp. the $(n-1)$-th player). In the first period, user n buys a single unit of one of the two technologies and commits to his choice in period $n+1$.[4] We denote player n's action set as $A_n = \{0, 1\}$ where $a_n = 1$ (resp. $a_n = 0$) corresponds to technology A (resp. B). We restrict our analysis

[4] Choice irreversibility can be introduced by assuming that it is prohibitively costly to switch, say, to technology A at time $n+1$ after having adopted technology B at time n. This assumption is quite standard in (dynamic) network adoption games. To our knowledge Farrell and Saloner (1985) are the only ones obtaining -rather than imposing- choice irreversibility. However they are admittedly unable to explain why this result arises (see Farrell and Saloner (1985), footnote 9 and Malin (2003))

to unsponsored technologies resulting in their supply at a price equal to zero.[5] Agents discount future payoffs via the factor $\beta \in (0,1)$.

2.1 Technology value

The technology value at time n is given by x_n, and can be thought of as player n's relative preference between the two technologies when no other agent shares the same base. We model the evolution of technology values through time as a random walk with barriers $-\alpha$ and α, where $\alpha > 0$, i.e.

$$x_n = x_{n-1} + \sigma\varepsilon_n \tag{1}$$

where $\sigma > 0$. The innovation ε is characterized by the density $g(\varepsilon)$:

$$g(\varepsilon) = \begin{cases} -1 & \text{prob. } q = 1-p \\ 1 & \text{prob. } p \end{cases} \tag{2}$$

where $p \in (0,1)$. The boundaries α and $-\alpha$ are partially reflecting. In other words, if the random walk is in state α at time n, the following period it can either go down to $\alpha - \sigma$ with probability q or stay at α with probability $p = 1 - q$.[6]

2.2 Individual preferences

Player's valuation of a technology reflects two components: the stand alone value and the network value. The former captures the utility the user derives if no other player adopts the same technology, while the latter is the benefit from interaction with other users.

2.2.1 Stand alone value

The $n-$th period stand alone value depends on the contemporaneous technology value x_n. After opting for one technology, generation n receives the stand alone value at period n. The time n

[5] The term unsponsored technologies was first used by Arthur (1989) to refer to technologies that are non-appropriable. The absence of property rights leads to entry in the market until marginal cost pricing condition is met, i.e. in our case until prices are zero.

[6] Similarly when $x_n = -\alpha$, the technology value can either jump up to $-(\alpha - \sigma)$ (with probability p) or stay at $-\alpha$ (with probability q) at time $n+1$.

stand alone value is therefore a function of the current technology value x_n and player n's chosen technology a_n according to

$$\pi_S = \pi_S\left(a_n, x_n\right) = \begin{cases} \alpha + x_n & \text{if } a_n = 1 \\ \alpha - x_n & \text{if } a_n = 0 \end{cases} \tag{3}$$

In words, players' relative preference for technology A over B is increasing in the technology value through an affine function.[7]

2.2.2 Network value

Network externalities assume two different aspects. On one hand, consumers may interact with each other because they use the same technology (direct network value). On the other, consumers benefit from the technologies adopted by previous generations, even without direct interaction (indirect network value).

Since each user lives for two periods, at each time n there are two generations active in the market. Thus player n's direct network value depends on the action chosen by his immediate predecessor $n-1$ and by his immediate successor $n+1$. We consider the direct network value to be linear in the number of users of the same technology. Further we assume that the two technologies are identical in terms of the direct network benefit they provide per unit of member in the base, $\nu_D > 0$. If player n and $n-1$ adopt the same technology, they receive a direct network benefit of ν_D. The same payoff is received by user n and $n+1$ if they coordinate on the same technology. Generation n's per-period direct network payoff π_D is given by:

$$\pi_D = \pi_D\left(a_n, a_{-n}\right) = \begin{cases} \nu_D a_{-n} & \text{if } a_n = 1 \\ \nu_D\left(1 - a_{-n}\right) & \text{if } a_n = 0 \end{cases} \tag{4}$$

where a_{-n} denotes the technology purchased by the other generation active in the market, i.e. $a_{-n} = a_{n-1}$ at time n and $a_{-n} = a_{n+1}$ at time $n+1$.

[7] Our choice for the barriers in the stochastic process (1) together with the payoffs in (3) imply that the stand alone value belongs to the interval $[0, 2\alpha] \in \Re^+$. This normalization provides individuals with a (weakly) positive utility from purchasing one of the two techologies in the absence of network benefits.

As for the indirect network value, we assume that technologies adopted by previous generations affect player n's choice through a summary statistic $h_n = h(a_{n-M}, ..., a_{n-2})$, $M \geq 2$, which describes the past behavior, or equivalently the history of technology adoption. We require the function $h(\cdot)$ to be real-valued, bounded, separable and increasing in each of its arguments such that the technology chosen by each generation from $n - M$ to $n - 2$ affects player n's utility.[8] Without loss of generality we let $h_n : \{0,1\}^{M-1} \rightarrow [0, H]$, where H is the value taken by the statistic h_n when all have predecessors chosen technology A, i.e. $H = h(1, ..., 1)$. The indirect network value then reads as

$$\pi_I = \pi_I(a_{n-M}, ..., a_{n-2}) = \begin{cases} \nu_I h_n & \text{if } a_n = 1 \\ \nu_I (H - h_n) & \text{if } a_n = 0 \end{cases} \tag{5}$$

where $\nu_I > 0$ describes the value attached to technology history.

2.2.3 Total payoffs

Adding up the stand-alone component (3) and the network components (4-5) gives player n's overall utility $u_n = u(a_n, a_{n-1}, a_{n+1}, h_n, x_n)$:

$$u_n = \begin{cases} \alpha + x_n + \nu_D (a_{n-1} + \beta a_{n+1}) + \nu_I h_n & \text{if } a_n = 1 \\ \alpha - x_n - \nu_D [(1 - a_{n-1}) + \beta (1 - a_{n+1})] + \nu_I (H - h_n) & \text{if } a_n = 0 \end{cases}$$

For the remainder of our analysis it is useful to rewrite player n's payoff in the following compact way:

$$u_n = \pi_S(a_n, x_n) + \pi_D(a_n, a_{n-1}) + \beta \pi_D(a_n, a_{n+1}) + \pi_I(a_n, h_n) \tag{6}$$

The terms on the RHS in (6) capture respectively the stand alone value, the direct network component -i.e. the sum of the payoff related to the existing base and to the (discounted) future base- and the indirect network component.[9]

[8] The assumption that h_n does not depend on the immediate predecessor action, a_{n-1}, is made without loss of generality. Suppose the indirect network value depends on a_{n-1} as well as on $(a_{n-M}, ..., a_{n-2})$ through some statistic $h(\cdot)$. This parametrization would be equivalent to another one in which the direct network value attached to a_{n-1} is different from the one attached to a_{n+1} because it is augmented by ν_I. Our main results would not be qualitatively altered. See also section 2.4 on this point.

[9] According to (6), despite player n lives for two periods, he receives the technology stand alone value only once.

2.3 Timing and Strategies

At time n, agent n is aware of the current technology valuation x_n and the predecessors' choices $a_{n-M}, ..., a_{n-1}$. A strategy for player n is thus a function $s_n = s(a_{n-1}, h_n, x_n) : \{0,1\} \times [0,H] \times [-\alpha, \alpha] \to \{0,1\}$. Player n chooses his action to maximize the expected payoff in (6). Letting $s_{n+1} = s(a_n, h_{n+1}, x_{n+1})$ denote the strategy of player $(n+1)$, the $n-$th user solves the following:

$$\max_{a_n \in A_n} E\left[u(a_n, a_{n-1}, a_{n+1}, h_n, x_n) \,|a_{n-1}, h_n, x_n; s_{n+1}\right] \tag{7}$$

From (7) it emerges that, when choosing which technology to purchase, player n takes into account the effect of his action in determining the future size of the network, which in turn affects player $n+1$ compatibility payoff. It is worth noting that the process (1) allows the technological valuation to be correlated through time, which turns out to be crucial in enabling player n to forecast the next generation's strategy after observing x_n. We allow user n to receive network benefits from the (expected) future base via (7), and thus we explicitly bring in a role for predicting future technology values.

2.4 Discussion

We now briefly discuss the modelization assumptions related to timing, technology values and players' payoffs.

- *overlapping generations.* The overlapping generations structure has been chosen for two reasons. First, OLG models are appropriate to study the choice of buying a durable good and holding it for several periods. Second, OLG models allow agents to form expectations over their finite lifetime. Shy (1996) considers an OLG setup as well, but like Arthur (1989) the future behavior of agents is not regarded as a determinant of consumers' choices.

This assumption is made for simplicity, and we could easily encompass a more articulated specification in which the utility depends on both x_n and x_{n+1} via, say, $\pi_S(a_n, x_n) + \beta\pi_S(a_n, x_{n+1})$. The main results would be unchanged, and the interested reader is referred to the working paper version of our work (see Colla and Garcia (2004)). A similar argument applies to the introduction of the (discounted) indirect network value, i.e. $\beta\pi_I(a_n, h_{n+1})$, in (6).

- *stochastic technology values.* We consider stochastic technology values as in Choi (1994) in order to capture the idea that the stand alone value at time $n+1$ will be revealed to players at the beginning of that period, such that x_{n+1} is a random variable from player n's standpoint at time n. Thus, we view the technology value as inherent to the technology itself, rather than being agent-specific. The step size $\sigma > 0$ in the technology process X (see (1)) can be thought of as a measure of (intertemporal) heterogeneity in technology valuation, and drives the randomness in agents' payoffs. As it has been mentioned in the Introduction, the literature on strategy and management provides an intuitive explanation for the stochastic technology values' assumption, known as the demand S-curve for technology improvements (see Adner (2003)). Alternatively, if we regard technologies as evolving through time, x_n measures the relative value of technologies' developments at time n.

- *correlated technology values.* The correlation through time of the technological valuation is crucial in enabling player n to forecast the next generation's action after observing x_n. Technology values are assumed to be independent through time in Arthur (1989), while the technology innovation -and thus the stand alone value- is given by a deterministic, rather than stochastic, process in Shy (1996). In the sequential move games of Arthur (1989) and Shy (1996) the intertemporal pattern of technology values does not play a relevant role, since users choose their action based on the installed base only. Unlike these works, we allow player n to receive network benefits from the (expected) future base in (7). Our specification for technology values is closely related to Oyama (2003), which employs a similar process to describe the pattern of fundamentals within an OLG speculative attack model. One can also explain the correlation through time in technology values resorting to taste shocks in users' preferences (see Macskasi (2002)). According to this interpretation, each generation tastes' are evolving, as to say that the relative preferences for the young generation at time n might differ from the tastes of the same generation when old at time $n + 1$. Note however that at each point in time both the young and the old generations agree on the technology value. This argument leads again to think of technology values as time specific.

- *affine stand alone value.* We assume that the stand alone depends on the technology value through an affine function. This formulation is fairly standard in the literature. Arthur (1989) considers two (classes of) agents with the former (resp. the latter) displaying a natural preference for technology A (resp. technology B). His formalization for the stand alone value can be obtained from ours restricting x to take only two values, say $x = \{-1, 1\}$, with equal probabilities. Our specification for π_S is analogous to Farrell and Saloner (1992) with the only difference that technology values belong to the interval $[-\alpha, \alpha]$ in our model rather than $[0, 1]$. Shy (1996) considers a richer specification for the stand alone value without restricting it to affine functions of x_n.

- *direct network value.* In line with much of the previous literature we consider the direct network value to be linear in the number of users of the same technology. Further we assume that the two technologies provide the same direct network benefit per unit of member in the base, $\nu_D > 0$. This assumption is quite standard in the literature (see Choi (1994) and Farrell and Saloner (1992)). Allowing for asymmetric network benefits as in Arthur (1989) can be easily incorporated into our framework without changing the main conclusions.

- *indirect network value.* The specification for π_I in (5) serves as a reduced form for more complex relationships between the installed base, i.e. the actions $a_{n-M}, ..., a_{n-2}$, and individual payoffs. Such indirect network externalities usually stem from the increase in the variety of complementary products available for each technology (see for example Clements and Ohashi (2005)). The minimal requirements we impose on the function $h(\cdot)$ allow for a rich variety of specifications for such indirect network externalities.[10]

[10] For example one might take the sum of predecessors' actions as summarizing past behaviour, and consequently set $h_n = \sum_{j=2}^{M} a_{n-j}$ and $H = M - 1$. In this case player n puts a weight equal to ν_I to each action chosen by generations from $n - M$ to $n - 2$. Alternatively, a weighted average of past actions can be employed to capture the benefit from indirect interaction with predecessors, i.e. $h_n = \sum_{j=2}^{M} \omega_j a_{n-j}$ with $\sum_{j=2}^{M} \omega_j = 1$, $\omega_j > 0$ for all $j = 2, ..., M$ and $H = 1$. One can then choose decaying weights, i.e. $\omega_j > \omega_{j+1}$, in order to capture the idea the technologies more recently chosen have a larger impact on generation n payoff.

3 Solving for an equilibrium

3.1 The benchmark model: no network externalities

If there are no network externalities, utility is simply given by the stand alone value (3). The maximization problem (7) therefore resumes to:

$$\max_{a_n \in A_n} \pi_S(a_n, x_n)$$

In the absence of network benefits, i.e. $\nu_D = \nu_I = 0$, agent n is indifferent between the two technologies when the technology value $x_n = \hat{x}$ solves:

$$\pi_S(1, \hat{x}) = \pi_S(0, \hat{x})$$

We refer to the technology value $\hat{x}$ as the pivotal point, that is the threshold above which the agent has a strict preference for technology A. Making use of (3) in the above equation gives $\hat{x} = 0$. Without network externalities the pivotal point is zero because actions are entirely determined in equilibrium by the technology value. If player n prefers technology A to B –this would occur when $x_n > 0$– then he opts for A.

3.2 The model with network externalities

We now analyze equilibrium strategies when both ν_D and ν_I are strictly positive. In order to solve for the equilibrium in the sequential move game outlined in section 2 we first consider the following:

Lemma 1 *Let the model parameters be such that*

$$2\alpha > \nu_D(1+\beta) + \nu_I H \ . \tag{8}$$

Then the space of technology values contains a region $[\bar{x}, \alpha]$ *where technology A is dominant, and a region* $[-\alpha, \underline{x}]$ *where technology B is dominant, i.e.*

$$a_n(x_n) = \begin{cases} 1 & \textit{if } x_n \geq \bar{x} = \frac{\nu_D(1+\beta)+\nu_I H}{2} \\ 0 & \textit{if } x_n < \underline{x} = -\bar{x} \end{cases}$$

The intuition behind condition (8) is as follows. Note from (3) that the maximal stand alone value is given by 2α, which would occur whenever the stocastic process in (1) reaches one barrier. The direct per-period network benefit is ν_D (from (4)) and the maximal indirect network benefit is $\nu_I H$ (from (5)). According to Lemma 1 there exist technology values for which the maximal stand alone valuation offsets the (discounted) benefits from coordinating on a network, i.e. adopting the installed technology ($\nu_D + \nu_I H$) given that it will be chosen by the immediate successor ($\beta\nu_D$). Lemma 1 ensures that some individuals would choose a technology regardless of the network size: technology A is dominant whenever x_n is above the critical value $\bar{x}$, whereas technology B is dominant if the technology value falls below $\underline{x}$. Thus condition (8) guarantees that the values $\bar{x}$ and $\underline{x}$ lie in the state space for the technology process X. In general, multiple equilibria would occur within the interval $[\underline{x}, \bar{x}]$.[11] Lemma 1 plays a key role in applying an iterated elimination argument, and thus solving for the unique equilibrium. Proposition 1 constitutes our main result.

Proposition 1 *Under (8) the game has a unique equilibrium, in which for all n*

$$s(a_{n-1}, h_n, x_n) = \begin{cases} 1 \text{ if } x_n \geq x^*(a_{n-1}, h_n) \\ 0 \text{ if } x_n < x^*(a_{n-1}, h_n) \end{cases}, \tag{9}$$

where $x^*(a_{n-1}, h_n)$ is a decreasing function of both a_{n-1} and h_n. Moreover, let the step size σ be sufficiently small such that:

$$\sigma < \bar{\sigma} = \nu_D \left(1 - \frac{\beta}{2}\right). \tag{10}$$

Then the cut-off points are given by:

$$x^*(0, h_n) = \frac{\nu_D (1 - \beta p) + \nu_I (H - 2h_n)}{2} \tag{11}$$

$$x^*(1, h_n) = \frac{-\nu_D (1 - \beta q) + \nu_I (H - 2h_n)}{2} \tag{12}$$

[11] This region is symmetric around zero, i.e. the pivotal point for dominant actions in the absence of network benefits. Note that the gap $\bar{x} - \underline{x}$ depends positively on the network benefits. The fact that $\bar{x} - \underline{x}$ increases in both ν_D and $\nu_I H$ stems from the fact that when individuals attach a large positive value to joining a network, a high stand alone component is needed in order to adopt technologies regardless of other users' choices. As a result both $\bar{x}$ and $\underline{x}$ would move away from zero the larger are ν_D, ν_I and H.

Proof. See the appendix. ■

According to Proposition 1, switching strategies are played at equilibrium. Within the global game literature this is a common finding due to strategic complementarities in (dynamic) games. Player n has an incentive to move to higher actions as soon as the successor raises his strategy from s_{n+1} to $s'_{n+1} > s_{n+1}$. The cut-off points $x^*(a_{n-1}, h_n)$ specify the technology values at which user n is indifferent between A and B. These cut-offs depend on the immediate predecessor's observed action, on the technology history and on the expected behavior of the immediate successor, and due to condition (8) they belong to the region $[\underline{x}, \bar{x}]$. Proposition 1 yields $x^*(0, h_n) > x^*(1, h_n)$, so that when technology A is highly valuable relative to B –this occurs when a player observes a relatively high value for x_n– it is going to be adopted regardless of the predecessors' choices. On the other hand when technology B is more valuable, player n is more likely to purchase technology A only if he observes his predecessor choosing A. Unlike condition (8), the inequality (10) does not play any role for the equilibrium uniqueness result. Thanks to condition (10), there exists at least one value of the technology process X within the two cut-off points. This rules out the admittedly uninteresting situation in which the technology values are such that the game jumps from one equilibrium to the other, i.e. outside the region $[x^*(1, h_n), x^*(0, h_n)]$, every few periods.

Consider the time during which all types fall into one of the dominance regions, say $x_n > \bar{x}$. In this case network benefits are not strong enough to observe A-lover individuals choosing the competing technology. When technology values fall into $[\bar{x}, \underline{x}]$ results from supermodular games allow to determine the unique equilibrium path. More specifically, when x_n is above $x^*(0, h_n)$ technology A is chosen regardless of the predecessors' actions (similarly, technology B is adopted for x_n below $x^*(1, h_n)$).[12]

[12] Model uncertainty –captured by the technology value heterogeneity σ– plays a role in determining the width of the region $[x^*(1, h_n), x^*(0, h_n)]$ but not $[\underline{x}, \bar{x}]$. When the uncertainty about future types is very large, the interval $[x^*(1, h_n), x^*(0, h_n)]$ is less effective in refining the dominance regions, i.e. $\lim_{\sigma\to\infty}[\bar{x} - x^*(0, h_n)] = \lim_{\sigma\to\infty}[x^*(1, h_n) - \bar{x}] = 0$. Other things equal, this refining ability vanishes with the future base importance. When individual n neglects the impact of his successor's action, i.e. for values of β close to zero, the equilibrium selection criterion has no power in eliminating the multiplicity of equilibria within $[\underline{x}, \bar{x}]$.

The interval $[x^*(1,h_n), x^*(0,h_n)]$ generates hysteresis, since player n's choice depends on the predecessors' actions (as well as the expectation of the successor's action) whenever x_n falls into the hysteresis band, i.e. the gap $x^*(0,h_n) - x^*(1,h_n) = \bar{\sigma}$. In other words, when $x_n \in [x^*(1,h_n), x^*(0,h_n)]$ individual n's choice is determined by his predecessors' actions, and equilibrium adoption is path dependent. When $\bar{\sigma}$ is small the likelihood of hysteresis is reduced, i.e. the probability of simultaneously observing a technology value falling into the interval $[x^*(1,h_n), x^*(0,h_n)]$ and a user choosing based on the past history is small.

We can perform the following comparative statics over the hysteresis band: 1) it narrows with the discount rate β, 2) it widens with the direct network benefit ν_D and 3) it is unaffected by the indirect network benefit ν_I as well as the probability p. High values for β mean that the importance of the future base is high relative to the installed base. Thus, individuals tend to disregard the predecessor's action and the switching points get closer to each other. On the other hand an increase in the direct network externality ν_D would increase the importance of the network component relative to the stand alone value in the individuals' expected utility. Future expected direct network values are discounted through β, implying that direct interaction with the predecessor becomes more important (in utility terms) with respect to interaction with the successor. As a result, player n attaches more importance to the technology adopted by the old generation thus widening the hysteresis band. These effects are summarized in figure 1-Panel A. Figure 1 shows a sample path for the random walk (1-2) with $N = 100$, $p = 1/2$, $x_0 = 0$, $\sigma = 0.5$, $\alpha = 4$ and $\nu_I = 0$; individual preferences are captured by $\beta = 0.5$ and $\nu_D = 5$. The hysteresis band narrows when the discount factor increases to $\beta = 0.95$ (see Panel A.2), and widens when the direct network benefit increases to $\nu_D = 6$ (see Panel A.3).

In order to explain the third finding, suppose that h_n is the arithmetic mean and at time n one has $h_n > H/2$, i.e. more than a half of the predecessors have chosen technology A. It follows from eqs. (11-12) that higher values for ν_I would move both cut-offs downwards by the same amount. This reflects the fact that technology A is more attractive since it takes a lower technology value x_n to have individuals choosing A. The comparative statics for p hinges on a similar argument. If p increases, the process for technology values is expected to move upwards with a higher probability as time goes by, so that there is a higher probability that the successor turns out to adopt technology A. Again, this lowers the minimum level of stand alone value required for the current player to choose technology A. An increase in p thus decreases both cut-offs by the same amount.

Finally, note from eqs. (11,12) that when the technology value is expected to be constant through time, i.e. $p = 1/2$, the cut-off points are symmetric around $\nu_I (H - 2h_n)/2$, i.e. the pivotal point in the absence of direct network externalities.[13] Further the cut-offs collapse to zero as the network externalities die out; in this case technologies are chosen in equilibrium depending on their stand alone value only. Similarly, when the technology value is expected to move away from its current value –because either $p > 1/2$ or $p < 1/2$–, the cut-off points converge to the pivotal point $\nu_I (H - 2h_n)/2$ as ν_D goes to zero.

4 Introducing Converters

4.1 Individual payoffs

We now consider the effect of converters enabling imperfect compatibility between technologies A and B. As in section 2, we focus on perfect competition, implying a null price for the converter. Let $r \in (0, 1)$ denote the compatibility of technology A with B (s is defined similarly). The stand alone component is not influenced by the existence of converters. Converters have an effect on the utility derived from networks, in that they allow agents to profit from the network even if no one else has chosen the same technology. Consider for example player n choosing technology A while both the previous and the next generations opt for technology B. Compatibility results in a payoff of $\nu_D r (1 + \beta)$ contrasting with a null network benefit in the absence of converters. Similarly player n receives $\nu_D s (1 + \beta)$ if he adopts technology B and both agent $n-1$ and $n+1$

[13] When $\nu_D = 0$ the maximization problem (7) reduces to $\max_{a_n \in A_n} \pi_S (a_n, x_n) + \pi_I (a_n, h_n)$. It is straightforward to check that in this case the pivotal point $\hat{x} = \nu_I (H - 2h_n)/2$ solves

$$\pi_S (0, \hat{x}) + \pi_I (0, h_n) = \pi_S (1, \hat{x}) + \pi_I (1, h_n).$$

choose A. Let $u_n^c = u^c(a_n, a_{n-1}, a_{n+1}, h_n, x_n)$ denote the individual payoff with compatibility.

Using the payoffs defined for the incompatibility case (see eqs. (3), (4) and (5)) gives:

$$u_n^c = \begin{cases} u(1, a_{n-1}, a_{n+1}, h_n, x_n) + \nu_D r((1-a_{n-1}) + \beta(1-a_{n+1})) & \text{if } a_n = 1 \\ u(0, a_{n-1}, a_{n+1}, h_n, x_n) + \nu_D s(a_{n-1} + \beta a_{n+1}) & \text{if } a_n = 0 \end{cases}$$

or equivalently:

$$u_n^c = u_n + \nu_D r a_n(1+\beta) + \nu_D(a_{n-1} + \beta a_{n+1})(s - a_n(r+s)) \tag{13}$$

Note that when both the compatibility levels are equal to zero one gets the $u_n^c = u_n$ and the previous results with incompatible technologies follow.

4.2 Equilibrium and interpretation

The equilibrium in the absence of external benefits ($\nu_D = \nu_I = 0$, see subsection 3.1) does not change with the introduction of converters as they act as network enhancers only. On the contrary, the equilibrium with $\nu_D > 0$ is different from the incompatibility case and is characterized in the following:

Proposition 2 *Let the model parameters satisfy restriction (8). Then the technology adoption game displays dominance regions. Technology A is dominant if* $x_n \geq \bar{x}_c = \bar{x} - r\nu_D(1+\beta)/2$ *and technology B is dominant if* $x_n < \underline{x}_c = \underline{x} + s\nu_D(1+\beta)/2$. *The game admits a unique equilibrium in which for all n:*

$$s_c(a_{n-1}, h_n, x_n) = \begin{cases} 1 & \text{if } x_n \geq x_c^*(a_{n-1}, h_n) \\ 0 & \text{if } x_n < x_c^*(a_{n-1}, h_n) \end{cases}$$

where the cut-off $x_c^*(a_{n-1}, h_n)$ *is decreasing in both* a_{n-1} *and* h_n. *Furthermore let the step size* σ *be sufficiently small according to:*

$$\sigma < \bar{\sigma}_c = \bar{\sigma} - \frac{\nu_D[r(1-\beta q) + s(1-\beta p)]}{2} \tag{14}$$

Then the cut-off points are given by:

$$x_c^*(0, h_n) = x^*(0, h_n) - \frac{\nu_D(r - \beta ps)}{2} \tag{15}$$

$$x_c^*(1, h_n) = x^*(1, h_n) + \frac{\nu_D(s - \beta qr)}{2} \tag{16}$$

Proof. See the appendix. ■

In the presence of converters the dominance regions are affected by the probability of upward movements p and the network benefits ν_D and ν_I along the same lines as the incompatibility case. An increase in either ν_D or ν_I would move $\underline{x}_c$ and $\bar{x}_c$ towards the barriers $-\alpha$ and α, thus shrinking the dominance regions. When technologies provide substantial network benefits, it takes higher stand alone values (and therefore higher technology values) in order to make individual opt without considering network externalities. Note that converters act in the opposite direction, that is an increase in either r or s (or both) would reduce the dominance regions. High values for r provide an individual opting for A with network payoffs even if other players choose B, thus making technology A more appealing and widening the region $[\bar{x}_c, \alpha]$. As it emerges comparing the dominance regions under compatibility and incompatibility (see Lemma 1), the presence of converters shrinks the interval in which no action is dominant, i.e. $[\underline{x}_c, \bar{x}_c] \subset [\underline{x}, \bar{x}]$. The reason behind this is that when technologies are compatible the gains from coordinating (i.e. three generations choosing the same technology) are reduced, since player n profits from some network externalities even if he chooses a different technology relative to players $n-1$ and $n+1$. The same argument implies that the hysteresis band narrows with compatible technologies. In fact from (14) one has $\bar{\sigma}_c < \bar{\sigma}$. Using the cut-off points in eqs. (15,16), we get the following comparative statics:

1. an increase in the discount rate β reduces the hysteresis band along the same lines as in section 3.2. However, in the presence of converters this gap reduces less than in the case without converters.[14] This is because agents derive utility from the installed base even if they buy a technology that has not been chosen by the previous generation;

2. when direct network payoffs are negligible, individuals switch around the pivotal point, namely $\nu_I \left(H - 2h_n\right)/2$;

3. converters affect cut-off points in an asymmetric fashion: higher values of s increase both $x_c^* \left(0, h_n\right)$ and $x_c^* \left(1, h_n\right)$ while an increase in r would lower both the switching points. This

[14] From (14) one has $\partial\bar{\sigma}_c/\partial\beta = \left(rq + sp - 1\right)\nu_D/2$ which is negative since $(p, r, s) \in (0,1)^3$.

finding has the following interpretation. Assume that, given predecessors' choices, player n is indifferent between the two technologies. Other things equal an increase in s makes technology B more attractive since user n achieves higher gains from compatibility with the competing technology A. As a consequence it would take a higher x_n, i.e. an individual that is relatively more prone to purchase technology A, to restore indifference. A similar reasoning would hold with respect to an increase in r. Note that despite this asymmetry, an increase in either of the compatibility parameters would narrow the hysteresis band $\bar{\sigma}_c$. Partial converters bring in network benefits that can be reaped when other players purchase the competing technology. A higher level of the one-way compatibility s has a higher impact in the utility function when the predecessor chose A than when he chose B. In order to restore indifference, the cut-off for $a_{n-1} = 1$ increase more than the cut-off for $a_{n-1} = 0$ for a given technology history h_n, thus narrowing the hysteresis band. A similar argument applies to changes in r.

The main point here is that, other things equal, converters decrease direct network benefits from coordination on the same technology and dominance regions widen. Consider symmetric converters first, i.e. $s = r$. Inspection of eqs. (15,16) then reveals that the cutoff points $x_c^*(0, h_n)$ and $x_c^*(1, h_n)$ are symmetric around $(H - 2h_n)/2$ as in the incompatibility case, and $[x_c^*(0, h_n), x_c^*(1, h_n)] \subset [x^*(0, h_n), x^*(1, h_n)]$. As a result hysteresis is less likely to occur with partial symmetric converters relative to the case of incompatible technologies (contrast Panel A.1 with Panel B.1 in figure 1, where we set $r = s = 0.2$). On the other hand asymmetric converters would drive individual choices towards the more compatible technology. In order to see this, compare Panel B.2 and B.3 in figure 1. Given a sample path for (1-2) and other exogenous parameters (see subsection 3.2), asymmetric converters with $r = 0.6$ and $s = 0.2$ (resp. $r = 0.2$ and $s = 0.6$) appears in Panel B.2 (resp. Panel B.3). Note that the level of overall compatibility $r + s$ is the same for both Panel B.2 and B.3, such that the difference $x_c^*(0, h_n) - x_c^*(1, h_n)$ does not change from one case to the other. Recall that $r > s$ corresponds to higher compatibility gains for technology A and note from Panel B.1 and B.2 that more users adopt technology A in the asymmetric converters case (similarly more users opt for technology B in Panel B.3 relative to Panel B.1).

5 Long run behavior and the expected time of adoption

5.1 Technology lock-in and limiting behavior

We investigate lock-in effects in our OLG setup where individuals explicitly take into account the actions of future generations when choosing between two competing technologies. As mentioned in the Introduction, in Arthur (1989) one technology emerges as dominant as time goes by. In other words there exists a time after which *all* players opt for the same technology. In our setup, due to the stochastic nature of the individual types, the emergence of a technology as dominant is related to: 1) the likelihood of the stochastic process for x_n hitting the barriers $\underline{x}$, $\bar{x}$, $x^*(0, h_n)$ and $x^*(1, h_n)$ (as well as their counterpart with converters) and 2) the impact of the underlying parameters on the mentioned barriers. As for the latter point, we have provided several comparative statics results in sections 3 and 4. The long run characterization of our adoption game is thus captured by the limiting behavior of the technology process (1-2) like in Kandori, Mailath and Rob (1993). Technology A locks in if and only if x_n is always above $x(0, h_n)$ for large n (similarly technology B locks in if and only if x_n is below $x(1, h_n)$). More formally, let the adoption probabilities of the two technologies be defined as $\pi_A = \lim_{n\to\infty} \Pr(x_n \geq x(0, 0))$ and $\pi_B = \lim_{n\to\infty} \Pr(x_n \leq x(1, H))$. Adoption probabilities with compatible technologies are defined similarly, i.e. with respect to the relevant cut-offs $x_c^*(0, 0)$ and $x_c^*(1, H)$, and denoted by $\pi_{A,c}$ and $\pi_{B,c}$. Then technologies lock in if and only if either $\pi_A = 1$ or $\pi_B = 1$ (lock-ins with compatible technologies are defined similarly).

Proposition 3 *No technology lock-in occurs in the long run, regardless of the presence of converters.*

Proof. See the appendix. ■

The idea behind Proposition 3 is that technology values in (1-2) hit any barrier with positive probability, regardless of the uncertainty about future values σ. As a consequence, no technology can emerge as dominant in the long run, and lock-ins can occur only temporarily.

We now consider the interplay between the parameters in our game and the long run probabilities of adopting each technology. This point is clearly related to the impact of such parameters on the cut-offs (see sections 3 and 4), since changes in the cut-offs affect the probabilities of adoption π_A and π_B.

Corollary 4 *i) π_A and π_B (and their counterparts with compatible technologies) increase with β and decrease with both ν_D and ν_I; ii) π_A and $\pi_{A,c}$ increase with p (resp. π_B and $\pi_{B,c}$ decrease with p). iii) $\pi_{A,c}$ increases in r and decreases in s (resp. $\pi_{B,c}$ increases in s and decreases in r); iv) $\pi_{A,c} > \pi_A$ and $\pi_{B,c} > \pi_B$ if and only if $r/s \in \left(\beta p, \dfrac{1}{\beta(1-p)}\right)$. Moreover for $p = 1/2$, v) $\pi_A = \pi_B = \pi$ and $\pi_{A,c} = \pi_{B,c} = \pi_c$ if and only if $s = r$*

Proof. See the appendix. ■

According to property *ii*) an increase in p increases the likelihood of adopting technology A. This is intuitive since higher values for p mean that technology A is more valuable, in that it provides higher stand alone values. Similarly, an increase in compatibility level r makes technology A more valuable, and thus increases the probability it becomes dominant in the long run (see property *iii*). Note from *iv*) that converters increase the adoption probabilities if and only if the ratio r/s belongs to the above interval. At a first glance this might seem to be in sharp contrast with findings in Arthur (1989) , where converters always increase adoption probabilities. However note that in Arthur (1989) consumers receive payoffs (both stand alone and network benefits) only upon purchase. This would correspond $\beta = 0$ in our setup, such that property v) implies that converters increase the probability of adoption, alike Arthur (1989). Finally for $p = 1/2$ the long run probabilities for the process (1-2) are all equal across states and as a consequence the adoption probabilities coincide.

5.2 Expected time of adoption

We now consider the length of time the random walk X in (1-2) takes to move from one state to the other. The aim here is to determine the expected time to observe individuals switching from one technology to the other. Consider incompatible technologies and assume that at time n the technology value is immediately below $x^*(1, h_n)$. We know from Proposition 1 that player n

would adopt technology B. The following period, player $n+1$ makes his choice comparing x_{n+1} with $x^*(0, h_{n+1})$. Since $h_{n+1} \leq h_n$ for $a_n = 0$, it follows that $x^*(0, h_{n+1})$ is always above x_{n+1}. Therefore technology B is chosen by player $n+1$, as well as the following generations until the process for technology values passes through $x^*(0,0)$. From now onwards it is technology A to be chosen until X crosses $x^*(1, H)$ and so on. Given that x_n is immediately below $x^*(1, h_n)$ we define by $m_B = m_B(h_n)$ the expected number of adopters of technology B. m_B is therefore related to the average number of periods the technology process X takes to move from $x^*(1, h_n)$ to $x^*(0,0)$. Similarly $m_A = m_A(h_n)$ is the expected number of agents choosing technology A given that the process x is immediately above $x^*(0, h_n)$, and corresponds to the average time the random process (1-2) takes to exit the band $[x^*(1, H), x^*(0, h_n)]$ after entering from $x^*(0, h_n)$. Knowledge of m_A and m_B is useful to determine how often we are likely to observe adopters switching from one technology to the other for a given technology history. With convertible technologies $m_{A,c}$ and $m_{B,c}$ are defined along the same lines and have a similar interpretation. Formulas to compute m_A and m_B (and their counterpart with compatibility) are given in the appendix. One might be interested in determining how compatible technologies affect the average time of adoption. For a given set of parameters $(p, \sigma, \alpha, \beta, \nu_D, \nu_I, H)$ and convertibility values (r, s) we say that compatible technologies decrease the likelihood of switching -or equivalently increase path dependence- whenever $m_{A,c} > m_A$ and $m_{B,c} > m_B$.

Proposition 5 *The introduction of symmetric converters reduces path dependence whenever* $r/s \in \left(\beta p, \dfrac{1}{\beta(1-p)}\right)$.

Proof. See the appendix. ■

This finding follows from the fact that converters reduce the hysteresis band, which in turn implies that the expected adoption time for both technologies cannot increase. This will be relevant in an extended version of our model with sponsored technologies and firms setting technology prices based on the expected demand schedule.

6 Discussion

We now briefly discuss the impact of alternative assumptions on our equilibrium outcomes. First of all equilibrium uniqueness is preserved under a generalization of the expected payoffs in (7). More specifically, linearity –albeit convenient– is not needed in order to preserve equilibrium existence, which is a consequence of expected payoffs exhibiting increasing differences (see Van Zandt and Vives (2003) and the proof of Proposition 1). A different specification of the payoffs would obviously imply different equilibrium cut-off points.

Second, the random walk specification (1-2) is not necessary to select a unique equilibrium. We could have used a different cumulative distribution function for the innovation ε –including for instance a continuous distribution with bounded support– as well as a random walk without barriers. We choose a binary distribution for the technology innovation for its simplicity and impose elastic barriers for the technology valuation in order to obtain bounded stand alone values like in the previous literature. What one needs for the equilibrium characterization is technology values exhibiting first order stochastic dominance, i.e. higher types of player n believe that successors are more likely to be of higher types as well.

Third, an important feature that must be imposed on the game is that it displays dominance regions, such that one can apply an iterative dominance argument and select a unique equilibrium. This means that payoffs should be specified in such a way that for some technology values the actions chosen by other players (via the installed and future base), play no role in determining player n's choice. The model in Arthur (1989) does not belong to this class: it is not true for all n that one action is optimal no matter the technology value and the history. This happens because the stand alone value is bounded but the network value is unbounded and increasing in the actions of all the predecessors.

7 Conclusion

We have analyzed technology adoption choices of agents characterized by a forward looking behavior. The existing literature does not encompass users getting utility from the purchased technology over their whole life time. Due to this assumption, agents take into consideration the installed base only, but do not form expectations of the future base. The OLG model allows us to consider agents that: 1) receive benefits in all periods of their permanence in a network and 2) take them into account when choosing the technology in the first period. Agents must therefore form expectations about future behavior. This feature, together with the strategic complementarities arising from the network effects, yields multiple equilibria in technology adoption. However, thanks to stochastic technology values, our game belongs to the class of Bayesian monotone games. In essence, even within our dynamic setting we show that the existence result of Van Zandt and Vives (2003) for static games still holds and we prove uniqueness. This is our main result: based on past observations, agents choose technologies through a unique switching strategy. A second result is that lock in cannot occur in our setup. The intuition is that, given the thresholds of the equilibrium strategy, it is always possible to find a technology value for which in any point in time agents switch from the most adopted technology to the least used one. Finally, we show that partially compatible technologies are characterized by larger adoption probabilities and lower path dependence.

The setup we consider lends itself to further extensions and modifications. For example, one can include agents living for more than two periods. In this case we would not expect the qualitative conclusions of our model to change. However, this extension would make the setup more realistic since at each point in time more than two generations are active in the market. Another promising direction is to introduce sponsored technologies produced by competing firms.

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8 Appendix

Proof. **(Lemma 1)** Let $\Delta_n = \Delta(a_{n-1}, a_{n+1}, h_n, x_n)$ denote the difference in player n's payoff when he switches between technology A and B:

$$\Delta_n = u(1, a_{n-1}, a_{n+1}, h_n, x_n) - u(0, a_{n-1}, a_{n+1}, h_n, x_n) \tag{17}$$

Using the payoffs in (6) we have

$$\Delta_n = -\nu_D (1+\beta) - \nu_I + 2x_n + 2\nu_I h_n + 2\nu_D (a_{n-1} + \beta a_{n+1}) \tag{18}$$

Let $\bar{\Delta}_n$ and $\underline{\Delta}_n$ denote respectively the value of Δ_n in (18) when both the users $n-1$ and $n+1$ choose the low and high action respectively, i.e. $\bar{\Delta}_n = \Delta(0, 0, h_n, x_n)$ and $\underline{\Delta}_n = \Delta(1, 1, h_n, x_n)$. From (18) one has:

$$\begin{aligned} \bar{\Delta}_n &= 2x_n - \nu_D (1+\beta) - \nu_I H \\ \underline{\Delta}_n &= 2x_n + \nu_D (1+\beta) + \nu_I H \end{aligned}$$

We now solve for the technology values making individuals indifferent between the two technologies. Let $\bar{x}$ (resp. $\underline{x}$) be the type that makes the individual n indifferent between choosing one of the two technologies when both the predecessor and the successor coordinate on technology A (resp. B), i.e. $\Delta(0, 0, 0, \bar{x}) = 0$ (resp. $\Delta(1, 1, H, \underline{x}) = 0$). Using the above expressions for $\bar{\Delta}_n$ and $\underline{\Delta}_n$ one gets $\bar{x} = \frac{\nu_D(1+\beta)+\nu_I H}{2}$ and $\underline{x} = -\bar{x}$. As is clear, $\bar{x} > \underline{x}$ and one needs to check that $\alpha > \bar{x}$ for both $\bar{x}$ and $\underline{x}$ to belong to $(-\alpha, \alpha)$. This is equivalent to require that condition (8) holds true. ■

Proof. **(Proposition 1)** With finite action space, monotone strategies can be represented by the cut-off technology values $x(a_{n-1}, h_n)$ where players switch from $a_n = 0$ to $a_n = 1$; moreover $x(.)$ is decreasing in a_{n-1} and h_n as in (9).

Step 1: existence of a greatest and least equilibrium in monotone strategies

Van Zandt and Vives (2003) (Theorem 1) set out minimal assumptions on agents' payoff function (u_n supermodular in a_n and with increasing differences in any pair of variables) as well as on beliefs (first order stochastic dominance) guaranteeing that there exist a greatest and least

Bayes-Nash equilibrium in monotone strategies. As is clear, the assumption about player types is fullfilled by the random walk (1-2) that we use for the technology process. We now show that the remaining conditions as layed out by Van Zandt and Vives are fulfilled in our model. Let $U_n = U(a_n, a_{n-1}, h_n, x_n; s_{n+1})$ be the expected utility (see (7)) of player n, when $n+1$ plays the monotone strategy s_{n+1} defined in (9). One can rewrite U_n as

$$\begin{aligned} U &= \Pr\left(x_{n+1} \geq x^*\left(a_n, h_{n+1}\right) | x_n\right) u\left(a_n, a_{n-1}, 1, h_n, x_n\right) + \\ & \quad \Pr\left(x_{n+1} \leq x^*\left(a_n, h_{n+1}\right) | x_n\right) u\left(a_n, a_{n-1}, 0, h_n, x_n\right) \end{aligned}$$

We now show that U_n has increasing differences in $(a_n, a_{n-1}), (a_n, h_n), (a_n, x_n)$ and (a_n, a_{n+1}).[15] To this end, we first show that u_n has the same properties.

Let $\Lambda_n = \Lambda(a_n, a_{n-1}, h_n, x_n)$ denote the difference in player n's payoff when player $n+1$ switches between technology A and B:

$$\Lambda_n = u\left(a_n, a_{n-1}, 1, h_n, x_n\right) - u\left(a_n, a_{n-1}, 0, h_n, x_n\right) \tag{19}$$

Using the payoffs in (6) we have

$$\Lambda_n = \nu_D \beta\left(2a_n - 1\right) \tag{20}$$

Inspection of (18-20) reveals that our payoff structure displays the following properties:

i) Δ_n is increasing in a_{n-1}, a_{n+1}, h_n and x_n

ii) Λ_n depends only on player's n action, i.e. $\Lambda(a_n, a_{n-1}, h_n, x_n) = \Lambda(a_n)$, and $\Lambda(1) > 0 > \Lambda(0)$.

While property *i)* is in fact equivalent to u_n having increasing differences in the relevant pair of variables (see Van Zandt and Vives (2003), pg. 21), property *ii)* is essential to establish increasing differences of the expected utility, U_n, within a dynamic monotone Bayesian game like the one we consider.[16] This property implies that there is strong complementarity between player

[15] In fact, increasing differences among any pair of arguments of U_n is equivalent to supermodularity in each argument since U_n is defined on the product of linearly ordered sets. Moreover it is straightforward to see that U_n is supermodular in a_n, which is another requirement for the monotonicity of the argmax of U_n.

[16] In fact, from property *i)* one only has that $\Lambda(1, a_{n-1}, h_n, x_n) > \Lambda(0, a_{n-1}, h_n, x_n)$.

n's action and his successor's, i.e. not coordinating brings disutility. With increasing differences only, coordinating increases the payoff of the players. However, not coordinating might not bring about losses. Thus, by means of property *ii*), player n has a strict incentive to coordinate with player $n+1$.

Let $D_n = D(a_{n-1}, h_n, x_n; s_{n+1})$ be the difference in player n's expected payoff from taking the high and the low action when agent $n+1$ plays according to (9):

$$\begin{aligned} D(a_{n-1}, h_n, x_n; s_{n+1}) &= U(1, a_{n-1}, h_n, x_n; s_{n+1}) - U(0, a_{n-1}, h_n, x_n; s_{n+1}) \\ &= \Pr\left(x_{n+1} \geq x_{n+1}\left(0, h_{n+1}\right) | x_n\right) \times \Delta\left(a_{n-1}, 1, h_n, x_n\right) + \\ &\quad \Pr\left(x_{n+1} < x_{n+1}\left(1, h_{n+1}\right) | x_n\right) \times \Delta\left(a_{n-1}, 0, h_n, x_n\right) + \\ &\quad \Pr\left(x_{n+1}\left(0, h_{n+1}\right) > x_{n+1} \geq x_{n+1}\left(1, h_{n+1}\right) | x_n\right) \times \\ &\quad \left[u\left(1, 1, h_n, x_n\right) - u(0, 0, h_n, x_n)\right] \end{aligned}$$

Therefore, showing that U_n has increasing differences in any pair of relevant arguments is equivalent to showing that $D\left(a_{n-1}, h_n, x_n; s_{n+1}\right)$ is increasing in a_{n-1}, h_n, x_n and a_{n+1}.

Letting G be the cumulative distribution function of the increment ε in the stochastic process for the technology values (see (2)), one can rewrite D_n as

$$\begin{aligned} D_n &= \Delta\left(a_{n-1}, 1, h_n, x_n\right) + \Lambda\left(0\right) G\left(\frac{x_{n+1}\left(0, h_{n+1}\right) - x_n}{\sigma}\right) \\ &\quad -\Lambda\left(1\right) G\left(\frac{x_{n+1}\left(1, h_{n+1}\right) - x_n}{\sigma}\right) \end{aligned} \tag{21}$$

As is clear, D_n is independent of a_{n+1}. Moreover:

- D_n is increasing in x_n from properties *i*) and *ii*), and G decreasing in x_n
- D_n is increasing in a_{n-1}, from properties *i*) and *ii*). Furthermore $h_{n+1} = h\left(a_{n+1-M}, ..., a_{n-1}\right)$ is increasing in a_{n-1} and $x_{n+1}\left(a_n, h_{n+1}\right)$ is decreasing in h_{n+1}, such that G is decreasing in a_{n-1}.
- D_n is increasing in h_n from properties *i*) and *ii*) and from h_{n+1} increasing in h_n.[17]

[17] In fact, suppose that h_n increases because any of the actions $a_{n+1-M}, ..., a_{n-2}$ increase. Then, h_{n+1} increases as well. If the increase in h_n is due to a higher value for a_{n-M} , then h_{n+1} is not affected.

Following Theorem 1 in Van Zandt and Vives (2003), the existence of a greatest and least equilibrium in monotone strategies obtains from the supermodularity of the expected utility function and the existence of dominance regions. In fact, $D(a_{n-1}, h_n, x_n; s_{n+1}) < 0$ if $x_n < \underline{x}$ and $D(a_{n-1}, h_n, x_n; s_{n+1}) > 0$ if $x_n > \underline{x}$, which provides us with the starting point for the greatest and least best reply.[18]

Step 2: equilibrium uniqueness

The final step of the proof consists of showing that the greatest and least equilibrium of our game coincide. Another alternative would be to show that the best-reply mapping is a contraction as in Levin (2001). Define $\overline{x}^{\infty}(a_{n-1}, h_n)$ as the limit of the sequence of iterated deletion of dominated strategies when player n observes a_{n-1} and h_n. The threshold $\overline{x}^{\infty}(a_{n-1}, h_n)$ defines the infimum technology value above which player n chooses technology A:

$$D(a_{n-1}, h_n, \overline{x}^{\infty}(a_{n-1}, h_n); s_{n+1}) \leq 0 \tag{22}$$

Similarly, let $\underline{x}^{\infty}(a_{n-1}, h_n)$ be the supremum of x_n below which player n chooses technology B, i.e.

$$D(a_{n-1}, h_n, \underline{x}^{\infty}(a_{n-1}, h_n); s_{n+1}) \geq 0 \tag{23}$$

Thus all strategies for player n have been deleted but the ones such that

$$s(a_{n-1}, h_n, x_n) = \begin{cases} 1 \text{ if } x_n \geq \overline{x}^{\infty}(a_{n-1}, h_n) \\ 0 \text{ if } x_n < \underline{x}^{\infty}(a_{n-1}, h_n) \end{cases}$$

where $\underline{x}^{\infty}(a_{n-1}, h_n) \leq \overline{x}^{\infty}(a_{n-1}, h_n)$. For a unique equilibrium to exist we must have that $\underline{x}^{\infty}(a_{n-1}, h_n) = \overline{x}^{\infty}(a_{n-1}, h_n)$.

Suppose player n observes $a_{n-1} = 1$ and h_n. Then inequality (22) becomes

$$\pi_s(1, \overline{x}^{\infty}(1, h_n)) + \pi_D(1, 1) + \pi_I(1, h_n) + \beta E[\pi_D(1, a_{n+1}) | \overline{x}^{\infty}(1, h_n)] \leq$$
$$\pi_s(0, \overline{x}^{\infty}(1, h_n)) + \pi_I(0, h_n) + \beta E[\pi_D(0, a_{n+1}) | \overline{x}^{\infty}(1, h_n)] \ ,$$

[18] For a graphical representation of the iterated elimination of dominated strategies see Frankel, Morris and Pauzner (2001).

that is

$$\alpha + \overline{x}^{\infty}(1, h_n) + \nu_D + \nu_I h_n + \beta p \nu_D \leq \alpha - \overline{x}^{\infty}(1, h_n) + \nu_I (H - h_n) + \beta \nu_D \ .$$

Solving the latter inequality for $\overline{x}^{\infty}(1, h_n)$ gives

$$\overline{x}^{\infty}(1, h_n) \leq \frac{-\nu_D (1 - \beta p) + \nu_I (H - 2h_n)}{2} \ .$$

The inequality (23) yields

$$\alpha + \underline{x}^{\infty}(1, h_n) + \nu_D + \nu_I h_n + \beta p \nu_D \geq \alpha - \underline{x}^{\infty}(1, h_n) + \nu_I (H - h_n) + \beta \nu_D$$

such that

$$\underline{x}^{\infty}(1, h_n) \geq \frac{-\nu_D (1 - \beta p) + \nu_I (H - 2h_n)}{2}$$

Since $\underline{x}^{\infty}(1, h_n) \leq \overline{x}^{\infty}(1, h_n)$ it follows that

$$\underline{x}^{\infty}(1, h_n) = \overline{x}^{\infty}(1, h_n) = x^*(1, h_n)$$

Following the same procedure for $a_{n-1} = 0$, we get

$$\underline{x}^{\infty}(0, h_n) = \overline{x}^{\infty}(0, h_n) = x^*(0, h_n)$$

where $x^*(0, h_n)$ is given in the main text. Thus a unique equilibrium exists. ■

Proof. (Proposition 2). The incremental payoffs for the compatibility case $\Delta_{n,c}$ and $\Lambda_{n,c}$ are defined along the same lines of (**??**,19). Using (13) they can be written in terms of their counterpart under incompatibility:

$$\Delta_{n,c} = \Delta_n + \nu_D r (1 + \beta) - \nu_D (r + s)(a_{n-1} + \beta a_{n+1}) \quad (24)$$

$$\Lambda_{n,c} = \Lambda_n + \beta \nu_D [s - (r + s) a_n] \quad (25)$$

The dominance regions can be determined as before since:

$$\begin{aligned} \bar{\Delta}_{n,c} &= \Delta_c (0, 0, 0, x_n) = \bar{\Delta}_n + r \nu_D (1 + \beta) \\ \underline{\Delta}_{n,c} &= \Delta_c (1, 1, H, x_n) = \underline{\Delta}_n - s \nu_D (1 + \beta) \end{aligned}$$

which yield $\bar{x}_c$ and $\underline{x}_c$ as in the main text. Since $\bar{x}_c < \bar{x}$ (and similarly $\underline{x}_c > \underline{x}$) it follows that condition (8) is sufficient to guarantee that the adoption game displays dominance regions with compatible technologies.

From (24-25) one has that $\Delta_{n,c}$ is increasing in a_{n-1}, a_{n+1}, h_n and x_n since Δ_n is increasing in the same variables (see property i) in the proof of Proposition 1) and $r+s<2$. Moreover

$$\Lambda_{n,c}(1, a_{n-1}, h_n, x_n) = \beta\nu_D(1-r) > 0 > -\beta\nu_D(1-s) = \Lambda_{n,c}(0, a_{n-1}, h_n, x_n)$$

so that $\Lambda_{n,c}(1) > 0 > \Lambda_{n,c}(0)$ and property ii) (see the proof of Proposition 1) holds. In order to show that the expected payoff U_c has increasing differences in any pair of variables, we have from (21) that

$$\begin{aligned} D_{n,c} &= \Delta_{n,c}(a_{n-1}, 1, h_n, x_n) + \Lambda_c(0)\, G\left(\frac{x_{n+1}(0, h_{n+1}) - x_n}{\sigma}\right) \\ &\quad -\Lambda_c(1)\, G\left(\frac{x_{n+1}(1, h_{n+1}) - x_n}{\sigma}\right) \end{aligned}$$

It is easy to see that the $D_{n,c}$ is increasing in x_n, a_{n-1}, h_n and independent of a_{n+1} from the analysis for D_n and the expressions (24-25). So there exists a greatest and least equilibrium in monotone strategies.

The proof of uniqueness closely mirrors the second step in the proof of Proposition 1. Suppose that player n observes $a_{n-1} = 1$ and h_n. The infimum technology value above which he chooses technology A is given by

$$D_c(a_{n-1}, h_n, \overline{x}_c^{\infty}(1, h_n); s_{n+1}) \leq 0$$

that is

$$\overline{x}_c^{\infty}(1, h_n) \leq \frac{-\nu_D\left[1 - s - \beta q(1-r)\right] + \nu_I(H - 2h_n)}{2}$$

The supremum of x_n below which player n chooses technology B is given by

$$D_c(a_{n-1}, h_n, \underline{x}_c^{\infty}(1, h_n); s_{n+1}) \geq 0$$

yielding

$$\underline{x}_c^{\infty}(1, h_n) \geq \frac{-\nu_D\left[1 - s - \beta q(1-r)\right] + \nu_I(H - 2h_n)}{2}$$

and uniqueness follows from $\underline{x}_c^\infty(1, h_n) \leq \bar{x}_c^\infty(1, h_n)$. Applying the same reasoning for the case $a_{n-1} = 0$ gives the cutoff $x_c^*(0, h_n)$ in the main text. Finally the hysteresis band is given by

$$x_c^*(0, h_n) - x_c^*(1, h_n) = \bar{\sigma} - \frac{\nu_D(r(1-\beta p) + s(1-\beta q))}{2}$$

■

Proof. (Proposition 3). Let S denote the state space for the process x. Without loss of generality we assume $\{0, \alpha\} \in S$ and let $\bar{n}$ denote the number of steps the process takes to move from 0 to α, i.e. $\bar{n} = \alpha/\sigma$. Therefore the (finite) state space S comprises $2\bar{n}+1$ elements and is given by $S = \{-\alpha, -(\alpha-\sigma), ..., -\sigma, 0, \sigma, ..., \alpha\}$. In what follows $s_i \in S$ denotes the i-th state, where the subscript i is an integer between 0 and $2\bar{n}$, i.e. $s_0 = -\alpha, ..., s_{\bar{n}} = 0, ..., s_{2\bar{n}} = \alpha$. We apply results for Markov chains to the random walk (1). The one-step transition matrix is given by:

$$P = \begin{bmatrix} p & q & 0 & \dots & 0 & 0 & 0 \\ q & 0 & p & & 0 & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ 0 & 0 & 0 & & q & 0 & p \\ 0 & 0 & 0 & \dots & 0 & q & p \end{bmatrix} \qquad (26)$$

with $p+q=1$. Given that at time n the Markov chain is in state s_i, P_{ij} gives the probability that the Markov chain moves to state s_j next period, i.e. $P_{ij} = \Pr(x_{n+1} = s_j | x_n = s_i)$ for $n \in N$. The Markov chain described by (26) is irreducible, aperiodic and regular. It follows that the n-step ahead matrix P^n converges as $n \to \infty$ to a positive matrix $\Pi = \mathbf{1}^\top \boldsymbol{\pi}$ (Çinlar (1975), Corollary 2.11).[19] The probability vector $\boldsymbol{\pi}$ is the unique solution to:

$$\boldsymbol{\pi} P = \boldsymbol{\pi} \qquad (27)$$

$$\boldsymbol{\pi}\mathbf{1}^\top = 1 \qquad (28)$$

$$\pi_j > 0, \; j = 0, ..., 2\bar{n} \qquad (29)$$

[19] In what follows boldface characters denote row vectors; for example $\mathbf{1}$ is the $1 \times (2\bar{n}+1)$ unity vector. π_j is the probability to reach state s_j as time goes to infinite, i.e. $\pi_j = \lim_{n\to\infty} P_{ij}^{(n)}$.

Since $\pi_j > 0, \forall j$, each element in the limiting matrix Π is strictly positive. This means that every state can occur with a positive probability as $n \to \infty$ and as a consequence technology lock-ins are ruled out in our game. This result is not affected by the compatibility between the two technologies, since the stationary distribution $\boldsymbol{\pi}$ is driven by the random process (1) only. ■

Proof. (Corollary 4). First of all we proceed in determining the probability vector $\boldsymbol{\pi}$. Using the transition matrix (26) the constraints in (27, 28) become:[20]

$$\begin{gathered} \pi_0 q + \pi_1 q = \pi_0 \\ \pi_0 p + \pi_2 q = \pi_1 \\ \pi_{i-1} p + \pi_{i+1} q = \pi_i, i = 2, ..., 2\bar{n} - 2 \\ \pi_{2\bar{n}-2} p + \pi_{2\bar{n}} p = \pi_{2\bar{n}} \\ \textstyle\sum_{i=0}^{2\bar{n}} \pi_i = 1 \end{gathered}$$

Letting $\rho = \frac{p}{q}$ the above system may be rewritten as:

$$\pi_1 = \rho \pi_0 \tag{30}$$

$$\pi_i = \pi_{i-1} \rho, i = 2, ..., 2\bar{n} - 1 \tag{31}$$

$$\pi_{2\bar{n}} = \pi_{2\bar{n}-1} \rho \tag{32}$$

$$\textstyle\sum_{i=0}^{2\bar{n}} \pi_i = 1 \tag{33}$$

By recursive substitution[21] one has $\pi_i = \pi_1 \rho^{i-1} = \pi_0 \rho^i$ for $i = 2, ..., 2\bar{n} - 1$, and $\pi_{2\bar{n}} = \pi_1 \rho^{2\bar{n}-1} = \pi_0 \rho^{2\bar{n}}$ such that $\sum_{i=0}^{2\bar{n}} \pi_i = \pi_0 \sum_{i=0}^{2\bar{n}} \rho^i$. Thus:

$$\begin{array}{ll} \pi_0 = (2\bar{n} + 1)^{-1} & \text{if } p = \frac{1}{2} \\ \pi_0 = \left(\frac{1 - \rho^{2\bar{n}+1}}{1 - \rho} \right)^{-1} & \text{if } p \neq \frac{1}{2} \end{array} \tag{34}$$

and the stationary distribution $\boldsymbol{\pi}$ is given by (34,30-32). Recall that by construction, for a given step size σ, the barriers $(\alpha, -\alpha)$ coincide with admissible states for the random walk process

[20] For the vector $\boldsymbol{\pi}$ to be uniquely determined as a solution for (27,28) one equation in the system (27) is redundant (or equivalently, the determinant of $(I - P)$ needs to be null). Thus we drop the $(2\bar{n} - 1)$-th equation in (27).

[21] When $\bar{n} = 1$, i.e. the state space for x is $S = \{-\alpha, 0, \alpha\}$, equations (31) are not defined. The stationary distribution is defined by equations (30,32,33) only.

$(s_{2\bar{n}}, s_0)$. On the other hand this might not occur for the cut-off points $x(0,0)$ and $x(1,H)$. Before determining the adoption probabilities, one has to determine the states for the random walk process in (1) corresponding to the cut-offs. We therefore consider s_{n_0} as the nearest state to $x(0,0)$, i.e. $n_0 = \{\min n \in (0, 2\bar{n}) : s_{n_0+1} > x(0,0)\}$. Similarly s_{n_1} is defined by $n_1 = \{\max n \in (0, 2\bar{n}) : s_{n_1-1} < x(1,H)\}$. As is clear from $x^*(0,0) > x^*(1,H)$ it follows that $n_1 < n_0$. It follows that the adoption probabilities are $\pi_A = \sum_{k=n_0}^{2\bar{n}} \pi_k$ and $\pi_B = \sum_{k=0}^{n_1} \pi_k$, or equivalently (adoption probabilities with compatible technologies are defined similarly, considering $n_{0,c}$ and $n_{1,c}$ instead of n_0 and n_1):

$$\begin{array}{c} \pi_A = \pi_0 \sum_{k=n_0}^{2\bar{n}} \rho^k \\ \pi_B = \pi_0 \sum_{k=0}^{n_1} \rho^k \end{array} \tag{35}$$

As it is obvious from (35), the probability of adopting technology A in the long run is decreasing in n_0, and similarly the probability of adopting technology B is increasing in n_1. For $p = 1/2$ one has $\rho = 1$ and $\pi_0 = (2\bar{n}+1)^{-1}$ from (34). The adoption probabilities (35) become $\pi_A = \dfrac{2\bar{n} - n_0 + 1}{2\bar{n}+1}$ and $\pi_B = \dfrac{n_1 + 1}{2\bar{n}+1}$. On the other hand for $p \neq 1/2$ equations (34,35) give:

$$\begin{aligned} \pi_A &= \left(1 - \rho^{2\bar{n}+1}\right)^{-1} \left(\rho^{n_0} - \rho^{2\bar{n}+1}\right) \\ \pi_B &= \left(1 - \rho^{2\bar{n}+1}\right)^{-1} \left(1 - \rho^{n_1+1}\right) \end{aligned}$$

From the expression for the cut-offs in (11,12,15,16) one has

$$\begin{aligned} x^*(0,0) &= \frac{\nu_D(1-\beta p) + \nu_I H}{2} \quad ; \quad x^*(1,H) = -\frac{\nu_D(1-\beta q) + \nu_I H}{2} \quad ; \\ x_c^*(0,0) &= \frac{\nu_D(1 - r - \beta p(1-s)) + \nu_I H}{2} \quad ; \quad x_c^*(1,H) = -\frac{\nu_D(1 - s - \beta q(1-r)) + \nu_I H}{2} \end{aligned}$$

Therefore properties (*i-iii*) easily follows taking the appropriate derivatives in the above expressions. Finally for $r/s \in \left(\beta p, \dfrac{1}{\beta(1-p)}\right)$, the cut-offs with compatible technologies are within $[x^*(1,H), x^*(0,0)]$, and v) obtains. Note that the above interval is always non-empty since $\dfrac{1}{\beta(1-p)}$ is strictly bigger than βp. For $p = 1/2$ the cut-offs $x^*(0,0)$ and $x^*(1,H)$ are symmetric around the origin, i.e. $x^*(0,0) = -x^*(1,H)$, it follows that $n_0 = 2\bar{n} - n_1$ which gives the first claim in property (v). Similarly $x_c^*(0,0) = -x_c^*(1,H)$ if and only if $s = r$. ■

Proof. (Proposition 5). For each state s_j let τ_j be the (function giving the) number of times that the process is in state s_j. The expected first passage time from state s_i to state s_j is given by $m_{ij} = E_i(\tau_j)$. For the Markov chain (26) the matrix $M = \langle m_{ij} > 0 \rangle$ is given by (see Kemeny and Snell (1976), chapter VII):

$$m_{ij} = \begin{cases} \frac{1}{\pi_i} & i = j \\ (4\bar{n}+1-i)\,i - (4\bar{n}+1-j)\,j & i > j \\ j\,(j+1) - i\,(i+1) & i < j \end{cases} \text{ if } p = \frac{1}{2} \tag{36}$$

$$m_{ij} = \begin{cases} \frac{1}{\pi_i} & i = j \\ \frac{1}{2p-1}\left(\frac{\rho^{2\bar{n}+1}\left(\rho^{-j}-\rho^{-i}\right)}{\rho-1} + j - i\right) & i > j \\ \frac{1}{2p-1}\left(j - i - \frac{\rho^{j}-\rho^{i}}{(\rho-1)\rho^{i+j}}\right) & i < j \end{cases} \text{ if } p \neq \frac{1}{2} \tag{37}$$

Fix a pair $(i,j) \in [1,2\bar{n}] \times [1,2\bar{n}]$ with $i > j$, such that m_{ij} is below the diagonal of M. Take another pair (k,l) with $k > l$, $k \geq i$ and $l \leq j$. Using (36,37) the difference $m_{kl} - m_{ij}$ becomes:

$$m_{kl} - m_{ij} = \begin{cases} (4\bar{n}+1)(k-i+j-l) - \left(k^2 - l^2\right) + \left(i^2 - j^2\right) & \text{if } p = 1/2 \\ \frac{1}{2p-1}\left[\frac{\rho^{2\bar{n}+1}\left(\rho^{-i}-\rho^{-k}+\rho^{-l}-\rho^{-j}\right)}{\rho-1} - (k-i+j-l)\right] & \text{if } p \neq 1/2 \end{cases}$$

Taking $k = i$ and $j > l$ gives:

$$m_{il} - m_{ij} = \begin{cases} [4\bar{n}+1-(j+l)]\,(j-l) & \text{if } p = 1/2 \\ \frac{1}{2p-1}\left[\frac{\rho^{2\bar{n}+1}\left(\rho^{-l}-\rho^{-j}\right)}{\rho-1} + l - j\right] & \text{if } p \neq 1/2 \end{cases}$$

Taking $k > i$ and $j = l$ gives:

$$m_{kj} - m_{ij} = \begin{cases} [4\bar{n}+1-(k+i)]\,(k-i) & \text{if } p = 1/2 \\ \frac{1}{2p-1}\left[\frac{\rho^{2\bar{n}+1}\left(\rho^{-i}-\rho^{-k}\right)}{\rho-1} + i - k\right] & \text{if } p \neq 1/2 \end{cases}$$

It follows that $m_{il} > m_{ij}$ for $j > l$ and $m_{kj} > m_{ij}$ for $k > i$. Therefore m_{ij} decreases with j and increases with i below the main diagonal.

Now consider: 1) the pair $(i,j) \in [1,2\bar{n}] \times [1,2\bar{n}]$ with $i<j$, i.e. m_{ij} is above the diagonal of M, and 2) the pair (k,l), $k<l$ with $k \leq i$ and $l \geq j$. From (36,37) one has:

$$m_{kl} - m_{ij} = \begin{cases} (l-j+i-k) + \left(l^2-k^2\right) - \left(j^2-i^2\right) & \text{if } p=1/2 \\ \frac{1}{2p-1}\left[(l-j+i-k) - \frac{\rho^{j+l}\left(\rho^i-\rho^k\right)+\rho^{i+k}\left(\rho^l-\rho^j\right)}{(\rho-1)\rho^{i+j+k+l}}\right] & \text{if } p \neq 1/2 \end{cases}$$

Proceeding as before consider $k=i$ and $j<l$:

$$m_{il} - m_{ij} = \begin{cases} (l-j)(1+j+l) & \text{if } p=1/2 \\ \frac{1}{2p-1}\left(l-j-\frac{\rho^l-\rho^j}{(\rho-1)\rho^{j+l}}\right) & \text{if } p \neq 1/2 \end{cases}$$

When $k<i$ and $j=l$ one has:

$$m_{kj} - m_{ij} = \begin{cases} (i-k)(1+i+k) & \text{if } p=1/2 \\ \frac{1}{2p-1}\left(i-k-\frac{\rho^i-\rho^k}{(\rho-1)\rho^{i+k}}\right) & \text{if } p \neq 1/2 \end{cases}$$

Therefore m_{ij} increases in j and decreases in i above the main diagonal. For a given model parametrization, the cut-off points are obtained using (11,12). Let these states be denoted by s_u and s_l respectively for the upper $(x^*(0,0))$ and the lower $(x^*(1,H))$ cut-off. Note that $s_l < s_u$ since $l<u$. Similarly let $s_{u,c}$ and $s_{l,c}$ be the corresponding states in the presence of convertible technologies obtained via (15,16). It follows that the expected number of adopters is $m_A = m_{ul}$ and $m_B = m_{lu}$. We know that converters reduce the hysteresis band, i.e. $s_{u,c} - s_{l,c} < s_u - s_l$. Moreover it is easy to show that $[x_c^*(1,H), x_c^*(0,0)] \subset [x^*(1,H), x^*(0,0)]$ whenever $r/s \in \left(\beta p, \dfrac{1}{\beta(1-p)}\right)$. It follows that it cannot occur simultaneously that: 1) $m_{lu,c}$ is above and to the right of m_{lu} and 2) $m_{ul,c}$ is below and to the left of m_{ul}. Since m_{lu} and m_{ul} are respectively above and below the main diagonal, it follows that it cannot be that $m_{A,c} > m_A$ and $m_{B,c} > m_B$ simultaneously. ■

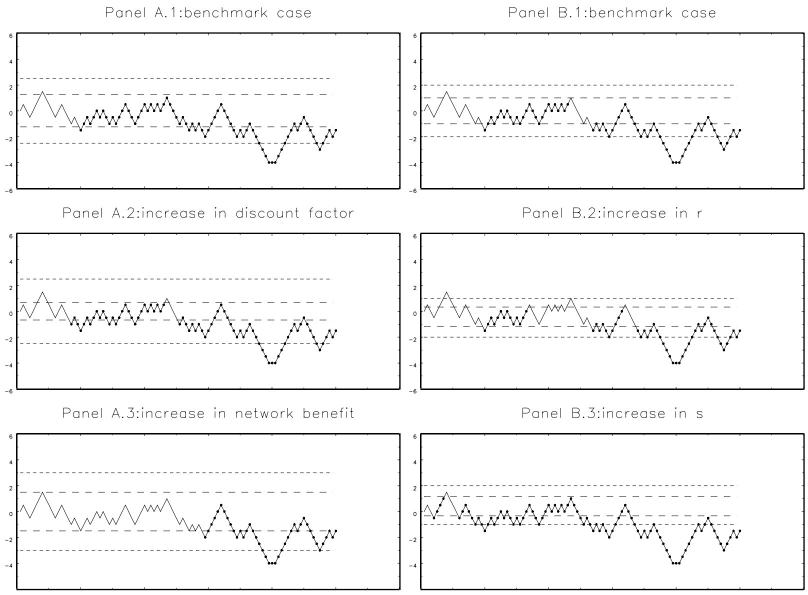

FIGURE 1: TECHNOLOGY ADOPTION PATTERN. Several technology adoption patterns are depicted. Panels A.1,A.2 and A.3 refer to incompatible technologies, while Panels B.1,B.2 and B.3 consider compatible technologies. In each Panel the solid line corresponds to one sample path for the process x_n. Black circles indicate individuals choosing technology B. The dashed lines display the cut-off points and the small-dashed lines the dominance regions limits. The following parameterization has been used: $N = 100$, $p = 1/2$, $x_0 = 0$, $\sigma = 0.5$ and $\alpha = 4$ (technology value process); $\beta = 0.5$ and $\nu = 5$ (individual preferences); $r = s = 0.2$ (compatibility values). Panel A.1 (resp. B.1) is the benchmark case with incompatible technologies (resp. compatible technologies). Panels A.2,A.3,B.2 and B.3 carry out some comparative statics exercises. Panel A.2 considers an increase in the individual discount factor ($\beta = 0.95$); Panel A.3 displays the effects of an increase in the network benefit ($\nu = 6$). Panels B.2 and B.3 consider asymmetric converters ($r = 0.6$ and $s = 0.6$ respectively).