Constitutions as Self-Enforcing Redistributive Schemes^{*}

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Abstract

We present a model of a fiscal constitution (i.e., a transfer scheme between income classes) that is self-enforcing against a background in which predatory activities ('revolutions') are feasible. In this environment, a constitution self-enforces by structuring society's interests in such a way that non-compliance necessarily results in a revolution which society would rather avoid.

Author Keywords: Anarchy; Constitutions; Redistribution;

Self-Enforcement

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1 Introduction

There is a long tradition in political thought (harking back at least to Rousseau ?) which views constitutions as 'social contracts' by which society obliges itself to follow certain rules in order to attain the 'common good' and/or ensure 'civilized coexistence'. In somewhat more modern and technical terms, one would say that constitutions are 'social' commitment devices for achieving efficiency (efficiency in a broad sense, i.e., concerning not only a better allocation of resources but also avoidance of wasteful conflict). However, the question as to how this commitment is achieved (how the 'social contract' is enforced), is often begged.

The problem of ensuring compliance with a constitutional contract goes beyond the usual problem of ensuring compliance with the terms of a private contract. An obvious difference is that normally a private contract is enforced by a third party, a 'court'. When it comes to constitutional contracts, however, the authority of any such 'court' would itself have to derive from a constitutional arrangement. Clearly, the only way to escape such circular reasoning is to try and model constitutional authority as 'self-enforcing'.

The task is then to figure out how exactly a constitution self-enforces? Yet again, it would seem that the issues involved in the self-enforcement of constitutional arrangements go beyond the issues that arise when considering the self-enforcement of agreements between private parties in a lawful environment. In our view, there are two key considerations in this respect. 1) Constitutional agreements must be self-enforcing starting out from a situation of anarchy, i.e., a situation in which agents can 'expropriate' each other -otherwise we would again be presupposing the existence of some sort of authority ¹. Thus, the problem here is not only one of generally not being able to commit, but, specifically, of preventing expropriation in a situation of anarchy. 2) The fact that constitutional agreements typically involve huge number of agents introduces an important 'collective action' dimension absent in self-enforcing agreements involving only 2 (or a few) agents (a point emphasized by Kolmar 2000). The impact on enforcement of this 'collective action' dimension plays, in our view, a crucial role².

In this paper we present a formal model through which we try to elucidate how exactly constitutions are implemented under these considerations (i.e., self-enforcement by means of coordination amongst a large number of agents, starting out from a situation of anarchy).

The *central constitutional problem* in our model will be for agents to commit to a transfer scheme ('a constitution'³) so as to balance effort incentives against conflict avoidance or 'revolution' (the tension between these two considerations arising because high levels of inequality lead to redistributive conflict, while inequality reduction via transfers distorts incentives to

¹The study of situations of anarchy was pioneered by Grossman (1991, 1994, 1995, 2002); see also Falkinger (1999), Hirshleifer (1995), Roemer (1985), Skaperdas, (1992).

²This impact is far from clear cut: It will make enforcement easier in as far as deviations will have to be coordinated if they are to be 'effective', though of course, it might it harder to implement the constitutional contract if individual deviations cannot be deterred. As will become apparent, in our model we take it for granted that individual deviations can be deterred.

³We take from the literature on so-called 'fiscal constitutions' (see Persson and Tabellini 1996, Azariadis and Galasso 2002) the idea to model constitutions as transfer schemes. This allows us to model in a straightforward way the trade-off between constitutional commitments and predation.

exert effort). Thus attention is focused on the trade-off between 'political efficiency' (conflict avoidance) and economic efficiency in the more traditional sense.

The key enforcement issue, however, will be to ensure **ex-post voluntary compliance** with constitutionally mandated transfers, as we will not exclude a priori transfers schemes which, even when complied with, result in a revolution. Neither is it a foregone conclusion that constitutions will be designed so as to avoid revolutions, since certain income distributions that are attainable via revolutions will not be attainable by means of transfers⁴.

Overall, the formalization offered here is firmly in the tradition of 'consensual' constitutional theorizing associated with Rawls (1999), as we will be assuming that all agents are identical at the moment of reaching a constitutional agreement (and that, hence, constitutional agreements are efficient), and that constitutional agreements are reached 'behind a veil of ignorance' (that is, agents agree on a transfer scheme *before* they choose efforts and an income lottery conditional on effort choice plays out).

In such a setup, self-enforcement through coordination is achieved as follows: After agents agree on a constitution, they choose efforts anticipating that, after the income lottery conditional on efforts plays out (which introduces idiosyncratic uncertainty but no aggregate uncertainty), the constitutionally mandated transfers will actually take place and will result in the desired distribution of income. For these expectations to be valid, it must

⁴Of course, given that revolutions are costly, it will generally be desirable to have peaceful outcomes. In any case, the interaction between the various factors influencing this tradeoff makes it hard to predict which compromises between conflict avoidance and incentives will be struck.

be that the effort choices result in an income distribution (prior to transfers) that makes it both feasible and desirable for the transfers to take place. This will require not only that those benefiting from the transfers have an incentive to rebel should the transfers not be forthcoming, but also that those paying the transfers do not prefer a revolution to complying with the constitution.

We find, first of all, that constitutional equilibria indeed exclude revolutionary expropriation. In other words, constitutions here turn out to be inherently non-violent. Since actual constitutions do not seem to allow for violence, this seems a minimal requirement any halfway satisfactory model of constitutions should meet (though it bears repeating that this is not a foregone conclusion in our setup). We find further that the conditions ensuring compliance with the constitution exclude regressive redistribution. The descriptive relevance of this feature is certainly far less obvious. Thinking of constitutions broadly as 'implicit social contracts' as we do here, it seems that this bias towards progressive redistribution is present in many cases if not most.

The rest of the paper is concerned with assessing to what extent effort efficiency (understood as the optimal level of effort in the absence of conflict) and conflict avoidance can be simultaneously attained by a self-enforced constitution. We look at two scenarios, one with positive effort externalities (the 'productive' case); the other with negative effort externalities (the 'rentseeking' case). We take each to represent a fundamental form of competition in society: Positive externalities arise in societies in which 'emulative' or 'constructive' competition prevails, while negative effort externalities obtain in societies in which 'rat-race' competition prevails⁵.

It turns out that in the former scenario, effort efficiency is fully precluded by conflict-avoidance considerations, while in the latter there is no systematic relationship between effort efficiency and conflict-avoidance.

At an intuitive level, the main insight from this paper is that constitutions enforce themselves by structuring society's interests in such a way that non-compliance necessarily results in a revolution which society would rather avoid⁶. Or putting it more succinctly (if somewhat inaccurately): Constitutions prevent revolutions by means of revolutions. This represents a substantial departure from the (legal) conventional wisdom that regards constitutions as the antithesis of revolution.

Two papers that basically deal with the same problem and pose it in similar terms are Bös and Kolmar 2002, and Grossman 2004. However, while these papers also embed their discussion of constitutional self-enforcement in a situation of anarchy, they define the content of constitutions differently, and, as a consequence, both end up viewing self-enforcement differently. Bös and Kolmar 2003 model constitutions as redistributive rules just as we do, but their central constitutional dilemma is how to achieve input redistribution today - desirable because of productivity differences, while avoiding redis-

⁵'Constructive' competition is, of course, the form of competition economics usually focuses on. 'Rat race' competition, on the other hand, has only more recently attracted attention, mainly in the so called 'rent-seeking' literature (see, for example, C. K. Rowley 1988). The distinction, though, goes back at least to Marshall (1920, p.8).

⁶The term 'society' refers to the fact that in our hybrid cooperative/non-cooperative model, the agents of constitutional change are coalitions, rather than individuals. See the discussion in section 3.

tributive conflict in the future. Grossman 2004 models constitutions more realistically than either us or Bös and Kolmar, namely as decision procedures (while maintaining the perfect transferability assumption) but focuses exclusively on conflict avoidance (thus there is no trade-off between economic and political efficiency).

Motivating redistribution via effort externalities represents an innovation relative to existing approaches to redistribution, which focus on risk-sharing (á la Rawls), equity (see Kolm 1996 for an overview of the issues and references), or pure conflict avoidance (see the seminal work of Grossman 1994a, 1995; also, Falkinger 1999).

The paper is structured in the following way. Section 2 presents the model. Section 3 discusses the stylizations used in the model. Section 4 characterizes the equilibrium concept. In section 5 we compare constitutional equilibria with power equilibria, and discuss the roles of constitutions in our set-up. Section 6 discusses some historical examples. Section 7 concludes the paper, summarizing the main results.

2 The Model

There is a unit interval of agents indexed by i. There is a single good in the economy. An agent's utility is defined over levels of consumption of this good (as there are no savings, consumption will coincide with income) and effort levels, and takes the separable form

$$U_i(y,e) = y - v(e)$$

Effort e generates disutility v(e), with v' > 0, v'' > 0. Consumption y enters linearly, i.e., we are assuming risk neutrality⁷.

Agents' incomes will be generated according to the following stochastic technology. Individual effort $e \in [0, \infty]$ will be invested in an activity which the following period yields, identically and independently across agents, a random output $y \in \{y^H, y^L\}$, with $y^H > y^L \ge 0$. We refer to those agents who obtain a high income as 'winners', and denote them by the index w, while we refer to those who obtain a low income as 'losers', and denote them by the index l.

The probability that the activity results in high output (the probability of success), p, depends on the individual effort, e, as well as on the average level of effort in the economy, e^{avg} . We assume that p(0, .) = 0, and $p_1(e, e^{avg}) > 0$, $p_{11}(e, e^{avg}) < 0$, $p_{22}(e, e^{avg}) \le 0$ for e > 0. Here p_j denotes the derivative of $p(e, e^{avg})$ with respect to the *j*th. argument (j = 1, 2), and p_{jk} is the cross derivative of $p(e, e^{avg})$ with respect to the *k*th. and *j*th. arguments (k, j = 1, 2).

We will consider both the case in which $p_2(e, e^{avg}) < 0$ (the **'rent-seeking' case**), the case in which $p_2(e, e^{avg}) > 0$ (the **'productive' case**), and the case in which $p_2(e, e^{avg}) = 0$ (the **'no externalities' case**). In each case, we will assume that $p_{12}(e, e^{avg}) < |p_{11}(e, e^{avg})|^8$.

We assume that, in absence of distortions induced by transfers (to be

⁷If we worked with risk-averse agents instead, we would be introducing an additional trade-off between risk-sharing and incentives, thus unneccessarily complicating the analysis.

⁸This is a sufficient condition for uniqueness of equilibrium.

discussed further down), exerting an interior level of effort is efficient⁹, so

$$[p_1(e^*, e^*) + p_2(e^*, e^*)](y^H - y^L) = 0 \Rightarrow e^* \in (0, \overline{e})$$

Agents will make two types of decisions here: An individual decision (choosing effort), and various kinds of "collective decisions". The game proceeds in multiple stages.

To start with, agents agree on a 'constitution' (to be defined promptly). This is a 'collective' decision. However, since agents are ex-ante identical, it will be made unanimously, and we will not model it explicitly. In the next stage, efforts will be chosen simultaneously. As a consequence of these decisions, a certain distribution of income will result. We refer to it as **interim 1 distribution**.

Definition 1 A constitution is a mapping τ (.) specifying a transfer to the individual as a function of that individual's interim 1 income level¹⁰.

Given an interim 1 distribution, agents have to collectively decide whether to implement or not the constitutionally mandated transfers. We assume that this collective decision must be taken by 'consensus'. Rather than model this 'consensus' directly, we will incorporate an additional condition into our equilibrium concept, namely, 'status quo proofness'. We postpone defining

⁹This assumption does not have substantial implications beyond simplifying the analysis by excluding corner cases in which equilibrium efforts turn out to be efficient despite the presence of effort externalities

¹⁰This corresponds to the notion of fiscal constitution in Persson and Tabellini (1996). In principle, one could define a constitution more generally as a mapping from income levels **and** identities into transfer lotteries.

this concept more precisely until after we finish describing the timeline of events.

The distribution of income in place after the decision to implement (or not) the constitutionally mandated transfers, we will refer to as **interim 2 distribution**. If transfers take place, then a new distribution of income results, and the interim 2 distribution will differ from the interim 1 distribution. Note that if no transfers take place, Interim 2 distribution coincides with Interim 1. Given the interim 2 distribution, agents must then make a final collective decision to 'revolt' or not.

The following diagram illustrates the timeline of the game,

		Interim Distribut	Interim Distribution 1		ion 2	Final Distiribution	
Constitutional Agreement	Effort Decisions	Output Realizations	Constin Manda Transfe		or	olution	

To fill in the gaps in the above description, we first have to define what we mean by a revolution. A **revolution** takes place when a coalition of agents $C \subseteq [0, 1]$ decides to expropriate those outside the coalition. We follow Acemoglu and Robinson (2000) in assuming that a coalition succeeds in expropriating those outside the coalition if and only if the coalition is of at least size r^c . Also, following the same authors, we assume that any revolutionary movement leads to a proportional loss of $1-\lambda$ in aggregate output.

For simplicity, we will consider only two coalitions, the richest and the poorest. We will assume that if a coalition involves only part of an income segment, then every member of that income segment is equally likely to belong to the coalition. More importantly, we introduce the following assumption,

Assumption Amongst all coalitions for whom it pays to revolt, the one which gets to do so is the poorest.

This is a simplifying assumption, and while we think that this assumption describes well many revolutionary episodes in which the poorest have played a protagonic role, we concede that it is not obvious how it applies to situations in which the rich have instigated coupes d'etat (note, however, that such episodes rarely represent attempts to expropriate the poor; rather, they are a reaction of the rich to an attempt on the part of the poor to expropriate them –as such coupes d'etat instigated by the rich might reasonably be regarded as part of the spiral of violence which follows an initial expropriatory round by the poor, rather than as revolutions in their own right).

A key feature of the technology of revolution is how the 'boot' is distributed after a successful revolution. Here, we consider **populist revolutions**, i.e., revolutions that distribute the 'boot' (i.e., the total product outside the rebel coalition net of the loss in output) evenly amongst the members of the rebel coalition¹¹.

Further, we will assume that revolutions take the form of 'revolutionary spirals', i.e., that once a revolutionary expropriation along the lines just sketched takes place, others will follow until the resulting income distribution is revolution-proof.

¹¹Alternatively, one could think of revolutions in which post revolutionary incomes must be equalized -socialist revolutions?

We define now three key concepts: *Renegotiation-proofness, revolutionproofness* and *status quo-proofness.*

- **Definition 2** A distribution of income is **revolution-proof** if there is no coalition of size r^c such that all its members derive a higher income from *one round* of expropriation than from this distribution. A **revolution-proof constitution** is one that mandates transfers that result in a revolution-proof final distribution of income¹².
- **Definition 3** A constitution specifying strictly positive transfers from rich to poor is **progressive status quo-proof** if the interim 1 distribution of income is not revolution-proof for the poor, and the rich prefer the interim 2 distribution resulting from the constitutionally mandated transfers to revolution at the interim 1 distribution. A constitution specifying non-zero transfers from poor to rich is **regressive status quo-proof** if it does not pay for the poorest coalition to revolt but it does pay for the richest coalition to do so.

Status quo proofness simply means that the Interim 1 distribution of income (the "status-quo") is not sustainable: In the case of progressive transfers it requires, 1) That the poor rather revolt than put up with the refusal to transfer resources on the part of the rich, and 2) That the rich rather comply with the mandated transfers than risk a revolution. The case of regressive

¹²Revolution proofness seems closely related to the coalitional core approach. That approach lets the value of a coalition depend on the whole partition, rather than only on its membership. This allows one to model external effects amongst coalitions. Here the value of a coalition and its complement are not independent (if a rebel coalition succeeds, its complement gets 0 value).

transfers, which will turn out to be irrelevant, is similar. Namely, we require that the poor put up with interim 2 distribution rather than risk violent expropriation by the rich, and, moreover, that the rich can credibly threaten to revolt if the poor refuse to comply with the mandated transfers.

This notion captures the idea that transfers can only take place consensually (that is, consensus amongst income $classes^{13}$). It also introduces a form of sequential rationality by ensuring that agents will not refuse to go along with the constitution ex-post.

Definition 4 A constitution is renegotiation-proof in a subset of the set of feasible constitutions if it is efficient amongst all the constitutions in that subset.

It is worth noting that we are using the term 'renegotiation' to refer to the requirement that the ex-ante agreement is efficient, rather than to an expost renegotiation of the agreed transfers (a form of renegotiation we allow only in a very restricted form).

We are now in a position to define what we mean by 'constitutional equilibrium':

Definition 4 A constitutional equilibrium is a vector of efforts $e(i)_{i \in [0,1]}$, and constitution, τ , such that

i) If τ is different from zero, then it is status quo-proof

ii) τ is renegotiation-proof in the subset of feasible constitutions satisfying i)

 $^{^{13}\}mathrm{Note}$ that we are begging the question of how a consensus within an income class is reached. .

iii) e(i) maximizes *i*'s utility under the prevailing constitution, given the effort decisions of other agents.

iv) τ is satisfies the constraint

$$\int_{[0,1]} \left\{ p\left(e\left(i\right), e^{avg}\right) \tau\left(y^{H}\right) + \left(1 - p\left(e\left(i\right), e^{avg}\right)\right) \tau\left(y^{L}\right) \right\} di = 0$$

with

$$e^{avg} = \int_{[0,1]} e\left(i\right) di.$$

Condition i), as mentioned before, not only captures the idea that nonviolent transfers require consensus, but also embodies a form of sequential rationality. By requiring that the interim 1 income distribution be such that, if transfers should not take place (contrary to what was expected), a revolution ensues, it underpins the presumption that the agreed upon transfers will take place. This presumption, in turn, underlies individual effort decisions.

It is worth noting that we are not requiring the constitution to result in a final distribution which is revolution proof. Further, it should be noted that since we have a continuum of identical agents, all will choose the same effort level e in any constitutional equilibrium. Thus the constraint under iv) in the preceding definition can be written more succintly as

$$p(e, e^{avg})\tau(y^{H}) + (1 - p(e, e^{avg}))\tau(y^{L}) = 0$$

Finally, as a benchmark, we need to say what happens if no constitution is agreed upon.

Definition 6 A **power equilibrium** is a vector of efforts $e(i)_{i \in [0,1]}$ such that each e(i) maximizes *i*'s expected utility, given the efforts of the remaining agents.

Clearly, power equilibria can be non-revolution proof (and will often be). Also, constitutional equilibria are by construction always at least as good as power equilibria (note that any outcome attainable in a power equilibrium can also be attained in a constitutional setup -simply by setting $\tau = 0$). Finally, under the assumption that p(0, .) = 0, there always exist power equilibria (indeed, $e = 0 = e^{avg}$ is always a power equilibrium, though perhaps not revolution-proof), and, hence, constitutional equilibria.

3 Characterizing Constitutional Equilibria

3.1 Characterizing Revolution-Proofness

Under our definition of constitutions, any income distribution that results from implementing constitutionally mandated transfers consists of at most 2 income levels, namely $y^L + \tau (y^L)$ (losers' income), and $y^H + \tau (y^H)$ (winners' income).

To obtain the conditions for a post-transfer income distribution to be populist revolution-proof (PR-Proofness), we have to consider two cases: First, we have to consider the case when e, p, and r^c are such that $(1 - p(e, e)) > r^c$. I.e. the case when there are more than enough losers to form a successful revolutionary coalition. The income to be expropriated is that of winners,

$$p(e,e)\left(y^{H}+\tau\left(y^{H}\right)\right)$$

plus that of losers outside the revolutionary coalition,

$$\left[\left(1-p\left(e,e\right)\right)-r^{c}\right]\tau\left(y^{L}\right)$$

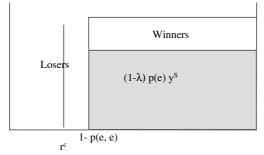
The condition for revolution-proofness is then that the sum of these two totals should not exceed the cost of making a revolution,

$$(1-\lambda) p(e,e) y^H$$

i.e., that

$$(1 - \lambda) p(e, e) y^{H} \ge [(1 - p(e, e)) - r^{c}] \tau (y^{L}) + p(e, e) (y^{H} + \tau (y^{H}))$$

The diagram below illustrates:



Here we are measuring the mass of winners from the right. It is immediate that when

$$\tau\left(y^{L}\right) = \tau\left(y^{H}\right) = 0$$

the condition is automatically satisfied.

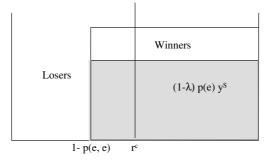
Consider now the case when $(1 - p(e, e)) < r^c$. Now there are not enough losers to build a successful revolutionary coalition. Some winners have to take part in the revolution if a rebel coalition of size r^c is to be attained. The expropriable income is now only that of the winners who remain outside the rebel coalition, i.e.,

$$(1 - r^c) \left(y^H + \tau \left(y^H \right) \right)$$

Thus, in this case, the condition for revolution-proofness is given by

$$(1 - \lambda) y^{H} p(e, e) \ge (1 - r^{c}) \left(y^{H} + \tau \left(y^{H} \right) \right)$$

The diagram below illustrates:



Note that now, when

$$\tau\left(y^{L}\right) = \tau\left(y^{H}\right) = 0$$

the condition for revolution-proofness is not necessarily satisfied. Whether it is, will depend on the size of the shaded rectangle to the left of r^c relative to the unshaded rectangle to the right of r^c .

Summarizing, we have:

Proposition 3 An income distribution $\{\tau(y^L), y^H + \tau(y^H)\}$, such that $\tau(y^L) \leq y^H + \tau(y^H)$,¹⁴ is revolution-proof iff

$$[1-\lambda] y^H p(e,e) \ge$$

$$[1 - \max\{1 - p(e, e), r^{c}\}] \tau(y^{L}) + [\max\{1 - p(e, e), r^{c}\} - r^{c}](y^{H} + \tau(y^{H}))$$

3.2 The Status-Quo Proof Constraint

With $y^L = 0$, status-quo proofness reduces to non-revolution-proofness of the interim income distribution.

¹⁴It can be shown that whenever $\tau(0) > y^S + \tau(y^S)$ is feasible, then $\tau(0) = y^S + \tau(y^S)$ is feasible. Hence, it is without loss of generality to concentrate on the case with $\tau(0) \le y^S + \tau(y^S)$.

As pointed out, as a consequence of corollary 3, the status quo proof constraint reduces to the requirement that the interim 1 income distribution is not revolution-proof (provided the constitution specifies non-trivial transfers). In other words, immediately after incomes are realized but before constitutionally mandated transfers take place, the cost of making a revolution should be smaller than the expropriable income.

As in the above subsection, we can distinguish two cases. The case when $e, p, \text{ and } r^c$ are such that $(1 - p(e, e)) > r^c$. I.e. the case when there are more than enough losers to form a successful revolutionary coalition. In this case, all income will be expropriable (i.e., $p(e, e) y^H$). Hence, status-quo proofness requires

$$p(e, e) y^{H} > (1 - \lambda) p(e, e) y^{H}$$

which is evidently satisfied.

If $(1 - p(e, e)) < r^c$, there will not be enough losers to build a successful revolutionary coalition consisting only of losers. Expropriable income in this case corresponds therefore to that of winners not belonging to the revolutionary coalition (i.e. $(1 - r^c) y^H$). The condition for status-quo proofness is then

$$(1 - r^c) y^H \ge (1 - \lambda) p(e, e) y^H$$

The following proposition summarizes both conditions.

Proposition 4 An interim distribution is status-quo proof iff

$$p(e,e) \lambda \ge r^c - \min\left\{1 - p(e,e), r^c\right\}$$

3.3 Constitutions are Inherently Non-Violent

We show that a non-revolution proof constitution mandating non-trivial transfers cannot be renegotiation proof and status-quo proof at the same time.

Proposition 5 If $r^c \ge \lambda$, there are no renegotiation proof, status quo proof, and non-revolution proof constitutions.

For the proof of this proposition, the reader is referred to the appendix. The previous proposition has some interesting corollaries:

- **Corollary 1** Constitutional equilibria are strictly superior to non-revolutionproof power equilibria.
- **Corollary 2** The constraint that the parties making the transfers should not prefer revolution to compliance is never binding.

Proof. The party making the transfers cannot be indifferent between the distribution that obtains from revolution and the one that obtains via transfers in a constitutional equilibrium, as the previous argument showed a way of obtaining a feasible distribution that is strictly Pareto superior to any post-revolutionary distribution and that induces exactly the same effort. \blacksquare

The last corollary means that we do not need to characterize the post revolutionary distributions, something that greatly eases the analysis.

3.4 Ruling Out Regressive Constitutions

We start by ruling out the possibility of regressive transfers in a constitutional equilibrium.

Proposition 2 Given an income distribution, if it does not pay for the poorest coalition to revolt, then it does not pay for any other coalition to do so.

Proof. Since for any given an income distribution the cost of a revolution is the same regardless of which revolutionary coalition forms, if the expropriated net income (i.e., the income of those not in the revolutionary coalition minus the cost of revolution) is non-positive when the poorest coalition forms, it has to be strictly negative when a richer coalition forms. \blacksquare

As an immediate corollary of this, we have that

Corollary 3 No constitution can be regressive status-quo-proof.

Since no efficiency gains can be attained via regressive transfers, we will normalize and set $y^L = 0$.

3.5 Constitutionally Stable Equilibrium

We have established that constitutional equilibrium will not allow for revolutions (Proposition 1). Here we add this requirement explicitly, as this facilitates the discussion (hence the word 'stable' in the equilibrium designation).

Since effort decisions are taken independently by each individual, an equilibrium must also satisfy an incentive compatibility constraint (IC).

A constitutionally stable equilibrium solves the following program,

$$\max_{e,\tau(y^H),\tau(y^L)} U\left(e,\tau\left(y^H\right),\tau\left(y^L\right)\right) = p\left(e,e^{avg}\right) u\left(y^H + \tau\left(y^H\right)\right) +$$

 $(1 - p(e, e^{avg})) u(\tau(y^L)) - v(e)$

i) (*Populist Revolution – Proofness*)

ii) (Status Quo – Proofness)

iii)
$$p_1(e, e^{avg}) u (y^H + \tau (y^H)) - p_1(e, e^{avg}) u (\tau (y^L)) - v'(e) = 0$$
 (*IC*)
iv) $(1 - p(e, e^{avg})) \tau (y^L) + p(e, e^{avg}) \tau (y^H) = 0$ (*BC*)
v) $y^L + \tau (y^L), y^H + \tau (y^H) \ge 0; e \in [0, \overline{e}]$

 $vi) e = e^{avg}$

Note that we are using a first order condition to characterize incentive compatibility ('first order approach'). This is justified since $p_1 > 0$ and $p_{11} < 0$.

The feasible set can be shown to be compact, and necessary and sufficient condition for it to be non-empty is that $r^c > \lambda$ (which guarantees that there are distributions that are revolution proof). Since the objective is continuous, this program will have a solution.

4 Constitutional Equilibria and Power Equilibria

How do the various aims of constitutions interact? In order to try and answer this question, we compare constitutional equilibria with power equilibria. We start by looking at the productive case, in which constitutions have only a **stabilizing role**, as no revolution-proof constitution attains first best efforts (i.e., 'effort efficiency'). Second, we look at the set-up without spillovers in order to illustrate what we call the **pure coordinating role** of constitutions. Finally, in the rent-seeking case, we look at the relation **between (effort)** efficiency and stability.

4.1 The Set-Up Without Revolutions: First Best versus Equilibrium Efforts

This section establishes a simple benchmark result to be used in describing stable constitutional equilibria. We characterize first-best efforts, both in the "productive" and in the "rent-seeking case", and compare them with equilibrium efforts in the absence of revolutionary technology.

First-best efforts are given by the solution to

$$e^{*} = \arg\max_{e} p(e, e) \left[y^{H} - y^{L} \right] - v(e)$$

Symmetric equilibrium efforts are given by

$$\overline{e} = \arg\max_{e} \ p\left(e,\overline{e}\right) \left[y^{H} - y^{L}\right] - v\left(e\right)$$

- **Proposition 5** *i)* If $p_{12} < |p_{11}|$ and $p_2 > 0$, then both first best effort, e^* , and symmetric-equilibrium effort, \overline{e} , are unique and such that $e^* > \overline{e}$.
 - ii) If $p_{12} < |p_{11}|$ and $p_2 < 0$, then both first best effort, e^* , and symmetric equilibrium effort, \overline{e} , are unique and such that $e^* < \overline{e}$.

Proof. Uniqueness: The first-order condition for first-best efforts is given by

$$[p_1(e,e) + p_2(e,e)] [y^H - y^L] = v'(e)$$
(1)

The first-order condition for symmetric equilibrium effort is given by

$$p_1(e,e) \left[y^H - y^L \right] = v'(e)$$
 (2)

Differentiating the LHS of condition 1), we obtain

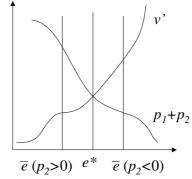
$$[p_{11}(e,e) + p_{22}(e,e) + 2p_{12}(e,e)] [y^H - y^L]$$
(1')

Differentiating the LHS of condition 2), we obtain

$$[p_{11}(e,e) + p_{12}(e,e)] [y^H - y^L]$$
(2')

For both conditions, we obtain v''(e) when we differentiate their RHS.

Since v''(e) > 0, while $p_{11} < 0$, $p_{22} < 0$ and $p_{12} < |p_{11}|$, both e^* and \overline{e} must be unique. If the solution \overline{e} to equation 2) is plugged into equation 1), the LHS of this latter equation will exceed (or be below) the RHS, as $p_2 > 0$ (< 0). The following diagram illustrates,



Hence, the remaining claims follow.

These results are intuitive and familiar from the study of coordination games¹⁵. The productive case corresponds to 'positive spillovers': In equilibrium, there is too little effort as agents do not internalize the positive

 $^{^{15}\}mathrm{See}$ Cooper 1999.

effect of their effort on others. The 'rent-seeking' case corresponds to 'negative spillovers'. In equilibrium, there is too much effort as agents do not internalize the negative effects of their effort on others.

4.2 The Role of Constitutions

4.2.1 The Productive Case: A Pure Stabilizing Role

In the productive case $(p_2 > 0)$, we established that, in a non-political setup without transfers, equilibrium effort was lower than the efficient one. Therefore, to attain efficiency gains, i.e., to incentivate additional effort, transfers from the poor to the rich would be called for. However, as argued above (Corollary 3), a constitution specifying such transfers is never stable. Hence, we have the following proposition,

Proposition 6 In the productive case, the first-best level of efforts cannot be attained via constitutional transfers.

From the definition of constitutional equilibrium, a stable constitution will require transfers from the rich to the poor (if any). Hence, a constitution will distort effort away from the first best level. Still, such a distorting constitution might be superior to a power equilibrium, as it might be the only means to avoid a costly revolution (costly not only because part of the economy's income is directly lost in the wake of revolutionary violence, but also because the expectation that income will be fully or partially expropriated will depress effort even more).

4.2.2 No Spillovers Case: A Pure Coordinating Role

We look here at the borderline situation in which there are no effort externalities, i.e., we assume that $p_2 = 0$. Then it is immediate that

Proposition 7 First best effort e^* is attained at a stable constitutional equilibrium iff $\tau(y^H) = \tau(y^L) = 0.$

In other words, efficiency requires here only that effort be fully rewarded, i.e., that winners are able to keep all of their income. The only issue is whether a situation without transfers is stable. If we take this to be the case, we have, however, the following result,

Proposition 8 If the first-best effort level e^{*} can be achieved at a constitutionally stable equilibrium, it can be achieved at a revolution-proof power equilibrium.

Does this mean that constitutions are superfluous under these conditions? The answer is no. The following example illustrates:

Example: Assume that at e^* the interim distribution is revolution-proof. Since p(0, .) = 0, we have that $1 - p(0, 0) > r^c$. This means that if a revolution takes place, winners will be fully expropriated, as there will be more than enough losers to form a rebel coalition of size r^c .

Now, if at e^* the interim distribution is revolution-proof (which we assumed), this will be the unique constitutionally stable equilibrium. By the preceding proposition, this effort level will also be sustainable as a power equilibrium. The point we want to emphasize is that there will exist **other** **power equilibria** besides the one that sustains efficient effort. In particular, under the previous assumption, it will be a power equilibrium to select zero effort. To see this, note that if an agent expects all others to select zero effort, then that agent will foresee that the resulting interim 1 distribution will not be revolution-proof. This implies, that should this agent turn out a winner in the income lottery, he or she would be fully expropriated in the ensuing rebellion (as $1 - p(0,0) > r^c$). But then, it clearly does not pay to exert positive effort.

If instead the agent expected others to exert e^* , it would pay for him or her to do so as well. In this case, the interim distribution of income will include a large number of winners, and it will not prove possible to form a rebel coalition that can fully compensate its members using the revolutionary boot. But then, it pays to exert the optimal effort level.

Finally, note that besides the two power equilibria mentioned (the efficient one and the zero effort one) there could be quite a few others. In all equilibria a revolution would take place, with some winners invariably included in the revolutionary coalition. But since the probability of any individual winner taking part in the revolutionary coalition is $\frac{r^c - (1 - p(e^R, e^R))}{p(e^R, e^R)}$, (where e^R is the corresponding effort level), is necessarily strictly less than one, e^R will always be below the efficient level.

What the constitutional process does is simply to select an equilibrium. This is what we refer to as the **pure coordination role of constitutions**. By coordinating effort decisions in the first period, the constitutional decision leads to a revolution-proof distribution of income in the second period, which, in turn, supports agents' effort choices. By the way, constitutional decisions are reached unanimously, as all agents are ex-ante identical. In this sense, the constitution is self-enforcing (we discuss self-enforcement further in Section 6).

Proposition 9 In the case with $p_2 = 0$, a constitutionally stable equilibrium either attains effort efficiency or requires excess transfers, i.e., too much redistribution relative to what would have induced first best efforts.

Proof. If the effort efficient allocation is feasible, i.e., satisfies all the constraints of program A, then, trivially, it is the unique constitutional equilibrium.

If the effort efficient allocation is not feasible, then parametrize transfers by the difference in post-transfer incomes between losers and winners, $d \in [0, y^H]$, and define, for any given revolutionary technology $\{r^c, \lambda\}$, a critical effort level $e^c(d)$ such that any higher effort (at transfer levels corresponding to d) results in a revolution-proof distribution, while any lower effort does not. This function must be increasing¹⁶.

Start out from the (first-best) efficient effort level and the level of transfers that elicits such effort level, say d^* . In the case with p(0, .) = 0, $d^* = y^H - y^L$. Hence, adjustment has to be downwards, since by corollary 3, regressive transfers cannot be sustained in a constitutional equilibrium.

¹⁶As d falls, the proportion of total product within the revolutionary coalition is higher, and hence, the net boot falls. If at, say, level d' the critical effort was e(d') (the net boot was exactly 0), at transfers corresponding to a lower d, the corresponding distribution yields a strictly negative net boot. Since the net boot increases as e falls, c.p., the new critical effort level must be below the previous one.

Note that this last proposition and Corollary 1 establish an **endogenous bias towards progressive redistribution** in this sort of economies. A little reflection will suffice to see that this bias is a direct consequence of assuming that the cost of revolution is independent of the composition of the rebel coalition. In assuming this, we were trying to capture the intuition that political power is distinct from economic power. Allowing for feedback between the rebel coalition's income and the cost of revolution would most likely lead one to qualify the previous conclusion.

4.2.3 The Rent-Seeking Case: Effort Efficiency versus Stability

One should not conclude, though, that whenever the efficient effort level can be supported in a constitutionally stable equilibrium, it can be supported as a power equilibrium as well. The "rent seeking" case offers an example of a situation in which it is possible to sustain effort efficiency via transfers though not in their absence, i.e., in a power equilibrium, regardless of whether it is revolution-proof or not.

Proposition 10 Efficiency can never be achieved in a power equilibrium in "rent-seeking" situations.

Proof. Clearly, a non-revolution proof power equilibrium can only sustain sub-optimal effort. Assume that the power equilibrium is revolution-proof. From conditions 1) and 2) in Subsection 3.1, it is immediate that the equilibrium effort level can never be optimal. \blacksquare

Even though transfers could in principle attain efficiency, the status-quo and revolution-proofness constraints prevent this from always being possible. Intuitively, status-quo proofness requires that efficient effort not be too high, for else the interim 1 distribution will be revolution-proof. Revolutionproofness, on the other hand, might require transfers that exceed those leading to efficient effort. In the rent seeking case, even if we assume that $p_1 > |p_2|$, there could be too much or too little redistribution. The argument in the proof of proposition 8 fails in this case as now $d^* < y^H - y^L$, which implies that reducing level of transfers starting from d^* no longer requires regressive transfers.

Note the dual role the threat of revolution plays here: On the one hand, the revolution-proof constraint forces progressive redistribution in the first place. On the other hand, the status-quo constraint relies on this very same threat of revolutionary violence to enforce these transfers¹⁷.

5 Discussion of Some Salient Features of the Model

As our modelling, by its nature, must be highly aggregated, it is unavoidable to have, so to say, 'black boxes' lodged at various junctures in the model. In what follows, we discuss what we consider to be the three most important such 'black boxes': The constitution as a redistributive scheme, the formation of coalitions, and finally, the dynamics of revolutions .

¹⁷By the way, it is the absence of a direct cost of revolution that forces Bös and Kolmar (2003) to rely on infinite repetition in order to enforce transfers (they only have an indirect cost of expropriation arising from the expenditure of resources on power).

5.1 Constitutions as Redistributive Schemes

Here we have chosen to model constitutions as redistribution schemes. In doing so, we are following a widespread practice in the political economy literature (for example, Persson and Tabellini, 1996, and Azariadis and Galasso, 2002, among others). Admittedly, modelling constitutions in this way is not particularly realistic as, clearly, real-world constitution are to a great extent an specification of procedures governing policy making.

It seems to us that a 'procedural' constitution in which what is ultimately at stake is fully divisible and transferable differs from a fiscal constitution only in that the latter directly specifies a distribution, while the former does so only indirectly¹⁸. Both, in the end, reduce to redistributive schemes.

As the emphasis here is not so much on constitutional design as on selfenforcement, we feel that skipping the details of how a constitution translates into an actual distribution is not such a serious omission (so long as a constitution is understood as a set of procedures for arriving at decisions concerning fully divisible and transferable magnitudes), though, of course, eventually this issue would have to be addressed¹⁹

In this light, 'constitutions' in this paper should be thought of broadly

¹⁸However, it is worth noting that most constitutions include commitments that rather directly impinge on the legal feasibility of implementing redistributive schemes (for example, clauses specifying a social role for private property or limiting the state's ability to expropriate or regulate economic activity –a clear illustration of this can be found in the 16th amendment of the American Constitution, introduced in 1913 in order to make income tax legislation constitutional after the Supreme Court had struck down such legislation).

¹⁹An example of such a procedural redistributive constitution is Grossman (2004).

as reduced form representations of the overall legal order ('implicit social contrats' as in Skyrms ?), rather than narrowly as constitutions in the literal sense. By the same token, one should not interpret the redistributive schemes in our 'constitutions' narrowly as pure transfer schemes of the kind that have become commonplace in advanced societies, but instead interpret them broadly as reduced form representations of the net effect of the legal system on the distribution of wealth via all sorts of channels (not only pure transfer schemes, but also, for example, the provision of free public goods and the enforcement of property rights).

5.2 'Collective' Decisions

Another strategic modelling decision concerns how to model collective action (as evidently, political action is most of the time collective, i.e., coordinated). We have decided to follow the conventional approach in cooperative games, and not deal explicitly with the issue (thus our model is a hybrid cooperative/non cooperative creature). Thus, when we postulate that coalitions act in unison, we are not necessarily assuming that their members agree unanimously (we do assume that transfers can only take place if all **coalitions** agree unanimously, though this seems quite natural when operating against an anarchic background). For example, in our set-up it is obvious that any individual called upon to cede income would be better off by deviating and refusing to do so (as there is a continuum of agents, the aggregate outcome would not be affected). The implicit assumption is that there are (unmodelled) arrangements in place that prevent individuals from doing so. Note that this paper is not about how **individuals** might be prevented from deviating from an established norm (there are many arrangements that fulfill this role, ranging from courts to social values²⁰), but rather it asks how groups might be forced to do things that in principle seem to go against their interests. This is the sense in which this paper is not about private but social contracts (i.e., constitutions) –this focus on social contracts is what, in turn, justifies our emphasis on self-enforcement.

Closely related is the issue of ex-post 'renegotiation'. We allow only for limited ex-post 'renegotiation' of constitutional agreements, in that we assume that at the interim 1 stage there is either full compliance with the mandated transfers or none at all. Why not assume instead that the winners in the income lottery are able to unanimously agree on a counterproposal that gives the losers just enough to make them indifferent between mounting a revolution and accepting this counteroffer ? Since such a proposal (if accepted by the losers) will make every winner strictly better off (the revolution proofness constraint need not be binding in the case with negative effort externalities), agreement on it should be easy. Actually, this is a much less 'natural' scenario than might appear at first. It implicitly assumes that the coalition of winners is in a position to make 'take-or-leave-it' offers to losers. But losers might not believe that winners will risk a revolution if they reject the winners' counteroffer. In a richer, open-ended renegotiation procedure, they might well expect a more favorable offer in that case. And if losers

²⁰One can even come up with fully individualistic stories to justify compliance by individuals. For example, we could have a psychological equilibrium (see Geneakopolos et al. 1989) in which individuals are happy to transfer money so long and only so long as they know that all others are complying. Thus, we can construct a (psychological) game with two equilibria only, full compliance and no compliance.

believe that winners are bluffing, they will have every reason to insist on the implementation of the constitutionally mandated transfers, thus effectively reducing renegotiation options to full compliance versus no compliance, as we have it^{21} .

5.3 Revolutionary Spirals

We assume that revolutions take the form of 'revolutionary spirals', i.e., that once a revolutionary expropriation takes place, others will follow until the resulting income distribution is revolution-proof. Many important revolutionary episodes (e.g., the French Revolution, the Mexican Revolution) seemed to have followed this spiralling pattern, with successive rounds of expropriation/restoration until finally the situation settled, more out of exhaustion than anything else. Exactly how and why such a pattern arises is, of course, an interesting question²², but in principle out of the scope of a highly aggregated exercise in modelling as ours.

Thus we will limit ourselves here to one remark concerning what might appear to be myopic behavior in our description of revolutions as spirals. Note that in the course of a revolution, groups of agents revolt whenever one

²¹Even if one grants the winners the right to make take-it-or-leave-it offers, allowing renegotiation proposals to treat agents within an income class differently will suffice for there to be in general plenty of revolution-proof distributions that make every winner strictly better off. However, not all winners will rank those in the same way. This raises doubts as to the ability of winners to reach agreement on a counterproposal. In fact, in such a scenario there appear to be obvious incentives for a subset of winners to counter

any renegotiation proposal by one that grants its members higher incomes.

 $^{^{22}\}mathrm{A}$ related paper is Gershenson and Grossman 2000.

round of expropriation results in a better income distribution for them, and thus apparently 'fail to foresee' the revolutionary spiral that such an expropriation might unleash and which might very well result in a final income distribution which they do not prefer to the original one. This 'short horizon' feature is endemic to attempts at predicting cooperative decisions (the most prominent example is offered by the solution concept of 'core', which allows a coalition to block even if it can itself be blocked). As in those cases, since it is collective decisions that are being described, the myopia interpretation is by no means the only reasonable one. For example, one could instead interpret the posited behavior as the outcome of some sort of dynamic 'prisoners dilemma' situation.

6 Some Historical Evidence

The central insight from the formalization presented here is that constitutional order is maintained because it itself structures the interests of society in such a way that society fears the consequences of this order coming unstuck.

The Mexican Revolution is in our view a good example of how fear of revolution helped maintain the status quo -the central feature of the status quo being the permanence of one party, the PRI party, in power for almost 70 years. The Mexican Revolution resembled nothing more than an almost unstoppable spiral of destruction (in fact, a process so destructive and violent that none of its leaders survived it). After some sort of acceptable status quo was attained around the middle of the 1930's with the succesful entrenchment of the PRI party (back then called National Revolutionary Party, rather than Revolutionary Institutional Party - what PRI stands for in Spanish- which is how it eventually, and tellingly, came to be named), none of the major players wanted to risk a rerun of the experiences of the two preceding decades. Those major players eventually adquired a very formal institutional existence in the so called 'sectors' which formed the ruling party (in the times of Lázaro Cárdenas there were 4 of them, labour, agrarian, popular and military; later on, only three, labour, agrarian, and popular -these sectors seem the closest real-life counterparts to the collective actors appearing in our model). Actually, the real constitution in Mexico, rather than the official document, would seem to have been the commitment of these sectors to the procedures and values of the ruling party (first and foremost, obedience to the president). Note the interesting 'boots-strap' character of the whole construction: The sectors came to be at the same time that the ruling party. The 'constitution' structured the interests of society in such a way that those very same interests feared the breakdown of the constitution -as in our formalization. The progressive redistributive bias of the regime that emerged after the Revolution is easy to document in its early decades (agrarian reform, oil nationalization, social security system), though it diluted steadily as time passed.

While the Mexican case is the one we are most familiar with, it would seem that this fear of revolution is also key to the stability of modern China. In China this fear seems to have allowed the Communist Party to keep a tight control of political life even in the midst of substancial economic change (and, at the time of Tianmen demonstrations, substantial political turmoil as well). Perhaps the Chinese case offers an even clearer illustration of the logic of our model, as no political actor was allowed to operate outside the Communist party at all -unlike what happend in the Mexican case. The progressive redistributive bias of the Chinese post revolutionary regime is again, if anything, even clearer than that of the Mexican regime²³.

Though our formalization does not take this explicitly into account, we think it indirectly helps explain the fact that long periods of stability follow traumatic revolutions. The point is that these episodes help cement the fear of revolution which according to our formalization is the key to postrevolutionary stability. It is easy to think of examples along these lines: Besides Mexico after the 1930's, and China following the Communist takeover, we could mention Spain, following the Civil War, India following the chaos of Partition, Russia after the end of the civil war that followed the Bolshevik Revolution, Germany following hyperinflation in the Weimar Republic, Nazism and World War II, etc.

While it is hard to relate precisely a highly aggregated model to specific historic episodes, we think the basic insight of the model stands well against a reasonable interpretation of the historical evidence.

7 Conclusions

Kolmar (1998) argues that constitutions must be self-enforcing and suggests that coordination is the key to this self-enforcement. Grossman (2004) and Bös and Kolmar (2003) present models of self-enforcing constitutions that

²³Actually, it seems hard to think of examples of regressive constitutions. The regime established after military coup de etats come to mind, but most of those were rather short lived and are perhaps better understood as early stages of revolutionary spirals.

incorporate such ideas. These papers add an important ingredient by embedding their stories of constitutional self-enforcement in situations of anarchy. The present work is a complimentary attempt to incorporate this way of thinking into a formal model in order to explore how coordination might lead to commitment in a self-enforcing manner -starting out from a situation of anarchy.

We argue that a constitution enforces itself by structuring society's interests in such a way that non-compliance necessarily results in a revolution which society would rather avoid. Or putting it more succinctly (if somewhat inaccurately): Constitutions prevent revolutions by means of revolutions.

Specifically, we show how general agreement on a transfer scheme results in collective effort decisions which generate an income distribution such that an eventual refusal to implement the agreed upon transfer scheme would trigger a revolution everyone prefers to avoid. Thus agents are lead to comply with the transfer scheme agreed upon, validating their individual effort decisions (which were predicated upon the implementation of the transfer scheme).

Appendix

Proposition 5 If $r^c \ge \lambda$, there are no renegotiation proof, status quo proof, and non-revolution proof constitutions.

Proof. Take a non-revolution proof constitution τ_o mandating non-trivial transfers, $\{\tau_o(y^H), \tau_o(y^L)\}$. These transfers must result in an interim 2 distribution with income levels $\{y^H + \tau_o(y^H), y^L + \tau_o(y^L)\}$ and which is

not revolution-proof. Representing this interim 2 distribution by a mapping $y(\tau_o) : [0,1] \rightarrow \{y^H + \tau_o(y^H), y^L + \tau_o(y^L)\}$, and adopting the convention of ordering agents by their incomes, from lowest to highest, for $y(\tau_o)$ not to be revolution-proof, we must have that

$$\int_{[r^{c},1]} y(i) \, di > (1-\lambda) \int_{[0,1]} y(i) \, di$$

Now work backwards. The revolutionary spiral at the interim 2 distribution will lead to a final distribution with income levels $\{y_i\}_{i=1}^n$, $n \in \{2, 3\}$, which, by construction, must be revolution-proof. This final distribution (and only this) determines the effort level e_o . Representing this final distribution again by a mapping $y_F : [0,1] \to \{y_k\}_{k=1}^n$, $n \in \{2,3\}$, for this distribution to be revolution proof, we must have

$$\int_{[r^{c},1]} y_{F}(i) \, di \le (1-\lambda) \int_{[0,1]} y_{F}(i) \, di$$

Next, we will sketch a procedure to construct another distribution, one which can be achieved with the transfer technology, elicits the same effort level, is also revolution-proof and which moreover is status-quo proof and Pareto superior to the original distribution.

Starting out from the post-revolutionary distribution y_F , redistribute the income accruing to winners (all who obtained an interim 1 income of y^H) equally amongst all of them; do the same for losers (all those that obtained an interim 1 income of y^L). That is, obtain a new distribution $y_1 : [0, 1] \rightarrow \{\overline{y}_F^{LI1}, \overline{y}_F^{WI1}\}$, with

$$\overline{y}_{F}^{LI1} = \int_{\left\{i \in [0,1] | y_{I1}(i) = y^{l}\right\}} y_{F}(i) \, di$$

and

$$\overline{y}_{F}^{WI1} = \int_{\{i \in [0,1] | y_{I1}(i) = y^{W}\}} y_{F}(i) \, di$$

where y_{I1} is the mapping representing the interim 1 distribution associated with effort level e_o . The distribution y_1 satisfies

$$y_{1}(i) = \begin{cases} \overline{y}_{F}^{LI1} & if \quad y_{I1}(i) = y^{L} \\ \\ \\ \overline{y}_{F}^{WI1} & if \quad y_{I1}(i) = y^{W} \end{cases}$$

This redistribution does not change in any way the effort incentives, since agents are assumed risk neutral, and the income level for each class (winners, losers) is the certainty equivalent of the post-revolutionary distribution within each class (which could have been uneven within each class as these classes are defined with reference to the interim 1 distribution). Also, the resulting distribution is attainable via transfers from an interim 1 distribution, as it consists of two income classes only. And finally, it is revolution-proof as well, since it is more equal than the previous one.

The easiest way to prove this last point is to show that

$$\int_{[r^{C},1]} y_{F}(i) \, di \ge \int_{[r^{c},1]} y_{1}(i) \, di \tag{(1)}$$

Since agents are ordered in a distribution y_F from lowest to highest, we have that,

$$\int_{[r^{C},1]} y_{F}(i) \, di \ge \int_{[0,1]} y_{F}(i) \, di$$

This implies that

$$\int_{[r^{C},1]\cap\{i\in[0,1]|y_{I1}(i)=y^{l}\}} y_{F}(i) \, di \ge \int_{\{i\in[0,1]|y_{I1}(i)=y^{l}\}} y_{F}(i) \, di = \overline{y}_{F}^{LI1}$$

Similarly for \overline{y}_F^{WI1} .Now,

$$\int_{[r^{C},1]} y_{F}(i) d =$$

$$\int_{[r^{C},1] \cap \{i \in [0,1] | y_{I1}(i) = y^{l}\}} y_{F}(i) di + \int_{[r^{C},1] \cap \{i \in [0,1] | y_{I1}(i) = y^{W}\}} y_{F}(i) di$$

$$\geq \overline{y}_{F}^{LI1} + \overline{y}_{F}^{WI1}$$

$$\geq \int_{[r^{C},1] \cap \{i \in [0,1] | y_{I1}(i) = y^{l}\}} \overline{y}_{F}^{LI1} di + \int_{[r^{C},1] \cap \{i \in [0,1] | y_{I1}(i) = y^{W}\}} \overline{y}_{F}^{WI1} di$$

But the last term in the preceding expression is just

$$\int_{\left[r^{C},1\right]}y_{1}\left(i\right)di$$

Thus we have established inequality (1).

Now, take all the product lost in the original revolutionary spiral, denoted by c, and add it back, dividing it equally amongst all agents. That is, construct a new distribution y_2 such that

$$y_{2}(i) = \begin{cases} \overline{y}_{F}^{LI1} + c & if \quad y_{I1}(i) = y^{L} \\ \\ \\ \overline{y}_{F}^{WI1} + c & if \quad y_{I1}(i) = y^{W} \end{cases}$$

This keeps the difference between income classes from y_1 unchanged. It can be easily shown that effort only depends on the difference between certainty equivalents for each interim 1 income class (losers, winners) in the final distribution (here, $\overline{y}_F^{WI1} - \overline{y}_F^{LI1}$). Hence, since the previous step leaves this difference unchanged, it must continue to induce the same effort e_o . Besides, an equal distribution across all agents must be revolution-proof -since $r^c \geq \lambda$ (if $\lambda > r^c$, nothing is), so the new distribution must also be revolution proof. If we have

$$\int_{[r^{c},1]} y_{1}(i) \, di \leq (1-\lambda) \int_{[0,1]} y_{1}(i) \, di$$

and

$$(1 - r^c) c \le (1 - \lambda) c$$

then clearly

$$\int_{[r^{c},1]} y_{1}(i) \, di + (1-r^{c}) \, c = \int_{[r^{c},1]} \left(y_{1}(i) + c \right) \, di \le (1-\lambda) \int_{[0,1]} \left(y_{1}(i) + c \right) \, di$$

It is also status quo proof: To see this, note that the interim 1 distribution remains unchanged (as effort does not change, and there is no aggregate uncertainty). Further, if winners preferred to go along with the original transfer scheme, i.e., $y(\tau_o)$, they must be willing to go along with the transfer scheme that would result in y_2 (starting out from the interim 1 distribution associated with effort e_o), as it gives them a higher income than the one they obtained following compliance in the original revolutionary constitutional scenario. Since this new transfer scheme is feasible (by construction), and leads to a Pareto superior final distribution, the original non-revolution proof transfer scheme could not have been renegotiation proof.

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