Extended Abstract of: Conventions in a Spatial Environment

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Abstract

We consider an $n \times n$ evolutionary coordination game played in a finite discrete spatial space where the number of agents increases over time. Each location can only sustain a limited number of agents. Agents can migrate to an adjacent location. An agent is matched to play the game with opponents in her neighborhood. The fitness of an agent is increasing in the average payoff, the number of possible matches, and decreasing in the number of opponents at the same location as the agent. If the migration procedure is such that the number of agents is unimodal distributed in a population, the total population will be portioned into isolated sub-populations where each sub-population will be in a Nash-equilibrium. The agents will eventually be forced to interact with other sub-populations as the total population grows. If there exist an equilibrium that is both risk-dominant and efficient, this equilibrium always will prevail. The slower the absolute growth is the more likely is it that an efficient equilibrium will prevail.

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1 Introduction

The existence of multiple equilibria in games has long posed an obstacle as well as a challenge for game theory in the pursuit of creating predictive models. In evolutionary game theory much of the attention has been on equilibrium selection in coordination games, in particular on models that satisfies the risk-dominance criteria introduced by Harsanyi & Selten (1988). Among the first

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models supporting the risk-dominance criteria were Foster & Young (1990), Kandori et al. (1993), and Young (1993). The driving force behind their results is that each agent has a small probability to change strategy in every period and that each agent in every period is matched up with all other agents in the population. Under a general Darwinian dynamic, this results in that the population eventually will adopt the risk dominant equilibrium (RD).

The main objection against this approach has been about longevity of risk dominated equilibrium before a sufficiently large number of mutations take place and the population converges to RD. As an answer to this criticisms, Ellison (1993) introduced a spatial environment into this framework and showed that by replacing the uniform matching process with one where agents instead are matched up with nearby located opponents the convergence towards RD is much faster in a spatial environment. Nevertheless, a spatial environment does not necessarily imply that the population converges to RD as shown by Robson (1993).

Lately, the main focus has shifted towards models supporting the Pareto-dominant equilibrium (PO). One of the first models in this category were Robson (1993); by assuming the population being partitioned into a finite set of isolated sub-populations where each sub-population grow independently according to a general Darwinian dynamic, he showed that if the population become extinct at fixed intervals and re-populated by groups randomly drawn from the previous generation, the Pareto-dominant equilibrium will prevail in the long run. consider a model where each equilibrium is present in a isolated sub-population. Oechssler (1999) consider a model where each equilibrium is represented in an isolated sub-population (group). By allowing the agents to choose the actions and group he shows in a setting similar to Kandori et al. (1993) that the population converges to the Pareto-dominant equilibrium even if there exist some interactions between groups (viscosity). Later, Ely (2002) show in a similar setting that even if the mobility of agents to change location is arbitrarily slow, the population converges to the Pareto-dominant equilibrium.

1.1 Outline of the Model

In the equilibrium selection models in games with multiple symmetric equilibria, two competing solution concepts can be identified: the risk-dominant equilibrium and the efficient equilibrium. Where as both these solution concepts clearly has its merits, one could question how realistic it is to assume that people always one or the other solution.

For example, consider the simple 2×2 symmetric coordination game as depicted in the Figure 1 below.

	R	E
R	P, P	$P-\varepsilon$, 0
E	0, $P-\varepsilon$	$P+\varepsilon$, $P+\varepsilon$

Figure 1:

The risk-dominant equilibrium yields P and the efficient equilibrium yields $P+\varepsilon$. If an "efficient" agent is matched with a "risk-dominant" agent, the "efficient agent" receives 0 where as the "risk-dominant" agent gets $P-\varepsilon$. That is, if ε is close to zero, the "risk-dominant" agent earns approximately just as much as the efficient outcome regardless whether the agent is matched up with a "risk-dominant" agent or another "efficient" agent, where as the efficient agent has a significant payoff decrease if he is matched up with a "risk-dominant" agent. Hence, a small probability to become matched up with "risk-dominant" opponent is sufficient to make the efficient action yield a lower expected payoff than the risk dominant action and thus rendering the efficient equilibrium unlikely to survive in environment where there is a positive probability that a "risk-dominant" agent could emerge.

Now instead consider the game in Figure 2.

	R	E
R	$P+\varepsilon$; $P+\varepsilon$	0 ; P
E	P ; 0	$2P$; $2P$

Figure 2:

The risk-dominant equilibrium yields $P + \varepsilon$ and the efficient equilibrium yields $2P$. If ε is close to zero, the "risk-dominant" agent earns approximately just as much as the efficient outcome regardless whether the agent is matched up with a "risk-dominant" agent or another "efficient" agent. However, the efficient equilibrium yields almost twice as high payoff as the risk-dominant equilibrium. Consequently, if the "efficient" agents some how could isolate themselves sufficiently well from the "risk-dominant" agents, the "efficient" agents have potential to earn a higher payoff than the "risk-dominant" agents.

Intuitively, games like in left-hand figure above seem more likely to yield a risk-dominant equilibrium where as games like the right-hand figure seem more likely to yield a efficient equilibrium. In an attempt to capture this intuition and perhaps somehow elucidate which circumstances favors different equilibrium, we analyze a general symmetric coordination game in richer setting than normal where spatial context with an increasing population is added and where the game is played by rationally bounded players. More precisely:

The game is player by a finite number of agents living in a finite discrete homogenous space where agents in each period are able to migrate from their present location to an adjacent location. The agents are in each period matched against all other agents within a fixed proximity from their own location (called neighborhood). An agent's payoff is given as the average outcome of the tournament matching within her neighborhood. Each location has limited resources, resulting in a decrease in fitness for the agents at a location as the number of agents at this location grows. The fitness is increasing in the number of matches in each period. The fitness of an agent is given as function of the number of matches in each period, number of agents at the same location, and the payoff.

The actions agents take are assumed to be "hardwired" and can only be changed through a fixed rare occurring mutation. Agents are assumed to be myopic, meaning that the agents information set are restricted to know how their fitness would change if they where to migrate from their present location to any of the neighboring locations.

As commonly assumed in evolutionary models, we let behavior yielding a higher fitness become more frequent in the population the following period. Less conventionally, we assume the population to grow in numbers as time goes by. Offspring are assumed to be born at the same location as its parent. Since the population is growing in number, agents will eventually migrate from their original location to a less populated area. We restrict our attention to migration procedures resulting in that the density of the population becomes unimodal distributed around the center of a population.

If the absolute growth is sufficiently slow it can under our assumption be shown that agents mutating in a population homogenous in strategies will migrate away from the population, which will lead to the creation of new sub-populations. In this manner the total population will be portioned into isolated sub-populations where each sub-population will be in a Nash-equilibrium. This process will continue until the entire location space is filled with so far isolated sub-populations.

If the location space is sufficiently large, each feasible Nash-equilibrium in the game will be represented in at least one sub-population. As the location space gradually will become more populated, the sub-populations will eventually be forced to interact with each other. If there exist a Nash-equilibrium that is efficient and not risk-dominated by any other Nash-equilibrium, it can be shown that this equilibrium always will prevail and eventually spread all over the location space. That is, in this model, risk-dominance (or efficiency) by itself is not sufficient to ensure that a Nash-equilibrium constitutes a long-run equilibrium.

It can also be shown that an efficient equilibrium is more likely to prevail if the absolute growth is slow. More general: the slower the absolute growth is the bigger will the difference in sizes be between an efficient and a less efficient equilibrium. This result can loosely be interpreted as if hard evolutionary pressure benefits more efficient equilibria.

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