PRIVATE INFORMATION AND NONBINDING ARBITRATION*

Ram Orzach† Stephen J. Spurr‡

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Abstract

This paper analyzes a procedure called mediation, that is really a form of nonbinding arbitration, and is widely employed in cases filed in State and federal courts in the U.S. Under the existing rules, a party who rejects an award proposed by the mediator is liable for sanctions unless the rejection turns out to be justified, i.e., unless the trial verdict is more favorable to the rejecting party than the mediation award. This penalty is designed to minimize the frequency of trial, by inducing both parties to accept the mediation award.

We consider two alternative procedures. In the first procedure a party is liable for sanctions if, and only if, the trial verdict reveals that she knowingly provided false information to the mediator. This procedure may be costly to implement because of difficulties of proof. In the second alternative, a party is liable only (a) if she accepts the mediator's award and (b) the trial verdict is further from her claim than the other party's claim. This procedure is easier to implement than the first one, but has less expected benefit. In comparison to the existing practice, both our procedures have a lower frequency of trial, and provide an ex ante gain to both parties.

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†Visiting Assistant Professor at Tulane University

‡Professor of Economics, Wayne State University. Address all correspondence to Stephen J. Spurr, Department of Economics, 2115 FAB, Wayne State University, Detroit, MI 48202; e-mail: sspurr@wayne.edu.

1. Introduction

In recent years there have been many initiatives prompted by recognition of the cost of resources involved in the trial of a legal case. One response has been increasing interest in, and implementation of, various methods of alternative dispute resolution (ADR). These methods include judicial settlement conferences, early neutral evaluation, arbitration, mediation, abbreviated trial procedures, and even programs such as rent-a-judge. In all these methods an impartial third party provides the parties with an opinion or information, in order to resolve the dispute or at least expedite its settlement.

This paper considers changes in a procedure that is often called mediation, but is really a form of nonbinding arbitration widely employed in the U.S., in both State and federal courts.[1] In the courts that have mandatory programs, the plaintiff must submit her claim to nonbinding arbitration (hereinafter called "mediation") before she can request a trial. Typically the mediation hearing is much briefer and more informal than a trial; the usual rules of evidence do not apply, and the formalities of civil procedure are not observed. After the hearing the mediator proposes an award; each party must then decide independently whether to accept or reject it.[2] If both parties accept, the case is resolved, and the defendant pays the plaintiff the amount of the award. If, however, either party rejects, the case proceeds on toward trial.[3]

Although there is variation across different courts, the usual practice is that a penalty is imposed on a party who rejects the mediation award, if the party does not do better at trial than she would have done by accepting the award. In State courts in Michigan, for example, a

[1] As of 1999, statutes or court rules provided for court-annexed mediation in thirty-three States and twenty-two federal district courts. Goldberg et al. (1999). Participation in court-annexed mediation is generally required in State courts, but is optional in most federal courts. Developments, Harvard Law Review (2000).

[2] The usual rule is that each party must submit its response to the mediation award without knowing the response of the other party.

[3] In reality, if the mediation award is not accepted, the case may still be, and often is, settled before trial. For simplicity our model ignores this possibility.

party who rejects the mediation award is liable for the post-mediation expenses of the opposing party, unless the trial verdict is more favorable to the rejecting party by a margin of more than ten per cent.[4] The idea behind this rule is that a party's act of rejecting the mediation award is unjustified unless the award is substantially worse for her than the expected outcome of trial. Consequently, unless the award is substantially worse, the party should bear the full social costs of her decision to reject it.

In our model, the mediator has no private information, but each party does.[5] Our analysis is motivated by the idea that the social cost of litigation can be minimized if the parties, each of whom usually has private information, are given incentives to provide accurate information to the mediator. Under these conditions the mediation award is more likely to be an accurate evaluation of the claim, and is thus less likely to be rejected and lead to trial. Therefore we present two procedures that will give the parties an incentive to provide the mediator with accurate information.

The First Procedure: The Truthful Reporting System

Under the first procedure (different from the current system) that we examine, a penalty is imposed on a party who is found (by the outcome of trial) to have lied to the mediator. In the event of a trial, and only then, the true value of the plaintiff's claim will be revealed, and it will also be apparent whether either party has made a misrepresentation to the mediator. The payoff to adopting this procedure is that there will be a lower frequency of trial than under the conventional scheme.

[4] The verdict is considered "more favorable" to a plaintiff who rejects if it is more than ten per cent above the mediation award, and more favorable to a defendant who rejects if it is more than ten per cent below the mediation award. MCL 600.4921(2) (2003).

[5] Models with two-sided informational uncertainty are generally considered less tractable than those with one-sided uncertainty, Farmer and Pecorino (1989), but there are some results on the choice between settlement and litigation. See Schweizer (1989) and Daughety and Reinganum (1994).

This procedure requires the exercise of at least some discretion (see the discussion in section 4.2 below), and thus cannot be reduced to a ministerial function, which is all that is required under the current system.

The Second Procedure: A Self-Enforcing System

Under the second, alternative procedure, which can be implemented as a clerical function, each party's statement to the mediator about the value of the claim is expressed as a dollar amount. Under this procedure, a penalty is imposed on a party, i.e., she is required to pay the trial expenses of the other party, if (1) she accepts the mediation award, and (2) her report of the claim's value is farther from the trial verdict than the report of the other party.[6]

The rationale for this procedure is that under the current system, a party who succeeds in misleading the mediator with a self-serving report will certainly accept the mediation award. This would not be the case under our second procedure. Under our second procedure a party plays a mixed strategy (sometimes reporting falsely and sometimes accurately). In the case where he reports falsely, and from the report of the other party or the mediation award he suspects there is a good chance the case will go to trial, he is better off rejecting the mediation award to avoid the penalty. This rejection reduces his incentive to make a false report in the first place, and this consequence drives our results. If both parties have an incentive to make an accurate report, the mediation award will be accurate and both parties will accept it.

In comparing the three alternatives – the current system and our two alternatives – one should notice that the amount of the punishment is the same in each one. Our two procedures only change the rule as to when the punishment is imposed.

[6] There is one other condition: a party must also pay the penalty if his behavior is inconsistent. Thus the defendant is penalized if he first offers more than the mediation award, and then rejects the award. Similarly, the penalty is imposed on the plaintiff if she proposes less than the mediation award and then rejects the award.

2. The Literature

The idea that the mediator or arbitrator could obtain information about the ideal settlement from the parties' offers has been examined by a number of researchers. A major contribution was made by Stevens (1966) who proposed a procedure that is called final-offer arbitration.[7] In this procedure each party simultaneously makes a formal offer, and the arbitrator must choose one of the offers as the settlement. The idea is that each party, knowing that the arbitrator will choose the offer that is closer to the arbitrator's ideal settlement value, has an incentive to make an offer close to that value. Thus the gap between the parties' offers will be reduced, and the arbitrator will gain information about the ideal settlement from those offers.[8]

Farber (1980) and Chatterjee (1981) independently developed models of final-offer arbitration, positing a two-person game of incomplete information in which the parties know the probability distribution of the arbitrator's view of a fair settlement. In these models the arbitrator's views concerning a fair settlement are unaffected by the parties' offers.[9]

Gibbons (1988) develops a model that extends the analysis of Farber (1980) to include learning. The arbitrator's objective is to minimize the difference between the actual settlement and the true settlement value. The arbitrator receives a noisy signal about the true value, and another noisy signal is received by both parties, i.e., each party receives the same signal. The arbitrator is able to infer the parties' private information perfectly from their offers, and can

[7] This idea had, however, been discussed informally before Stevens' paper was published. Stern et al. (1975), at 113, n.7.

[8] It should, however, be noted that other work suggests that the parties' offers may not be close approximations of their view of the median of the arbitrator's preferences. Brams and Merrill (1983) focused on the degree of convergence of the offers of the two parties under different assumptions about the arbitrator's preferences, i.e. different probability distributions. They found that when there are Nash equilibria in pure strategies, the parties' offers are, for most common distributions, symmetric around the median, but separated from one another by two or more standard deviations. Chatterjee (1981) also found a tendency for the parties' offers to diverge under final-offer arbitration, under certain assumptions about the distribution of the arbitrator's preferences.

[9] Crawford (1982) subsequently showed that some of the findings in Farber's (1980) paper were incorrect.

use this information together with his own signal to compute a posterior belief about the true settlement value. He then chooses the offer that is closer to this value.

Zeng (2003) proposed a change in the rules of final-offer arbitration, in which the mechanism of second-price auctions gives the two parties an incentive to submit correct information to the arbitrator, who has a prior notion of a fair settlement: if the offers diverge, the arbitration settlement is determined by the loser's offer.

Samuelson (1991) also analyzes final-offer arbitration, but in his model each disputant has private information - information unavailable to the other side or the arbitrator. In equilibrium, the arbitrator learns from the final offers of the disputants. Samuelson considers how well final-offer arbitration performs in arriving at the true value relative to the benchmark of complete information.

Bernstein (1993) points out that a party may deliberately present a weak case in order to avoid sanctions for rejecting the ADR award. If, for example, a defendant makes a half-hearted presentation to the mediator, the mediation award will be high, and it will not be difficult for the defendant to do better at trial.

A work that is close to ours in spirit is by Spier (1994). She first analyzes a sequential game with one-sided private information: the plaintiff knows the value of his claim, and the uninformed defendant makes one offer to the plaintiff before trial, on a take-it-or-leave-it basis. Spier derives the defendant's optimal offer under the American rule and under Rule 68,[10] and determines the conditions under which Rule 68 increases the probability of settlement. Spier also uses the revelation principle (Myerson (1979)) to derive a payoff mechanism that maximizes the settlement rate.

[10] Rule 68 of the Federal Rules of Civil Procedure provides that if a defendant makes a formal settlement offer to the plaintiff, which the plaintiff refuses, the plaintiff must pay the costs incurred after the offer was made unless the judgment at trial is more favorable than the offer.

We develop an extensive-form game in which the settlement mechanism is implemented by the mediator. The mediator is a player who has his own objective function – to minimize trials – but his strategies are restricted: he cannot propose a settlement that is inconsistent with the reports of both parties. The fee shifting rules are relatively easy to implement and arguably have some moral and legal justification, i.e., a penalty is imposed on a party who is found to have misled the mediator[11] Our proposed system improves the expected payoff of both parties given their private information (at least in the weak sense).

Our assumption that the mediator does not have private information is based on what we believe are the realities of most civil litigation in the federal courts or state courts of general jurisdiction.[12] Indeed, if the mediator actually had such information, that would normally be grounds for disqualification.[13] We examine the objective of mediation below after the discussion of Proposition 1.

[11] There are many ways in which the law currently imposes sanctions on a party who gratuitously increases the costs of litigation. When A requests B to admit the truth of any matter in the course of pretrial discovery, and B refuses without a good reason for doing so, the court may require B to compensate A for the expense of proving those facts (Rule 37(c), Federal Rules of Civil Procedure). Another federal rule authorizes the courts to impose sanctions on parties who file frivolous papers, such as those which make allegations without evidentiary support, or denials that are unwarranted and unreasonable (Rule 11(c), Federal Rules of Civil Procedure). Sanctions under this rule are most often applied to plaintiffs who file frivolous claims; in such cases the court typically requires the plaintiff to pay the defendant's legal expenses. Finally, in many State courts the plaintiff will not receive prejudgment interest, i.e. interest from the date of filing the claim to the date of judgment, if she is deemed to have prolonged the litigation by refusing a "reasonable" settlement offer (Carroll (1983), cited by Spier (1994)).

[12] To be sure there are situations involving arbitration or mediation where the assumption that the arbitrator or mediator has private information is viable. In final-offer arbitration, it is quite reasonable to assume the arbitrator knows more about his own preferences than the parties do. In labor disputes intermediaries are often chosen because of their knowledge of the industry. In the context of litigation, it can happen that a mediator is more familiar with the applicable law, or industry custom, than the parties, especially if the parties are not represented by lawyers. Thus our model, in which the mediator has no private information, is best suited to civil litigation in which the stakes are substantial and each side is represented by lawyers, the usual situation in the federal courts or state courts of general jurisdiction.

[13] See, e.g., Michigan Compiled Laws 600.4905(4).

3. The Model

Consider a three player game. The first player is the plaintiff (P) who brings an action for damages caused by negligence of the second player, the defendant (D). However, there is uncertainty both about the amount of damages and liability. This uncertainty is described by a probability space (Ω, p) where Ω is a set of a finite number of states of nature[14] and p is a probability measure on Ω. Once the state of nature is realized, the amount of damages is determined by the mapping $v : \Omega \rightarrow R$ (i.e. if $\omega_j \in \Omega$ is the true state of nature then the damage is $v(\omega_j)$ in dollars). Before going to trial, P and D appear before the third player, the mediator (M), in an attempt to settle the case without incurring the cost of a trial.

We consider a private information game in which players P and D hold private information regarding damages and liability. We represent the private information of player $i \in \{P, D\}$ as a partition[15] Π_i of Ω (i.e. if $\omega_j \in \Omega$ is the true state of nature, then player i will observe $\pi_i \in \Pi_i$ which contains ω_j).

We posit three states of nature, since this is the minimum number of states that is required to enable either P or D to have differential information (if there were only two states, either (a) one player would have always better information than the other, or (b) both players would always have the same information). We consider a "symmetric" case. In a symmetric information

[14] The assumption of a finite number of states is just for simplicity. An alternative would be to assume a continuous set, but that the information set of each player is finite.

[15] In our model, as in Savage's model of differential information (Savage (1954)), a player's private information is represented by a partition of the space of states of nature. The use of partitions is common in decision theory, information systems, and models of the value of information. To analyze private information game theory uses either partitions (see for example, Chapter 4 in Laffont (1990)) or the alternative approach suggested by Harsanyi (1967). Harsanyi represents agents' private information by a set of types, and takes the set of states of nature to be the cross product of the sets of agents' types. These two approaches have been shown to be equivalent (see, for example, Jackson (1993) and Vohra (1999)). However, partitions provide a "visual" way to see whether one agent has better information than another. In our model, the use of partitions rather than types makes it easier to determine which player has better information, given the true state of nature. For this reason, the use of partitions is common in models in which more than one person has private information ex ante (for example, see Aumann (1976)).

case, the information provided by each party influences the mediator's decision. In a sufficient "non-symmetric" case, the mediator will be biased toward one party almost without regard to quality of the information provided to him by the other party, and it is hard to "justify" such an outcome. Therefore, we consider the Minimal Differential Symmetric Case. This case is also sufficient for us to show that the existing system increases the players' incentives to misrepresent the truth, and as a result more cases go to trial than necessary.

Let us now define the Minimal Differential Symmetric Case (MDSC). Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$ with a corresponding probability distribution $(1/3, 1/3, 1/3)$, meaning that there are three states of nature with equal probability. Without loss of generality[16] assume that $v(\omega_1) = 0$, $v(\omega_2) = \frac{1}{2}X$ and $v(\omega_3) = X$ meaning that if the true state is ω_1, then the defendant is not liable (i.e. $v(\omega_1) = 0$). If the true state is ω_2, then damages equal $\frac{1}{2}X$; this can be interpreted as a case where the defendant is liable, but damages are relatively low. We set $v(\omega_2) = \frac{1}{2}X$ for reasons of symmetry. Finally, if the true state is ω_3 then the damages are X (i.e. $v(\omega_3) = X$). This can be interpreted as a case in which the defendant is liable, and damages are relatively high.

The partition of the plaintiff is $\Pi_P = \{(\omega_1, \omega_2)(\omega_3)\}$, so that if the true state is ω_1, player P observes $\pi_P(\omega_1, \omega_2)$. If the true state is ω_2 then player P observes the same $\pi_P(\omega_1, \omega_2)$, which means that player P cannot distinguish between the two states of nature ω_1, ω_2 (i.e., the plaintiff cannot determine whether the defendant is liable if the damages are not large). If the true state is ω_3 then player P observes $\pi_P(\omega_3)$, which means that player P can distinguish between ω_3 and (ω_1, ω_2) (in this case the plaintiff knows that the defendant is liable for \$$X$). Player D's partition is $\Pi_D = \{(\omega_1)(\omega_2, \omega_3)\}$; thus the defendant knows whether he is liable, but cannot

[16] the generality of the symmetric case where $v(w_2) = \frac{v(w_1)+v(w_3)}{2}$. We set $v(\omega_1) = 0$, and could normalize by setting $v(\omega_3) = 1$, but we prefer to set the cost of the trial equal to 1.

determine the exact amount of damages.[17] As previously noted, an alternative way to formulate this model would be with different types, following Harsanyi.[18]

The game consists of 5 stages (in some cases it ends after 4 stages):

1) In the first stage, Nature selects the true state ω_o and players P and D observe their partition that contains ω_o. As mentioned previously, the mediator M has no independent information, but knows the structure of Π_P and Π_D. Our assumption that the the mediator has no private information differentiates our paper from most of the literature. The mediator is an expert in the field and knows the partition of the players and the probability distribution, but does not have private information.

2) In the second stage players P and D simultaneously and independently make their first report to M as a function of their partition $\pi_{i,}$ $i \in \{P, D\}$. This statement can be represented by a dollar value, or report on any π_i or report on any symbol. Let S_P^1 be the first statement of P, $S_P^1 : \Pi_P \rightarrow \{s_P^1\}$. Let S_D^1 be the first statement of D, $S_D^1 : \Pi_D \rightarrow \{s_D^1\}$.

3) In the third stage the mediator announces his award A_M in dollar value. Let A_M be a function from the statements of P and D to $M's$ award, i.e., $A_M : S_P^1 \times S_D^1 \rightarrow R$. The mediation award depends only on the information provided to the mediator and his knowledge about the partition and the distribution.

4) In the fourth stage players P and D each make their second decision, S_P^2 and S_D^2 respectively, simultaneously and independently. Let $S_P^2 : \Pi_P \times S_P^1 \times S_D^1 \times A_M \rightarrow \{Y, N\}$ be player P's second decision where Y accepts the mediation award and N rejects it, given

[17] One could also posit a model with the partition $\Pi_P = \{(\omega_1)(\omega_2, \omega_3)\}$ and $\Pi_D = \{(\omega_1, \omega_2)(\omega_3)\}$, but the results of this case are similar.

[18] In a Harsanyi representation, the plaintiff would have private information represented by two types - high damages or low damages. The defendant would also have private information represented by two types - liable or not liable. The types would be correlated, resulting in three states of nature rather than four. This assumption seems more challenging to model but also more realistic than an assumption of independence, since it is relatively unlikely that the plaintiff had information of high damages and the defendant of no liability. We will discuss the correlation assumption in the conclusion.

that each player has heard the other player's first statement in the second stage.[19] Let S_D^2 : $\Pi_D \times S_P^1 \times S_D^1 \times A_M \rightarrow \{Y, N\}$ be the equivalent for player D . If both players accept the award, the game ends and player P receives A_M \$ from player D. In this case Player M benefits from having solved the case; we represent this as a payoff of 0. Therefore, the payoff of the three players can be represented as $(A_M, -A_M, 0)$. If either or both of the first two players responds with $S_i^2 = N$ the game moves to the fifth stage.

5) In the fifth stage the first two players appear in court. We assume that the court can identify the true[20] state of nature ω_o and will award damages of $v(\omega_o)$. However appearing in court will involve a cost C (filing fees, attorney fees, etc.) for both players P and D. Without loss of generality let $C = 1$ for each player[21]. The main objective of this paper is to analyze how different allocation schemes for trial costs affect the outcome of this game. Let B_P and B_D be the allocation of the cost between P and D respectively. We restrict B_i to be $B_i \in \{0, 1, 2), i \in \{P, D\}$, and to cover the costs we require that $B_P + B_D = 2$. This setup implies that either each litigant covers his own cost, or one of them covers the cost for both. Player M's payoff at this stage is -1. Thus the payoffs at this stage are $(v(\omega_o) - B_P,\ -v(\omega_o) - B_{D,}\ -1)$. The values of B_P and B_D will be determined by equations 4.1, 4.7, and 4.10 below, for the current system, the first procedure, and the second procedure respectively.

We can interpret the expected absolute payoff of M as the probability of going to trial. Thus M maximizes utility by minimizing the likelihood of trial. M is trying to maintain a reputation as an effective mediator, but at the same time and as will be shown later, M's strategy is very

[19] S_P^1 is not redundant and has a major role in the procedures we consider.

[20] Similar results could be obtained if the court could only determine the true state with high probability.

[21] Since we set $v(\omega_1) = 0$, we have the freedom to set $C = 1$, and for reasons of symmetry, it should be the same for P and D.

restricted as he cannot award damages that are inconsistent with the statements of P and D (For example, if P asks for damages of X, and D claims that damages are $\frac{1}{2}X$, then M cannot make an award of 0).

4. Results

This paper will compare three methods of "fee shifting" or allocation of the total costs of trial between P and D: the current system, the truthful reporting system and the self-enforcing system. We analyze these three systems in subchapters 4.1, 4.2, and 4.3, respectively.

4.1. The Current System

A) Under the current system, one party (P or D) bears all the costs of trial only if he rejects the mediator's award but fails to do better at trial than he would have by accepting the mediator's award, by some specified amount (in some jurisdictions, for example, the player must do more than 10% better than the mediation award.)[22] Otherwise, each player bears his own trial costs, namely 1. For simplicity in this paper we model the current system as imposing a penalty unless the rejecting party does better than the mediation award. Formally, let $B_P^M(A_M \times S_P^2 \times S_2^2 \times v(\omega_o))$ be the allocation B_P under this policy, where the superscript C indicates that the allocation is made under the current system.

$$B_P^C(A_M \times S_P^2 \times S_D^2 \times v(\omega_o)) = \left\{ \begin{array}{ll} 2 & : S_P^2 = N \ , \ S_D^2 = Y \ \text{ and } \ A_M \geq v(\omega_o) \\ 0 & : S_P^2 = Y \ , \ S_D^2 = N \ \text{ and } \ A_M \leq v(\omega_o) \\ \quad 1 & : \ \text{otherwise} \end{array} \right\} \tag{4.1}$$

This will determine $B_D^C(A_M \times S_P^2 \times S_D^2 \times v(\omega_o))$ as $B_P^C + B_D^C = 2$.

Let us next consider the strategies of P, D and M.

Since the partitions of P and D contain only two elements (recall that $\Pi_P = \{(\omega_1, \omega_2)(\omega_3)\}$ and $\Pi_D = \{(\omega_1)(\omega_2, \omega_3)\}$), without loss of generality let $S_P^1 : \Pi_P \to \{h, l\}$ where h represents

[22] This is the rule in Michigan. M.C.L. 600.4919 (see Spurr (2000)). For the rules in other States, see Bernstein (1993), at 2294.

high and l low damages. Let $S_D^1 : \Pi_D \rightarrow \{d, a\}$ where d represents denying liability and a represents admitting liability. Players P and D can use mixed strategies. Consider the following strategy for P.

$$\begin{aligned} S_P^1(\pi_P(\omega_1, \omega_2)) &= \left\{ \begin{array}{ll} l & : \alpha \\ h & : 1-\alpha \end{array} \right\} \\ S_P^1(\pi_P(\omega_3)) &= h \end{aligned} \tag{4.2}$$

where $\alpha \in [0, 1]$. We will interpret these strategies as follows: if Player P's partition is $\pi_P(\omega_1, \omega_2)$, he will report l with probability α, and h with probability $1 - \alpha$. Note that reporting l is in effect reporting $\pi_P(\omega_1, \omega_2)$ (truthful information), and reporting h is in effect reporting $\pi_P(\omega_3)$ (false information). As for $S_P^1(\pi_P(\omega_3))$ we will not use a mixed strategy as $S_P^1(\pi_P(\omega_3) = l$ is weakly dominated (in this setup) by the strategy $S_P^1(\pi_P(\omega_3)) = h$ if the mediation award is weakly monotonically increasing with respect to S_P^1.

As for player D, we will restrict the set of his strategies to the following:

$$\begin{aligned} S_D^1(\pi_2(\omega_1)) &= d \\ S_D^1(\pi_2(\omega_{2,}\omega_3)) &= \left\{ \begin{array}{ll} d & : \beta \\ a & : 1-\beta \end{array} \right\} \end{aligned} \tag{4.3}$$

where $\beta \in [0, 1]$. The restriction on $S_D^1(\pi_2(\omega_1))$ will not change the result.[23]

We assume that $M's$ decision rule is determined as follows: if the information provided by P and D enables M to determine the most likely state of nature, he will award that amount. If, on the other hand, the most likely state cannot be determined from that information, M will choose the alternative that minimizes the probability of trial.

[23] An alternative model could assume that, even if the mediation award is rejected by one or both parties, the case would go to trial only if P elected to pursue it. In such a model D might be inclined to overstate damages, to inflate the mediation award. D's motive for doing so would be to ensure that he would do substantially better than the mediation award at trial, and thereby avoid the penalty of paying $P's$ trial costs. See Bernstein (1993).

Lemma 1. *In the MDSC if the mediator chooses the state of nature with the highest probability, then*

$$A_M(S_P^1 = h, S_D^1 = a) = X \tag{4.4}$$

$$A_M(S_P^1 = l, S_D^1 = a) = \frac{1}{2}X \tag{4.5}$$

$$A_M(S_P^1 = l, S_D^1 = d) = 0 \tag{4.6}$$

proof: see appendix.

A mediator who desires to maintain his reputation, by choosing the most likely state of nature, would follow these principles. For an intuitive explanation, consider (4.4). For player P, reporting h is equivalent to reporting damages of X, and he can distinguish between damages of X and $\frac{1}{2}X$, while player D admits liability (a), but he cannot distinguish between damages of $\frac{1}{2}X$ and X. Thus M should accept P's report and award X. For 4.6, the intuitive explanation is similar: player P asks for l but he cannot distinguish between 0 and $\frac{1}{2}X$ while player D declares d (which is equivalent to 0) and can distinguish between $\frac{1}{2}X$ and 0. As for 4.5, this statement is equivalent to a report of $\frac{1}{2}X$ by each player. Therefore M should award $\frac{1}{2}X$.

Lemma 2. *In the MDSC if the statements made by the parties to the mediator are $S_P^1 = h, S_D^1 = d$, then he cannot determine the state of nature with the highest probability without knowing α and β .*

proof: see appendix.

Here it must be true that one of the players has misrepresented his information, so the mediator must exercise his judgment. To determine the state of nature with the highest probability, we must know the ratio of α to $1-\beta$ (the truthfulness of the plaintiff relative to the defendant).

If P makes false reports with higher probability than D, so that $\alpha < 1 - \beta$, then the mediator will realize that ω_1 is the state of nature with the highest probability and award 0. However, this will give Player D a greater incentive to make a false report, and vice versa for Player P. Therefore, there is no way we can sensibly restrict the strategy of the mediator in this case before calculating the equilibrium of this game. Hence, we will compare all three possible pure strategies of the mediator in this case.[24]

Lemma 3. *In the MDSC under the current system if the mediator's award is* $A_M(S_P^1 = h, S_D^1 = d) = \frac{1}{2}X$ *and* $X > 6$*, then the only equilibrium given our restricted strategies has the following properties :*

1) The plaintiff's first report is h regardless of his partition (equivalent to $\alpha = 0$).

2) The defendant's first report is d regardless of his partition (equivalent to $1 - \beta = 0$).

3) The plaintiff accepts the mediator's award iff his partition is $\pi_P(\omega_1, \omega_2)$.

4) The defendant accepts the mediator's award iff his partition is $\pi_D(\omega_2, \omega_3)$.

5) The probability of trial is 2/3

proof: see appendix.

Let us consider the case where X is large, since if X is small the plaintiff might decide to avoid trial for fear of a negative return, in the event the recovery is less than the trial costs of both parties. In the case where X is large, the plaintiff will not have this concern, and the only motive that would induce the parties to settle, rather than go to trial, is to save the costs of trial. If X is large and $A_M(S_P^1 = h, S_D^1 = d) = \frac{1}{2}X$, the mediator will award the intermediate value.

[24] To reduce the dimensionality of the problem, we rule out mixed strategies for the mediator.

Lemma 4. *In the MDSC under the current system, suppose the mediator's award is* $A_M(S_P^1 = h, S_D^1 = d) = 0$ *and* $X > 6$. *Then the only equilibrium given our restricted strategies has the following properties :*

1) The defendant's first report is d regardless of his partition (equivalent to $1 - \beta = 0$).

2) The plaintiff's first report is irrelevant, as the mediator's award is always $A_M(S_P^1 = \cdot, S_D^1 = d) = 0$.

3) The plaintiff never accepts the mediator's award .

4) The probability of trial is 1.

A formal proof of the existence and uniqueness of equilibrium is similar to the proof of lemma 3. Intuitively, as the mediator "believes" the defendant and awards $A_M(S_P^1 = \cdot, S_D^1 = d) = 0$ the defendant has an incentive to report $S_D^1 = d$. Therefore, the result of the mediation process is 0 . If $X > 6$ then the plaintiff has a positive expected profit from rejecting the mediator's award even when his partition is $\pi_P(\omega_1, \omega_2)$. The expected profit is $0.5(0 - 2) + 0.5(0.5X - 1)$ If. $X > 6$, the expected profit > 0. Therefore the case will always go to trial.

Lemma 5. *In the MDSC under the current system, suppose the mediator's award is* $A_M(S_P^1 = h, S_D^1 = d) = X$ *and* $X > 6$. *Then the only equilibrium given our restricted strategies has the following properties :*

1) The plaintiff's first report is h regardless of his partition (equivalent to $\alpha = 0$).

2) The defendant's first report is irrelevant as the mediator's award is always $A_M(S_P^1 = h, S_D^1 = \cdot) = X$.

3) The defendant never accepts the mediator's award .

4) The probability of trial is 1.

A formal proof of the existence and uniqueness of equilibrium is similar to the proof of lemma 3. Intuitively as the mediator "believes" the plaintiff and awards him $A_M(S_P^1 = h, S_D^1 = \cdot) = X$ the plaintiff has an incentive to report $S_P^1 = h$. Therefore, the result of the mediation process is X . If $X > 6$ then the defendant has a positive expected profit from rejecting the mediator's award even when his partition is $\pi_D(\omega_2, \omega_3)$ (which is $0.5(-0.5X - 2) + 0.5(-X - 1) = -0.75X - 1.5$). Therefore, the case will always go to trial.

Corollary 1. *Regardless of the strategy of M, and regardless of their private information, P and D will report high damages and no liability, respectively.*

Proposition 1. *In the MDSC under the current system, if $X > 6$, a mediator whose objective is to minimize the probability of trial should choose $A_M(S_P^1 = H, S_D^1 = d) = \frac{1}{2}X$. This policy will lead to a 2/3 chance that the process will end in trial.*

Since the parties are already in court and have reached the stage of mediation, we will assume they have exhausted all their opportunities for out-of-court settlement; thus we assume that without mediation the probability of trial is 1. Consequently the mediation process reduces the number of cases that go to court even under the current system.

As in a model of final-offer arbitration, one might argue that a mediator without private information is superfluous. However, we would argue that the mediator has a critical role. In our model, mediation is the last alternative to trial; thus we assume that the parties would not reach a settlement through further bargaining outside of mediation. Secondly, without a mediator the two parties are bargaining over the set of all X_S s.t. $X_S \in [X_1, X_2]$ where X_1 (which may be 0) is what the defendant has agreed to pay, while the plaintiff demands X_2. The mediator provides a "focal point" that restricts the choices of both parties to a single point. If they do not both accept this amount, they must go to trial.[25] Thus the importance of a good mediator is not determined by whether he has private information, but by the fact that he can choose a point that the parties are not likely to refuse.

4.2. The truthful reporting system

B) Under our first alternative, the "truthful reporting" system, one party (player P or D) bears all the costs of trial if the outcome reveals that the amount he reported to the mediator was not consistent with his partition (private information) i.e., he was "lying." An example will show how this proposed rule works. Suppose $v(\omega_o) = 0$. Then $\Pi_P = \{\omega_1, \omega_2\}$. If P reports $0.5X$ to the mediator, and the case is tried, yielding a verdict of 0, then P is not subject to any penalty since he did not lie; he reported an amount that was consistent with his partition. Thus the proposed system does not require P or D to guess the outcome of the trial, just to avoid misleading the mediator. If the outcome of the trial reveals that both players were lying, each bears his own cost. If it shows that both players were telling the truth, each bears his own cost. Formally, let $B_P^T(A_M \times S_P^1 \times S_D^1 \times v(\omega_o))$ be the allocation B_P under this policy.

$$B_P^T(S_P^1 \times S_D^1 \times v(\omega_o)) = \left\{ \begin{array}{ll} 2 & : f_P(s_P^1, \omega_o, \Pi_P) = 0 \text{ and } f_D(s_D^1, \omega_o, \Pi_D) = 1 \\ 0 & : f_P(s_P^1, \omega_o, \Pi_P) = 1 \text{ and } f_D(s_D^1, \omega_o, \Pi_D) = 0 \\ 1 & : \text{otherwise} \end{array} \right\} \tag{4.7}$$

Where $f_P(s_P^1, \omega_o, \Pi_P) = 0$ indicates that, based on the trial verdict, $P's$ first period report was false, given his partition Π_P; $f_P(s_P^1, \omega_o, \Pi_P) = 1$ indicates that $P's$ report was not false, and similarly for $f_D(s_D^1, \omega_o, \Pi_D)$. The idea behind the function f is straightforward, but admittedly it cannot be reduced to a ministerial function. Nonetheless we believe this procedure can readily be implemented, by replacing a penalty for rejection of the mediation award with a penalty for providing false or incomplete information to the mediator, or to the other party during pretrial

[25] Our conjecture is that if the model allowed bargaining after mediation and before trial, the parties would still have a mutual gain from mediation, since it provides a "focal point" that determines the allocation of the costs of trial and establishes a new point of reference for subsequent bargaining.

discovery. In many cases each party will have private information; thus there are a number of ways in which this procedure can be applied.[26]

We will consider only the case where $A_M(S_P^1 = H, S_D^1 = d) = \frac{1}{2}X$. The mediator awards the intermediate value. If the truthful reporting procedure outperforms the current system in this case, then this procedure is shown to be better. Under the current system, assuming the mediator wants to minimize the probability of trial, he will choose $A_M(S_P^1 = H, S_D^1 = d) = \frac{1}{2}X$, and the probability of a trial is then 2/3. If we can show that with this decision rule, the probability of trial under the truthful system is less than 2/3, there are two possibilities: either this rule is optimal under the truthful system, or there is an alternative rule for which the probability of trial is even lower under the truthful system. In either case the truthful system outperforms the current system.

Lemma 6. *In the MDSC under the truthful reporting system, suppose the mediator's award is $A_M(S_P^1 = H, S_D^1 = d) = \frac{1}{2}X$ and $X > 6$. Then the only equilibrium given our restricted strategies has the following properties:*

1) The plaintiff's first report is $S_P^1(\pi_P(\omega_1, \omega_2)) = L$ with probability $\frac{X-2}{X}$ and $S_P^1(\pi_P(\omega_1, \omega_2) = H$ with probability $\frac{2}{X}$ (i.e., $\alpha = \frac{X-2}{X}$).

2) The defendant's first report is $S_D^1(\pi_D(\omega_2, \omega_3)) = a$ with probability $\frac{X-2}{X}$ and $S_D^1(\pi_D(\omega_2, \omega_3) = d$ with probability $\frac{2}{X}$.

3) The probability of trial is $\left(\frac{2}{3} \cdot \frac{X}{X+4}\right)$.

[26] In a medical malpractice case, for example, the plaintiff usually has better information about the severity of his injury, while the defendant physician has a better idea whether his conduct met the customary standard of medical practice in the area. At the mediation hearing the mediator could ask the defendant a question designed to reveal the degree of the defendant's negligence, and ask the plaintiff to disclose the severity of his injury. Both parties would know that the court, whether judge or jury, would make explicit findings of fact on these issues. If there was a disparity between the party's response to the mediator and the court's finding, the party would be liable for sanctions.

The proof (which employs the same logic as lemma 3) is set forth in the appendix .

In the truthful reporting system, P and D play mixed strategies and are each reporting truthfully $\frac{X-2}{X}$ of the time. For example, if $X = 10$, so that $v(\omega_3)$ is ten times the cost of a trial (five times the total cost for both parties), then the plaintiff reports truthfully in $\frac{4}{5}$ of the cases. The probability of trial falls from $\frac{2}{3}$ to 0.47619.

4.3. The Self-Enforcing System

Under the second alternative procedure, which we call the "self-enforcing" system each party's statement to the mediator about the value of the claim must be expressed as a dollar amount. Therefore the adjustment strategies are

$$\begin{aligned} S_P^1(\pi_P(\omega_1, \omega_2)) &= \left\{ \begin{array}{cc} 0.5x & : \quad \alpha \\ x & : 1-\alpha \end{array} \right\} \\ S_P^1(\pi_P(\omega_3))) &= x \end{aligned} \tag{4.8}$$

and

$$\begin{aligned} S_D^1(\pi_2(\omega_1)) &= 0 \\ S_D^1(\pi_2(\omega_{2,}\omega_3)) &= \left\{ \begin{array}{cc} 0 & : \quad \beta \\ 0.5x & : 1-\beta \end{array} \right\} \end{aligned} \tag{4.9}$$

Lemmas 1 and 2 can be similarly adjusted.

Formally, let $B_P^S(A_M \times S_P^2 \times S_D^2 \times v(\omega_o))$ be the allocation B_P under the self-enforcing policy, where the superscript S indicates self-enforcement. Then

$$B_P^S(S_P^1 \times S_D^1 \times S_P^2 \times S_D^2 \times v(\omega_o)) = \left\{ \begin{array}{cl} 2 & : S_P^1 \leq A_M \text{ and } S_P^2 = N \text{ and } S_D^2 = Y \\ 0 & : S_D^1 \geq A_M \text{ and } S_D^2 = N \text{ and } S_P^2 = Y \\ 2 & : S_P^2 = Y \text{ and } S_P^1 - V(\omega_o) > V(\omega_o) - S_D^1 \\ 0 & : S_D^2 = Y \text{ and } V(\omega_o) - S_D^1 > S_P^1 - V(\omega_o) \\ 1 & : \text{otherwise} \end{array} \right\} \quad (4.10)$$

The purpose of the first two rules is to prevent a party from manipulating the system by rejecting a mediation award that is more favorable to him than the amount he proposed.

This will determine $B_D^S(S_P^1 \times S_D^1 \times S_P^2 \times S_D^2 \times v(\omega_o))$ as $B_P^S + B_D^S = 2$.

As before, we consider only the case where $A_M(S_P^1 = H, S_D^1 = d) = \frac{1}{2}X$.

Lemma 7. *In the MDSC under the self-enforcing system, suppose the mediator's award is* $A_M(S_P^1 = H, S_D^1 = d) = \frac{1}{2}X$ *and* $X > 6$. *Then the only equilibrium given our restricted strategies has the following properties:*

1) The plaintiff's first report is $S_P^1(\pi_P(\omega_1, \omega_2)) = 0.5x$ with probability $\frac{X-2}{X}$ and $S_P^1(\pi_P(\omega_1, \omega_2) = x$ with probability $\frac{2}{X}$ (i.e., $\alpha = \frac{X-2}{X}$).

2) The defendant's first report is $S_D^1(\pi_D(\omega_2, \omega_3)) = 0.5x$ with probability $\frac{X-2}{X}$ and $S_D^1(\pi_D(\omega_2, \omega_3) = 0$ with probability $\frac{2}{X}$.

3) The probability of trial is $\left(\frac{2}{3} \cdot \frac{X^2+2X+2}{X^2+4X}\right)$.

The proof (which employs the same logic as lemma 3) is set forth in the appendix .

In the self-enforcing system, P and D play mixed strategies and are each reporting truthfully $\frac{X-2}{X}$ of the time - the same rate that occurs under the truthful reporting system.. For example, if $X = 10$, so that $v(\omega_3)$ is ten times the cost of a trial (five times the total cost for both parties), then the plaintiff reports truthfully in $\frac{4}{5}$ of the cases. The probability of trial falls from

$\frac{2}{3}$ under the current system to 0.5809 in the self-enforcing system. But this is less than the trial probability of 0.47619 achieved by the truthful reporting system.

5. The Payoffs to the Parties under the Current and Proposed Systems

Since the objective of the legal system is not just to minimize the number of cases going to trial (at least not officially) we should also compare the payoffs to the parties under the current system and the two proposed systems.

Lemma 8. *In the MDSC if $X > 6$ the expected payoff of the plaintiff given his partition $\pi_P(\omega_3)$ is $X - 1$ under the current system compared to an expected payoff of $X - \frac{X-4}{X+4}$ under the truthful system. Under the self-enforcing system it is $X - \frac{X^2-X}{X^2+4X}$. The expected payoff of the plaintiff given his partition $\pi_P(\omega_1, \omega_2)$ is $\frac{1}{4}X - \frac{1}{2}$ under all three systems.*

proof: see appendix.

Lemma 9. *In the MDSC if $X > 6$ the expected payoff of the defendant given his partition $\pi_D(\omega_1)$ is -1 under the current system compared to an expected payoff of $-\frac{X-4}{X+4}$ under the truthful system. Under the self-enforcing system it is $-\left(\frac{X^2-X}{X^2+4X}\right)$.The expected payoff of the defendant given his partition $\pi_D(\omega_2, \omega_3)$ is $\left(-\frac{3}{4}X - \frac{1}{2}\right)$ under all three systems.*

The proof of Lemma 9 is similar to that of Lemma 8.

It should be noted that the entire welfare improvement of going from the current system to either one of the proposed systems is captured by the party who has better private information, given the true state, or equivalently who has the stronger case. That is, the party whose position is stronger has a higher expected payoff under either of the two alternative procedures;

the plaintiff is strongly better off if the true state of nature is ω_3, while the defendant is strongly better off if the true state is ω_1.

6. Conclusion

We have analyzed a sequential game in which each party has private information, and the mediator must acquire all his information from the statements made by the parties. In this game the mediator is a player, whose objective is to minimize the frequency of trial. All players maximize expected utility at each stage of the game. In our model the types are not independent,[27] a feature that makes the game harder to solve but may be more realistic.

In the Minimal Differential Symmetric Case that we consider, there is a lower frequency of trial under the two procedures that provide incentives for truthfulness than under the current system, that rewards acceptance of the mediation award. Under the current system, the probability of trial in equilibrium is $\frac{2}{3}$, while it is $\left(\frac{2}{3} \cdot \frac{X}{X+4}\right)$ under the truthful reporting system and $\left(\frac{2}{3} \cdot \frac{X^2+2X+2}{X^2+4X}\right)$ under the self-enforcing system . These results are counterintuitive, since under the current system a penalty is imposed directly on the action (rejecting the mediation award) that makes a trial possible. Also, each party stands to gain by replacing the current system with one of our procedures.

The results are driven by the fact that the current system is myopic. It turns out that the best way to ensure that both parties accept the mediation award is, not by penalizing them for rejecting it, but rather by giving them an incentive to give accurate information to the mediator in the first place. When the mediation award is more accurate, the parties have less to gain by going to trial, and will be deterred from doing so by the costs of trial, even if, as here, those costs are low relative to the value of claims.

[27] For example, if the plaintiff knows that the state is w_3, he knows the defendant is liable.

As noted previously, we believe our first procedure, the truthful reporting system, can readily be implemented by replacing the penalty for rejection of the mediation award with a penalty for providing false or incomplete information to the mediator, or to the other party during pretrial discovery.[28] However, if this procedure is deemed impractical because of difficulties involved in proving that a party made false statements, then the second procedure we analyze, the self-enforcing system, also reduces the probability of trial and provides ex ante gains to both parties.

[28] In a products liability case, for example, the defendant usually has much better knowledge of how the product was manufactured, while the plaintiff knows how he was using the product when he was injured (under the rules of product liability law, the defendant would not be liable if the victim were misusing the product, unless the misuse was "foreseeable.") Here the mediator could ask the defendant a specific question designed to reveal whether the product was defective, and ask the plaintiff exactly how he was using the product. Again, sanctions would be imposed on a party if the court's finding on the issue was different from the party's response to the mediator.

7. Appendix

In this appendix we provide complete proofs for the propositions concerning the current system and the truthful reporting system. We do not yet have proofs for the self-enforcing system; these will be added soon. It should be noted that the proofs for the self-enforcing system will be quite similar to those provided for the truthful reporting system.

Comment: throughout this appendix we will denote the plaintiff by P, the defendant by D, the mediator by M, and the mediation award by A_M

7.1. Proof of Lemma 1

Consider first the following statements made to M by P and D: $S_P^1 = h, S_D^1 = a$. By Bayes' Rule the posterior probabilities that M assigns to the state of nature given that he observes the strategies S_P^1 and S_D^1 described in 4.2 and 4.3 are $P(\omega_1|S_P^1 = h, S_D^1 = a) = 0$, $P(\omega_2|S_P^1 = h, S_D^1 = a) = \frac{1-\alpha}{2-\alpha}$ and $P(\omega_3|S_P^1 = h, S_D^1 = a) = \frac{1}{2-\alpha}$ .Therefore since M will select the state[29] with the (weakly) higher probability for every α , it should be ω_3, which implies an award of $A_M(S_P^1 = h, S_D^1 = a) = X$.

Secondly, consider $S_P^1 = l, S_D^1 = a$. By Bayes' Rule, $P(\omega_1|S_P^1 = h, S_D^1 = a) = 0$, $P(\omega_2|S_P^1 = h, S_D^1 = a) = 1$ and $P(\omega_3|S_P^1 = h, S_D^1 = a) = 0$. Therefore ω_2 is the state with the highest probability, and M should award $A_M(S_P^1 = l, S_D^1 = a) = \frac{1}{2}X$.

Finally, consider $S_P^1 = l, S_D^1 = d$. By Bayes' Rule the posterior probabilities are $P(\omega_1|S_P^1 = l, S_D^1 = d) = \frac{1}{1+\beta}$, $P(\omega_2|S_P^1 = l, S_D^1 = d) = \frac{\beta}{1+\beta}$, and $P(\omega_3|S_P^1 = l, S_D^1 = d) = 0$ Therefore ω_1 is

[29] In our setup M can only award the value of one of the states of nature; he cannot award, for example, $\frac{3}{4}X$. However, it appears that this restriction reduces the probability that the parties will go to trial in this case. A formal proof is not provided. But the motivation is that if the true state of nature is ω_3 and M awards, let's say, $\frac{3}{4}X$, then P will certainly reject (since he can recognize that the state is ω_3) and so the case will go to trial. If the state is not ω_3 , P will not take the case to trial, but then D is better off doing so.

the state with the highest probability, and M should award $A_M(S_P^1 = l, S_D^1 = d) = 0$. ■

7.2. Proof of Lemma 2

Here it must be true that one of the players has misrepresented his information, so M must exercise his judgment. Using Bayes' Rule we see that $P(\omega_1|S_P^1 = h, S_D^1 = d) = \frac{1-\alpha}{1-\alpha+2\beta-\alpha\beta}$, $P(\omega_2|S_P^1 = h, S_D^1 = d) = \frac{\beta-\alpha\beta}{1-\alpha+2\beta-\alpha\beta}$ and $P(\omega_3|S_P^1 = h, S_D^1 = d) = \frac{\beta}{1-\alpha+2\beta-\alpha\beta}$. In order to determine the state of nature with the highest probability, M must know the ratio of α to $1-\beta$. ■

7.3. Proof of Lemma 3

We will analyze this game by backward induction (it is not a subgame refinement as in this game there is no subgame, but it will make the analysis easier).

Let us start with $S_P^2 : \Pi_P \times S_P^1 \times S_D^1 \times A_M \rightarrow \{Y, N\}$. This expression represents P's response to the mediation award A_M , where Y denotes acceptance of A_M and N rejection of it. Observe first that

$$S_P^2(\cdot, \cdot, \cdot, A_M = X) = Y \tag{7.1}$$

is a dominant strategy as P cannot get more than X and $S_P^2(\cdot, \cdot, \cdot, A_M = X) = N$ guarantees that P's payoff will not exceed $X - 1$. Next observe that

$$S_P^2(\pi_P(\omega_1, \omega_2), S_P^1 = l,\ S_D^1 = a,\ A_M = \frac{1}{2}X) = Y \tag{7.2}$$

Using Bayes' Rule, we can conclude that P's posterior probability for ω_2 is $P_P(\omega_2|\pi_P(\omega_1, \omega_2), S_P^1 = l, S_D^1 = a) = 1$. If he accepts the award, P is guaranteed a payoff of $\frac{1}{2}X$. If , however, P rejects the award, he will receive at most $\frac{1}{2}X - 1$. Next

$$S_P^2(\pi_P(\omega_3), S_P^1 = h, S_D^1 = d, A_M = \frac{1}{2}X) = N \tag{7.3}$$

Note that P will be awarded X at trial, and therefore in the worst case his payoff will be $X-1$. If, on the other hand, he accepts the mediation award, he is certain to receive $\frac{1}{2}X$ since, as we will see below (7.12), D will accept the mediation award under these conditions, as $X > 6$ and therefore $X - 1 > \frac{1}{2}X$.

The following case requires more extensive analysis:

Claim 1 $\quad S_P^2(\pi_P(\omega_1, \omega_2), S_P^1 = h,\ S_D^1 = d,\ A_M = \frac{1}{2}X) = Y$

proof of claim 1 We must compare two expected payoffs:

(1) Let $EU_P[S_P^2 = Y \mid (\pi_P(\omega_1,\omega_2), S_P^1 = h, S_D^1 = d, A_M = \frac{1}{2}X)]$ be the expected payoff of P if he accepts $A_M = \frac{1}{2}X$, given his partition $\pi_P(\omega_1, \omega_2)$ and that the second period demands of P and D are $S_P^1 = h, S_D^1 = d$ respectively. Observe that

$$\begin{aligned} EU_P[S_P^2 &= Y|(\pi_P(\omega_1,\omega_2), S_P^1 = h, S_D^1 = d, A_M = \frac{1}{2}X)] = \qquad (7.4) \\ P_P(\omega_1|\pi_P(\omega_1,\omega_2), S_D^1 &= d)[(1-\beta_2)(\frac{1}{2}X) + \beta_2(0-1) + \\ P_P(\omega_2|\pi_P(\omega_1,\omega_2), S_D^1 &= d)[(1-\beta_3)(\frac{1}{2}X) + \beta_3(\frac{1}{2}X)] \end{aligned}$$

where $\beta_2 \in [0,1]$ is the probability that D rejects A_M when his partition is $\pi_D(\omega_1)$ and $\beta_3 \in [0,1]$ is the probability that D rejects A_M when his partition is $\pi_D(\omega_2,\omega_3)$. This expected payoff is the sum of the expected outcomes for ω_1 and ω_2. The expected outcome for ω_1 is the probability that P assigns to ω_1 given that he observed $S_D^1 = d$, multiplied by $\frac{1}{2}X$ if D accepts A_M,an event with probability $1-\beta_2$, and by $(0-1)$ (recalling that $C = 1$) if D rejects A_M and the case goes to trial, an event with probability β_2. A similar analysis applies to ω_2 , except that D's probability of rejecting $A_M = \frac{1}{2}X$ is now β_3.

(2) Let $EU_P[S_P^2 = N \mid (\pi_P(\omega_1,\omega_2), S_P^1 = h, S_D^1 = d, A_M = \frac{1}{2}X)]$ be the expected payoff if

$P's$ response is N , so that the case goes to trial with certainty. Observe that

$$\begin{aligned} EU_P[S_P^2 &= N|\ (\pi_P(\omega_1,\omega_2), S_P^1 = h, S_D^1 = d, A_M = \frac{1}{2}X)] = \quad (7.5) \\ P_P(\omega_1|\pi_P(\omega_1,\omega_2), S_D^1 &= d)[(1-\beta_2)(0-2) + \beta_2(0-1)] + \\ P_P(\omega_2|\pi_P(\omega_1,\omega_2), S_D^1 &= d)[(1-\beta_3)(\frac{1}{2}X-2) + \beta_3(\frac{1}{2}X-1)] \end{aligned}$$

In this case the payoff depends on whether the outcome is ω_1 or ω_2 , which generate the trial verdicts of 0 and $\frac{1}{2}X$ respectively, and on whether D accepts or refuses A_M. In either case P has rejected A_M and does not do better at trial. If D accepted A_M, then P must pay the trial costs of both, but if both rejected A_M then each pays his own costs.

It is easy to see that irrespective of the values of β_2,β_3 and $P_P(\omega_1|\pi_P(\omega_1,\omega_2)), S_D^1 = d)$ 7.4 is greater than 7.5 therefore $S_P^2(\pi_P(\omega_1,\omega_2)), S_1^1 = h, S_2^1 = d, A_M = \frac{1}{2}X) = Y$.

End of proof of claim 1

Claim 2 If D plays $S_D^1(\pi_2(\omega_{2,}\omega_3)) = d$ (see 4.3) with $\beta = \frac{2}{0.5X-1}$ then $S_P^2(\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0)$ is a mixed strategy. If $\beta > \frac{2}{0.5X-1}$ then $S_P^2(\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0) = N$. if $\beta < \frac{2}{0.5X-1}$ then $S_P^2(\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0) = Y$

proof of claim 2 Let $EU_P[S_P^2 = Y \mid (\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0)]$ be P's expected payoff if he accepts $A_M = 0$. Obviously D will accept A_M. Therefore, $EU_P[S_P^2 = Y \mid (\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0] = 0$

The alternative strategy for P is to reject A_M, which yields an expected payoff of

$$\begin{aligned} EU_P[S_P^2 &= N|\ \pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0)] = \quad (7.6) \\ P_P(\omega_1|\pi_P(\omega_1,\omega_2)), S_D^1 &= d)(0-2) + P_P(\omega_2|\pi_P(\omega_1,\omega_2)), S_D^1 = d)(\frac{1}{2}X-1) \end{aligned}$$

In the event of ω_1 P receives 0 and must pay 2, the trial cost of both sides. In the event of ω_2 he will receive $\frac{1}{2}X$ and pay only his cost of 1 (as his rejection of A_M was justified). By Bayes' Rule and 4.3 , $P_P(\omega_1|\pi_P(\omega_1,\omega_2), S_D^1 = d) = \frac{1}{1+\beta}$ and $P_P(\omega_2|\pi_P(\omega_1,\omega_2)), S_D^1 = d) = \frac{\beta}{1+\beta}$. Hence:

$$EU_P[S_P^2 = N|\ (\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0)] = \frac{1}{1+\beta}[\beta(\frac{1}{2}X - 1) - 2] \tag{7.7}$$

For $\beta = 0$ (which means that D always reports truthfully) P is better off accepting A_M. But for $\beta = 1$ (which means that D always reports falsely) P is better off rejecting A_M.

P will be indifferent and play a mixed strategy only if

$$\frac{1}{1+\beta}[\beta(\frac{1}{2}X - 1) - 2] = 0 \tag{7.8}$$

(which means $EU_P[S_P^2 = N|\ (\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0)] = EU_P[S_P^2 = Y \mid (\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0)])$ which leads to

$$\beta = \frac{2}{0.5X - 1} \tag{7.9}$$

We do not claim that D plays a mixed strategy with $\beta = \frac{2}{0.5X-1}$. We also do not claim that $S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0)$ is a mixed strategy. We claim only that if D plays $S_D^1(\pi_2(\omega_{2,}\omega_3)) = d$ with $\beta = \frac{2}{0.5X-1}$, then P adopts a mixed strategy for $S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0)$. Let α_1 be the probability that $S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0) = Y$

$$P(S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0) = Y) = \alpha_1 \tag{7.10}$$

However, we will show that D will always report falsely, so that $S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0) = N$. This exercise will eliminate the mixed strategy equilibrium.

End of proof of claim 2

Next let us analyze $S_D^2 : \Pi_D \times S_P^1 \times S_D^1 \times A_M \to \{Y, N\}$. Recall that this expression represents

D's response to the mediator's award. Observe first that

$$S_D^2(\bullet,\bullet,\bullet,A_M = 0) = Y \tag{7.11}$$

$$S_D^2(\pi_D(\omega_2,\omega_3),S_P^1 = l,\ S_D^1 = a,\ A_M = \frac{1}{2}X) = Y \tag{7.12}$$

$$S_D^2(\pi_D(\omega_1),S_P^1 = h, S_D^1 = d, A_M = \frac{1}{2}X) = N \tag{7.13}$$

The argument is similar to those of 7.1, 7.2 and 7.3 .

Claim 3 $S_D^2(\pi_D(\omega_2,\omega_3),S_P^1 = h,\ S_D^1 = d,\ A_M = \frac{1}{2}X) = Y$

The proof is similar to the proof of claim 1.

Claim 4 if P employs $S_P^1(\pi_P(\omega_1,\omega_2)) = l$ with $\alpha = \frac{0.5X-3}{0.5X-1}$ then $S_D^2(\pi_D(\omega_2,\omega_3),S_P^1 = h,S_D^1 = a,A_M = X)$ is a mixed strategy If $\alpha > \frac{0.5X-3}{0.5X-1}$ then $S_D^2(\pi_D(\omega_2,\omega_3),S_P^1 = h,S_D^1 = a,A_M = X) = Y$,and if $\alpha < \frac{0.5X-3}{0.5X-1}$ then $S_D^2(\pi_D(\omega_2,\omega_3),S_P^1 = h,S_D^1 = a,A_M = X) = N$.

The proof is similar to that of claim 2, as β is equivalent to $1-\alpha$. (observe that $1-\alpha = 1-\frac{0.5X-3}{0.5X-1} = \frac{2}{0.5X-1}$).

Claim 5 P does not play a mixed strategy; he will be deceptive at every opportunity $S_P^1(\pi_P(\omega_1,\omega_2) = h$ namely $(\alpha = 0)$. Therefore $S_D^2(\pi_D(\omega_2,\omega_3),S_P^1 = h,S_D^1 = a,A_M = X) = N$.

proof of claim 5 P will play the mixed strategy $S_P^1(\pi_P(\omega_1,\omega_2) = l$ with probability α and $S_P^1(\pi_P(\omega_1,\omega_2) = h$ with probability $1-\alpha$ only if the expected payoff from h is the same as the expected payoff from l, namely $EU_P[S_P^1(\pi_P(\omega_1,\omega_2) = h] = EU_P[S_P^1(\pi_P(\omega_1,\omega_2) = l]$

Let us first consider $EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) = h]$. Now if the state of nature is ω_1 then $S_D^1 = d$ (4.3) which leads to $A_M = \frac{1}{2}X$ (the condition of lemma 4 $A_M(S_P^1 = h,S_D^1 = d) = \frac{1}{2}X$) which leads to $S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = h,\ S_D^1 = d,\ A_M = \frac{1}{2}X) = Y$ and $S_D^2(\pi_D(\omega_1), S_P^1 = h,\ S_D^1 = d,$ $A_M = \frac{1}{2}X) = N$ which will generate a payoff of $0-1$.

Now if the state of nature is ω_2 then $S_D^1 = d$ with probability β and $S_D^1 = a$ with probability $1-\beta$ (4.3) .If $S_D^1 = d$ then $A_M = \frac{1}{2}X$ and both P and D accept A_M. If $S_D^1 = a$ then $A_M = X$ which P accepts and D accepts with probability β_1 and rejects with probability $1-\beta_1$. Therefore

$$\begin{aligned} EU_P[S_P^1(\pi_P(\omega_1,\omega_2) &= h] = P_P(\omega_1|\pi_P(\omega_1,\omega_2)[0-1] + \\ & P_P(\omega_2|\pi_P(\omega_1,\omega_2)[\beta(\frac{1}{2}X) + (1-\beta)\{\beta_1(X) + (1-\beta_1)(\frac{1}{2}X-1)\}] \end{aligned} \tag{7.14}$$

as $P_P(\omega_1|\pi_P(\omega_1,\omega_2)) = 0.5$,

$$EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) = h] = 0.5[(-2 + \frac{1}{2}X + \frac{1}{2}X\beta_1 - \frac{1}{2}X\beta_1\beta + \beta + \beta_1 - \beta_1\beta)] \tag{7.15}$$

similarly

$$\begin{aligned} EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) &= l] = P_P(\omega_1|\pi_P(\omega_1,\omega_2)[\alpha_1(0) + (1-\alpha_1)(0-2)] + \\ & P_P(\omega_2|\pi_P(\omega_1,\omega_2)[\beta\{\alpha_1(0) + (1-\alpha_1)(\frac{1}{2}X-1)\} + (1-\beta)(\frac{1}{2}X)] \end{aligned} \tag{7.16}$$

The sequence of events is as follows.

$$\begin{array}{lll} \text{If } \omega_1 & \Rightarrow & S_D^1 = d \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y \text{ and } \left\{ \begin{array}{c} S_P^2 = Y : \alpha_1 \Rightarrow 0 \\ S_P^2 = N : 1-\alpha_1 \Rightarrow 0-2 \end{array} \right\} \\ \text{If } \omega_2 & \Rightarrow & \left\{ \begin{array}{c} S_D^1 = d : \beta \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y \text{ and } \left\{ \begin{array}{c} S_P^2 = Y : \alpha_1 \Rightarrow 0 \\ S_P^2 = N : 1-\alpha_1 \Rightarrow \frac{1}{2}X - 1 \end{array} \right\} \\ S_D^1 = a : 1-\beta \Rightarrow A_M = \frac{1}{2}X \Rightarrow S_D^2 = Y \text{ and } S_P^2 = Y \Rightarrow \frac{1}{2}X \end{array} \right\} \end{array}$$

7.16 can be reduced to

$$EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) = l] = 0.5[(-2 + \frac{1}{2}X - \frac{1}{2}X\alpha_1\beta - \beta + 2\alpha_1 + \beta_1\alpha_1)] \tag{7.17}$$

Since $\beta = \frac{2}{0.5X-1}$ (see 7.9). we know that for all β_1, $EU_P[S_P^1(\pi_P(\omega_1,\omega_2) = h] > EU_P[S_P^1(\pi_P(\omega_1,\omega_2) = l]$ implying $\alpha = 0$. Therefore $S_D^2(\pi_D(\omega_2,\omega_3), S_P^1 = h, S_D^1 = a, A_M = X) = N$.

End of proof of claim 5

Claim 6 D does not play a mixed strategy; he is deceptive all the time. $S_D^1(\pi_D(\omega_2,\omega_3) = d$ namely ($\beta = 0$); therefore $S_P^2(\pi_p(\omega_1,\omega_2), S_P^1 = l, S_D^1 = a, A_M = 0) = N$.

The proof of claim 6 is similar to the proof of claim 5.

We have proved that there is no equilibrium for a mixed strategy. Next we will prove that there is no equilibrium for a pure truthful strategy.

Claim 7 There is no pure strategy truthful equilibrium under the current system.

proof of claim 7 Consider the following strategy for P : $S_P^1(\pi_P(\omega_1,\omega_2)) = l$, $S_P^1(\pi_P(\omega_3)) = h$. Obviously D will not choose to take the case to trial; therefore $S_D^2(\pi_D(\omega_2,\omega_3), S_P^1 = h, S_D^1 = a, A_M = X) = Y$. However, given this strategy of D , P is better off deviating to $S_P^1(\pi_P(\omega_1,\omega_2)) = h$, as in the event of ω_2 this strategy generates an outcome of x and in the event of ω_1 it generates an outcome of -1. These outcomes may be compared to those of the truthful strategy, which are $0.5X$ and 0 , respectively. Since P assigns the same probability to ω_1 and ω_2, and $X > 6$, P is better off deviating. Similar arguments will eliminate the truthful strategy for D.

End of proof of claim 7

A similar argument will eliminate the equilibrium in which P uses a purely truthful strategy and D uses a deceptive strategy, and vice versa. The next claim will prove the existence of a pure strategy deceptive equilibrium.

Claim 8 There is a pure strategy deceptive equilibrium under the proposed system.

proof of claim 8 Consider the following strategies for P and D: $S_P^1(\pi_P(\omega_1, \omega_2)) = S_P^1(\pi_P(\omega_3)) = h$, $S_D^1(\pi_2(\omega_1)) = S_D^1(\pi_2(\omega_{2,}\omega_3)) = d$.

Suppose P deviates in the first statement by reporting $S_P^1(\pi_P(\omega_1, \omega_2)) = l$, i.e., he reports low damages if his partition is $\pi_P(\omega_1, \omega_2)$ (the rest of his strategy stays the same)

According to M's strategy, $A_M = 0$. But if $A_M = 0$, P is better off rejecting A_M, since that will give him an expected value of $-1.5 + 0.25X$, compared to 0 if he accepts. Therefore, a deviation will yield a payoff of $-1.5 + 0.25X$ compared to the payoff of $-0.5 + 0.25X$ under the equilibrium strategy. We conclude that P will not deviate from the deceptive strategy. The same analysis applies to D.

End of proof of claim 8.

To complete the proof of lemma 1 we need to calculate the probability of trial. P and D will report h and d regardless of their partition (private information), so $A_M = \frac{1}{2}X$ regardless of the state of nature ω_i . Now P will reject A_M if the true state of nature is ω_3 and D will reject A_M if the true state is ω_1. Therefore the only case that will not go to trial is ω_2 and the probability of this state is 1/3. Thus the probability of trial is 2/3. ■

7.4. Proof of Lemma 6

The proof is similar to that of Lemma 3 so we will just concentrate on the calculation, and on the different rule for sharing trial costs that leads to a different result. Observe that

$$S_P^2(\cdot, \cdot, \cdot, A_M = X) = Y \tag{7.18}$$

$$S_P^2(\pi_P(\omega_1, \omega_2), S_P^1 = l,\ S_D^1 = a,\ A_M = 0.5X) = Y \tag{7.19}$$

$$S_P^2(\pi_P(\omega_3), S_P^1 = h, S_D^1 = d, A_M = 0.5X) = N \tag{7.20}$$

The reasoning is similar to that of 7.1, 7.2 and 7.3.

claim 6.1 $S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = h,\ S_D^1 = d,\ A_M = \frac{1}{2}X) = Y$

The proof of claim 6.1 is similar to the proof of claim 1.

claim 6.2 If D plays $S_D^1(\pi_2(\omega_{2,}\omega_3)) = d$ (see 4.3) with $\beta = \frac{2}{X}$ then $S_P^2(\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0)$ is a mixed strategy. If $\beta > \frac{2}{X}$ then $S_P^2(\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0) = N$. if $\beta < \frac{2}{X}$ then $S_P^2(\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0) = Y$.

proof of claim 6.2 Let $EU_P[S_P^2 = Y \mid \pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0)]$ be the expected payoff for P if he accepts $A_M = 0$. Obviously D will accept A_M. Therefore, $EU_P[S_P^2 = Y \mid (\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0] = 0$

The alternative strategy for P is to reject A_M, which yields an expected payoff of

$$\begin{aligned} EU_P[S_P^2 &= N| \ (\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0)] = \\ P_P(\omega_1|\pi_P(\omega_1,\omega_2)), S_D^1 &= d)(0-1) + P_P(\omega_2|\pi_P(\omega_1,\omega_2)), S_D^1 = d)(\frac{1}{2}X) \end{aligned} \quad (7.21)$$

In the event of ω_1 player P receives 0 and must pay 1. The reason is that both parties report truthfully, so each bears his own cost. In the event of ω_2 he will receive $\frac{1}{2}X$ and will not pay his trial cost as D reports deceptively (and misleads M). By Bayes' Rule and 4.3 , $P_P(\omega_1|\pi_P(\omega_1,\omega_2), S_D^1 = d) = \frac{1}{1-\beta}$ and $P_P(\omega_2|\pi_P(\omega_1,\omega_2)), S_D^1 = d) = \frac{\beta}{1-\beta}$. Hence:

$$EU_P[S_P^2 = N| \ (\pi_P(\omega_1,\omega_2)), S_P^1 = l, S_D^1 = d, A_M = 0)] = \frac{1}{1-\beta}[\beta\frac{1}{2}X - 1] \quad (7.22)$$

For $\beta = 0$, which means that D always reports truthfully, P is better off accepting A_M. But for $\beta = 1$, which means that D always reports falsely, P is better off rejecting A_M. P will be indifferent and play a mixed strategy only if $\frac{1}{1-\beta}[\beta\frac{1}{2}X - 1] = 0$ (which means $EU_P[S_P^2 =$

$N| \ (\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = R, D_M = 0)] = EU_P[S_P^2 = Y \mid (\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = R, D_M = 0)])$ which leads to

$$\beta = \frac{2}{X} \tag{7.23}$$

Let α_1 be the probability that $S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0) = Y$ namely

$$P(S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0) = Y) = \alpha_1 \tag{7.24}$$

(We will show that this is the equilibrium outcome in the proposed system).

End of proof of claim 6.2

Next let us analyze $S_D^2 : \Pi_D \times S_P^1 \times S_D^1 \times A_M \to \{Y, N\}$. Observe $S_D^2(\cdot,\cdot,\cdot, A_M = 0) = Y$, $S_D^2(\pi_D(\omega_2,\omega_3), S_P^1 = l,\ S_D^1 = a,\ A_M = \frac{1}{2}X) = Y$ and $S_D^2(\pi_D(\omega_1), S_P^1 = h, S_D^1 = d, A_M = \frac{1}{2}X) = N$ The following case is similar to claim 6.1.

claim 6.3 $S_D^2(\pi_D(\omega_2,\omega_3), S_P^1 = h,\ S_D^1 = d,\ A_M = \frac{1}{2}X) = Y$

The proof of claim 6.3 is similar to the proof of claim 6.1.

claim 6.4 If P employs $S_P^1(\pi_P(\omega_1,\omega_2)) = l$ with $\alpha = \frac{X-2}{X}$ then $S_D^2(\pi_D(\omega_2,\omega_3), S_P^1 = h, S_D^1 = a, A_M = X)$ is a mixed strategy. If $\alpha > \frac{X-2}{X}$ then $S_D^2(\pi_D(\omega_2,\omega_3), S_P^1 = h, S_D^1 = a, A_M = X) = Y$, and if $\alpha < \frac{X-2}{X}$ then $S_D^2(\pi_D(\omega_2,\omega_3), S_P^1 = h, S_D^1 = a, A_M = X) = N$.

The proof of claim 6.4 is similar to the proof of claim 6.2.

claim 6.5 In equilibrium, P plays a mixed strategy; he plays $S_P^1(\pi_P(\omega_1,\omega_2) = l$ with probability $\alpha = \frac{X-2}{X}$. Therefore D plays $S_D^2(\pi_D(\omega_2,\omega_3), S_P^1 = h, S_D^1 = a, A_M = X) = Y$ with probability $\beta_1 = \frac{6X-8}{(X-2)(X+4)}$.

proof of claim 6.5 P will play the mixed strategy $S_P^1(\pi_P(\omega_1,\omega_2) = l$ with probability α and $S_P^1(\pi_P(\omega_1,\omega_2) = h$ with probability $1-\alpha$ only if the expected payoff from h is the same as the expected payoff from l, i.e.,$EU_P[S_P^1(\pi_P(\omega_1,\omega_2) = h] = EU_P[S_P^1(\pi_P(\omega_1,\omega_2) = l]$

Let us first consider $EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) = h]$. Now if the state of nature is ω_1 then $S_D^1 = d$ (4.3) , which leads to $A_M = \frac{1}{2}X$ (from the condition of the lemma $A_M(S_P^1 = h, S_D^1 = d) = \frac{1}{2}X$), which in turn leads to $S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = h$, $S_D^1 = d$, $A_M = \frac{1}{2}X) = Y$ and $S_D^2(\pi_D(\omega_1), S_P^1 = h$, $S_D^1 = d$, $A_M = \frac{1}{2}X) = N$ which will generate a payoff of $0-2$.

If, on the other hand, the state of nature is ω_2 then $S_D^1 = d$ with probability β and $S_D^1 = a$ with probability $1-\beta$ (4.3). If $S_D^1 = d$ then $A_M = \frac{1}{2}X$ and both P and D accept the mediation award. if $S_D^1 = a$ then $A_M = X$ which P accepts and D accepts with probability β_1 and rejects with probability $1-\beta_1$. Therefore

$$\begin{aligned} EU_P[S_P^1(\pi_P(\omega_1,\omega_2) &= h] = P_P(\omega_1|\pi_P(\omega_1,\omega_2))[0-2] + \\ & P_P(\omega_2|\pi_P(\omega_1,\omega_2))[\beta(\frac{1}{2}X) + (1-\beta)\{\beta_1(X) + (1-\beta_1)(\frac{1}{2}X-2)\}] \end{aligned} \tag{7.25}$$

As $\beta = \frac{2}{X}$ and $P_P(\omega_1|\pi_P(\omega_1,\omega_2)) = 0.5$, we get

$$EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) = h] = 0.5[(-1 + (X-2)(0.5+\frac{2}{X})\beta_1 + (X-2)(0.5-\frac{2}{X})] \tag{7.26}$$

similarly

$$\begin{aligned} EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) &= l] = P_P(\omega_1|\pi_P(\omega_1,\omega_2))[\alpha_1(0) + (1-\alpha_1)(0-1)] \\ & P_P(\omega_2|\pi_P(\omega_1,\omega_2))[\beta\{\alpha_1(0) + (1-\alpha_1)(\frac{1}{2}X)\} + (1-\beta)(\frac{1}{2}X)] \end{aligned} \tag{7.27}$$

The sequence of events is as follows:

$$\begin{aligned} \text{If } \omega_1 &\Rightarrow S_D^1 = d \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y \text{ and } \left\{ \begin{array}{c} S_P^2 = Y : \alpha_1 \Rightarrow 0 \\ S_P^2 = N : 1-\alpha_1 \Rightarrow 0-1 \end{array} \right\} \\ \text{If } \omega_2 &\Rightarrow \left\{ \begin{array}{c} S_D^1 = d : \beta \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y \text{ and } \left\{ \begin{array}{c} S_P^2 = Y : \alpha_1 \Rightarrow 0 \\ S_P^2 = N : 1-\alpha_1 \Rightarrow \frac{1}{2}X \end{array} \right\} \\ S_D^1 = a : 1-\beta \Rightarrow A_M = \frac{1}{2}X \Rightarrow S_D^2 = Y \text{ and } S_P^2 = Y \Rightarrow \frac{1}{2}X \end{array} \right\} \end{aligned}$$

As $\beta = \frac{2}{X}$ 7.27 can be reduced to

$$EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) = l] = 0.25[X-2] \tag{7.28}$$

implying $EU_P[S_P^1(\pi_P(\omega_1,\omega_2) = h] = EU_P[S_P^1(\pi_P(\omega_1,\omega_2) = l]$ which is equivalent to

$0.5[(-1 + (X-2)(0.5 + \frac{2}{X})\beta_1 + (X-2)(0.5 - \frac{2}{X})] = 0.25[X-2].$

which leads to the conclusion that

$$\beta_1 = \frac{6X-8}{(X-2)(X+4)} \tag{7.29}$$

End of proof of claim 6.5

claim 6.6 D plays a mixed strategy: he plays $S_D^1(\pi_D(\omega_2,\omega_3) = d$ with probability $\beta = \frac{2}{X}$, and P therefore plays $S_P^2(\pi_p(\omega_1,\omega_2), S_P^1 = l, S_D^1 = a, A_M = 0) = Y$ with probability $\alpha_1 = \frac{6X-8}{(X-2)(X+4)}$.

The proof of claim 6.6 is similar to the proof of claim 6.5.

We have established the existence of a mixed strategy equilibrium. For uniqueness we must prove that pure strategy equilibria do not exist in the proposed system. Claim 6.7 will prove that there is no pure strategy truthful equilibrium, and claim 6.8 will prove that there is no pure strategy deceptive equilibrium.

claim 6.7 There is no pure strategy truthful equilibrium under the proposed system.

The proof of claim 6.7 is similar to the proof of claim 7. A similar argument will eliminate the equilibrium with the purely truthful strategy for P and the deceptive strategy for D, and vice versa.

claim 6.8 There is no pure strategy deceptive equilibrium under the proposed system.

We will set forth the proof in detail, as it emphasizes the difference between the existing (myopic) system and the proposed system.

proof of claim 6.8 Consider the following strategies for P and D : $S_P^1(\pi_P(\omega_1,\omega_2)) = S_P^1(\pi_P(\omega_3)) = h$, $S_D^1(\pi_2(\omega_1)) = S_D^1(\pi_2(\omega_2,\omega_3)) = d$.

Observe next that the second decisions of P are:

$S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l,\ S_D^1 = a,\ A_M = \frac{1}{2}X) = Y$

$S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = h,\ S_D^1 = d,\ A_M = \frac{1}{2}X) = Y$

$S_P^2(\pi_P(\omega_1,\omega_2), S_P^1 = l, S_D^1 = d, A_M = 0) = N.$

Suppose P deviates in the first statement by reporting $S_P^1(\pi_P(\omega_1,\omega_2)) = l$, i.e., he reports low damages if his partition is $\pi_P(\omega_1,\omega_2)$ (the rest of his strategy stays the same)

The sequence of events will then be as follows:

$$\begin{aligned}
&\text{If } \omega_1 \quad \Rightarrow \quad S_D^1 = d \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y\ ,\ S_P^2 = N \Rightarrow 0-1 \\
&\text{If } \omega_2 \quad \Rightarrow \quad S_D^1 = d \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y\ ,\ S_P^2 = N : 1-\alpha_1 \Rightarrow \frac{1}{2}X
\end{aligned}$$

In the event of ω_1 no one lies; therefore each party bears his own cost, namely 1. In the event of ω_2, D lies; therefore he bears all the cost (i.e., P pays 0 costs for the trial). Therefore,

$$EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) = l] = P_P(\omega_1|\pi_P(\omega_1,\omega_2)[0-1] + P_P(\omega_2|\pi_P(\omega_1,\omega_2)[\frac{1}{2}X] = 0.25X - 0.5$$

If P does not deviate (so that $S_P^1(\pi_P(\omega_1,\omega_2)) = h$) then the sequence of events is

$$\begin{aligned}
&\text{If } \omega_1 \quad \Rightarrow \quad S_D^1 = d \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y\ ,\ S_P^2 = N \Rightarrow 0-2 \\
&\text{If } \omega_2 \quad \Rightarrow \quad S_D^1 = d \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y S_P^2 = N : 1-\alpha_1 \Rightarrow \frac{1}{2}X - 1
\end{aligned}$$

Now in the event of ω_1, P lies, and therefore bears all the costs of trial, so he pays 2 . In the event of ω_2, both lie therefore each party his own costs (namely 1). Therefore,

$$EU_P[S_P^1(\pi_P(\omega_1,\omega_2)) = h] = P_P(\omega_1|\pi_P(\omega_1,\omega_2)[0-2] + P_P(\omega_2|\pi_P(\omega_1,\omega_2)[\frac{1}{2}X-1] = 0.25X - 1.5$$

As $0.25X - 0.5 > 0.25X - 1.5$, in the proposed setup P will not adopt the pure strategy $S_P^1(\pi_P(\omega_1,\omega_2)) = h$. A similar analysis applies to D.

End of proof of claim 6.8

To complete the proof of lemma 6 we must calculate the probability of trial. We will use the following claim to calculate the expected payoffs for P and D, which are needed for lemmas 7,8, and 9.

claim 6.9 The probability of trial is $\frac{2}{3}\frac{X}{X+4}$

proof of claim 6.9 Let us find the expected payoff of each of the three players given the events ω_1, ω_2 , and ω_3. Since the expected payoff to player M is minus the probability of trial, we determine the probability of trial by finding his expected payoff.

First consider the case that the true state is ω_1. As stated previously D will deny liability with probability 1 (so $S_D^1 = d$). Consider the following event:

$$\left\{ \begin{array}{c} S_p^1 = l : \alpha = \frac{X-2}{X} \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y \text{ and } \left\{ \begin{array}{c} S_P^2 = Y : \alpha_1 = \frac{6X-8}{(X-2)(X+4)} \Rightarrow (0,0,0) \\ S_P^2 = N : 1-\alpha \Rightarrow (-1,-1,-1) \end{array} \right\} \\ S_P^1 = h : 1-\alpha = \frac{2}{x} \Rightarrow A_M = 0.5X \Rightarrow S_D^2 = N \text{ and } S_P^2 = Y \Rightarrow (-2,0,-1) \end{array} \right\}$$

Let $EU_i(\omega_j)$ be the expected payoff of player i given the true state is ω_j in this equilibrium, where $i \in \{P, D, M\}$, and $j \in \{1,2,3\}$. Therefore,

$$EU_P(\omega_1) = \frac{X-2}{X}(1 - \frac{6X-8}{(X-2)(X+4)})(-1) + \frac{2}{X}(-2) = -\frac{X-4}{X+4} - \frac{4}{X} \tag{7.30}$$

$$EU_D(\omega_1) = \frac{X-2}{X}(1 - \frac{6X-8}{(X-2)(X+4)})(-1) = -\frac{X-4}{X+4} \tag{7.31}$$

$$EU_M(\omega_1) = \frac{X-2}{X}(1 - \frac{6X-8}{(X-2)(X+4)})(-1) = -\frac{X-4}{X+4} - \frac{2}{X} \tag{7.32}$$

From equation 7.32 we determine that the probability of trial in the event of ω_1 is $\frac{X-4}{X+4}+\frac{2}{X}$.

If the true state is ω_2 both P and D will play mixed strategies.

1) D will deny liability with probability $\frac{2}{x}$ (namely $S_D^1 = d$), so the sequence of events is:

$$\left\{ \begin{array}{c} S_p^1 = l : \alpha = \frac{X-2}{X} \Rightarrow A_M = 0 \Rightarrow S_D^2 = Y \text{ and } \left\{ \begin{array}{c} S_P^2 = Y : \alpha_1 = \frac{6X-8}{(X-2)(X+4)} \Rightarrow (0,0,0) \\ S_P^2 = N : 1-\alpha_1 \Rightarrow (0.5X, -0.5X-2, -1) \end{array} \right\} \\ S_P^1 = h : 1-\alpha = \frac{2}{X} \Rightarrow A_M = 0.5X \Rightarrow S_D^2 = Y \text{ and } S_P^2 = Y \Rightarrow (0.5X, -0.5X, 0) \end{array} \right\}$$

2) D will admit liability with probability $\frac{x-2}{x}$ (namely $S_D^1 = a$), so the sequence of events is:

$$\left\{ \begin{array}{c} S_p^1 = l : \alpha = \frac{X-2}{X} \Rightarrow A_M = 0.5X \Rightarrow S_D^2 = Y \text{ and } S_P^2 = Y \Rightarrow (0.5X, -0.5X, 0) \\ S_P^1 = h : 1-\alpha = \frac{2}{X} \Rightarrow A_M = X \Rightarrow S_P^2 = Y \text{ and } \left\{ \begin{array}{c} S_D^2 = Y : \beta_1 = \frac{6X-8}{(X-2)(X+4)} \Rightarrow (X, -X, 0) \\ S_D^2 = N : 1-\beta_1 \Rightarrow (0.5X-2, -0.5X, -1) \end{array} \right\} \end{array} \right\}$$

Therefore,

$$EU_P(\omega_2) = \frac{1}{2}X - \frac{4X-16}{X(X+4)} \tag{7.33}$$

$$EU_D(\omega_2) = -2\frac{X-4}{X+4} - \frac{(X-2)^2}{2X} - \frac{2}{X} - \frac{12X-16}{X(X+4)} - \frac{4(X-4)}{X(X+4)} \tag{7.34}$$

$$EU_M(\omega_1) = -\frac{X-4}{X+4} \tag{7.35}$$

From equation 7.35 we determine that the probability of trial in the event of ω_2 is $\frac{X-4}{X+4}$.

Lastly, consider the case that the true state is ω_3 . As stated above, P will report h with probability 1 (so $S_P^1 = h$),

$$\left\{ \begin{array}{c} S_D^1 = a : \alpha = \frac{X-2}{X} \Rightarrow A_M = X \Rightarrow S_P^2 = Y \text{ and } \left\{ \begin{array}{c} S_D^2 = Y : \beta_1 = \frac{6X-8}{(X-2)(X+4)} \Rightarrow (X, -X, 0) \\ S_D^2 = N : 1-\beta_1 \Rightarrow (x-1, -X-1, -1) \end{array} \right\} \\ S_D^1 = d : 1-\alpha = \frac{2}{X} \Rightarrow A_M = 0.5X \Rightarrow S_P^2 = N \text{ and } S_D^2 = Y \Rightarrow (X, -X-2, -1) \end{array} \right\}$$

Therefore after some algebra,

$$EU_P(\omega_3) = X - \frac{X-4}{X+4} \tag{7.36}$$

$$EU_D(\omega_3) = \frac{6X-8}{X+4} - \frac{X(X-4)}{X+4} - \frac{X-4}{X+4} - \frac{2(X+2)}{X} \tag{7.37}$$

$$EU_M(\omega_3) = -\frac{X-4}{X+4} - \frac{2}{X} \tag{7.38}$$

From equation 7.38 we determine that the probability of trial in the event of ω_3 is $\frac{X-4}{X+4} + \frac{2}{X}$.

Let $P(c)$ be the probability of going to trial in this equilibrium. Thus $P(c) = \sum_{i=1,2,3} P(\omega_i)|EU_M(\omega_I)|$ as $P(\omega_i) = \frac{1}{3}$, after some calculation,

$$P(c) = \frac{2}{3}\frac{X-4}{X+4} \tag{7.39}$$

■

7.5. Proof of Lemma 7

First, let us calculate P's expected payoff, which we denote by $EU_P^T(\Omega)$ in the proposed system. Thus $EU_P^T(\Omega) = \sum_{i=1,2,3} \frac{1}{3} EU_P(\omega_i)$ We have the values of $EU_P(\omega_1)$, $EU_P(\omega_2)$ and $EU_P(\omega_3)$ from the proof of claim 6.9; see 7.30 , 7.33 and 7.36 respectively. Therefore

$$EU_P^T(\Omega) \ = \frac{1}{2}X - \frac{4X}{6(X+4)} \tag{7.40}$$

Similarly, we can calculate D's expected payoff, which we denote by $EU_D^T(\Omega)$ in the proposed system.

$$EU_D^T(\Omega) \ = -\frac{1}{2}X - \frac{1 + \frac{X-4}{X+4}}{3} \tag{7.41}$$

It is easy to see that under the current system, the payoff for the three players in the event of ω_1 is $(-1, -1, -1)$. In the event of ω_2 it is $(0.5X, -0.5X, 0)$; in the event of ω_3 it is $(X-1, -X-1, -1)$. Therefore $EU_P^M = \frac{1}{2}X - \frac{2}{3}$ and $EU_D^M = -\frac{1}{2}X - \frac{2}{3}$. ■

7.6. Proof of Lemma 8

Let us first consider the case where P's partition is $\pi_P(\omega_3)$:

In the current system the expected payoff for P if his partition is $\pi_P(\omega_3)$ is $X - 1$, as the case always goes to trial and the trial's reward is X but P must pay his trial costs.

As for the proposed system, let $EU_P^T(\pi_P(\omega_3))$ be P's expected payoff given that his partition is $\pi_P(\omega_3)$. Therefore, $EU_P^T(\pi_P(\omega_3)) = EU_P(\omega_3)$, and from 7.36, $EU_P^T(\pi_P(\omega_3)) = X - \frac{X-4}{X+4}$.

Next consider the case where P's partition is $\pi_P(\omega_1, \omega_2)$:

In the current system P's expected payoff is $\frac{1}{2}(-1) + \frac{1}{2}(\frac{1}{2}X)$. The first term applies to the event ω_1 , as the case goes to trial and the verdict is 0 but P must pay his trial costs. The second term applies to the event ω_2 ; in this case P and D accept $A_M = \frac{1}{2}X$.

As for the proposed system, let $EU_P^T(\pi_P(\omega_1, \omega_2))$ be P's expected payoff given that his partition is $\pi_P(\omega_1, \omega_2)$. Then $EU_P^T(\pi_P(\omega_1, \omega_2)) = \frac{1}{2}EU_P(\omega_1) + \frac{1}{2}EU_P(\omega_2)$, and from 7.30 and 7.33, we get $EU_P^T(\pi_P(\omega_1, \omega_2)) = \frac{1}{4}X - \frac{1}{2}$. ■

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