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Assigning Rights and the Exchange of Assets with Interdependent Values

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Abstract

We study the assignment of property rights and compensation rules to facilitate the exchange of valuable assets when investors are differentially informed about their value. We find that when assets are divisible and investors' values are interdependent it is optimal to endow each party with some initial rights over the assets. This has important implications for the design of optimal dissolution of partnerships, the sale of real property, the protection of intellectual property and avoiding conflicting uses of common property.

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1 Introduction

Imagine a setting where two investors prepare for the eventual exchange of a divisible asset. The parties value the asset similarly and each only knows his expected valuation to begin with. In this paper we examines how one should optimally allocate initial rights over the asset when just one of investors eventually observes the asset values for both parties. This issue is important in the dissolution of partnerships, the sale of real property, the protection of intellectual property and in resolving conflicting uses of common property.

1.1 A motivating example:

We illustrate the main issues with this simple example. There are two risk neutral investors I (the informed) and U (the uninformed). I and U wish to eventually divide total assets of Q between themselves to maximize their joint surplus. To begin the investors only know the expected valuation of their assets. Should I receive assets $q \in [0, Q]$, his surplus will be $wv(q)$ while $U's$ surplus will be $\rho(w) v(Q-q)$ from consuming the remaining assets, $Q - q$. The surplus functions $v(\cdot)$ are increasing and concave in the assets consumed. The respective weights, w and $\rho(w)$ for I and U are functions of the initially unknown parameter w, with mean w_E which is distributed by $G(w)$ with $G'(w) = g(w) > 0$ for $w \in [a, b]$. The weights vary together as we assume $\rho(w) = w_0 + \rho(w - w_0)$ where $w^0 \in [a, b]$ and $\rho \in (0, 1)$. This implies the weights are equal at $w = w_0$ and that U values the assets more (less) than I for relatively low (high) w values.

I eventually observes w and learns how the assets can best be divided to maximize total surplus. But prior to learning w the agents agree to an initial asset division of $(q_o, Q - q_0)$ between I and U. This division is important because it affects $I's$ incentives to recommend efficient trade of assets after he learns the relative valuations of the parties. The primary issue is if I can be trusted to recommend the efficient division of assets or will he act instead to maximize his personal gain from exchange. If I observes that U values the assets more, will he recommend that U receive a greater division of assets and if so what compensation will I demand to transfer these additional assets to U? Alternatively, if I learns that he values the assets more than U what payment will he offer to acquire the assets and will it be enough to compensate U for the exchange.?

These issues reveal the importance of initial rights allocations in fostering efficient investment and exchange. In setting initial entitlements is it best to grant complete control of the assets to one of the parties? If the informed investor is best suited to deploy the asset should he be vested with complete control of the assets initially? What effect on the informed investor's willingness to trade does the initial provision of rights have?. To investigate these questions we describe a process for resolving the allocation of asset problem below.

2 A mechanism for efficient distribution of assets.

Once I observes w the exchange of assets is governed by a mechanism that offers a menu of options

$$m(w) \equiv (\Delta(q(w)), \tau(w)) \tag{2.1}$$

for I to select from. The menu specifies net exchanges from the initial allocation, $\Delta q(w) \in [-q_0, Q - q_0]$ and a payment, $\tau(w)$ for I as a function of his report of w. Denote

$$\begin{aligned} w\Delta v^I(\Delta q(w'), q_0) &\equiv w[v(q_0 + \Delta q(w')) - v(q_0)] \\ \rho(w)\Delta v^U(\Delta q(w'), q_0) &\equiv \rho(w)[v(Q - q_0 - \Delta q(w')) - v(Q - q_0)] \end{aligned} \tag{2.2}$$

as the increase in weighted surplus I and U receive from trade of $\Delta q(w')$ given w. Without loss of generality we restrict attention to direct message menu's of type $m(w)$ which also satisfy

$$\pi(w) \equiv w\Delta v^I(\Delta q(w), q_0) + \tau(w) \geq w\Delta v^I(\Delta q(w'), q_0) + \tau(w') \text{ for all } w, w' \tag{IC$_I$}$$

$$w\Delta v^I(\Delta q(w), q_0) + \tau(w) \geq 0 \text{ for all } w \tag{IR$_I$}$$

and

$$E_w\left[\rho\left(w\right)\Delta v^U\left(\Delta q\left(w\right),q_0\right)-\tau\left(w\right)\right]\geq 0 \tag{IR$_U$}$$

Condition (IC_I) requires I truthfully report w. The participation of I and U are guaranteed by conditions (IR_I) and (IR_P) respectively.

The incentive compatibility constraint for I limits the allocations that one can implement.

Lemma 1: *Necessary and sufficient conditions for (IC_I) are that (a) $\pi'\left(w\right)=\Delta v^I\left(\Delta q\left(w\right),q_0\right)$ and (b) $\Delta q'\left(w\right)\geq 0$ for almost all w.*

Proof : The proof of all formal results appear in the Appendix

Condition (b) turns out to have particular importance in the determining the asset trades that can be implemented when I is privately informed of value. In equilibrium I will be induced to acquire more assets the greater his asset value. This means that if I discovers his value for assets is high, pretending to have a low value to reduce his acquisition payments will be costly since the amount of increase I receives will be decrease when he understates his value. This restricts $I's$ ability to profit from his private information.

Suppose a menu $m\left(w\right)=\left\{\tilde{q}\left(w\right)-q_0,\tau\left(w\right)\right\}$ is offered to implement the asset exchange $\tilde{\Delta}\left(q\left(w\right)\right)=\tilde{q}\left(w\right)-q_0$ where $\tilde{q}\left(w\right)$ is non decreasing. Define

$$\begin{aligned}\Phi(q_0,\Delta\tilde{q}\left(w\right)) &= \int_a^b w\Delta v^I\left(\Delta\tilde{q}\left(w\right),q_0\right)+\rho\left(w\right)\Delta v^U\left(\Delta\tilde{q}\left(w\right),q_0\right)dG \\ &+\int_a^{\hat{w}^-(q_0,\tilde{q}(w))} G\left(w\right)\Delta v^I\left(\Delta\tilde{q}\left(w\right),q_0\right)dw \\ &-\int_{\hat{w}^+(q_0,\tilde{q}(w))}^b (1-G\left(w\right))\Delta v^I\left(\Delta\tilde{q}\left(w\right),q_0\right)dG\end{aligned} \tag{2.3}$$

where $\hat{w}^{-}(q_0, \tilde{q}(w)) \equiv \sup\{w \mid \Delta\tilde{q}(w) < 0\}$ and $\hat{w}^{+}(q_0, \tilde{q}(w)) \equiv \inf\{w \mid \Delta\tilde{q}(w) > 0\}$.[1]The expression for $\Phi(q_0, \tilde{q}(w))$ in (2.3) is the expected surplus generated by exchange of $\Delta\tilde{q}(w)$ (given by the first term) net of $I's$ information rents from privately observing w (appearing in the last two terms). The significance of $\Phi(q_0, \tilde{q}(w))$ is reported in the following:

Proposition 1: *It is possible to implement the asset exchange* $\Delta\tilde{q}(x)$ iff $\Phi(q_0, \tilde{q}(w)) \geq 0$

The uninformed investor will only agree to the exchange if he expects to break even at least. The compensation available to U consists of the total surplus net of $I's$ information rents. This must be positive for U to participate.

The surplus maximizing allocation one can implement is determined by solving requires the following problem:

$$\max_{\{q_o, \Delta q(w)\}} \int_a^b wv(q_0) + \Delta q(w)) + \rho(w) v(Q - q_0 - \Delta q(w)) dG$$

$$s.t.\ \ (i)\, \Phi(q_0, \Delta q(w)\ ) \geq 0 \ \text{ and } \ (ii)\ \ \Delta q(w) \ \text{ is increasing} \quad \text{(P)}$$

In proceeding to solve this problem we identify as a benchmark the efficient allocation as a defined by $q^*(w) = \arg\max_q p^I(w) v(q) + p^U(w) v(Q - q)$ which is unique and non decreasing given our assumptions with,

$$q^*(w) = \left\{ \begin{array}{c} 0 \\ \in (0, Q) \\ Q \end{array} \right\} \text{ if } wv'(q^*(w)) - \rho(w) v'(Q - q^*(w)) \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} 0$$

Further we assume,

Condition (M): $G(w)/g(w)$ is increasing and $(1 - G(w))/g(w)$ is decreasing

[1] If $\Delta\tilde{q}(w) \leq 0$ for all w let $\hat{w}^{+} = b$. If $\Delta\tilde{q}(w) > 0$ for all w let $\hat{w}^{-} = a$.

This is a standard monotonicity condition typically employed to insure a separating solution to hidden information allocation problems.

The solution to (P) is characterized in the following Proposition

Proposition 2 *Let $g(w)$ satisfy Condition M. Let $\lambda \geq 0$ be the multiplier corresponding to constraint (ii) in $[P]$. The optimal allocation satisfies:*

(a) *The initial distribution of assets is $q_0 = q^*(w_E)$*

(b) If $w < w_E$

$$\left(w + \frac{(\lambda)}{1+\lambda}\frac{G(w)}{g(w)}\right) v'(q_0 + \Delta q(w)) - \rho(w)\, v'(Q - q_0 - \Delta q(w)) \geq 0$$

$$(= 0 \text{ if } \Delta q(w) < 0)$$

(c) If $w > w_E$

$$\left(w - \frac{(\lambda)}{1+\lambda}\frac{1-G(w)}{g(w)}\right) v'(q_0 + \Delta q(w)) - \rho(w)\, v'(Q - q_0 + \Delta q(w)) \leq 0$$

$$(= 0 \text{ if } \Delta q(w) > 0)$$

The solution (P) summarized in Proposition 2 is illustrated in Figures 1 and 2 The most important feature of the solution is property (a) which addresses the crucial question of how to design initial rights to optimally distribute assets. The parties should initially assign rights coinciding with the optimal allocation for w_E ,the expected weight for I. The rationale for selecting $q_0 = q^*(w_E)$ is that it maximizes the excess surplus available after exchange to insure $U's$ participation. To see how this works, consider what happens to $\Phi(q_0 = q^*(w_E), \Delta q(w))$ as we change q_0 from it's optimal level. An increase in q_o changes

Φ at the rate of

$$\begin{aligned}\frac{d\Phi\left(q^*\left(w_E\right),\Delta q\left(w\right)\right)}{dq_0} &= E_w\rho\left(w\right)v'\left(Q-q^*\left(w_E\right)\right) \\ &-\int_a^{w_E}(w+\frac{G(w)}{g(w})v'(q^*\left(w_E\right)dG \\ &-\int_{w_E}^{b}(w-\frac{1-G(w)}{g(w})v'(q^*\left(w_E\right)dG\end{aligned}$$

where the first term indicates the expected marginal increase in $U's$ surplus and the second two terms represent the net surplus from $I's$ reallocation after netting out the addition in rent that I retains. After integrating by parts and collecting and simplifying terms we have,

$$\begin{aligned}\frac{d\Phi\left(q^*\left(w_E\right),\Delta\tilde{q}\left(w\right)\right)}{dq_0} &= \left[\rho w_E+\left(1-\rho\right)w_0\right]v'\left(Q-q^*\left(w_E\right)\right)-w_Ev'(q^*\left(w_E\right) \\ &\left\{\begin{array}{c}\leq \\ = \\ \geq\end{array}\right\}0 \text{ as } q^*\left(w_E\right)\left\{\begin{array}{c}=0 \\ \in(0,Q) \\ =Q\end{array}\right\}\end{aligned} \tag{2.4}$$

Equation (2.4) implies that $\Phi\left(q_0,\Delta\tilde{q}\left(w\right)\right)$ is maximized at $q^*\left(w_E\right)$. Either $q_0=q^*\left(w_E\right)$ is at an interior optimum where the marginal increase in net surplus is zero, or q_0 is at a corner where it is not possible to vary it so as to increase net surplus any further. This division reduces the constraints on the exchange of assets and thereby allows for the greatest surplus to be achieved.

The initial distribution $q^*\left(w_E\right)$ will depend on the marginal value of assets (the concavity of the surplus functions) and on the relative expected surplus weights. If assets are essential $\left(v'\left(0\right)=\infty\right)$ then $q^*\left(w_E\right)\in\left(0,Q\right)$ and each investor will be allocated some portion of the assets initially. $I's$ portion will increase the greater his relative weight. So if assets are more valuable to U on average, he'll be granted a larger initial share to begin with . Interestingly,

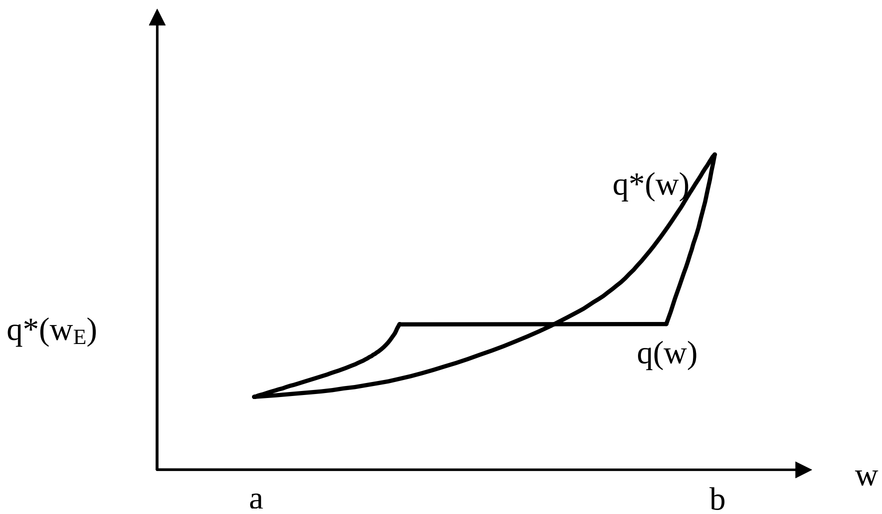

Figure 1: Asset Allocation

it's the relative importance of the assets, not which party is better informed that determines the initial property division between the parties.

Properties (b) and (c)describe how the final asset allocation differs from q_0. When $U's$ compensation constraint is not binding, the first best allocation is implemented with $\Delta q(w) = q^*(w) - q_o$. Figure 1 illustrates this possibility. The allocation begins with the optimal expected division, with I recommending adjustments from there to reach the efficient division $q^*(w)$ once he learns w.

The prospects for achieving the efficient allocation depend on how $I's$ and $U's$ preference co vary. It is generally possible to implement the first best asset allocation if ρ is small so that there is little interdependence in preferences . The gains from exchanging assets is greatest so that both investors can receive sufficient compensation to insure their participation. In extreme instances where $\rho = 0$ and there is no interdependence the initial assignment of assets is unimportant, as the first first best is always achieved. For instance when $q_0 = q^*(b)$

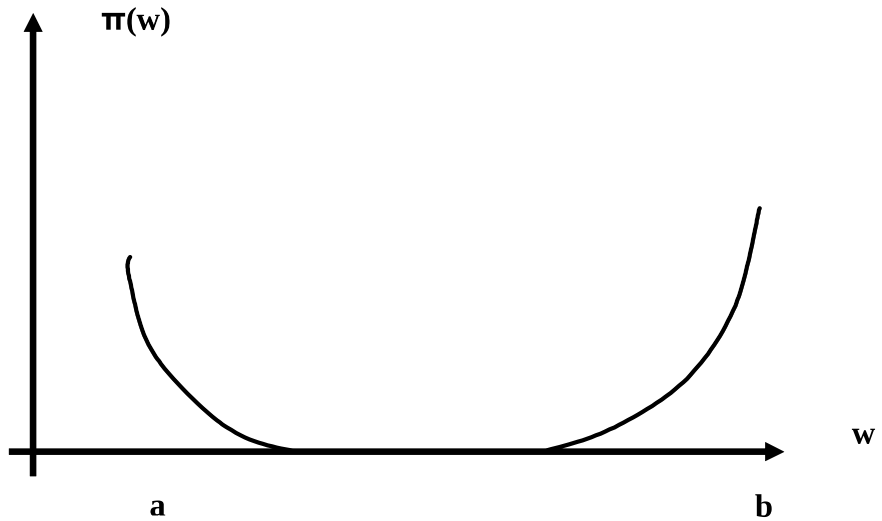

Figure 2: Profits for Informed Investor

there is sufficient surplus remaining after implementing $\Delta q^*(w)$ to compensate U.

$$\begin{aligned}
\Phi(q^*(b), \Delta q^*(w)) &= \int_a^b \left(\left(w + \frac{G}{g}\right) \Delta v^I (q^*(b), \Delta q^*(w)) + w^0 \Delta v^U (q^*(b), \Delta q^*(w))\right) dG \\
&= \left[G(w)\left[w\Delta v^I \ (q^*(b), \Delta q^*(w)) + w_0 \Delta v^U (q^*(b), \Delta q^*(w))\right]\right)\Big]_{w=a}^{w=b} \\
&\quad + \int_a^b (G(w) - G(w)) \Delta v^I (q^*(b), \Delta q^*(w)) \, dw \\
&= 0 \qquad (2.5)
\end{aligned}$$

The concavity of the surplus function is another factor affecting the parties' ability to reach efficient allocations. When surplus is linear in assets so that $v'(q) = 0$ we have the optimal $q_0 = 0$ provided $w_E \leq w_0$ and $q_0 = Q$ provided $w_E > w_0$ .It follows that if $\rho(w)$ is increasing in the linear surplus case that when $q_0 = 0$ we have

$$\begin{aligned}\Phi(0, \Delta q^*(w)) &= \int_a^{w_0} w - \rho(w) - \frac{(1-G(w))}{g(w)} dG \\ &= w_0(1-G(w_0)) - \int_{w_0}^{b} f(w)\, dG \\ &= \int_{w_0}^{b} (w_0 \;\; - \rho(w))\, dG < 0 \qquad (2.6)\end{aligned}$$

And similarly when $q_0 = Q$ we have

$$\begin{aligned}\Phi(Q, \Delta q^*(w)) &= w_0 \int_a^{w_0} \left(-w + \rho(w) - \frac{G(w)}{g(w)} \right) dG \\ &= w_0 G(w_0) - \int_a^{w_0} \rho(w)\, dG \\ &= \int_a^{w_0} (w_0 \;\; - \rho(w))\, dG < 0 \qquad (2.7)\end{aligned}$$

Consequently it is not possible to implement the first best. These observations are summarized in the following Corollary:

Corollary 1:

(ii) If $\rho = 0$ *it is possible to achieve the first best allocation*

(ii) If $\rho > 0$ *and surplus is linear in assets, it is not possible to achieve the first allocation of asset*s.

When the first best is not available it is because the asset allocation is constrained to generate enough surplus to insure participation of the U investor. In these cases, illustrated in Figures 1 and 2, properties (b) and (c) indicate that asset allocations are not adjusted for states w close to w_E. It is only for w sufficiently different from w_E that asset divisions are adjust adjusted towards their efficient levels. To understand this feature consider the incentives for I to recommend reallocations when w is less than w_E. I is tempted to exaggerate the value of the assets to obtain higher payment for transferring assets to U. Therefore reducing the amount I is sell to U when he claims a high value close w_E will discourage I from overstating his value. When I discovers that w is greater than w_E he has the opposite incentive to understate value to reduce the amount he must pay U to acquire more assets. Here it It is optimal to reduce the additional assets I receives to dissuade him from understating value. $I's$ ability to profit from private information about value is limited by reducing the adjustments in asset allocation he implements. The different amounts of rent that I earns decreases as $U's$ compensation constraint binds more as Figure 2 illustrates.

3 Implications

The proceeding analysis prescribes initial entitlements over assets to promote efficient allocations when one party learns about relative valuations. The design of initial rights places the parties in a bargaining position that minimizes the advantage of the informed investor to profit from his superior knowledge. The informed investor is delegated the responsibility of recomending changes in asset allocation to increase joint surplus in response to new information he observes. The control of assets is primarily vested with the party most likely to value them the most. Thus there is a dichotomy of initial entitlements with the right to propose changes vested with the investor who is most knowledgeable and the predominant ownership of assets vested with the party most likely to benefit from their use.

Our findings have significant implications for designing commercial transactions and for protecting intellectual property and assigning liability in nuisance settings. Regarding commercial and market transactions our analysis implies that the initial consignment of assets should primarily reside with the party most likely to use them in subsequent transactions, whereas decisions to adjust inventories in response to market demand should reside with the party (most likely the retailer) who is best able to gauge the product demand in regional markets. The power to adjust allocations should rest with the party who is most knowledgeable about market conditions, whereas the initial assignment of inventories for sale should reside with the either the manufacturer or retailer depending on who is most likely to value the product more highly.

With regards to assigning intellectual property protection, our analysis implies that initial rights resembling a mandatory license or a negotiable liability rule may be desirable. The inventor should retain control rights in settings where he is best informed about the product's value in different uses. The initial rights to use the property should be determined on the basis of which party is most likely to benefit from its use. The most knowledgeable investor and the most likely benefactor of a new discovery needn't not coincide. Rights to employ a new invention and rights to control its use might reside with different individuals.

Our findings suggest that an inventor might be obliged to make some portion of his assets available initially to follow on improvers or entrepreneurs who develop new technologies for the market. Once the relative value of the novel technology is discovered, the inventor might decide to license additional portions of his technology to others or alternatively he might decide to buy out investors holding initial rights in order to increase his holding of a valuable asset.

Our entitlement system gives the informed investor an option to buy or sell initial rights once the relative values of the asset are revealed. When the inventor is informed this resembles a mandatory license committing the inventor to make some portion of his discovery available to subsequent users, unless he decides to reduce or to increase his control of assets conditional on the new information he obtains. When the informed investor is a follow on user, the option resembles a liability rule. The user is obliged to compensate the inventor for his use of the property, but he may modify his use provisions either by expanding or contracting his claims provided he provides adequate compensation to the inventor.