

Contests with Thresholds

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Abstract

Many important economics problems can be modeled as a contest. The contest literature includes among others Loury (1979), Tullock (1980), Lazear and Rosen (1981), Rosen (1981, 1986), Dixit (1987, 1999), Baik and Shogren (1992), Baye and Shin (1999), and Moldovanu and Sela (2001).

In this paper, we consider n -player contest (a 'la Tullock (1980)) where the contest designer imposes an aggregate threshold level. Players have commonly known budget constraints and (can) value the prize differently. If the total contribution of all n players does not match the total threshold level, the contest designer has a positive probability to keep the prize.

We analyze three cases: (1) the threshold level is not met; (2) the threshold level is met and budget constraints are not bidding; (3) the threshold level is met and some budget constraints are bidding.

We show that in the case (1), each player who values the prize above the threshold level, but does not have enough resources to match the threshold level spends all her resources. Even though this result looks obvious, it has interesting applications. As we all know, budget constraints can play a crucial role in economic transactions. The suggested mechanism allows a transaction between a seller and potential buyers even if (all) buyers (together) do not have enough resources to match the seller's reserve price. For example, museums can consider this mechanism – a contest (a *lottery* for expensive items) with the threshold, in order to raise funding. We demonstrate how the optimal threshold level should be selected, if the designer knows buyers' budget constraints and the distribution function of their private valuations for the prize.

We derive the equilibrium spending in the general case (2): n players, different private valuations, no budget constraints. Well known results of the equilibrium spending if all prizes are the same and/or if two players have different values are corollary of our result.

We describe the equilibrium spending if some budget constraints are bidding in case (3) and therefore characterize completely the players' equilibrium behavior in the contest with the threshold.

Our model has natural connections with lotteries. However, this is the first attempt to introduce the (soft) threshold level: the threshold level might not be met in which case the designer has a positive probability to keep the prize. Moreover, players can spend more than the threshold level in which case one player gets the prize for sure.

Our paper is also related to all-pay auctions. However, our mechanism works even if "the reserve price" (the threshold level) is not met.

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