Combining Expert Opinions*

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First draft: July 2002
This version: May 2004

Abstract

I analyze a model of advice with two perfectly informed experts and one decision maker. The bias of an expert is her private information. I show that consulting two experts is better than consulting just one. In the simple "peer review" mechanism, the decision maker receives just one report, and the second expert decides whether to block the first expert's report. A more rigid peer review process improves information transmission. Simultaneous consultation transmits information better than sequential consultation and peer review. However, peer review achieves significant information transmission, with the decision maker receiving only one report. There is an asymmetric equilibrium that is more efficient than the symmetric equilibrium. When given the chance to discover biases of experts, the decision maker may prefer not to do so.

*I thank my advisor, Larry Samuelson, for helpful discussions and continuous support. I am also grateful to Bill Sandholm for very useful suggestions. Thomas Gilligan, Eric Rasmusen, and seminar participants at Indiana University (BEPP), University of Wisconsin, and WEA Meeting in Taipei have provided helpful comments. Any remaining mistakes are my own.

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I. Introduction

People specialized in decision making rarely have specialized knowledge about things on which they make decisions. Either they lack the necessary talent to acquire the knowledge, or the opportunity costs for acquiring it are too high. They usually turn to experts for advice. This naturally brings about the issue of objectivity and credibility of advice from the experts. As well said by Samuel Butler, "opinions have vested interests just as men have." Experts have more specialized knowledge, but their preferences on issues may well be different from those of the decision maker.

There are ample cases in which this should be a real concern. If an investor asks an analyst about the stock of a company, the analyst tends to make suggestions that favor herself, depending on her positions in the stock and business relations to the company.[1] If the US President asks his cabinet members on policies about world affairs, they are likely to recommend policies that favor their own interests. If a CEO elicits advice on compensation schemes, each employee is likely to favor schemes that benefits her most.

A common way believed to potentially alleviate the problem is to introduce counterbalancing forces. For example, decision makers may seek second opinions. By asking multiple experts, it is conceivable that decision makers may be able to hear more diversified opinions, so as to get a better picture of the underlying situation and avoid blunders in the final decision.

However, there are difficulties facing a decision maker. An expert takes the presence of other experts into strategic consideration when providing advice. The decision maker must first listen to these experts, disentangle the strategic elements in the experts' advice, and choose the best action. As an outsider to experts' specialized field, the decision maker may not know the actual preferences of the experts. A second problem is that interactions among experts are complicated. Experts may strengthen, rebut, and fine tune each other's reports depending on their respective biases. The decision maker may not have the time or energy to listen to and handle all the pros and cons on an issue.

Thus, it is important to find the kind of communication mechanisms the decision maker should use to maximize information transmission efficiency given time and financial constraints.

In the business organization environment, lower level managers and employees have better information about consumer preferences, production technologies, de-

[1] In this paper, I refer the decision maker as "he," and each expert "she."

mand for staff, etc. A CEO has to choose a mechanism to combine opinions from employees. He does not necessarily know their preferences. The reason is two-fold: on the one hand, different individuals may have different biases, which are hard for top management to track; on the other hand, the issues to be dealt with are varied, people's biases may change depending on the issue at hand. If one views communication mechanism as inflexible, then it is important to have a mechanism that optimizes communication allowing uncertainty about experts' biases.

This paper studies efficiency of information transmission between experts and the decision maker. I compare various communication mechanisms on their ability to transmit information. Information transmission efficiency can be measured by the decision maker's expected utility. It turns out for the class of preferences I consider, before knowing the state, any expert ranks equilibria the same as the decision maker does.

In my model, there are *two* perfectly informed experts and a single decision maker. The experts may have biases on what decision to take, and their biases are unknown to both the decision maker and the other expert.

It is always an option for the decision maker to ask just one expert or ignore reports from one of the experts. On the other hand, when using information from both experts, he has various options.

One mechanism he can choose is to ask each expert *simultaneously.* Experts are aware of the existence of another expert, but they do not observe each other's report when making their own reports. Another option for the decision maker is to ask two experts *sequentially.* The first expert makes a report, and the second expert gets to observe it and then offer her own report. Hearing both reports, the decision maker makes a decision. I call these two mechanisms direct consultation mechanisms since the decision maker hears both experts' reports. I also consider a mechanism called *peer review.*

Peer review. By "peer review," I refer to mechanisms in which some experts do not report their own opinion, but decide whether they want to pass on other experts' opinion.

Consider the two-expert model. The decision maker asks the first expert to make a report, and let the second expert, the reviewer, decide whether she wants to reject it or not. If she accepts it, then the decision maker hears the original report. If she rejects it, however, then with some probability, the message fails to reach the decision maker. I interpret this probability as the rigidity of the peer review process. When the first expert's report is rejected, the decision maker receives a signal drawn from the random distribution endogenously generated by interactions of experts.

It is important to note that in the peer review mechanism the decision maker only receives one final report, as opposed to two reports in the direct consultation mechanisms. This mechanism is simpler for the decision maker and presumably less costly, since he receives and handles just one report. In some sense, he delegates part of the information solicitation task to the reviewer. In contrast, the direct consultation mechanisms generate two reports.

The consideration of the peer review mechanism is of interest since it captures some of the features of many communication and organization structures in reality. CEO's do not directly listen to reports of lower level employees. Lower level employees make reports to the middle level management, and what the executives get is a selective pool of reports that are filtered by middle level management. Similarly, the US president does not directly listen to field personnel, but learn information through his advisors.

Findings. Before an expert knows the underlying state, she has the same preference ordering over different equilibria as the decision maker. So communication effectiveness can be compared across different equilibria.

Under certain assumptions about the distribution of biases, having a more rigid peer review process improves communication if we consider symmetric equilibria. In symmetric equilibria, truthful reports are never rejected by reviewers, so the only effect peer review has is to deter the experts from lying and to keep false reports from reaching the decision maker when they do lie.[2]

Consulting two experts is always better than consulting just one. Consulting two experts simultaneously does the best among all three mechanisms. When comparing peer review with consulting two experts sequentially, there exist symmetric equilibria in which sequential consultation does better than peer review. However, there do exist equilibria in which the decision maker is worse off than under peer review.

Although the setup of the model is symmetric, there also always are asymmetric equilibria in the peer review case. In some asymmetric equilibria, the decision maker is better off than he is in symmetric equilibria.

Given the chance to discover what the biases of experts are, the decision maker may prefer not to do so.

The paper proceeds as follows. Section II introduces the basics of the model,

[2]Since the decision taken by the decision maker does not correspond exactly to the label of messages, I use the words "lie" and "misrepresentation" to mean any report by biased-experts that is different from what an unbiased expert would send.

describes communication mechanisms, and defines equilibria. Section III characterizes symmetric equilibria of the various communication mechanisms. Section IV compares the benefits and shortcomings of the three different mechanisms. Section V discusses extensions of the basic model. Section VI summarizes related literature. Section VII concludes and suggests further research.

II. The Model

I study a model with a decision maker and two experts. The decision maker takes an action that affects payoffs of both the decision maker and the experts. The decision maker's objective is to maximize the welfare of a society or an organization. The best action for him to take depends on some underlying state. The decision maker does not know the state, but experts do. Experts' preferences are not necessarily the same as those of the decision maker. Their preferences are unknown to the decision maker and to each other.

Let the set of feasible actions be the real numbers $\mathbf{R}$. There are three possible states of the world. The state space is $S = \{-1, 0, 1\}$. The best decision for the decision maker in state $s \in S$ is equal to s. Each state happens with probability $\frac{1}{3}$. There are also three types of experts from the set $X = \{-1, 0, 1\}$. So there exist "neutral" experts and experts with opinions to the two extremes. The prior distribution of experts' types is uniform on the three possible types. The state are both experts' biases are independent of one another. An expert of type $x \in X$ has bias

$$b_x = bx,$$

where $b \in (\frac{1}{2}, 1)$.[3] The meaning of "bias" will be made clear below. Experts do not know each other's biases a priori. Neither does the decision maker know the biases of the experts.[4]

[3] There exists a fully revealing equilibrium for all communication mechanisms above when $b \leq \frac{1}{2}$, in which every expert tells the truth and the decision maker takes the corresponding actions.

[4] One may argue that a decision maker should be able to infer biases of experts if he interacts often with experts. It is conceivable that a decision maker does not know their biases when their interactions start. How decision makers learns biases of experts is best modelled in a repeated game, such as in Morris (2001). As Morris shows, however, an expert may want to conceal his type, in order to get future benefits from the decision maker's decision. Another force working against learning by the decision maker is that he faces a variety of issues and a pool of different experts over time. Furthermore, as shown in Section V, the decision maker may prefer not to learn experts' biases even if given the chance to do it at no costs.

Both the experts and the decision maker have utility functions of the following form

$$u(y, s, \tilde{b}) = -(y - (s + \tilde{b}))^2. \quad (1)$$

where y is the action taken by the decision maker, s is the true state, and $\tilde{b}$ is the bias of the agent. For the decision maker $\tilde{b} = 0$, and for an expert of type $x \in X$, $\tilde{b} = b_x$. This utility function form is the same as the special case introduced by Crawford and Sobel (1982). It has the convenient property that for any agent with bias $\tilde{b}$, the ideal action is $s + \tilde{b}$. When there is uncertainty about the state, the most preferred action is always $E(s) + \tilde{b}$, the expected value of the state plus the bias. An expert with a positive bias always prefers the decision maker to take an action greater than the true state, and vice versa.

Using the terminology of Krishna and Morgan (2001b), $Y = [-1, 1]$ is the set of "rationalizable" decisions for the decision maker.[5] Except taking an action according to the report(s) he receives, the decision maker cannot commit to decision rules using any other device. In particular, he cannot use monetary transfers that are correlated with reports.

The assumption that the decision maker has to use communication mechanisms instead of committing to other decision rules is also significant. If he could use other commitment devices, he would let the expert and the reviewer report the underlying state, and give them prohibitive punishments for inconsistent reports (the "shoot them all" mechanism). This would guarantee full revelation to be an equilibrium, assuming that experts cannot collude with each other.

The decision maker asks for advice from experts. He can choose to ask just one expert. In the case where the decision maker asks both experts, he may choose among various mechanisms. In this paper, I consider the following three: simultaneous consultation, sequential consultation, and peer review.

Simultaneous Consultation

In this mechanism, two experts make simultaneous reports to the decision maker. They are aware of the existence of another expert, but they do not know the bias of the other expert, or what the other expert reports. Based on the two reports, the decision maker makes his decision. This is called *simultaneous consultation.*

Each expert is allowed to send a signal from a message space M. For clarity of notation, I label the experts A and B. Expert i's report is denoted m^i, $i = A, B$.

[5]Precisely, an action is *rationalizable* if it is an optimal action for some belief by the decision maker about the underlying state.

Define the strategy of expert i of type x as

$$m_x^i : S \to M, \ i = A, B \text{ and } x \in X.$$

In other words, $m_x^i(s)$ is what expert i of bias x would report if the state is $s \in S$. The decision maker's strategy is defined as

$$y : M \times M \to [-1, 1],$$

where $y(m^A, m^B)$ is the action taken by the decision maker when he receives the message pair (m^A, m^B).

Sequential Consultation

The decision maker asks both experts, but one is consulted first. The first expert makes a report, which is observed by the second expert. The second expert makes her report based on both the underlying state and the first expert's report. The decision maker hears *both* reports and bases his action on these two reports. This is called *sequential consultation.*

Similar to the notation for simultaneous consultation, I call the first expert "Expert A" and the second expert "Expert B." With a slight abuse of notation (since m_x^B has already been used above as a function with a single argument), their strategies can be defined respectively as

$$m_x^A : S \to M,$$
$$m_x^B : S \times S \to M.$$

For A, $m_x^A(s)$ is the report sent by an expert of type x when the true state is s; for B, $m_x^B(s, t)$ is the report sent by an expert of type x when the true state is s and expert A has reported t. The decision maker's strategy is

$$y : M \times M \to [-1, 1],$$

where $y(m^A, m^B)$ represents the action taken when the reports are respectively m^A and m^B.

Peer Review

In the *peer review* mechanism, one expert provides advice to a decision maker by making statements about the underlying state. However, the advice must pass through a reviewer (who is another expert) before reaching the decision maker. The reviewer decides whether to reject the advice or to accept it and pass it on to the decision maker. When she rejects it, she may or may not be able to successfully block the report from reaching the decision maker. For example, the decision maker may commit to investigating the original report with a certain probability. I interpret the success rate as the rigidity of the peer review process. When the reviewer is successful at blocking the report, the decision maker receives a random report coming from the endogenous distribution of signals eventually getting through to the decision maker. This distribution is generated by interactions between the experts in the model. The decision maker cannot distinguish whether the message he receives is an original report or a random report drawn after a rejection. He takes an action based on the message he eventually receives.

More explanation of this mechanism is in order. The equilibrium of this mechanism can be viewed as the steady state of a dynamic process. Experts make reports based on their knowledge of the state and biases, and reviewers decide whether to reject the report based on their knowledge of the state and biases. When a certain report is rejected and successfully blocked, the decision maker is more likely to receive other reports that are already "out there". On the other hand, the distribution of messages that are "out there" is a result of interactions between experts and reviewers.

The "peer review" mechanism is in some sense similar in spirit to mechanisms involving "veto power" in the literature. See Gilligan and Krehbiel (1987), Krishna and Morgan (2001a), and Dessein (2002) for details. In these mechanisms, the decision maker retains the power to "do nothing," that is, maintains the status quo. Results typically depend on the exogenously given value of the status quo. If one wants to evaluate the overall efficiency of a mechanism with veto power, a distribution over the status quo should be specified. The distribution is best modelled as endogenously generated by the mechanism itself. The peer review mechanism studied in this paper can be viewed as an attempt to achieve this goal. When the reviewer successfully blocks what the expert reports, the decision maker receives a random message, the distribution of which is endogenously generated.

Since the role of reviewers is performed by fellow experts, there are also the same three types of reviewers in X. Again, the expert and the reviewer do not know each other's types.[6]

[6] The significant assumption here is that the expert does not know the reviewer's type. In fact even if the reviewer does know the expert's type, the equilibria below remain so if we let the reviewer's equilibrium strategy be independent of the expert's type. However if the expert knows the reviewer's type, her incentive structure will change, which will affect her decision. The assumption that the

It is important to note that the decision maker does *not* observe the expert's report or the reviewer's decision. This is one major difference between this mechanism and the two mechanisms above. Thus he cannot make decisions based on the reviewer's rejection or acceptance decisions. The role of the second player is also different. Rather than making her own report, she affects the decision maker's decision by influencing what the decision maker receives.

Let $\alpha \in [0, 1]$ be the rigidity of the peer review process. Denote by Γ the distribution of messages eventually received by the decision maker in equilibrium. It will be shown that there are only a finite number of messages sent in equilibrium. So I use γ_m to indicate the probability of message $m \in M$ being received. If the expert's advice gets *rejected*, then with probability α the decision maker receives a message randomly drawn from the endogenous distribution Γ, and with probability $1 - \alpha$ the original advice gets through to the decision maker. When $\alpha = 1$, when the expert's advice is rejected it never gets to the decision maker. When $\alpha = 0$, the decision maker receives the advice of the expert, even if the reviewer rejects it. As α gets higher, it is increasingly hard to evade the peer review process.

Define the expert's strategy as

$$m_x : S \to M,$$

where $m_x(s)$ is the message sent by an expert of type $x \in X$ when the state is s. Let $\tilde{m}_x(s, t)$ indicate the probability with which an expert of type x sends message $t \in M$ when the true state is s. That is, if $m_x(s) = t$ then $\tilde{m}_x(s, t) = 1$, otherwise $\tilde{m}_x(s, t) = 0$ if we consider only pure strategies. Since the reviewer does not know the expert's type, her strategy can only be based on the true state and the message she receives from the expert. Define her strategy as

$$r_v : S \times M \to \{0, 1\},$$

where $r_v(s, t)$ indicates whether a reviewer of type v rejects message t when the state is s. The number 1 means rejection, and 0 means acceptance. The decision maker bases his decision on the message he receives as a result of the expert's advice and the reviewer's decision. The decision maker's strategy is thus defined as

$$y : M \to [-1, 1].$$

For convenience denote by y_m the action taken by the decision maker after hearing message $m \in M$.

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expert does not know the reviewer's type best fits situations of anonymous review or situations in which the expert does not interact frequently with the same reviewer. One may also want to consider the case that the expert *does* know the type of the reviewer

Equilibrium

I consider only pure strategy equilibria. An *equilibrium* is a strategy profile that satisfies the following conditions where they apply:

(EQ1) An expert of any type $x \in X$ sends the message that maximizes her expected utility in any state $s \in S$, given strategies of the other expert, the decision maker, and the distribution Γ.

(EQ2) Peer review: A reviewer of any type $v \in X$ rejects a message t in state s, i.e. $r_v^*(s,t) = 1$ if and only if rejection gives her higher expected utility than acceptance,[7] given the decision maker's strategy, and the distribution Γ. That is, $u(y_t, s, v) < \alpha u(y_t, s, v) + (1-\alpha)\sum_{t' \in M} \gamma_{t'} u(y_{t'}, s, v)$, which is equivalent to

$$u(y_t, s, v) < \sum_{t' \in M} \gamma_{t'} u(y_{t'}, s, v).$$

(EQ3) The decision maker takes action $y(m) = E(s|m)$ when receiving message(s) m, where m could either be a scalar or a two-dimensional vector.

(EQ4) Peer review: the distribution of messages generated from interactions between experts and reviewers is the same as the distribution from which a random message is drawn when a report is rejected and successfully blocked. Formally, this can be written as

$$\begin{aligned} \gamma_t \;=\; & \textstyle\sum_{s \in S} P_s \sum_{x \in X} P_x \tilde{m}_x(s,t) \sum_{v \in X} P_v [1 - r_v(s,t) + r_v(s,t)(1-\alpha)] \\ & + \textstyle\sum_{s \in S} P_s \sum_{x \in X} P_x \sum_{t' \in M} \tilde{m}_x(s,t') \sum_{v \in X} P_v r_v(s,t') \cdot \alpha \gamma_t. \end{aligned}$$

In the above equation P_s, P_x, and P_v stand for the probabilities of state s, type x expert, and type v reviewer occurring respectively. They are all equal to $\frac{1}{3}$ in this model. The first part is the probability of the event that experts report the message t and it gets through to the decision maker, and the second part is the probability of the event that experts report any message, the message gets rejected, and the randomly drawn message is t.

As in all cheap talk models, two issues arise. First, the meaning of messages. Since the decision space is $Y = [-1, 1]$, it is without loss of generality that I limit the message space M to $[-1, 1]$. The message space is rich enough for experts to reveal

[7] This requirement involves the idea of sequential rationality. Even if in some states a message is never sent in equilibrium, the reviewer still needs to compare the utilities from rejection and acceptance.

all their private information. I make the following assumption to reduce essentially identical equilibria into one.[8]

Monotonicity. The decision maker's strategy must be increasing in the message(s) he receives. The experts' reports should be increasing in the state and their biases.

The idea is that messages should have its natural meaning, that is, a high message should more likely indicate a higher state than a low message. A right-biased expert should be more likely to make a right-biased report than other types of experts. Any expert should be more likely to report a state to be high when it is indeed high.

Second, multiplicity of equilibria. In particular, a babbling equilibrium always exists. Following previous applications of the cheap talk model (Krishna and Morgan (2001b), Morris (2001)), I will focus on *informative* equilibria, meaning that the decision maker's equilibrium strategy satisfies the following assumption.

Informativeness. For each of its arguments, $y(m)$ strictly varies with it some of the time, unless one of the messages reveals the underlying state.

This prevents an expert's report from being completely uninformative whenever there is information to be revealed. For example, this restriction rules out the case in which $y(m^A, m^B) = f(m^B)$ for all $m^A \in M$ and some $m^B \in M$ as long as m^B is not exclusively sent in a single state.

In general, I consider the most informative equilibrium. For papers studying refinements of cheap talk equilibria, one may refer to Farrell (1993), Farrell and Rabin (1996), and Matthews, Okuno-Fujiwara, and Postlewaite (1991). The second paper provides a concise survey.

Before characterizing the equilibria, I look at whether full revelation is possible in equilibrium. Take the peer review mechanism, for example. Full revelation means that $y = s$ for every $s \in S$. Given that rejection results in messages being sent that are inconsistent with the underlying state, there cannot be any rejection on the equilibrium path. Thus full revelation could only happen if every expert tells the truth about the underlying state and truthful reports are always accepted. Thus the

[8]This assumption does not pose additional restrictions for simultaneous consultation and peer review. However it does for sequential consultation, but it is unlikely that equilibria that are ruled out by monotonicity are more informative than those that do satisfy monotonicity.

decision maker should take action $y = m$ when receiving message m. Also observe that in such cases the message $m = 0$ from an expert should always be accepted, because by rejecting 0 the reviewer will with probability α let the decision maker receive messages $-1, 0, 1$ with equal probabilities, which is the endogenous distribution of signals generated by the truth-telling strategy profile. The utility the reviewer receives is thus lower than that from acceptance regardless of the type of the reviewer, which is an implication of the quadratic loss utility function. Given this, reporting 0 when the state is -1 gives a type 1 expert higher payoff than reporting -1 because $b > \frac{1}{2}$. Thus I have the following result.

Proposition 1. *There does not exist a fully revealing equilibrium for any of the above mechanisms.*

Proof. See Appendix for the proof for peer review. Proofs for simultaneous consultation and sequential consultation are omitted. In the following sections, all the pure strategy symmetric equilibria will be characterized for these two mechanisms. None of the equilibria fully reveals information. □

Proposition 1 is a typical result of cheap talk models. It illustrates the difficulty of communication between informed experts and the uninformed decision maker when their objectives are not perfectly aligned with each other.

III. Symmetric Equilibrium

A pure strategy profile $(\hat{m}, (\hat{r},)\hat{y})$ is a "mirror image" of another strategy profile $(m, (r,)y)$ if for all $i = A, B$, $x, v \in X$, $s, t \in S$, and $m \in S$ or $S \times S$, the following conditions are satisfied where they apply:

(SE1) Simultaneous consultation: $m^i_x(s) = -\hat{m}^i_{-x}(-s)$;
Sequential consultation: $m^A_x(s) = -\hat{m}^A_{-x}(-s)$;
Peer review or consulting one expert: $m_x(s) = -\hat{m}_{-x}(-s)$.

(SE2) Sequential consultation: $m^B_x(s,t) = -\hat{m}^B_{-x}(-s,-t)$;
Peer review: $r_v(s,t) = \hat{r}_{-v}(-s,-t)$.

(SE3) $\hat{y}(m) = -y(-m)$.

(SE4) Peer review: $\gamma_t = \hat{\gamma}_{-t}$ for all $t \in M$.

An equilibrium is *symmetric* if and only if the equilibrium strategy profile is a mirror image of itself.

In fact, to define a symmetric equilibrium, conditions (SE3) and (SE4) are redundant as they follow from the equilibrium conditions (EQ3) and (EQ4) and symmetry conditions (SE1) and (SE2). Intuitively, in a symmetric equilibrium experts and reviewers of type 1 and -1 behave in a similar way. For example, if a reviewer of type 1 rejects report 0 when the true state is 1, then a reviewer of type -1 must reject report 0 when the true state is -1. Also experts and reviewers of type 0 treat state values -1 and 1 and report values -1 and 1 in a similar way. Note that given the above three conditions, there are certain facts that have to be true in a symmetric equilibrium. For example, $y(0,0) = y_0 = 0$, $m_0(0) = m_0^A(0) = 0$, etc. Furthermore, when I make descriptions or prove facts about symmetric equilibria, I need only consider behavior of experts (and reviewers) of types 0 and 1.

The crucial aspects of symmetric equilibria are that the resulting distribution of actions is symmetric and that experts' behavior regarding the messages that correspond to these actions are symmetric. The labelling of messages is not crucial. For example, one can relabel the message "0" to "0.01." This would violate the symmetry requirements but would not alter the essential properties of the equilibrium. Putting these extra symmetry restrictions on equilibria, however, would not reduce the number of essentially distinct symmetric equilibria precisely because one can rename "0.01" back to "0" to satisfy these restrictions.

Consulting One Expert

First I establish a result on the number of messages sent in equilibrium.

Lemma 1. *When the decision maker consults one expert,*

1. *If* $b \in (\frac{1}{2}, 1)$*, then no more than* 5 *messages are sent in pure strategy symmetric equilibria;*

2. *If* $b \in (\frac{3}{4}, 1)$*, then no more than* 3 *messages are sent in pure strategy symmetric equilibria.*

Proof. Symmetry requires that the message 0 is always sent in equilibrium by experts of type 0 in state 0. In state 1 experts of types 0 and 1 both want to send the message that induces the highest possible action from the decision maker in equilibrium since their most preferred action in $[-1, 1]$ is 1. For simplicity I call this message 1.

Similarly in state -1 experts of types 0 and -1 both want to send the message -1, which induces the lowest possible action from the decision maker in equilibrium.

Now the only possibility of an expert sending a positive message $m_{1,0}$ other than the highest message is that $m_{-1}(1)$ or $m_1(0)$ is equal to $m_{1,0}$. In fact it must be that both $m_{-1}(1)$ and $m_1(0)$ are equal to $m_{1,0}$. Otherwise this message would induce an action of 0 or 1, a contradiction. Thus $y(m_{1,0}) = \frac{1}{2}$. In order for $m_{-1}(1) = m_1(0) = m_{1,0}$ to be optimal we must have $b \leq \frac{3}{4}$. Otherwise an expert of type 1 would prefer 1 instead in state 0.

To summarize, there are no more than 5 messages in pure strategy symmetric equilibria. When $b > \frac{3}{4}$, there are no more than 3 messages. □

For the discussion of the rest of the section, I restrict the range of b, which gives the most salient set of results for the mechanisms.

Range of bias. $b \in [\frac{17}{21}, \frac{6}{7}]$.

Note that b is greater than $\frac{3}{4}$. Thus there are only 3 messages in equilibrium. More general values of b will be discussed in a later section.

Proposition 2. *When the decision maker consults only one expert, the only symmetric equilibrium is as follows:*
1) $m_0^*(s) = s$, $m_{-1}^*(s) = s - 1$ *if* $s \neq -1$, $m_{-1}^*(-1) = -1$, $m_1^*(s) = s + 1$ *if* $s \neq 1$, *and* $m_1^*(1) = 1$*;*
2) $y_m^* = (2/3)m$.
In equilibrium, the decision maker's expected payoff is $-10/27$.

Proof. See Appendix. □

In equilibrium, biased experts always misrepresent the state when possible. That is, an expert of type 1 reports state -1 as 0 and 0 as 1. They are able to do so since there are no forces to counteract or punish biased reports. In a sense, this is the worst that could happen to the decision maker in an informative equilibrium. It is imaginable that by introducing another expert, the situation can be improved.

When studying two-expert mechanisms, each expert is allowed to choose from a set of three messages. It can be shown that this does not pose constraints for these mechanisms given my assumption that $b \in [\frac{17}{21}, \frac{6}{7}]$. In particular, the experts can send messages in the set S.

Simultaneous Consultation

In this mechanism, each expert simultaneously sends a report to the decision maker. In addition to the symmetry conditions above, I add another symmetry condition.

Anonymity. $m_x^A(s) = m_x^B(s)$ for all $x \in X$ and $s \in S$.

The idea behind this condition is that an expert's reports are not affected by her labelling, but only by the underlying state and her bias. This captures the "simultaneity" nature of the mechanism. As a result, in equilibrium, the decision maker does not discriminate according to the source of messages. His decision is based only on the combination of message pairs, but not who provides which message.

Symmetry and anonymity put certain restrictions on the decision maker's strategies, which are straightforward to derive. These properties include $y(0,0) = 0$, $y(-1,1) = y(1,-1) = 0$, $y(m^A, m^B) = y(m^B, m^A) = -y(-m^A, -m^B)$ for all $m^A, m^B \in S$, etc.

The following lemma describes experts' equilibrium behavior.

Lemma 2. *When the decision maker consults two experts simultaneously, in equilibrium, the following must be true about experts' strategies: for $i = A, B$, $m_0^i(s) = m_s^i(s) = s$ for $s = -1$ and 1, and $m_0^i(0) = 0$.*

Proof. See Appendix. □

Intuitively, unbiased experts never try to distort information. A biased expert tells the truth about the state when the state is at the extreme in the same direction as the bias of the expert. There is no way for her to further shift the decision maker's decision in her favor, and she doe not want to report the other two messages that are unfavorable.

Since I only look for symmetric equilibria, when describing strategy profiles I only state what is necessary in uniquely identifying the strategy profile and leave the rest for deduction with the symmetry conditions. Now we define strategy profile (A):

(A1) For $i = A, B$, $m_0^i(s) = s$ for all $s \in S$, and $m_1^i(s) = s$ for $s = 1$ and $s + 1$ for $s = -1, 0$;

(A2) $y(0,0) = 0$, $y(0,1) = y(1,0) = 2/3$, $y(1,-1) = 0$, and $y(1,1) = 4/5$.

Proposition 3. *In the simultaneous consultation game, strategy profile (A) is the only pure strategy symmetric equilibrium satisfying anonymity. In this equilibrium, the decision maker's expected payoff is $-94/405$.*

Proof. See Appendix. □

Strategy profile (A) looks much like the equilibrium of the one-expert case, except that now two experts are present. Biased reports are sometimes balanced by the other expert of a different bias. For example, although a right-biased expert would still report state -1 as 0, it is offset by the other expert when the other expert is of bias -1 or 0, which happens with probability 2/3. When the decision maker receives the message pair $(0, -1)$ or $(-1, 0)$, he takes the action $-2/3$. On the other hand, in the one-expert mechanism, he takes action 0 when he receives the message 0, which is farther from his most preferred action -1 in state -1.

Sequential Consultation

In the sequential consultation mechanism, the second expert gets to observe what the first expert has reported. Thus the second expert's strategy depends on both the underlying state, and the first expert's report.

Now I solve for the symmetric equilibria of the sequential consultation game that satisfy the *informativeness* and *monotonicity* conditions. First I establish a lemma describing the experts' behavior in such equilibria.

Lemma 3. *When the decision maker consults two experts sequentially, the following must be true about experts' strategies in equilibrium:*

1. $m_0^A(1) = m_1^A(1) = 1$, *and* $m_0^A(0) = 0$;
2. $m_0^B(1, m^A), m_1^B(1, m^A) \in \arg\max_{m^B} y(m^A, m^B)$;
3. $m_0^B(0, 0) = 0$, $m_1^B(0, -1) \in \arg\max_{m^B} y(-1, m^B)$, *and* $m_1^B(0, 0) \in \arg\max_{m^B} y(0, m^B)$.

Proof. See Appendix. □

Monotonicity and informativeness require that the first expert reports 1 in state 1 if she is of type 1. If she does not, then monotonicity implies $m_x(s) = 0$ for all $x \in X$, which renders the first expert's reports uninformative, violating informativeness. Symmetry requires that the first expert reports 0 in state 0 when she is of type 0. When the true state is 1 and the expert is of type 1 or 0, the expert wants the action taken by the decision maker to be as close to 1 as possible, which translates into as large an action as possible since the decision maker's action must lie in $[-1, 1]$ to be rationalizable. The other results in the lemma follow similar lines of argument.

Let us define strategy profiles (B), (C), and (D). Strategy profile (B) is defined by

(B1) $m_0^A(s) = s$ for all $s \in S$, and $m_1^A(s) = s$ for $s = 1$ and $s + 1$ for $s = -1, 0$;

(B2) $m_1^B(-1, 0) = 0$, $m_0^B(s, 0) = s$ for $s \in S$, $m_1^B(1, 0) = 1$ for all $t \in S$, and expert B's other strategies do not matter as long as they make (B3) hold;

(B3) $y(0, 0) = 0$, $y(0, 1) = 2/3$, and $y(1, m^B) = \frac{2}{3}$ for all $m^B \in S$.

Strategy profile (C) is defined by

(C1) $m_0^A(s) = s$ for all $s \in S$, and $m_1^A(s) = s$ for $s = 1$ and $s + 1$ for $s = -1, 0$;

(C2) $m_1^B(-1, -1) \in \arg\max_{m^B} y(-1, m^B)$, $m_1^B(-1, 0) = 0$, $m_0^B(0, 0) = 0$, $m_0^B(0, -1), m_1^B(0, -1) \in \arg\max_{m^B} y(-1, m^B)$, $m_1^B(0, 0) = 1$, $m_0^B(1, 0) = m_1^B(1, 0) = 1$, and $m_0^B(1, t), m_1^B(1, t) \in \arg\max_{m^B} y(t, m^B)$ for $t = -1, 1$; furthermore, they must also ensure that (C3) hold;

(C3) $y(0, 0) = 0$, $y(0, 1) = 2/3$, $y(1, -1) = \frac{1}{2}$, $y(1, 0) \in [\frac{1}{2}, 2b - \frac{4}{5}] \cup \{\frac{4}{5}\}$ and $y(1, 1) = \frac{4}{5}$.

Strategy profile (D) is defined by

(D1) $m_0^A(s) = s$ for all $s \in S$, and $m_1^A(s) = s$ for $s = 1$ and $s + 1$ for $s = -1, 0$;

(D2) $m_1^B(-1, -1) \in \arg\max_{m^B} y(-1, m^B)$, $m_0^B(0, 0) = 0$, $m_0^B(0, -1), m_1^B(0, -1) \in \arg\max_{m^B} y(-1, m^B)$, and $m_0^B(1, t), m_1^B(1, t) \in \arg\max_{m^B} y(t, m^B)$ for $t = -1, 1$; furthermore, they must also ensure that (D3) hold;

(D3) $y(0, m^B) = 0$ for all $m^B \in S$, $y(1, -1) = 1/2$, $y(1, 0) \in [1/2, 2b - \frac{4}{5}] \cup \left\{\frac{4}{5}\right\}$, and $y(1, 1) = 4/5$.

Now comes the main proposition of this section.

Proposition 4. *Strategy profiles (B), (C), and (D) are the only symmetric equilibria of the sequential consultation game that satisfy the monotonicity condition. The decision maker's expected payoff is* $-26/81$ *in (B),* $-104/405$ *in (C), and* $-16/45$ *in (D).*

Proof. The proof is rather involved and so relegated to the Appendix. □

Now I examine the equilibria more closely. In equilibrium, expert A always distorts her report towards the direction she prefers if she is biased. For example, an expert of type 1 reports -1 as 0 and 0 as 1. If she is unbiased, then she simply reports the true state. Expert B makes her reaction based on her own bias, the underlying state and expert A's report. If expert A has not made a biased report, then expert B acts as if she were the first expert and sends a distorted report. In the case that the first report is biased, if expert B has the same bias as expert A, she may choose to further distort it, make a moderate report, or corrects the distortion by expert A it proves to be excessive (for example, reporting -1 as 1). If she is unbiased or if her bias is opposite to expert A's, then she chooses to offset the distortion by expert A if this option is available in equilibrium. For example, $m_0^B(0, -1) = 1$. That is, an expert of type 0 would like to report 1 if expert A has reported 0 as -1. She does not exactly "tell the truth" in the face of a biased report even though she herself is unbiased. Taking all this into account, the decision maker takes an action based on the message combination he receives.

Equilibrium (D) gives the lowest payoff among the three equilibria. The main reason is that $y(0, m^B) = 0$ for all m^B. Thus expert B loses the leverage to control what the decision maker does if expert A chooses to make the report 0. A biased expert A then has the liberty of misrepresenting extreme states as the middle state, which results in loss of information.

Notably, there are actually equilibria in which monotonicity is violated and in which the decision maker does even worse. In one such equilibrium, $y(1, -1) < y(0, -1)$. A positive-biased expert A reports all states as 1. Also, a positive-biased expert B reports state -1 as 1 given that expert A has honestly reported the state. This can be viewed as an extremely partisan debate, which not surprisingly does not transmit much information.

In equilibrium, it is possible for the second expert and the decision maker to infer the type of the first expert (or both experts in the case of the decision maker) although I assume that experts' types are known only to themselves. However this does not provide useful information for expert B or the decision maker. The only information that matters to their decision is the underlying state (for expert B) and the report(s) by the expert(s). If the decision maker knew the experts' types or the first expert knew the second expert's type *ex ante*, then the situation would be different.

When $\alpha = 0$, the reviewer is in effect nonexistent. The only equilibrium is thus the same as the one in the one-expert case. But as α increases, if rejection happens in equilibrium, and if the unbiased reviewer wants to reject the report, then it has the effect of increasing the decision maker's payoff. Peer review also has the potential of deterring experts from lying.

Formally, when the report of an expert is rejected by a reviewer, with probability $1-\alpha$, the report still gets to the decision maker; with probability α, the reviewer will draw a signal from the endogenous signal distribution over $\{-1, 0, 1\}$. Let us denote the distribution by $(\gamma_{-1}, \gamma_0, \gamma_1)$. Due to symmetry it can be represented as $(\gamma, 1-2\gamma, \gamma)$, where

$$\gamma \in (0, \frac{1}{2}).$$

Before solving for the equilibrium, I establish a result that measures the decision maker and an expert's expected utility from any strategy profile.

Proposition 5. *In any strategy profile, the decision maker's expected payoff is* $-\frac{2}{3} + \sum_m \gamma_m y_m^2$*, and his payoff in a symmetric equilibrium is* $-\frac{2}{3} + 2\gamma y_1^2$*.*
An expert's expected payoff before knowing the underlying state is $-\frac{2}{3} - b_x^2 + \sum_m \gamma_m y_m^2$*, and* $-\frac{2}{3} - b_x^2 + 2\gamma y_1^2$*, where* b_x *is her bias,* $x \in X$*.*

Proof. In any strategy profile, the decision maker's expected payoff is

$$U_0 = -\sum_s \sum_m P_{sm}(y_m - s)^2 = -\sum_s P_s s^2 - \sum_m \gamma_m y_m^2 + \sum_s \sum_m 2P_{sm} s y_m,$$

where U_0 indicates the expected utility of an unbiased agent. In the above equation, P_{sm} indicates the joint probability of the state being s, and the decision maker receiving message m, and P_s and γ_m are the corresponding marginal probabilities. Using the fact that $\sum_s P_{sm} s = \gamma_m y_m$, we have

$$U_0 = -\frac{2}{3} + \sum_m \gamma_m y_m^2.$$

In symmetric equilibria, we have $y_0 = 0$ and $y_{-1} = -y_1$, and the definition $\gamma = P(s = 1)$, and conclude

$$U_0 = -\frac{2}{3} + 2\gamma y_1^2.$$

The expected payoff for an expert of type x is

$$U_x = -\sum_s \sum_m P_{sm}(y_m - (s + b_x))^2 = U_0 - \sum_s P_s(b_x^2 + 2sb_x) + \sum_s \sum_m 2P_{sm} b_x y_m.$$

Now we may use the fact $\sum_s P_s s = 0$ and $\sum_m \gamma_m y_m = \sum_m \gamma_m E(s|m) = E(s) = 0$, and conclude

$$U_x = U_0 - b_x^2.$$

Hence the desired statement. □

One conclusion we can draw from the above proposition is that ranking of equilibria by the decision maker and that by the experts are the same. This is true even after the experts learn their biases, and thus true before the experts learn their biases. Note that the crucial conditions in proving this, namely $\sum_s P_s s = 0$ and $\sum_m \gamma_m y_m = 0$, are just the statements that the probability-weighted average of states and actions are equal to the expected value of the state. The value 0 is not crucial here. This condition can thus be generalized to a variety of circumstances in which the decision maker and the experts have quadratic loss utility functions.

Another observation one can make is that the first term in the expression, $-\frac{2}{3}$, is just the expected utility of the decision maker when he takes action 0 no matter what the state is. Thus the second term in the expression measures the gain in expected utility from communication with experts. In the light of the discussion of the previous paragraph, the second term is therefore the measure of communication effectiveness between experts and the decision maker.

Now we define a pure strategy profile (E).

(E1) $m_0^*(s) = s$ for all $s \in S$, and $m_1^*(s) = s$ for $s = 1$ and $s + 1$ for $s = -1, 0$;

(E2) $r_0^*(s,t) = 1$ if $(s,t) = (-1,1), (0,-1), (0,1)$ or $(1,-1)$, and 0 otherwise; $r_1^*(s,t) = 1$ if $(s,t) = (-1,1), (0,-1)$ or $(1,-1)$, and 0 otherwise;

(E3) $y_1^* = \frac{2(27-4\alpha)}{9(9-2\alpha)}$;

(E4) $\gamma = \frac{9-2\alpha}{27-4\alpha}$.

Now I characterize the equilibrium of the peer review mechanism under the assumption $b \in [\frac{17}{21}, \frac{6}{7}]$.

Proposition 6. *For the peer review mechanism, strategy profile (E) is the only symmetric equilibrium of the game.*

Remark: Within this equilibrium, whenever biased reports are rejected, a reviewer of bias 0 rejects them. Higher α means higher rejection rate of biased reports, which benefits the decision maker. The following corollary states this fact.

Corollary 6.1. *The decision maker's expected utility in the symmetric equilibrium is nondecreasing in α.*

Proof. By Proposition 5, the decision maker's expected utility in strategy profile (E) is equal to

$$U_0 = -\frac{2}{3} + 2\gamma y_1^2$$

Note in strategy profile (E) $\gamma y_1 = \frac{2}{9}$. Thus $\frac{\partial U_0}{\partial \alpha} = \frac{4}{9}\frac{\partial y_1}{\partial \alpha} > 0$. Thus the highest payoff of the decision maker is achieved when $\alpha = 1$. In this case $y = \frac{46}{63}$, and $U_0 = -\frac{194}{567}$. □

This implies that under my model assumptions, when designing a peer review mechanism, there should not be ways around the peer review process. If any report must pass through a reviewer first, then we may achieve the most informative equilibrium given this mechanism.

Since proving the Proposition helps provide insight to how experts and reviewers behave in equilibrium, I put the proof in the main text.

Consider any reviewer of type $v \in X$. In equilibrium $r_v^*(s,t) = 1$ if and only if

$$\begin{aligned} &-\alpha\gamma((y_1^* - (s+b_v))^2 - \alpha\gamma(y_{-1}^* - (s+b_v))^2 \\ &-\alpha(1-2\gamma)(0-(s+b_v))^2 - (1-\alpha)(y_t^* - (s+b_v))^2 \quad > -(y_t^* - (s+b_v))^2. \end{aligned}$$

But this inequality is equivalent to

$$\gamma((y_1^* - (s+b_v))^2 + (-y_1^* - (s+b_v))^2) + (1-2\gamma)(0-(s+b_v)^2) < (y_t^* - (s+b_v))^2. \quad (2)$$

First, the following lemma establishes a relationship between the behavior of a reviewer and that of an expert of the same type.

Lemma 4. *In any equilibrium (symmetric or otherwise), if $r_v^*(s,t) = 1$, then $\tilde{m}_v^*(s,t) = 0$, i.e., $m_v^*(s) \neq t$.*

Proof. Since $r_v^*(s,t) = 1$, Equation (2) must hold. This implies that $-(y_t^* - (s+b_v))^2 = u(y_t^*, s, b_v) < \max_{t' \in S} u(y_{t'}^*, s, b_v)$. Let $\tilde{t} = \arg\max_{t' \in S} u(y_{t'}^*, s, b_v)$. Then if an expert of type v sends the message $\tilde{t}$ when the state is s, her expected payoff is at least $(1-\alpha)u(y_{\tilde{t}}^*, s, b_v) + \frac{\alpha}{3}\sum_{m \in S} u(y_m^*, s, b_v)]$, which is greater than $(1-\alpha)u(y_t^*, s, b_v) + \frac{\alpha}{3}\sum_{m \in S} u(y_m^*, s, b_v)]$, but the second expression is greater than $u(y_t^*, s, b_v)$ by Equation (2). Since the expert's expected payoff from sending message t is a convex combination of this expression and $u(y_t^*, s, b_v)$, the expert is strictly better off sending message $\tilde{t}$. Hence $\tilde{m}_v^*(s,t) = 0$.
Note that I have not used symmetry in the above proof. So Lemma 4 applies to all equilibria of the game, not just symmetric ones. □

The intuition behind the above lemma is as follows. In equilibrium, if a reviewer of a certain type rejects a message t in a state s, she prefers a random message from the endogenous distribution to the message t. So there must be another message $\tilde{t}$ that is strictly better than t for the reviewer. An expert of the same type as the reviewer also prefers $\tilde{t}$ to t if the message is to get through. Even if $\tilde{t}$ is rejected, the worst thing that can happen is a random message from the endogenous distribution, which is still better than t. Thus the expert should not report message t in state s.

Now I prove a result that says the only possible behavior of reviewers in symmetric equilibria is as described in $(E2)$, which also implies certain behavior from experts.

Lemma 5. *In a symmetric equilibrium,*

1. $r_0^*(s,t) = 1$ *if* $(s,t) = (-1,1),(0,-1),(0,1)$ *or* $(1,-1)$, *and* 0 *otherwise;* $r_1^*(s,t) = 1$ *if* $(s,t) = (-1,1),(0,-1)$ *or* $(1,-1)$, *and* 0 *otherwise;*

2. $m_0(s) = s$ *for all* $s \in S$ *and* $m_1(1) = 1$.

Proof. See Appendix. □

Observe that rejection results in the decision maker receiving a mixture between the original message and a random message drawn from the endogenous distribution. Thus rejection happens only if the reviewer prefers the random distribution of messages to the original message. The random distribution is never preferred by any expert/reviewer to the message 0 because agents dislike variance in our model and because of symmetry. Therefore, a reviewer never rejects the message 0 since rejecting would only make her worse off. At the same time, a reviewer of type 0 rejects messages 1 and -1 in state 0 and message 1 in state -1, since these messages are the worst for her to pass on to the decision maker. For similar reasons, a reviewer of type 1 rejects the message -1 when the state is 0 or 1 since it is her least preferred message. These arguments do not depend on the size of the bias. However, the argument for type 1 not rejecting -1 in state -1 and rejecting 1 in state -1 does depend on the fact that her bias is not very large.

Proof. (of Proposition 6) By Lemmas 4 and 5, $m_1(-1) \neq 1$ and $m_1(0) \neq -1$. Thus what is left to be determined is whether $m_1(0) = 0$ or 1 and whether $m_1(-1) = -1$ or 0.
To summarize, the following must be true:

(i) $m_0(s) = s$ for all $s \in S$ and $m_1(1) = 1$.

(ii) $r_0(s,t) = 1$ for $(s,t) = (-1,1)$, $(0,-1)$, $(0,1)$, and $(1,-1)$, and $r_0(s,t) = 0$ otherwise; $r_1(s,t) = 1$ for $(s,t) = (-1,1)$, $(0,-1)$, and $(1,-1)$, and $r_1(s,t) = 0$ otherwise.

Before proceeding, we define $\theta = 1 - \frac{2}{3}\alpha$. It is decreasing in α and goes from 1 to $\frac{1}{3}$ as α goes from 0 to 1. Observe that since when the decision maker receives no messages, a signal is randomly drawn from the endogenously generated distribution $(\gamma, 1-2\gamma, \gamma)$, the following must be true:

$$\begin{aligned}
\gamma &= P(s=1)[P(x=0,1) + P(x=1)(1-\tilde{m}_1(-1,0))] \\
&\quad + P(s=0)P(x=1)\tilde{m}_1(0,1)[P(v=1) + P(v=-1,0)[(1-\alpha)+\alpha\gamma]] \\
&\quad + P(s=0)P(x=-1)\tilde{m}_{-1}(0,-1)(P(v=0,1)\alpha\gamma) \\
&= \frac{1}{3}[\frac{2}{3} + \frac{1}{3}(1-\tilde{m}_1(-1,0))] + \frac{1}{3} \times \frac{1}{3}\tilde{m}_1(0,1)[\frac{1}{3} + \frac{2}{3}[(1-\alpha)+\alpha\gamma]] \\
&\quad + \frac{1}{3}\tilde{m}_{-1}(0,-1)(\frac{2}{3}\alpha\gamma) \\
&= (\frac{1}{3} - \frac{\tilde{m}_1(-1,0)}{9}) + \frac{\tilde{m}_1(0,1)}{9}(1 - \frac{2}{3}\alpha(1-2\gamma))
\end{aligned}$$

I used the fact $\tilde{m}_{-1}(0,-1) = \tilde{m}_1(0,1)$ by symmetry. From the above equation I get

(†) $\qquad \gamma = \frac{1}{9}(1 - \frac{2}{3}\alpha(1-2\gamma))\tilde{m}_1(0,1) - \frac{\tilde{m}_1(-1,0)}{9} + \frac{1}{3}.$

Note that

(††) $\qquad y_1 = \frac{P(s=1,m=1)}{P(m=1)} = \frac{\frac{1}{3} - \frac{\tilde{m}_1(-1,0)}{9}}{\gamma}.$

Given the reviewer's strategies, the decision of an expert of type 1 should be characterized by the following comparisons.

(1) Since neither 0 nor -1 is ever rejected in state -1, $m_1(-1) = 0$ if $u(0,-1,b) \geq u(-y_1,-1,b)$, which is equivalent to $|0-(-1+b)| \leq |-y_1-(-1+b)|$. Thus $m_1(-1) = 0$ if

$$y_1 \geq 2(1-b).$$

(2) Since 0 is not rejected in state 0, the utility of an expert of type 1 is

$$u(0,0,b) = -(0-(0+b))^2 = -b^2$$

if $m_1(0) = 0$. On the other hand, as shown in the proof of Lemma 5 the expected payoff of the expert from reporting 1 is

$$-[(1-\frac{2}{3}\alpha)(y_1-b)^2 + \frac{2}{3}\alpha(2\gamma {y_1}^2 + b^2)]$$

Thus by comparing the two, we get $m_1(0) = 0$ is optimal if

$$(1 - \frac{2}{3}\alpha(1-2\gamma))y_1^2 - 2b(1 - \frac{2}{3}\alpha)y_1 \geq 0$$
$$\Leftrightarrow \quad y_1 \geq \frac{2b(1-\frac{2}{3}\alpha)}{1-\frac{2}{3}\alpha(1-2\gamma)}$$

Since $0 \leq \gamma \leq 1/2$ and $0 \leq \alpha \leq 1$, it is easy to see that $1 - \frac{2}{3}\alpha > 0$ and $1 - \frac{2}{3}\alpha(1-2\gamma) > 0$. But when $2b(1-\frac{2}{3}\alpha) > 1 - \frac{2}{3}\alpha(1-2\gamma)$, $m_1(0) = 0$ is impossible since $y_1 \in [0, 1]$.

(3) As shown in the proof of Lemma 5, in order for $r_1(-1,-1) = 0$ to be optimal we need

$$y_1 \leq \frac{2(1-b)}{1-2\gamma}.$$

By the no fully revealing equilibrium result in Proposition 1, we only need to consider three cases: (a) $m_1(-1) = 0$ and $m_1(0) = 0$; (b) $m_1(-1) = 0$ and $m_1(0) = 1$; (c) $m_1(-1) = -1$ and $m_1(0) = 1$.

(a) It can also be represented as $\tilde{m}_1(-1, 0) = 1$ and $\tilde{m}_1(0, 1) = 0$.
By Equation (†), $\gamma = 2/9$.
By Equation (††) we have $y_1 = (1/3 - 1/9)/(2/9) = 1$.
According to condition (3) in this proof, we need $1 \leq \frac{2(1-b)}{5/9}$. This requires $b \leq \frac{13}{18}$, which is not satisfied by our assumed b values.

(b) This case can be represented by $\tilde{m}_1(-1, 0) = 1$ and $\tilde{m}_1(0, 1) = 1$.
By Equation (†), $\gamma = \frac{\frac{2}{9}+\frac{1}{9}\theta}{1-\frac{2}{9}(1-\theta)}$.
By Equation (††), $y_1 = \frac{2}{9}/\gamma$.
According to condition (1) in this proof, we need $y_1 \geq 2(1-b)$, which translates into $(1 - b - \frac{2}{9})\theta \leq 1 - 2(1-b) - \frac{2}{9}$. This holds since $\theta \leq 1$ and $b \geq 2(1-b)$ for our assumed values of b.
In this case $1 - \frac{2}{3}\alpha(1-2\gamma) = 9(\gamma - \frac{2}{9})$. Substituting this into condition (2) in this proof, we get $9y_1(\gamma - \frac{2}{9}) \leq 2b\theta$. Using $\gamma y = \frac{2}{9}$, we get

$$b\theta^2 + (2b - \frac{5}{9})\theta - \frac{4}{9} \geq 0.$$

The L.H.S. is strictly increasing in θ since $2b - \frac{5}{9} > 0$. Now substituting $\theta = \frac{1}{3}$ into the L.H.S., we get $\frac{7}{9}b - \frac{17}{21} \geq 0$, which is true for $b \geq \frac{17}{21}$. Hence condition (2) is always satisfied if $b \geq \frac{17}{21}$.

Now we check condition (3). We need $\frac{2}{9} \cdot \frac{1-2\gamma}{\gamma} \leq 2(1-b)$, which simplifies into

$$\theta \geq \frac{1}{3(1-b)} - 2.$$

This inequality holds for any $\theta \in [\frac{1}{3}, 1]$ if $b \leq \frac{6}{7}$.

So we get that **strategy profile (E)** is an equilibrium if $b \in [\frac{17}{21}, \frac{6}{7}]$.

(c) This case can be represented by $\tilde{m_1}(-1, 0) = 0$ and $\tilde{m_1}(0, 1) = 1$.
By Equation (†), $\gamma = \frac{\frac{1}{3}+\frac{1}{9}\theta}{1-\frac{2}{9}(1-\theta)}$.
By Equation (††), $y_1 = \frac{1}{3}/\gamma$.
By condition (1) of this proof, we need $y_1 \leq 2(1-b)$, which requires $\frac{1}{3}(\frac{7}{9}+\frac{2}{9}\theta) \leq (1-b)(\frac{2}{3}+\frac{2}{9}\theta)$, an impossible statement since $1-b < \frac{1}{3}$. So this is not an equilibrium strategy profile.

Summarizing the above arguments proves the proposition. □

In the peer review mechanism, letting the reviewer have more power never hurts the decision maker if we consider the most informative equilibrium. Hence peer review mechanisms should be designed such that the review process cannot be evaded by experts.

IV. Comparisons

In the one expert case, the decision maker gets an expected payoff of $-10/27$. We have seen that all equilibria of mechanisms with two experts give strictly higher payoff than this, confirming the intuition that seeking second opinions improves information transmission.

It is also interesting to compare payoffs of the equilibria of two-expert mechanisms. Peer review is a simple mechanism in which the expert gets only one final report. If peer review dominates direct consultation of *two* experts, one can certainly make a strong case in support of using peer review as an effective communication mechanism between experts and decision makers.

There is a major difference between direct consultation mechanisms and peer review. In simultaneous consultation and sequential consultation, although biased reports will be counteracted by other reports, they do get seen by the decision maker. In this sense, they do not prevent experts from misrepresenting information. They improve information transmission through finely partitioning the decision space so as to lower the cost of misrepresentation to the decision maker. However in peer review, biased experts worry about their reports being rejected by a reviewer that has biases different from theirs, in which case there will be uncertainty in the decision maker's decision. Since experts dislike uncertainty, they will withhold biased reports if the cost of rejection to them is too high. Note that existence of different biases is crucial for peer review to work for the decision maker.

In my model, the unique equilibrium in simultaneous consultation gives the decision maker an expected payoff of $-94/405$. In the most informative equilibrium (C) in sequential consultation, the decision maker earns an expected payoff of $-104/405$. In the unique equilibrium (E) of peer review with $\alpha = 1$, the decision maker gets an expected payoff of $-194/567$. Peer review does worse than both simultaneous consultation and sequential consultation in the most informative equilibrium. However the message regarding sequential consultation is mixed. Direct sequential consultation gives the highest information efficiency, but unlike the other two mechanisms, it generates multiple equilibria. In particular, strategy profile (D) gives the decision maker an expected payoff of $-16/45$, which is lower than the expected payoff generated by peer review. If one is confident that the most informative equilibrium will occur, then he should favor sequential consultation over peer review. If not, peer review may be a better choice.

To summarize, the following is the result of the comparisons:

Result of comparisons. *Considering the most informative equilibrium, the ranking of information transmission efficiency of the three mechanisms is(from the highest to the lowest): 1. simultaneous consultation; 2. sequential consultation; 3. peer review. However, there exists an equilibrium in sequential consultation in which the decision maker is worse off than he is under peer review.*

In the table below, I calculate the magnitude of the improvement in communication from the uninformative equilibrium. The first column in the table is the payoffs from equilibria of different mechanisms. The second column reflects the magnitude of improvement from the babbling equilibrium, as a percentage of the improvement by the most informative equilibrium – the simultaneous consultation equilibrium. Since von Neumann-Morgenstern expected utility is unique up to affine transformations, these numbers accurately reflect the welfare comparisons between different equilibria. From the table, one can see that all two-expert mechanisms do better than the one-expert mechanism. Also, peer review achieves about $\frac{3}{4}$ of the information transmission by simultaneous consultation.

	Payoff	Improvement Percentage
Babbling	$-2/3$	0
One Expert	$-10/27$	68.2
Peer Review	$-194/567$	74.7
Sequential-B	$-26/81$	79.5
Sequential-C	$-104/405$	94.3
Sequential-D	$-16/45$	71.6
Simultaneous	$-94/405$	100

In this paper, I model peer review as interactions between experts that are not transparent to the decision maker, with the final signal as the only thing observed by the decision maker. This corresponds to situations in which the decision maker just receives one unified recommendation, instead of knowing each expert's opinion and what they think about each other's opinion. For example, a politician would only choose a prominent economic theory for his use, without knowing how economists have promoted the theory to prominence. A CEO often receives only one report concerning a new project instead of having competing reports or knowing how the final report is formed from preliminary versions by lower management.

It is reasonable to expect that monitoring interactions among experts is not costless. This is especially true when the decision maker is unfamiliar with the profession of experts. Time and financial constraints may prevent decision makers from listening to multiple opinions on an issue. Peer review, on the other hand, gives the decision maker an option in which he does not have to monitor all the interactions. Coming back to the examples, the politician is better off having counterbalancing forces in his advice seeking process, but it may not pay for him to follow what is really going on in the process if it proves costly. A CEO is better off having lower management peer reviewing each other, but it may not be in his best interest to deal with the opinions of all the lower management. If the sequential consultation game is caught in a "bad" equilibrium, then the decision maker would be much better off to use peer review instead.

V. Discussions

Generalized Biases

In the discussion in previous sections, attention has been focused on biases in the range $[\frac{17}{21}, \frac{6}{7}]$. On the other hand, there exist values of b such that 5 messages are sent in the one-expert mechanism. As long as $b \leq \frac{3}{4}$, the following strategy profile constitutes an equilibrium. Everything else is the same at that in strategy profile (A), except that there are now a pair of new messages. For illustrative purposes, I call them "$\frac{1}{2}$" and "$-\frac{1}{2}$" respectively. Message $\frac{1}{2}$ is sent in state 0 by experts of type 1 and in state 1 by experts of type -1. When receiving message $\frac{1}{2}$, the decision maker takes action $\frac{1}{2}$, which is the expected value of the underlying state conditional on the message $\frac{1}{2}$ being received. Using a generalized version of Proposition 5, one can see that the decision maker's expected payoff is $-\frac{2}{3} + \sum_m \gamma_m y_m^2 = -\frac{1}{9}$, which is higher than any equilibrium above with three available messages.

Furthermore, given the appropriate values of b such that 5-message equilibria are possible, if we allow 5 messages to be sent in equilibrium, then having an additional expert as reviewer does not help the decision maker. It is not possible for a reviewer of type 1 to not reject a report of $-\frac{1}{2}$ in state 0. This results in actions far from the real state being taken in equilibrium due to the nature of the mechanism, which lowers the decision maker's expected utility.

Restrictions on the Set of Allowed Messages

In many situations, it is conceivable that the decision maker wants to limit the number of messages from which experts are allowed to choose. He may have varied decisions to make, which requires him to process a vast amount of information. Or, he may need to make the decision in a fixed time frame. These factors may prevent him from listening to nuanced description of each problem. As a result, he may just be able to afford to handle a brief report, or an "up or down" type recommendation. Thus, it is also interesting to look at how the mechanisms compare with each other when the message space is restricted to a subset of all possible messages.

For example, the US president makes the final decision on a wide range of issues. Ideally, he would like to retain the option of allowing experts to give detailed recommendations on each issue.[9] However, this option may be impossible or too costly, especially on issues of lesser importance. Therefore, he may consider limiting the set of messages his experts can send. Other examples include a commander who must make rapid decisions in a military campaign. Time constraint prevents him from listening to detailed analysis.

[9] It is not clear whether having the option would strictly benefit him. For example, Szalay (2002) shows that when it takes effort for the expert to acquire information it is sometimes necessary for the decision maker to limit the expert's message space in order to induce better effort on the part of the expert. However, as long as the decision maker can choose to not use the option of receiving nuanced reports, retaining that option at least does not hurt him.

Suppose the decision maker allows only three possible messages. I label the three messages as statements in the form of "the state is 1," "0," or "−1." This restriction does not alter the discussion in previous sections where my attention was limited to the case $b \in [\frac{17}{21}, \frac{6}{7}]$, since there are only three messages in equilibrium even without the restriction. However, for smaller bias values, there could be more than 3 messages in equilibrium when there are no restrictions. Therefore, comparisons between mechanisms may change when such restrictions are imposed.

Indeed, consider the case $b = \frac{2}{3}$. It can be shown that under the restriction to three messages, the ranking of the most informative equilibria of the three mechanisms is: sequential consultation, peer review, and then simultaneous consultation. Furthermore, sequential consultation again allows a less informative equilibrium that is worse than the unique informative equilibrium under peer review.[10] This can be viewed as lending some support for the adoption of the peer review mechanism.

Asymmetric Equilibrium

Although symmetric equilibria seem to be a natural choice for the game given the symmetric setup of the model, I shall also look at asymmetric equilibria of the game. In fact, there exist asymmetric equilibria in which the decision maker is better off than he is in the symmetric equilibrium. This somewhat counterintuitive result echoes with the result found in Admati and Pfleiderer (2001) under a different setup. I shall argue this in fact corresponds to phenomena observed in reality.

Due to the symmetric setup of the model, for any asymmetric equilibrium there always exists a "mirror image" of it that is also an equilibrium. Thus equilibria always exist in pairs, unless they are symmetric. This means without loss of generality, when there are two equilibria that are mirror images of each other I only have to look at one of them.

First, I shall prove properties that must be true about reviewers' decisions in any equilibrium of the game under the general setup.

Lemma 6. *In equilibrium, the following must be true for all $v, v' \in X$, $s \in S$, and $t \in M$, where $v \geq v'$:*

1. *If $y_t > 0$ then $r_v(s,t) = 1$ implies $r_{v'}(s,t) = 1$;*
2. *If $y_t < 0$ then $r_v(s,t) = 1$ implies $r_{v'}(s,t) = 1$;*

[10]The analysis is similar to that in previous sections. The construction of those equilibria is in an earlier version of this paper, and is available from the author upon request.

3. If $y_t = 0$ then $r_v(s,t) = 0$.

Proof. See Appendix. □

The intuition of the above lemma is that a negative-biased reviewer is more likely to reject a message that induces a positive action, compared with neutral and positive-biased reviewers. Note also if we change the statements into $r_v(s,t) = 0$, then the order of implication reverses, which expresses the same statement in terms of acceptance instead of rejection. This lemma is useful since it enables us to check at most two review decisions for any given state-message pair.

Based on the comment about "mirror images" of equilibria, henceforth I only look at cases in which $y_0^* \geq (y_1^* + y_{-1}^*)/2$, because its mirror image satisfies $y_0^* \leq (y_1^* + y_{-1}^*)/2$.

Now I focus my attention on the setup in Section III. That is, $b \in [\frac{17}{21}, \frac{6}{7}]$ and each expert is allowed to choose from a set of three messages. Again, for illustrative purposes, let $M = S$. Using similar notation to that in Section III, let γ_m indicate the probability of message m being received in equilibrium.

The following proposition states that it is possible to have asymmetric equilibria that are more informative than the symmetric ones. Let us first define strategy profile (F):

(F1) $m_{-1}(-1) = m_{-1}(0) = -1$, $m_{-1}(1) = 0$ and $m_0(s) = m_1(s) = s$ for all $s \in S$;

(F2) $r_{-1}(s,t) = 1$ for $(s,t) = (-1,0), (-1,1), (0,1)$ and $(1,-1)$, and 0 otherwise; $r_0(s,t) = 1$ for $(s,t) = (-1,1), (0,-1), (0,1)$ and $(1,-1)$, and 0 otherwise; $r_1(s,t) = 1$ for $(s,t) = (-1,1), (0,-1)$ and $(1,-1)$, and 0 otherwise;

(F3) $y_{-1} = -\frac{(1-4\alpha/27)/3}{1/3+(1-2\alpha/3)/9}$, $y_0 = \frac{(1-4\alpha/27)/9}{1/9+2(1-\alpha/3)/9}$, and $y_1 = 1 - 4\alpha/27$;

(F4) $\gamma_{-1} = \frac{1/3+(1-2\alpha/3)/9}{1-4\alpha/27}$, $\gamma_0 = \frac{1/9+2(1-\alpha/3)/9}{1-4\alpha/27}$, and $\gamma_1 = \frac{2/9}{1-4\alpha/27}$.

Proposition 7. *There exists α large enough such that strategy profile (F) (and its mirror image) is an equilibrium.*

Proof. See Appendix. □

What happens in the asymmetric equilibrium is that while two types of experts always tell the truth, the other type misrepresents the states whenever possible. A right-biased expert finds it not in her best interest to overstate the states. This is because the decision made by the decision maker is so close to the right-biased expert's most preferred action when she tells the truth, that she finds it unfavorable for her to make a biased report taking into account the possibility of being rejected.

This effect is enforced by large α values. However for a left-biased expert, the effect is in the reverse direction. Now she has extra incentives to lie since lying induces a much more favorable decision than telling the truth. When the true state is 0, type -1 expert's most preferred action is $-b$, however if she reports it as 0 the induced action would be positive. So she reports -1, which brings the decision much closer to her most preferred action, although it runs the risk of being rejected.

When $\alpha = 1$, by Proposition 5, the decision maker's payoff under the asymmetric equilibrium as prescribed by Proposition 7 is

$$-\frac{2}{3} + \sum_m \gamma_m y_m^2 = -\frac{2}{3} + (1 - \frac{4}{27})(\frac{2}{9} + \frac{1}{9} \cdot \frac{\frac{1}{9}}{\frac{1}{9} + \frac{2}{9} \cdot \frac{2}{3}} - \frac{1}{3} \cdot \frac{-\frac{1}{3}}{\frac{1}{3} + \frac{1}{9} \cdot \frac{1}{3}}) = -\frac{3083}{17010},$$

which is higher than his payoff $-194/567 = -\frac{5820}{17010}$ under the symmetric equilibrium.

When receiving 0, the decision maker actually takes a positive action. This means the "middle ground" is not exactly in the middle, but tilted towards one direction. The gain from having this message instead of an impartial one is that it reduces the amount of (in this case eliminates) misrepresentation by right-biased experts and it also reduces the loss from left-biased experts reporting state 1 as 0. The loss from having a biased 0 is more than offset by the reduction in the loss from experts misrepresenting the states. Thus the decision maker is better off under the above asymmetric equilibrium than he is under the symmetric equilibrium.

Morgan and Stocken (2003) show that analysts' stock recommendations are asymmetrically distributed on "sell," "hold," and "buy". However, in their model analysts have preferences that are biased towards pumping up stock prices.[11] In my model, biases of experts are evenly distributed around zero. This makes the equilibrium more counterintuitive. However, asymmetric equilibria are widespread in reality. If a movie is rated "average" by critics, normally people would think the movie is probably not very good. If it is rated downright "awful" then the movie is likely to be so. However, people tend to discount "two thumbs up" recommendations. There is no obvious reason to believe that on average critics have upward biases about the quality of movies or that movie quality is unevenly distributed. In light of the result of this section, people may actually benefit from this seemingly uninformative recommendation scheme even if the underlying distributions are symmetric.

[11]In their model, there do exist unbiased analysts. However, there do not exist analysts biased towards making stock prices lower.

Would a Decision Maker Prefer to Know the Bias of an Expert?

In this paper I have assumed throughout that the decision maker does not know experts' biases. An interesting question to ask is, were a decision maker given the option to discover the bias of an expert, would he like to know the bias of the expert? Assume that the decision maker cannot conceal his knowledge of the expert's bias. Again, I discuss biases within the range $[\frac{17}{21}, \frac{6}{7}]$ to make a comparison. Once a decision maker discovers an expert to be unbiased, then there will not be any communication problems. If a decision maker discovers that the expert is biased, the situation changes into one of the classic cheap talk model. Consider a positive-biased expert. I shall argue that there could be no information transmission between the expert and the decision maker. There could not be more than one message inducing a positive action, since the expert would always prefer to send the higher message in state 1, making that message the only message that could induce a positive action. This unique positive message must be strictly preferred by the expert in state 0 to any other message since her bias is greater than $\frac{1}{2}$. Thus this positive message must induce an action not more than $\frac{1}{2}$. Now in state -1, between reporting this "positive" message and an alternative message that ensures the action -1 be taken, the expert prefers to send this "positive" message. Hence there could not be any informative equilibria. Thus the decision maker's expected utility if he chooses to discover the bias of the expert is $2 \cdot \frac{1}{3} \cdot (-\frac{2}{3}) = -\frac{4}{9}$, which is lower than his expected utility $-\frac{10}{27}$ from strategy profile (A), in which he does not know the expert's bias. Hence even if he could discover the bias of the expert at zero costs, he would still prefer not to do so. In fact, this phenomenon is true in other settings. Li (2003) shows this is the case when the decision maker is uncertain about the direction of an expert's bias, regardless of the degree of uncertainty.

VI. Related Literature

My work is closely related to research using the cheap talk model to study one decision maker obtaining advice from multiple experts. They are usually applications and extensions of Crawford and Sobel (1982). Gilligan and Krehbiel (1989) study a model in which a committee of two experts with opposing biases simultaneously communicate with the decision maker. They are interested in comparing the "closed rule" and the "open rule" of making legislations. In the closed rule, bills cannot be amended after being submitted and other members can only make statements about the underlying state, while in the open rule the other committee members can submit their own amendments. They find that "closed rule" is information superior when the committee is composed of experts with heterogeneous preferences. For homogeneous committees, the result is mixed depending on the degree of preference divergence. Krishna and Morgan (2001a) show that with experts of heterogeneous preferences, full efficiency is attainable under the open rule but not under the closed rule. For homogeneous committees, closed rule is always information superior. The difference between the above two papers are that the latter select the most informationally efficient equilibria (among the equilibria they find) for each rule while the former does not. Austen-Smith (1993) also studies a model with two experts. He is concerned with comparing sequential consultation and simultaneous consultation. The state and signal spaces are binary and the experts are imperfectly informed. He finds that sequential consultation is superior to simultaneous consultation. Krishna and Morgan (2001b) examine sequential consultation with two experts with like biases and opposing biases and conclude that the latter conveys more information than having just one expert, but the former is not more informative than having one expert. But the order in which the experts speak also matters to the outcome. In particular, when a moderate is paired with an extremist, the moderate should be consulted second.

This paper is closest in spirit to the last paper cited above. However, biases of the expert and the reviewer are unknown to the decision maker in this paper. This paper extends the literature to cases in which the decision maker does not know the experts' biases, but would like to maximize expected utility based on his knowledge of the distribution of biases of the experts.[12] I also introduce a mechanism in which interactions between experts take the "expert-reviewer" form, and the interactions are unobservable to the decision maker.

Another line of research is concerned with the problem how reputation concerns of experts affect information transmission behavior. Sobel (1985) was the first to study the reputation building motives of informed experts when their interests may contradict the decision maker's. Morris (2001) studies an expert's instrumental concern for reputation, in the sense that reputation only matters because the expert wants to take part in the discussion of future decisions. In his model, experts are either biased (bad) or unbiased (good). There is only one possible direction of bias. In order to be perceived as a good advisor tomorrow, the expert chooses not to report the true state when reporting it hurts her reputation as an unbiased advisor. Ottaviani and Sørensen (2001b) study the reputation concerns of experts in the sense

[12]Cheap talk with uncertain biases is discussed by de Garidel-Thoron and Ottaviani (2000). But multiple experts with unknown biases have not been well studied.

that they want to be perceived as talented predictors of the true state. They conclude that full revelation of information is never an equilibrium. In the equilibrium at most two messages get sent, although agents are free to choose from a rich message space. Ottaviani and Sørensen (2001a) consider the problem of choosing the order for multiple experts to speak to optimize information transmission when experts have reputation concerns. They also address the problem of the order of speech, but the setting is different from Krishna and Morgan's.

There is also a literature on eliciting opinions under more general settings. Glazer and Rubinstein (1998) study how to implement the policy target based on the motive structure of experts. The conclusion is that when experts care only about the the correctness of the decision, the public target (minimizing the possibility of mistakes) cannot be implemented, while if experts care both about the correctness of the decision and whether their recommendations coincide with the final decision, then the public target can be implemented. Dewatripont and Tirole (1999) study the case when information collection is costly for the experts. They argue that having advocates of opposing causes is superior to having one impartial information collector working for both causes. Szalay (2002) is concerned with the design of contracts when the agent has to spend effort collecting information. He shows limiting the agent's freedom of choice may benefit the principal.

There has also been research showing the disadvantages of peer review under different contexts from this paper. For example, Föster (1995) uses an experiment to show the negative effect peer review has on the introduction of new research methods in academic publication, when the reviewer status depends on whether one performs well as a reviewee. Baliga and Sjostrom (2001) show self assessment is better than peer review at transmitting information about innovation projects when there are conflicts of interest between the reviewer and the reviewee.

VII. Concluding Remarks

In this paper, I have studied situations in which the decision maker consults multiple experts with uncertain biases. I compare three mechanisms: simultaneous consultation, sequential consultation, and peer review. Simultaneous consultation does better than the other two mechanisms. Peer review is a simple and less costly mechanism for the decision maker. It does achieve significant information transmission, sometimes better than sequential consultation.

In this paper, the state space and the expert type space are both discrete and the prior distributions are symmetric. However it is also interesting to see what happens if all experts have biases in the same direction. Which mechanism brings about more effective communication?

Another extension is the possibility of reviewers and experts having different distribution of biases. Suppose there are two populations of experts with the same information, but one population has higher variance in bias. An interesting question is to see which population should serve as reviewers, and which population as experts. In reality, decision makers may have better information about types of reviewers, but it is not clear whether it is optimal to assign those who have lower variances in biases as reviewers.

Appendix: Proofs

Proof. (of Proposition 1) Suppose there exists a fully revealing equilibrium. That is, in equilibrium $P(y = s|s) = 1$ for all $s \in S$. The informativeness assumption implies that for all $s \in S$,

$$P(m = s|s) = 1. \tag{3}$$

and for all $m \in S$,

$$y_m = m. \tag{4}$$

This implies that for all $s \in S$ and $x \in X$,

$$m_x(s) = s. \tag{5}$$

If $m_{x_0}(s_0) \neq s_0$ for some $x_0 \in X$ and $s_0 \in S$, then $P(m \neq s_0|s_0) \geq P(x = x_0)(1 - 2\alpha/3) = 1/3 - 2\alpha/9$. So $P(m = s_0|s_0) < 1$, and we find a contradiction. We also have that $r_v(s, s) = 0$ for all $v \in X$ and $s \in S$. Otherwise, Equation 3 will not hold either. Observe also that given Equation 4, $r_v(s, 0) = 0$ for all $v \in X$ and $s \in S$. The utility $-(0 - (s + b_v))^2$ from acceptance is always higher than the utility $-[(1 - \alpha)(0 - (s + b_v))^2 + \alpha \sum_{m \in S} (y_m - (s + b_v))^2]$ from rejection. Hence the message 0 is never rejected. Given this, if $s = 1$ type -1 expert's utility is $-(0-(1-2/3))^2 = -1/9$ from sending 0, and $-(1-(1-2/3))^2 = -4/9$ from sending 1. Thus $m_{-1}(1) = 1$ cannot be part of the equilibrium, contradicting Equation 5. Hence there is no fully revealing equilibrium. □

Proof. (of Proposition 2) Since we consider symmetric equilibria, $y_{-1} = -y_1$ and $y_0 = 0$. It must be that $y_1 > 0$ otherwise the equilibrium is uninformative. Let $y = y_1$ to save notation. Since an expert's utility function is as defined in Equation 1, we have $m_0^*(s) = s$ for all $s \in S$. This is because that y_s and s have the same sign implies $-(y_s - s)^2 < -(y_{s'} - s)^2$ for all $s, s' \in S, s \neq s'$.
I calculate $m_1^*(s)$ only and $m_{-1}^*(s)$ follows from symmetry. Note that $b_1 = b \in [\frac{17}{21}, \frac{6}{7}]$
First, $m_1^*(1) = 1$ since $u(1, 1, b) = -(y-(1+b))^2 > -(0-(1+b))^2 > -(-y-(1+b))^2$.
Second, $m_1^*(0) = 1$. Since $|0 - (0 + b)| > |y - (0 + b)|$, and $|-y - (0 + b)| > |0 - (0 + b)|$ for all $b > \frac{1}{2}$ and $y \in (0, 1]$, we have $u(1, 0, b) = -(y - (0 + b))^2 > -(0 - (0 + b))^2 > -(-y - (0 + b))^2$.
Finally, $m_1^*(-1) \neq 1$ because $b < 1$ implies $|0 - (-1 + b)| < |y - (-1 + b)|$. If $m_1^*(-1) = -1$ (by symmetry, $m_{-1}^*(1) = 1$) then $y = \frac{P(s=1)\cdot 1 + P(s=0,x=1)\cdot 0}{P(s=1)+P(s=0,x=1)} = (1/3)/(1/3 + 1/3 \times 1/3) = 3/4$. Thus $|0 - (-1 + b)| = 1 - b < b - \frac{1}{4} = |-y - (-1 + b)|$, which makes $m_1(-1) = -1$ not optimal. The inequality comes from the fact that

$b \geq \frac{17}{21} > \frac{5}{8}$. So only $m_1^*(-1) = 0$ can be part of the equilibrium strategy. In this case, $y = \frac{P(s=1,x\neq-1)\cdot 1 + P(s=0,x=1)\cdot 0}{P(s=1,x\neq-1)+P(s=0,x=1)} = (1/3 \times 2/3)/(1/3 \times 2/3 + 1/3 \times 1/3) = 2/3$, thus $|0-(-1+b)| = 1-b < b-\frac{1}{3} = |-y-(-1+b)|$. Again the inequality comes from the fact that $b \geq \frac{17}{21} > \frac{2}{3}$. Hence $m_1^*(-1) = 0$ is optimal. To summarize, $m_1^*(s) = s+1$ if $s \neq 1$, and $m_1^*(1) = 1$, and by symmetry $m_{-1}^*(s) = s-1$ if $s \neq -1$, $m_{-1}^*(-1) = -1$. In the above paragraph, I have shown that $y_1^* = 2/3$. Furthermore the decision maker's expected utility is

$$\begin{aligned} & -[P(s=1,x\neq-1)(y^*-1)^2 + P(s=-1,x\neq 1)(-y^*-(-1))^2 \\ & +P(s=1,x=-1)(0-1)^2 + P(s=-1,x=1)(0-(-1))^2 \\ & +P(s=0,x=1)(y_1^*-0)^2 + P(s=0,x=-1)(-y_1^*-0)^2] \\ = & -2[2/9 \times (1/3)^2 + 1/9 \times 1^2 + 1/9 \times (2/3)^2] \\ = & -10/27 \end{aligned}$$

□

Proof. (of Lemma 2) In any pure strategy equilibrium, symmetry implies $m_0^i(0) = 0$. To show the other statement, it is enough to consider $m_0^A(1)$ and $m_1^A(1)$.
First $m_1^A(1) = 1$ by informativeness and monotonicity. For an expert of type 0, it is always true that $u(y(s, m_x^B(1)), 1, 0) \geq u(y(t, m_x^B(1)), 1, 0)$ for all $s, t \in S, s \geq t$ and $x \in X$. Thus
(*) $\frac{1}{3}\sum_{x\in X} u(y(s, m_x^B(1)), 1, b_1) \geq \frac{1}{3}\sum_{x\in X} u(y(t, m_x^B(1)), 1, b_1)$, for all $s, t \in S, s \geq t$,
making $m_0^A(1) = 1$ a best response no matter what the other does. Now we check that this is the only possible best response in equilibrium.
First $m_0^A(1) = -1$ would violate monotonicity.
Suppose $m_0^A(1) = 0$. Due to symmetry and anonymity, $y(1,-1) = y(-1,1) = -y(1,-1)$, thus they must both be 0. Informativeness gives us $y(1,1) > 0 = y(-1,1)$, which in turn implies that either $y(1,1) > y(0,1)$ or $y(1,0) > y(0,0)$ must be true. Thus (*) holds strictly for $s = 1$ and $t = 0$, since $m_1^B(1) = 1$ and $m_0^B(1) = 0$. This contradicts $m_1^A(1) = 0$.
The above arguments prove the lemma. □

Proof. (of Proposition 3) Given Lemma 2, the only strategies left to be determined are $m_s^i(-s)$ and $m_s^i(0)$. Once they are determined, y can be derived from (EQ3). By symmetry and anonymity, it is enough to discuss $m_1^A(-1)$ and $m_1^A(0)$.
First I consider $m_1^A(0)$.

(a) $m_1^A(0) = -1$. This would violate monotonicity.

(b) $m_1^A(0) = 0$. By symmetry $m_{-1}^A(0) = 0$. Thus $\frac{1}{3}\sum_{x\in X} u(y(0, m_x^A(0)), 0, b_1) = u(y(0,0), 0, b_1)$. Thus we have
$\frac{1}{3}\sum_{x\in X} u(y(1, m_x^A(0)), 0, b_1) > \frac{1}{3}\sum_{x\in X} u(y(0, m_x^A(0)), 0, b_1)$,
unless $y(1,0) = y(0,0)$. But if $y(1,0) = y(0,0) = 0$, then it must be that $m_{-1}^A(1) \neq 0$, otherwise by Lemma 2 $y(1,0) = 1$. As shown above $y(1,-1) = 0$. But $m_1^A(-1) = 1$ and $m_1^A(-1) = -1$ would be worse responses than $m_1^A(-1) = 0$.
When $m_1^A(-1) = 1$, $y(1, m_{-1}^A(-1)) = y(1, m_0^A(-1)) = y(1,-1) = y(0,-1)$, while $y(1, m_1^A(-1)) = y(1,1) > y(0,1) > -1+b$. Since $-1+b$ is type 1 expert's most preferred action, $m_1^A(-1) = 0$ is better than $m_1^A(-1) = 1$.
When $m_1^A(-1) = -1$, $y(1,1) = 1$. Thus $y(-1, m_x^A(-1)) = y(-1,-1) = -1$ and $y(0,-1) = 0$, and the latter is closer to a type 1 expert's most preferred action $-1+b$ since $b > \frac{1}{2}$.

(c) $m_1^A(0) = 1$. Now we discuss $m_1^A(-1)$.

(i) $m_1^A(-1) = -1$. Given this, we can calculate the eventual probability distribution over signal pairs and the decision maker's optimal strategy. The following is a list:[13] The probabilities are calculated according to

(m^A, m^B)	$\text{Prob}(m^A, m^B, s = -1 + 0 + 1)$	$y(m^A, m^B)$
$(1,1)$	$0 + \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} + \frac{1}{3}$	$\frac{9}{10}$
$(0,1)$	$0 + \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} + 0$	0
$(1,-1)$	$0 + \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} + 0$	0
$(0,0)$	$0 + \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} + 0$	0

the experts' strategies. For example, $\text{Prob}(0,1,s=0) = P(s=0) \times P(x^A = 0) \times P(x^B = 1)$, because in state 0, only type 0 experts report 0 and only type 1 experts report 1. The probabilities and decisions for the omitted pairs can be inferred from symmetry and anonymity. Take $(-1,-1)$ for example. The probabilities should be derived from $(1,1)$,

[13] The notation $\text{Prob}(m^A, m^B, s = -1 + 0 + 1)$ means that in that column, probabilities of (m^A, m^B, s) are separated by "+" according to different s.

which are $\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$, and the decision should be $-y(1,1)=-\frac{9}{10}$. Given these we may check the optimality of strategies of experts. Now given $s=-1$, an expert of type 1 would earn $u(-\frac{9}{10},-1,b_1)$ if she chooses $m_1^A(-1)=-1$, or $u(0,-1,b_1)$ if she chooses $m_1^A(-1)=0$. The latter is higher since 0 is closer to $-1+b$ (her most preferred action when $s=-1$) than $-9/10$ is. So $m_1^A(-1)=-1$ is not optimal.

(ii) $m_1^A(-1)=0$. Again, the following is the table of probabilities and decisions: Now we check the optimality of $m_1^A(-1)=0$. I omit the calculation

(m^A,m^B)	$\text{Prob}(m^A,m^B,s=-1+0+1)$	$y(m^A,m^B)$
$(1,1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}$	$\dfrac{4}{5}$
$(0,1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}$	$\dfrac{2}{3}$
$(1,-1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0
$(0,0)$	$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$	0

here, but it turns out for $t=-1$ and 1
$\frac{1}{3}\sum_{x\in X}u(y(0,m_x^A(0)),-1,b_1)>\frac{1}{3}\sum_{x\in X}u(y(t,m_x^A(0)),-1,b_1)$.
Thus $m_1^A(-1)=0$ is optimal. Note that this is exactly **strategy profile (A)**. The expected payoff of the decision maker when $s=-1$ or $s=-1$ is

$$\begin{aligned}
& -[P(x^A=1or0,x^B=1or0)\times(y(1,1)-1)^2 \\
& +2\times P(x^A=-1,x^B=1or0)\times(y(0,1)-1)^2 \\
& +P(x^A=-1,x^B=-1)\times(y(0,0)-1)^2] \\
= & -[\frac{2}{3}\times\frac{2}{3}\times(\frac{4}{5}-1)^2+2\times\frac{1}{3}\times\frac{2}{3}\times(\frac{2}{3}-1)^2+\frac{1}{3}\times\frac{1}{3}\times(0-1)^2] \\
= & -\frac{361}{2025}.
\end{aligned}$$

His expected payoff when $s=0$ is

$$\begin{aligned}
& -[2\times P(x^A=1,x^B=1)\times(y(1,1)-0)^2 \\
& +2\times P(x^A=0,x^B=1or-1)\times(y(0,1)-1)^2 \\
= & -[2\times\frac{1}{3}\times\frac{1}{3}\times(\frac{4}{5}-0)^2+2\times\frac{1}{3}\times\frac{2}{3}\times(\frac{2}{3}-0)^2] \\
= & -\frac{688}{2025}.
\end{aligned}$$

Thus his expected payoff is $-\frac{1}{3}(\frac{361}{2025}+\frac{688}{2025}+\frac{361}{2025})=-\mathbf{\frac{94}{405}}$.

(iii) $m_1^A(-1) = 1$. The following is the table of probabilities and decisions:

(m^A, m^B)	$\text{Prob}(m^A, m^B, s = -1 + 0 + 1)$	$y(m^A, m^B)$
$(1,1)$	$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}$	$\frac{1}{2}$
$(0,1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0
$(1,-1)$	$\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}$	0
$(0,0)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0

Now we check the optimality of $m_1^A(-1) = 1$. Note that $u(y(0,1), -1, b_1) = u(0, -1, b_1) > u(1/2, -1, b_1) = u(y(1,1), -1, b_1)$ and $u(y(0,-1), -1, b_1) = u(0, -1, b_1) = u(y(1,-1), -1, b_1)$. Thus $m_1^A(-1) = 0$ is strictly better than $m_1^A(-1) = 1$. Hence $m_1^A(-1) = 1$ is not optimal.

Summarizing the above arguments proves us the proposition. □

Proof. (of Lemma 3) By symmetry $m_0^A(0) = 0$, $m_0^B(0,0) = 0$ and $y(0,0) = 0$. Due to monotonicity and the rationalizability requirement for y (that is $y \in [-1,1]$), I have $m_0^B(1, m^A), m_1^B(1, m^A) \in \arg\max_{m^B} y(m^A, m^B)$. By monotonicity $m_1^A(1) = 1$, otherwise $m_x^A(s) = 0$ for all $x \in X$ and $s \in S$, making the first expert's reports uninformative, violating the informativeness condition.
Now I show Part 3. Note that by monotonicity $y(1,1) \geq y(1,0) \geq y(0,0) = 0$ yet in state 0 the most preferred action for an expert of type -1 is $-b$. Thus $y(1,-1)$ is as good as or better than $y(1,0)$ and $y(1,1)$ for her. Hence $m_{-1}^B(0,1) \in argmin_{m^B} y(1, m^B)$. To show that $m_1^B(0,0) \in \arg\max_{m^B} y(0, m^B)$, note that $y(0,1) \geq y(0,0) = 0 \geq y(0,-1)$. Thus $y(0,1)$ is closer to b than $y(0,0)$ and $y(0,-1)$ since $b > \frac{1}{2}$. Hence $m_1^B(0,0) \in \arg\max_{m^B} y(0, m^B)$.
To show that $m_0^A(1) = 1$ I show that the other two messages are impossible.

First, $m_0^A(1) = -1$ would violate monotonicity.

Second, $m_0^A(1) = 0$ (hence $m_0^A(-1) = 0$ by symmetry). Then by monotonicity $m_1^A(-1) = 0$ or 1.

1. Now consider the case $m_1^A(-1) = 0$.

    - $m_1^A(0) = 0$. Then $y(1, m^B) = 1$ for all $(1, m^B)$ that are sent in equilibrium since $m^A = 1$ only when $s = 1$. Given this $m_0^A(1) = 0$ could be optimal only if $y(0,1) = 1$ and $m_{-1}^B(1,0) = 1$. But the latter is impossible since $y(0,0)$ is closer to $1 - b$ than $y(0,1) = 1$.

- $m_1^A(0) = 1$. First $m_1^B(-1,0) = 1$ only if $y(0,1) = y(0,0) = 0$, since in state -1 a type 1 expert's most preferred action is $-1+b < 0$. By symmetry $y(0,-1) = 0$. Monotonicity implies $y(1,-1) \geq 0$. Thus $m_0^A(1) = 0$ guarantees an action of 0 from the decision maker, while $m_0^A(1) = 1$ guarantees nonnegative actions from the decision maker and strictly positive action some of the time due to informativeness. This implies that $m_0^A(1) = 1$ is better than $m_0^A(1) = 0$.
  If $y(0,1) > y(0,0)$ then $m_1^B(-1,0) = 0$ or -1. We need $m_1^B(1,0) = m_0^B(1,0) = 1$, thus $y(0,1) \geq 4/5$. This has two implications. The first one is $m_1^B(-1,0) = 0$ since $b > \frac{3}{5}$, which implies $y(0,1) = \frac{4}{5}$. The second one is that by monotonicity $y(1,1) \geq 4/5$. On the other hand, $y(1,m^B) \leq \frac{3}{4}$ for any m^B if $m_v^B(0,1) = m^B$ for some $v \in X$. But as proven above $m_0^B(1,1), m_1^B(1,1) \in \arg\max_{m^B} y(1,m^B)$. Thus $y(1,m) \geq \frac{4}{5}$ for all $m \in \arg\max_{m^B} y(1,m^B)$, which leaves the only possibility $\max_{m^B} y(1,m^B) = 1$. Hence $m_0^A(1) = 0$ induces actions $y(0,1) = \frac{4}{5}$ with probability $\frac{2}{3}$ and 0 with probability $\frac{1}{3}$, while $m_0^A(1) = 1$ induces actions $\max_{m^B} y(1,m^B) = 1$ with probability $\frac{2}{3}$ and another nonnegative action with probability $\frac{1}{3}$. This means that $m_0^A(1) = 1$ is a better response.

2. Now we consider the case $m_1^A(-1) = 1$. We get $m_1^A(0) = 1$ by monotonicity. Thus

$$\sum_{m^B} P(m^B|m^A = 0)y(0,m^B) = \sum_{m^B} P(m^B|m^A = 0)y(0,m^B) = 0.$$

   Since $m^A = 1$ happens in state -1, by monotonicity $y(1,-1) < 0$ and $y(0,-1) < 0$. Thus $m_{-1}^B(-1,1) = m_0^B(-1,1) = -1$ and $m_1^B(0,-1) = 1$. We can also conclude that $m_1^B(-1,0) = 0$ or -1 since $y(0,1)$ is positive and hence farther from $-1+b$ than $y(0,0) = 0$.
   If $m_1^B(-1,0) = -1$ then by the proven parts of this lemma $y(0,-1) = -3/4$, which makes $m_1^B(-1,0) = -1$ not optimal since $y(0,0) = 0$ is closer to $-1+b$. So $m_1^B(-1,0) = 0$ and $y(0,-1) = -2/3$. But this implies $y(1,-1) \geq -2/3$ and $y(-1,-1) \leq -2/3$ by monotonicity. These imply $m_1^B(-1,0) = 0$ since a type 1 expert's most preferred action in state -1 is $-1+b$, which lies in the interval $(-\frac{1}{3}, 0)$. We can also conclude $m_1^B(1,-1) = m_0^B(1,-1) = 1$.
   As proven above $m_1^B(1,1), m_0^B(1,1) \in \arg\max_{m^B} y(1,m^B)$. We separate our discussion into two different cases.

Case 1. $y(1,0) = y(1,1) \geq \frac{2}{3}$.

We claim $m_0^B(0,1) = -1$. Suppose to the contrary $m_0^B(0,1) = 0$ or 1. We have proved that $m_1^B(0,1) = 0$ or 1. This would cause $P(s = 1|(m^A, m^B) = (1,0)$ or $(1,1)) \leq \frac{2}{5}$, contradicting the assumption $y(1,0) = y(1,1) \geq \frac{2}{3}$.
We also need $m_1^B(-1,1) = -1$, otherwise $y(1,0) \leq \frac{2}{5}$.
Now if $m_{-1}^B(1,1) = 0$ or 1, then $m_0^A(1) = 1$ is a better response than $m_0^A(1) = 0$. Since the former ensures that the decision maker takes the action $y(1,0) = y(1,1) \geq \frac{2}{3}$, while the latter induces actions $y(0,0=0)$ with probability $\frac{1}{3}$ and $y(0,1) = \frac{2}{3}$ with probability $\frac{2}{3}$.
If $m_{-1}^B(1,1) = -1$ then $y(1,-1) = -\frac{1}{3}$, and $y(1,0) = y(1,1) = \frac{2}{3}$. But in this case $m_1^A(1) = 1$ would be a worse response than $m_1^A(1) = 0$, a contradiction. The former induces actions $y(1,-1) = -\frac{1}{3}$ with probability $\frac{1}{3}$ and $y(1,0) = y(1,1) = \frac{2}{3}$ with probability $\frac{2}{3}$, while the latter induces the same distribution of actions described in the previous paragraph.

Case 2. $y(1,1) > y(1,0)$. This implies $m_1^B(1,1) = m_0^B(1,1) = 1$, $m_1^B(-1,1) \neq 1$, and $m_0^B(0,1) \neq 1$. We may also conclude that $m_{-1}^B(1,1) \neq 1$ since $y(1,1) \geq \frac{2}{3}$, $y(1,0) \geq 0$, and a type -1 expert's most preferred action in state 1 is $1 - b < \frac{1}{3}$. Now we consider the two possibilities:

- $m_1^B(0,1) = 0$. This implies that $y(1,1) = 1$. If $m_{-1}^B(1,1) = -1$ then $y(1,0) = 0$ since $y(1,0) \geq 0$ by monotonicity. But then $m_{-1}^B(1,1) = 0$ is better than $m_{-1}^B(1,1) = -1$ since $y(1,-1) < 0$ and $b < 1$. Thus $m_{-1}^B(1,1) = 0$. Now $m_0^A(1) = 1$ is a better response than $m_0^A(1) = 0$ since the former induces $y(1,0) \geq 0$ with probability $\frac{1}{3}$ and $y(1,1) = 1$ with probability $\frac{2}{3}$, while the latter induces $y(0,0) = 0$ with probability $\frac{1}{3}$ and $y(0,1) = \frac{2}{3}$ with probability $\frac{2}{3}$.
- $m_1^B(0,1) = 1$. This implies $y(1,1) = \frac{2}{3}$.
  If $m_{-1}^B(1,1) = -1$ then $m_0^B(0,1) = -1$, since $m_0^B(0,1) = 0$ implies $y(1,0) = 0$, and $m_{-1}^B(1,1) = 0$ would be a better response than $m_{-1}^B(1,1) = -1$. Thus $y(1,-1) < 0 = y(0,0)$ and $y(1,1) = y(0,1)$, and $m_1^A(1) = 1$ would not be optimal. A contradiction.
  If $m_{-1}^B(1,1) = 0$ then $m_0^A(1) = 1$ is a better response than $m_0^A(1) = 0$ and constitutes a contradiction unless $y(1,0) = 0$. To make $y(1,0) = 0$ we need $m_1^B(-1,1) = 0$, and $y(1,0) = 0$ implies $m_0^B(0,1) = 0$. Thus the second expert's report is completely uninformative.

The above arguments show that the only possibility is $m_0^A(1) = 1$. □

Proof. (of Proposition 4) According to Lemma 3, the only parts of expert A's strategy

left to be determined are $m_1^A(-1)$ and $m_1^A(0)$.
I proceed by considering all possible combinations.

Case 1. $m_1^A(-1) = -1$ and $m_1^A(0) = 0$. No matter what expert B does, the state is perfectly revealed to the decision maker. But given this $m_1^A(-1) = -1$ is not optimal since 0 is closer than -1 is to her most preferred action $-1 + b > -\frac{1}{2}$.

Case 2. $m_1^A(-1) = -1$ and $m_1^A(0) = 1$. In this case since $m^A = 0$ only happens when $s = 0$, $y(0, m^B) = 0$ for all m^B such that $m_v^B(0, 0) = m^B$ for some $v \in X$. Now in order for the second expert's report to be informative, we need $y(1, 1) > y(1, -1)$. This implies $m_0^B(0, 1) \neq 1$. Note that $y(1, 1) \geq \frac{3}{4}$ since $P(s = 1|m^A = 1) = \frac{3}{4}$ and $s \geq 0$ when $m^A = 1$. Thus $y(-1, -1) \leq -\frac{3}{4}$.
Now we find a contradiction since $m_1^A(-1) = 0$ is a better response than $m_1^A(-1) = -1$. The former induces action 0 by the decision maker, while the latter induces $y(-1, -1) \leq -\frac{3}{4}$ with probability $\frac{2}{3}$. The difference in expected utility is thus greater than

$$-(0 - (-1 + b))^2 - [-\frac{2}{3}(-\frac{3}{4} - (-1 + b))^2] = -\frac{1}{3}(b^2 - 5b + \frac{23}{8}) > 0$$

for all $b \geq \frac{2}{3}$. Hence $m_1^A(-1) = 0$ is better.

Case 3. $m_1^A(-1) = 0$ and $m_1^A(0) = 0$. In this case $y(1, m^B) = 1$ for all m^B such that $m_v^B(1, 1) = m^B$ for some $v \in X$. In order for the second expert's report to be informative, we need $y(0, 1) > y(0, 0) > y(0, -1)$. Thus $m_1^B(0, 0) = 1$ since $b > \frac{1}{2}$. We may also conclude $m_{-1}^B(-1, 0) = m_0^B(-1, 0) = -1$.

Now we only need to determine $m_1^B(-1, 0)$. First $m_1^B(-1, 0) \neq 1$ since $y(0, 1) > y(0, 0) > 0 > -1 + b$. If $m_1^B(-1, 0) = -1$, then $y(0, -1) = -\frac{1}{2}$. But $m_1^B(-1, 0) = -1$ would not be optimal since $b > \frac{3}{4}$ implies $-1 + b$ is closer to $y(0, 0)$ than $y(0, -1)$.

If $m_1^B(-1, 0) = 0$, then $y(0, -1) = -\frac{2}{5}$. Thus $m_1^B(-1, 0) = 0$ is optimal if $b \geq \frac{4}{5}$, which is satisfied by our assumed values of b. Now we compare the expected utility of a type 1 expert from $m_1^A(0) = 0$ and $m_1^A(0)=1$. By reporting the former, the expert induces actions $y(0, -1) = -y(0, 1)$, $y(0, 0)$, and $y(0, 1)$ with equal probabilities. By reporting the latter, she induces actions $y(1, m_v^B(0, 1))$ with equal probabilities for $v = -1, 0, 1$. In order for $m_1^A(0) = 0$ to be optimal, we need the difference in expected utility to be nonnegative. That is,

$$-\frac{1}{3}[(b - y(0, 1))^2 + (b - 0)^2 + (b - (-y(0, 1)))^2] - (-\frac{1}{3}) \sum_v (b - y(1, m_v^B(0, 1)))^2 \geq 0.$$

We need appropriate off-equilibrium behavior by the second expert and the decision maker in order for the above condition to be satisfied. Note that

$$\left|b - y(1, m_1^B(0,1))\right| \leq |b - y(1,1)| = 1 - b,$$

and that

$$\left|b - y(1, m_v^B(0,1))\right| \leq \max_{m^B} \left|b - y(1, m^B)\right| \leq \max\{b, b - y(1,-1)\}.$$

Thus it is necessary that either

$$-\frac{1}{3}[3b^2 + 2y(0,1)^2] - (-\frac{1}{3})[(1-b)^2 + 2b^2] \geq 0 \text{ and } y(1,-1) \geq 0$$

or

$$-\frac{1}{3}[3b^2 + 2y(0,1)^2] - (-\frac{1}{3})[(1-b)^2 + 2(b - y(1,-1))^2] \geq 0 \text{ and } y(1,-1) \leq 0.$$

The first inequality is impossible since $b > 1 - b > 0$. The second inequality on the other hand would imply $y(1,-1) \in [-\frac{2}{5}, 0]$ since $y(0,-1) \leq 0$. But an expert B of type -1 would prefer $y(1,-1)$ to $y(1,1) = 1$ in state 1 since it is closer to her most preferred action $1 - b$, due to our assumption $b \in [\frac{17}{21}, \frac{6}{7}]$. It is now a contradiction since I have shown above that $y(1, m^B) = 1$ for all m^B such that $m_v^B(1,1) = m^B$ for some $v \in X$.

Case 4. $m_1^A(-1) = 0$ and $m_1^A(0) = 1$. In this case $y(1, m^B) \geq 0$ for all m^B such that $m_v^B(s,1) = m^B$ for some $v \in X$ and $s \in S$.

In order to ensure that the second expert's reports be informative we must have $y(0,-1) < y(0,0)$ (which implies $y(0,0), y(0,1)$ by symmetry) or $y(1,-1) < y(1,1)$.

1. $y(0,-1) < y(0,0)$.
   This immediately implies $m_1^B(0,0) = 1$, $m_1^B(1,0) = 1$, and $m_0^B(1,0) = 1$. We also know $m_1^B(-1,0) \neq 1$ since $y(0,0)$ is better than $y(0,1)$ in state -1 for an expert of type 1. From these facts we conclude $y(0,1) \geq \frac{2}{3}$. Hence $m_1^B(-1,0) = 0$ since a type 1 expert's most preferred action in state -1 is $-1 + b > -\frac{1}{3}$, which is closer to $y(0,0) = 0$ than to $y(0,-1)$. We conclude $y(0,1) = \frac{2}{3}$.
   If $y(1,-1) = y(1,0) = y(1,1)$, they must all be equal to $\frac{2}{3}$. Thus it does not matter what the $m_v^B(s,1)$ are as long as they are such that $y(1, m^B) = \frac{2}{3}$ for all m^B such that $m_v^B(s,1) = m^B$ for some $s \in S$ and $v \in X$. It remains to check whether $m_1^A(-1) = 0$ and $m_1^A(0) = 1$ are optimal. The strategy $m_1^A(-1) = 0$ is optimal since $m_1^A(-1) = 0$ induces actions $y(0,-1) = -\frac{2}{3}$ with probability $\frac{2}{3}$ and $y(0,0) = 0$ with probability $\frac{1}{3}$, while $m_1^A(-1) = -1$ induces action $y(-1, m^B) = -\frac{2}{3}$ for sure, and 0 is closer than $-\frac{2}{3}$ to the expert's favorite

action $-1+b$ in state -1. The strategy $m_1^A(-1)=1$ is even worse since it induces action $\frac{2}{3}$ for sure, which is worse than all the actions mentioned above. The strategy $m_1^A(0)=1$ is optimal since $m_1^A(0)=1$ induces action $y(1,m^B)=\frac{2}{3}$ for sure, while $m_1^A(0)=0$ induces actions $y(0,-1)=-\frac{2}{3}$, $y(0,0)=0$, and $y(0,1)=\frac{2}{3}$ with equal probabilities, and 0 and $-\frac{2}{3}$ are farther than $\frac{2}{3}$ from her most preferred action b in state 0. This strategy profile thus constitutes an equilibrium since every player is playing his or her best responses. Note that this is exactly **strategy profile (B)**. In any strategy profile, the decision maker's expected payoff can be written

$$\sum_s \sum_{m^A} \sum_{m^B} P(s,m^A,m^B)u(y(m^A,m^B),s,0),$$

where $P(s,m^A,m^B)$ is the probability of the state-messages triple (s,m^A,m^B) occurring in equilibrium. Thus the decision maker's payoff in strategy profile (B) is

$$\begin{aligned} & -2\cdot\tfrac{1}{3}\{\tfrac{2}{3}(\tfrac{2}{3}-1)^2+\tfrac{1}{3}[\tfrac{2}{3}(\tfrac{2}{3}-1)^2+\tfrac{1}{3}(0-1)^2]\} \\ & -\tfrac{1}{3}\{2\cdot\tfrac{1}{3}(\tfrac{2}{3}-0)^2+\tfrac{1}{3}[2\cdot\tfrac{1}{3}(\tfrac{2}{3}-0)^2+\tfrac{1}{3}\cdot 0]\} \\ = & -\tfrac{26}{81} \end{aligned}$$

Now we consider the possibility $y(1,-1)<y(1,1)$. Note that $y(1,1)=y(1,0)$ if the message pair $(1,1)$ is not sent in equilibrium, since otherwise we would have $m_1^B(1,1)=1$. Similarly $y(1,-1)=y(1,0)$ if the message pair $(1,-1)$ is not sent in equilibrium, since otherwise we would have $m_{-1}^B(0,1)=-1$. Now we separate our discussion into two cases according to the number of different actions $y(1,m^B)$.

- $y(1,1)=y(1,0)$ or $y(1,-1)=y(1,0)$ (but not both).
  If $y(1,1)=y(1,0)$, it is possible that $(1,1)$ or $(1,0)$ is never sent, but it does not matter to our discussion since we may replace them with each other without changing the essential strategy profile. Now we have $m_{-1}^B(1,1)=-1$ and $m_{-1}^B(0,1)=m_0^B(0,1)=-1$. Hence $y(1,-1)\leq\frac{1}{2}$, which implies $m_1^B(0,1)\neq-1$ since $y(1,1)$ is closer than $\frac{1}{2}$ is to a type 1 expert's most preferred action b in state 0. Thus we get $y(1,1)=y(1,0)=\frac{4}{5}$, $y(1,-1)=\frac{1}{2}$. The case $y(1,0)=y(1,-1)$ is similar. Now we need to check the optimality of $m_1^A(-1)=0$ and $m_1^A(0)=1$. Note that $m_1^A(-1)=0$ induces actions $y(0,-1)=-\frac{2}{3}$ with probability $\frac{2}{3}$ and $y(0,0)$ with probability $\frac{1}{3}$, that $m_1^A(-1)=-1$ induces actions $y(-1,-1)=-\frac{4}{5}$ with probability $\frac{2}{3}$ and $y(-1,1)=-\frac{1}{2}$ with probability $\frac{1}{3}$, and that $m_1^A(-1)=1$ induces action

$y(1,-1) = \frac{1}{2}$ for sure. Thus $m_1^A(-1) = 0$ is better than $m_1^A(-1) = -1$ since a type 1 expert prefers $-\frac{2}{3}$ to $-\frac{4}{5}$ and 0 to $-\frac{1}{2}$ in state -1, due to our assumption $b \in [\frac{17}{21}, \frac{6}{7}]$. The difference in expected utility between $m_1^A(-1) = 0$ and $m_1^A(-1) = 1$ is

$$-\frac{1}{3}[2(-\frac{2}{3} - (-1+b))^2 + (0 - (-1+b))^2] - (\frac{1}{2} - (-1+b))^2 = \frac{199}{36} - \frac{17}{3}b,$$

which is positive as long as $b < \frac{199}{204}$. Thus $m_1^A(-1) = 0$ is better than $m_1^A(-1) = 1$. Second, $m_1^A(0) = 1$ induces actions $y(1,1) = \frac{4}{5}$ with probability $\frac{1}{3}$ and $y(1,-1) = \frac{1}{2}$ with probability $\frac{2}{3}$, while $m_1^A(0) = 0$ induces actions $y(0,1) = \frac{2}{3}$, $y(0,0) = 0$, and $y(0,-1) = -\frac{2}{3}$ with equal probabilities. Thus $m_1^A(0) = 1$ is a better response than $m_1^A(0) = 0$ since a type 1 expert prefers $\frac{4}{5}$ to $\frac{2}{3}$ and $\frac{1}{2}$ to any nonpositive action, due to our assumption $b \in [\frac{17}{21}, \frac{6}{7}]$. Thus the strategy profile constitutes an equilibrium. Note that it is **strategy profile (C)**. In this strategy profile, the decision maker's expected payoff is

$$\begin{aligned} & -2 \cdot \tfrac{1}{3}\{\tfrac{2}{3}[\tfrac{2}{3}(\tfrac{4}{5} - 1)^2 + \tfrac{1}{3}(\tfrac{1}{2} - 1)^2] + \tfrac{1}{3}[\tfrac{2}{3}(\tfrac{2}{3} - 1)^2 + \tfrac{1}{3}(0 - 1)^2]\} \\ & -\tfrac{1}{3}\{2 \cdot \tfrac{1}{3}[\tfrac{2}{3}(\tfrac{1}{2} - 0)^2 + \tfrac{1}{3}(\tfrac{4}{5} - 0)^2] + \tfrac{1}{3}[2 \cdot \tfrac{1}{3}(\tfrac{2}{3} - 0)^2 + \tfrac{1}{3} \cdot 0]\} \\ = & -\tfrac{104}{405} \end{aligned}$$

- $y(1,1) > y(1,0) > y(1,-1)$.
  This implies $m_1^B(1,1) = m_0^B(1,1) = 1$ and $m_{-1}^B(0,1) = m_0^B(0,1) = -1$. Therefore $y(1,-1) \leq \frac{1}{2}$, which implies that $m_1^B(0,1) \neq -1$.
  If $m_1^B(0,1) = 0$ then we need $|y(1,0) - b| \leq |y(1,1) - b|$, which implies $y(1,0) \geq 2b - y(1,1) \geq 2b - 1 > \frac{1}{2}$. The last inequality sign is due to our assumption that $b \in [\frac{17}{21}, \frac{6}{7}]$. This requires $m_{-1}^B(1,1) = 0$, since $m_1^B(1,1) = m_0^B(1,1) = 1$. But given $y(1,0) > \frac{1}{2}$, $m_{-1}^B(1,1) = 0$ is not optimal since $y(1,-1) \in [0, y(1,0))$.
  If $m_1^B(0,1) = 1$, then $y(1,0) = 1$ as long as $(1,0)$ is sent in equilibrium, which would violate $y(1,0) < y(1,1)$.
  Thus $(1,0)$ is never sent in equilibrium, and this case collapses into strategy profile (C).

2. $y(0,0) = y(0,1) = 0$ and hence $y(1,-1) < y(1,1)$ by informativeness. The analysis is similar to that above, and the only possible equilibrium involves $y(1,1) = \frac{4}{5}$ and $y(1,-1) = \frac{1}{2}$.
   It remains to check the optimality of $m_1^A(0) = 1$ and $m_1^A(-1) = 0$. The strategy $m_1^A(0) = 1$ induces actions $y(1,1) = \frac{4}{5}$ with probability $\frac{1}{3}$ and $y(1,-1) = \frac{1}{2}$ with

probability $\frac{2}{3}$, while $m_1^A(0) = 0$ induces action 0 for sure. All positive actions are preferred to 0 since $b > \frac{1}{2}$. Hence $m_1^A(0) = 1$ is optimal. The strategy $m_1^A(-1) = 0$ induces the action 0 for sure, $m_1^A(-1) = -1$ induces actions $y(-1,-1) = -\frac{4}{5}$ with probability $\frac{2}{3}$ and $y(-1,1) = -\frac{1}{2}$ with probability $\frac{1}{3}$, and $m_1^A(-1) = 1$ induces the action $y(1,-1) = \frac{1}{2}$ for sure. Among all these actions 0 is a type 1 expert's most preferred action in state -1. Thus $m_1^A(-1) = 0$ is optimal.

Note that this is exactly **strategy profile (D)**. In this strategy profile, the decision maker's expected payoff is

$$\begin{aligned} & -2 \cdot \tfrac{1}{3}\{\tfrac{2}{3}[\tfrac{2}{3}(\tfrac{4}{5} - 1)^2 + \tfrac{1}{3}(\tfrac{1}{2} - 1)^2] + \tfrac{1}{3}(0 - 1)^2\} \\ & -\tfrac{1}{3}\{2 \cdot \tfrac{1}{3}[\tfrac{2}{3}(\tfrac{1}{2} - 0)^2 + \tfrac{1}{3}(\tfrac{4}{5} - 0)^2] + \tfrac{1}{3} \cdot 0\} \\ = \; & -\tfrac{16}{45} \end{aligned}$$

Case 5. $m_1^A(0) = 1$ and $m_1^A(-1) = 1$.

Again for any m^B such that $(0, m^B)$ is sent in equilibrium, we have $y(0, m^B) = 0$. This implies that $y(0,1) = 0$. Otherwise $m_1^B(0,0) = 1$ would be true as $b > \frac{1}{2}$. Now monotonicity requires $y(1,-1) \geq 0$ since $y(0,-1) = 0$. In order for expert B's opinion to be informative, we need $y(1,-1) < y(1,1)$. So $m_v^B(-1,1) \neq 1$ for any $v \in X$ since $b < 1$. Now $y(1,-1) \geq 0$ is impossible since $m_1^B(1,1)$ and $m_0^B(1,1)$ both belong to $\arg\max_{m^B} y(1, m^B)$.

Summarizing the above arguments gives us the proposition. □

Proof. Proof (of Lemma 5) Using Equation (2), I show $r_0^*(1,-1) = r_0^*(-1,1) = r_0^*(0,1) = r_0^*(0,-1) = 1$ and $r_1^*(1,-1) = r_1^*(0,-1) = 1$. Recall that the reviewer wants the decision to be as close to her most preferred action as possible. In all these cases, $|y_t^* - (s + b_v)| = \max_{t' \in S} |y_{t'}^* - (s + b_v)|$, and there exists $\tilde{t} \in S$ such that $|y_t^* - (s + b_v)| > \left|y_{\tilde{t}}^* - (s + b_v)\right|$. For example, when $v = 0, s = 0, t = 1$, $|y_1^* - (0 + 0)| = |-y_1^* - (0 + 0)| > |0 - (0 + 0)|$.

On the other hand, I can use Equation (2) to show that $r_0^*(s,s) = 0$ and $r_1^*(1,1) = r_1^*(0,1) = 0$ because in these cases $|y_t^* - (s + b_v)| = \min_{t' \in S} |y_{t'}^* - (s + b_v)|$

Simplifying Equation (2) further using condition (SE-3), I have $r_v^*(s,t) = 1$ if and only if

$$2\gamma(y_1^*)^2 + (s + b_v)^2 < (y_t^* - (s + b_v))^2. \tag{6}$$

Since $y_0^* = 0$ and $\gamma > 0$, I get that for $t = 0$, L.H.S. > R.H.S. for all s, v. Thus it must be that $r_v(s,0) = 0$ for all $v \in X, s \in S$.

Since 0 is never rejected, I conclude $m_0(0) = 0$. Now I argue $m_0(1) = m_1(1) = 1$. First by Lemma 4 and the results proven above they are not equal to -1. By reporting 0 in state 1, an expert of type $x \in 0, 1$ receives an expected payoff of

$$(0 - (1 + b_v))^2.$$

When reporting 1 in state 1, a reviewer of type 0 or 1 would accept it, but a reviewer of type -1 may reject it. Thus the expert's expected payoff is greater than or equal to

$$\begin{aligned} & -P(v = 0, 1)(y_1^* - (1 + b_v))^2 - P(v = -1) \textstyle\sum_{t \in S}(y_t^* - (1 + b_v))^2 \\ = \quad & -\tfrac{2}{3}(y_1^* - (1 + b_v))^2 - \tfrac{1}{3}(2\gamma(y_1^*)^2 + (1 + b_v)^2). \end{aligned}$$

Now the difference in utility between reporting 1 and reporting 0 is at least

$$2 \cdot \frac{2}{3}(1 + b_v)y_1^* - \frac{2}{3}(1 + \gamma)(y_1^*)^2.$$

This expression is positive since $b_v \geq 0$, $y_1^* \in (0, 1]$, and $\gamma < \frac{1}{2}$. Hence $m_1(1) = m_0(1) = 1$. I have thus finished the proof of part 2.

The only review decisions left to check are $r_1(-1, 1)$ and $r_1(-1, -1)$. Substituting $v = 1,\ s = -1,\ t = 1$ into Equation 6, we get the condition for $r_1(-1, 1) = 1$ to be

$$0 \quad < \quad (1 - 2\gamma)(y_1^*)^2 + 2(1 - b)y_1^*$$

This inequality is always true for any $y_1^* > 0$, so $r_1(-1, 1) = 1$.

Now for $r_1(-1, -1)$, by Equation 6

$$0 \quad < \quad (1 - 2\gamma)(y_1^*)^2 - 2(1 - b)y_1^*$$

The inequality holds only if

$$y_1^* > \frac{2(1 - b)}{1 - 2\gamma}.$$

We now show that $r_1(-1, -1) = 0$. Suppose $r_1(-1, -1) = 1$ instead. By Lemma 4, we have $m_1(-1) \neq -1$. Since we have shown above $r_1(-1, 1) = 1$, again by Lemma 4 we have $m_1(-1) = 0$.

Now we only need discuss $m_1(0)$. By Lemma 4 it can only be either 0 or 1 since we have shown $r_1(0, -1) = 1$.

To make sure $m_1(0) = 0$ we must require that an expert A of type 1 gets higher expected utility from reporting 0 than reporting 1 in state 0. Reporting 0 guarantees an expected utility of $-(0 - b)^2$, while reporting 1 induces rejection from reviewers of types 0 and -1 and acceptance from reviewers of type 1, and results an expected utility of $-\frac{2}{3}[(1 - \alpha)(y_1^* - b)^2 + \alpha(2\gamma(y_1^*)^2 + b^2)] - \frac{1}{3}(y_1^* - b)^2$. Thus the difference in utility between reporting 1 and reporting 0 is $2b(1-\frac{2}{3}\alpha)y_1^* - [1-\frac{2}{3}\alpha(1-2\gamma)](y_1^*)^2$. We need the above expression to be less than or equal to 0, which translates into

$$y_1^* \geq \frac{2b(1-\frac{2}{3}\alpha)}{1-\frac{2}{3}\alpha(1-2\gamma)}.$$

In this strategy profile

$$\begin{aligned}\gamma &= P(s=1)P(x=0 \text{ or } 1)[P(v=0 \text{ or } 1)+P(v=-1)(1-\alpha+\alpha\gamma)] \\ &\quad +P(s=-1)P(x=0 \text{ or } -1)P(v=1)\alpha\gamma \\ &= \tfrac{1}{3}\times\tfrac{2}{3}[\tfrac{2}{3}+\tfrac{1}{3}(1-\alpha(1-2\gamma))] \\ &= \tfrac{1}{9}+\tfrac{1}{9}[1-\tfrac{2}{3}\alpha(1-2\gamma)]\end{aligned}$$

We define $\theta = 1-\frac{2}{3}\alpha$. Thus $\alpha = \frac{3}{2}(1-\theta)$ and $\theta \in [\frac{1}{3},1]$. From the above expression we derive $1-\frac{2}{3}\alpha(1-2\gamma) = 9\gamma - 1$ and $\gamma = \frac{\frac{1}{9}(1+\theta)}{1-\frac{2}{9}(1-\theta)}$. Furthermore $y_1^*\gamma = P(s=1, m=1) - P(s=-1, m=1) = \frac{1}{9}(1+\theta)$. So in order for $m_1(0)=0$ to be optimal, we need $y_1^* \geq \frac{2b\theta}{9\gamma-1}$, which simplifies into

$$\theta \leq \frac{\frac{2}{9}}{2b-\frac{7}{9}} < \frac{1}{3}.$$

The last inequality comes from our assumption that $b \in [\frac{17}{21}, \frac{6}{7}]$ (hence $b > \frac{13}{18}$). We have found a contradiction since $\theta \in [\frac{1}{3}, 1]$.
If $m_1(0) = 1$, then

$$\begin{aligned}\gamma &= P(s=0)P(x=1)[P(v=1)+P(v=0 \text{ or } -1)(1-\alpha+\alpha\gamma)] \\ &\quad +P(s=0)P(x=-1)P(v=0 \text{ or } 1)\alpha\gamma \\ &\quad +P(s=1)P(x=0 \text{ or } 1)[P(v=0 \text{ or } 1)+P(v=-1)(1-\alpha+\alpha\gamma)] \\ &\quad +P(s=-1)P(x=0 \text{ or } -1)P(v=1)\alpha\gamma \\ &= \tfrac{1}{3}\times\tfrac{1}{3}[1-\tfrac{2}{3}\alpha(1-\gamma)]\tfrac{1}{3}\times\tfrac{2}{3}[\tfrac{2}{3}+\tfrac{1}{3}(1-\alpha(1-2\gamma))] \\ &= \tfrac{1}{9}+\tfrac{2}{9}[1-\tfrac{2}{3}\alpha(1-2\gamma)]\end{aligned}$$

From this we obtain $\gamma = \frac{\frac{1}{9}(1+2\theta)}{1-\frac{4}{9}(1-\theta)}$. Again $\gamma y_1^* = \frac{1}{9}(1+\theta)$. Substituting this into the condition for $r_1(-1,-1) = 1$, we have

$$\theta \leq \frac{\frac{1}{3}-2(1-b)}{4(1-b)-\frac{1}{3}}$$

if $\theta \leq \frac{8}{9}$. But when $b \in [\frac{17}{21}, \frac{6}{7}]$ (hence less than $\frac{13}{15}$), the expression on the right hand side is less than $\frac{1}{3}$, contradicting the requirement $\theta \in [\frac{1}{3}, 1]$.
Hence $r_1(-1,-1) = 0$ in equilibrium.
Summarizing the above arguments gives us Lemma 5. □

Proof. (of Lemma 6) The expected utility from acceptance is $-(y_m - (s + b_v))^2$, and that from rejection is $-[(1 - \alpha)(y_m - (s + b_v))^2 + \alpha(y_m) \sum_{m'} \gamma_{m'}(y_{m'} - (s + b_v))^2$, where $\gamma_{m'}$ indicates the probability of message m' being received in equilibrium. The difference in expected utility between rejection and acceptance is

$$-\alpha[\sum_{m'} \gamma_{m'}(y_{m'} - (s + b_v))^2] - (y_m - (s + b_v))^2 = -\alpha[\sum_{m'} \gamma_{m'} y_{m'}^2 - y_m^2 + 2y_t(s + b_v)].$$

In deriving the above equality, I have used the fact that $\sum_{m'} \gamma_{m'} y_{m'} = 0$. Thus holding everything else fixed, an increase in v increases b_v, which decreases the difference in utility if $y_m > 0$. This makes it less likely for the expert to reject the message. The contrary is true if $y_m < 0$. Hence the first two statements.
If $y_m = 0$ then the difference in utility is always negative unless $y_{m'} = 0$ for all $m' \in M$, which is not informative and which implies rejection is meaningless. Thus the expert should always accept a message m inducing the action 0. □

Proof. (Sketch of Proof of Proposition 7) First, it is clear that (F1) and (F2) imply (F3) and (F4). For example,

$$\begin{aligned}\gamma_0 &= P(s = 1)P(x = -1) + P(s = 0)[P(s = 0, 1)(P(v = 0, 1) + P(v = -1)(1 - \alpha + \alpha\gamma_0)) \\ &\quad + P(x = -1)P(v = 0, 1)\alpha\gamma_0] \\ &= \tfrac{1}{3} \cdot \tfrac{1}{3} + \tfrac{1}{3}[\tfrac{2}{3}(\tfrac{2}{3} + \tfrac{1}{3}(1 - \alpha + \alpha\gamma_0) + \tfrac{1}{3} \cdot \tfrac{2}{3}\alpha\gamma_0)]\end{aligned}$$

Solving the equation for γ_0 yields

$$\gamma_0 = \frac{\frac{1}{9} + \frac{2}{9}(1 - \frac{1}{3}\alpha)}{1 - \frac{4}{27}\alpha}.$$

Then I can use $\gamma_0 y_0 = E(s|m = 0) = \frac{1}{9}$ to solve for y_0. Similar procedures apply to the messages -1 and 1. A fact worth noting is that $\gamma_1 y_1 = \frac{2}{9}$ and $\gamma_{-1} y_{-1} = -\frac{1}{3}$.

Now I check the optimality of the reviewer's decisions. Consider any $r_v(s, m)$, where $v \in X$, $s \in S$, and $m \in M$. Using the result in the proof of Lemma 6, the difference in utility between rejection and acceptance is

$$-\alpha[\sum_{m'} \gamma_{m'}(y_{m'} - (s + b_v))^2] - (y_m - (s + b_v))^2 = -\alpha[\sum_{m'} \gamma_{m'} y_{m'}^2 - y_m^2 + 2y_t(s + b_v)].$$

Furthermore, many results can be obtained by using Lemma 6. I put down the proof of review decisions that are crucial to the proposition, and the proof of other review decisions follow similar procedures.

Now I consider the optimality of $r_{-1}(1,1)=0$. The difference in utility between rejection and acceptance is

$$\begin{aligned} & -\alpha[\tfrac{2}{9}y_1+\tfrac{1}{9}y_0-\tfrac{1}{3}y_{-1}-y_1^2+2y_1(1-b)] \\ = & -\alpha y_1[\tfrac{2}{9}+\tfrac{1}{9}\cdot\frac{\frac{1}{9}}{\frac{1}{9}+\frac{2}{9}(1-\frac{1}{3}\alpha)}+\tfrac{1}{3}\cdot\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{9}(1-\frac{2}{3}\alpha)}-(1-\tfrac{4}{27}\alpha)+2(1-b)] \end{aligned}$$

Note for $\alpha=1$, the above expression is transformed into

$$-\alpha y_1[\frac{2}{9}+\frac{1}{9}\cdot\frac{3}{7}+\frac{1}{3}\cdot\frac{9}{10}-\frac{23}{27}+2(1-b)],$$

which is negative as long as $b<\frac{3247}{3780}$. Our assumed b values satisfy this condition since $\frac{6}{7}=\frac{324}{378}<\frac{3247}{3780}$. By continuity, for α close to 1, the difference in utility must be negative, which implies $r_{-1}(1,1)=0$. That $r_0(1,1)=r_1(1,1)=0$ comes from the fact that y_1 is the reviewer's most preferred action in state 1. This is true irrespective of what α is.

The proof of other review decisions can be similarly done.

Finally I check the optimality of the expert's reports. First $m_0(1)=m_1(1)=1$, $m_0(-1)=m_{-1}(-1)=-1$, and $m_{-1}(1)=0$ since the messages correspond to the expert's most preferred action, and are never rejected. Second, by Lemma 4, $m_{-1}(0)=-1$ and $m_0(0)=0$ since a reviewer of the same type as the expert rejects the other two available messages. Last, I need to determine whether $m_1(0)=0$ and $m_1(-1)=-1$ is optimal.

I show that they are optimal at $\alpha=1$, and appeal to continuity to show that they are optimal for large enough α. When $\alpha=1$, $y_1=\frac{23}{27}$, $y_0=\frac{3}{7}y_1$, and $y_{-1}=-\frac{9}{10}y_1$. Before proceeding, $m_1(0)\neq-1$ and $m_1(-1)\neq 1$ due to Lemma 4 and the fact that $r_1(0,-1)=r_1(-1,1)=1$.

In state 0, if an expert of type 1 reports 0, she receives utility $-[\frac{2}{3}(y_0-b)^2+\frac{1}{3}[(1-\alpha)(y_0-b)^2+\alpha\sum_m\gamma_m(y_m-b)^2]]$, while if she reports 1, she receives utility $-[\frac{1}{3}(y_1-b)^2+\frac{2}{3}[(1-\alpha)(y_1-b)^2+\alpha\sum_m\gamma_m(y_m-b)^2]]$. When $\alpha=1$, the difference in utility between sending 0 and sending 1 is

$$\begin{aligned} & \tfrac{1}{3}[(y_1-b)^2+\textstyle\sum_m\gamma_m(y_m-b)^2-2(y_0-b)^2] \\ = & \tfrac{1}{3}[y_1^2-2by_1+\tfrac{2}{9}y_1+\tfrac{1}{9}y_0-\tfrac{1}{3}y_{-1}-2y_0^2+4by_0] \\ = & \tfrac{1}{3}y_1[y_1-2b+\tfrac{2}{9}+\tfrac{1}{9}\cdot\tfrac{3}{7}-\tfrac{1}{3}\cdot\tfrac{-9}{10}-2(\tfrac{3}{7})^2y_1+4b\cdot\tfrac{3}{7}] \\ = & \tfrac{1}{3}y_1[\tfrac{31}{49}y_1-\tfrac{2}{7}b+\tfrac{2}{9}+\tfrac{1}{21}+\tfrac{3}{10}] \\ > & 0. \end{aligned}$$

The first inequality uses the fact $\sum_m\gamma_m y_m=0$, and the last inequality uses the fact that $b\leq 1$. Thus $m_1(0)=0$ is optimal at $\alpha=1$.

In state 1, following similar procedures to that of the previous paragraph, I obtain that the difference in utility for an expert of type 1 between sending -1 and 0 in state -1 is

$$\begin{aligned}\frac{1}{3}y_1[(\frac{9}{49}-\frac{243}{100})y_1+(\frac{6}{7}+\frac{27}{5})(1-b)+\frac{4}{9}+\frac{2}{21}+\frac{3}{5}]\\ > \quad 0.\end{aligned}$$

The inequality is gotten by substituting $y_1 = \frac{23}{27}$ and using my assumption that $b \leq \frac{6}{7}$. Hence $m_1(-1) = -1$ is optimal at $\alpha = 1$.

To conclude, there exists α large enough such that strategy profile (F) constitutes an equilibrium. □

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