The Kuhn - Tucker Theorem and Resource Allocation Games

A. Garnaev, A. Solovyev, T. Szigyarto

Department of Computer Modelling and Multiprocessor Systems,
Faculty of Applied Mathematics and Control Processes,
St Petersburg State University,
Universitetskii prospekt 35,
Peterhof, St Petersburg 198504, Russia
e-mail: agarnaev@rambler.ru

Abstract

In this paper an application of the Kuhn - Tucker Theorem to two resource allocation games is considered. The first one considered in the paper is a non-zero sum two-sided investment allocation game based on a market share model. Two firms, say 1 and 2, compete against each other in n independent markets. The total sales potential of each market V_i ($i \in [1, n]$) is fixed and known. We assume that already A_i, B_i and D_i efforts to market i are located by firm 1, firm 2 and some third party. These value are fixed and known to both players. If firm 1 and firm 2 allocates x_i and y_i efforts to market i, respectively, then the share of market i getting by firm 1 and 2 is $V_i(x_i + A_i)/(x_i + A_i + y_i + B_i + D_i)$ and $V_i(y_i + B_i)/(x_i + A_i + y_i + B_i + D_i)$. Also, we assume that budget of firm 1 and 2 is X and Y. If firm 1 (2) allocates $\sum_{i=1}^{n} x_i$ ($\sum_{i=1}^{n} y_i$) resources then the investment cost is $C_1 \sum_{i=1}^{n} x_i$ ($C_2 \sum_{i=1}^{n} y_i$) where $C_j > 0$ for $j = 1, 2$. The aim of each firm is to plan its budget so that to maximize the total firm profit minus the investment cost. In this game the Nash equilibrium are derived and numerical examples are given. It is shown that in some specific cases the Nash equilibrium is unique. Also the case of one firm game and the Stackelberg equilibrium are investigated.

The second game considered in this paper as an application of the Kuhn - Tucker Theorem is a generalization of the Sakaguchi resource allocation game on an integer interval $[1, n]$. Two players (Player 1 and 2) want to find an immobile object hidden at one of n points. Each point $i \in [1, n]$ is characterized by a detection parameter λ_i (μ_i) for Player 1 (Player 2) such that $p_i(1 - \exp(-\lambda_i x_i))$ ($p_i(1 - \exp(-\mu_i y_i))$) is the probability that Player 1 (Player 2) discovers the hidden object with amount of search effort x_i (y_i) applied at point i where $p_i \in (0, 1)$ is the probability that the object is hidden at point i. Player 1 (Player 2) undertakes search by allocating the total amount effort $X(Y)$. The payoff for Player 1 (Player 2) is 1 if he detects the object but his opponent does not. If both players detect the object each of them gets 1/2. If the player does not find the object than his payoff is 0. In the Sakaguchi game player's payoffs are 0 if they both find the object. In this paper we show that introducing a natural assumption about possibility of sharing the hidden object by the players when it is found by all of them but not only one allows to escape the problem of solution of the overdetermined system of equations which arises when we are looking for the Nash equilibrium using the Kuhn-Tucker Theorem. This assumption about sharing the object reduces the problem of finding the Nash equilibrium to an unambiguous solvable system of non-linear equations. Also the case where the cost search presents is investigated.

Key words: Resource Allocation Game, Nash Equilibrium
