Relative performance of two simple incentive mechanisms in a public good experiment*

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25th November 2004

Abstract

The paper reports on experiments designed to compare the performance of two incentive mechanisms in public goods problems. One mechanism rewards and penalizes deviations from the average contribution of the other agents to the public good (tax-subsidy mechanism). Another mechanism allows agents to subsidize the other agents' contributions (compensation mechanism). It is found that both mechanisms lead to an increase in the level of contribution to the public good. The tax-subsidy mechanism allows for good point prediction of the average level of contribution. The compensation mechanism predicts the level of contributions less reliably.

Keywords: public goods, voluntary provision, incentive mechanisms.

JEL: H42, D62

*The authors are grateful to the Leverhulme Trust for funding this project through CMPO and to Nick Feltovich for his valuable comments. We would also like to thank Ken Binmore, Rosemarie Nagel and participants at the CMPO seminar, the LAMETA internal seminar, the Universitat Pompeau Fabra microeconomics seminar and the International Meeting of Economic Science Association in Amsterdam for helpful discussions.

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1 Introduction

Many problems in relation with the organization of society revolve around giving right incentives to people so that their individual motivations are in line with the general interest. The "invisible hand" successfully gives such incentives in a world of perfect competition. But it fails to do so in realistic departures from this stylized framework. For instance, in the presence of public goods, since agents can benefit from others' contributions to such goods without contributing themselves, individually rational decisions generally fall short of Pareto optimal levels, a phenomenon referred to as the free-rider problem.

The free-rider problem has come to be viewed as the canonical representation of collective action problems. Environmental issues like pollution reduction and biodiversity protection offer only but two examples from many such issues, that can be found in other fields like public economics, macroeconomics, international economics... This has created a great deal of interest among social scientists, who have put much effort to increase our knowledge of the problem. Later on, they have also reacted by engineering theoretical mechanisms to restore Pareto optimality. Meanwhile, laboratory experiments have continuously provided a precious help in both testing theories and achieving a better understanding of free-rider behaviors and the efficiency of suggested solutions.

The goal of this paper is to discuss, from an experimental point of view, the relative merits or weaknesses of two mechanisms proposed in the theoretical literature to solve the free-rider problem in public goods situations. One mechanism rewards and penalizes deviations from the average contribution of the other agents to the public good: this is the *tax-subsidy mechanism* proposed by Falkinger (1996). The other one introduces a pre-play stage where agents contemplate the possibility to offer subsidies to one another before contributions are decided: this two-stage mechanism is referred to as the *compensation mechanism* (the term has been coined by Varian (1994a, 1994b) who himself builds on earlier works of Guttman (1978, 1985, 1987), Moore and Repullo (1988), Danziger and Schnytzer (1991)).

Within the set of existing theoretical mechanisms, those two mechanisms share the merit of simplicity and, therefore, are more likely to be applied in real life situations. And there are two additional reasons to focus on them.

Firstly, they rest on two different (some would even say opposite) views of the public sector intervention. The *tax-subsidy mechanism* could be called a “coercion” solution since, once in place, agents cannot escape it and can no longer enjoy the same level of utility by keeping unchanged their behaviors. Also, it clearly assumes a higher degree of involvement of the public authorities than usually required by market institutions. By contrast, the *compensation mechanism* could be termed a ”liberal” solution: it does respect the agents’ freedom of choice as agents can escape the mechanism; keeping unchanged their behaviors won’t reduce their utility levels. Though former actions are no longer in equilibrium as agents can increase their utility by appropriate unilateral deviations. It is also less demanding as far as public authorities involvement is concerned. It is more coasian in spirit for it requires state interventions only to protect property rights and to guaranty enforceability of agents’ agreements. With this coercion/liberal distinction, we have in mind something more precise than a general philosophic remark. Recent experimental and field evidence suggest that externally imposed rules tend to "crowd out" endogenous cooperative behaviors (see Ostrom (2000) pp 147-148 and the references therein). Social norms like trust, fairness and reciprocity might explain the persistent propensity of subjects to contribute to the public good somehow above the full free-rider level, i.e. agents may be intrinsically motivated to behave cooperatively. The imposition of external rules or explicit incentive mechanisms to force cooperation may actually have distorting effects on agents’ behavior and crowd out their intrinsic motivation. The degree of crowding out may depend on the design of the mechanism: in the presence of a strongly coercive mechanism, cooperation may be sustained even if internal norms are wiped out. But with a weak external rule, it seems that the intrinsic motivation can be substantially eroded while the temptations to deviate from cooperation cannot be punished sufficiently by use of the external rule, possibly resulting in the worst scenario; intrinsic and extrinsic motivations would be imperfect substitutes. To the extent that the liberal solution could be considered a weak external rule, one would expect it to be less efficient than the coercion solution.

Secondly, the tax-subsidy mechanism produces a near supermodular game, a technical property which is theoretically sufficient to ensure convergence to Nash equilibria under a wide set of learning dynamics. This is important for subjects are boundedly rational: they generally start somewhere off the equilibrium path and do not necessarily converge to it. In other words, the predictive power of theory is stronger in supermodular games. Whether the exact form of the compensation mechanism we used is supermodular is an open question[1]. Existing laboratory experiments somehow confirm the relevance of the supermodularity property for convergence (see Chen and Gazzale

[1]Chen and Gazzale (2003) have investigated this property for a generalized version of

(2003) for a systematic study of this topic).

In the simple framework we have chosen for the experiment, abstracting from convergence issues, those two mechanisms theoretically produce (roughly) the same outcome. The next step is to check how well they perform with subjects. The two mechanisms have already been separately tested in the laboratory into two different contexts. Overall, those two simple solutions have surprisingly good performances, in comparison with other theoretical mechanisms already tested[2]. Andreoni and Varian (1999) have assessed the merits of the compensation mechanism in a prisoners' dilemma experiment. Without the mechanism, only one third of the subjects chose to cooperate. The introduction of the subsidy stage increased cooperation from one third to two thirds. Falkinger et al (2000) studied the impact of the tax-subsidy mechanism in several public good environments, varying group size and payoff functions. They report that the mechanism causes an immediate and large shift toward the efficient level of public good. They also report that the Nash equilibrium is an unusually good predictor for behaviors under the mechanism.

The originality of our paper is twofold. First, it is to test experimentally the simplest version of the compensation mechanism (that of Danziger and Schnytzer (1991)) in a public good framework with interior dominant strategies, which despite its simplicity is a bit more complicated than the prisoners' dilemma game used in Andreoni and Varian (1999). Here our contribution is also (modestly) theoretical: our framework does not meet the sufficient assumptions required by Danziger and Schnytzer (1991) for the existence of an equilibrium. So existence must be established; this is done in Section 2.2.2. Second, it is to offer the possibility to compare, from an empirical point of view, the two mechanisms, by running them in the same framework. Our findings are as follows: *i*) we confirm Falkinger et al. result, namely that the initial levels of contributions are unusually high; our environment differs from theirs as we use a sample of subjects from a different population (University of Bristol students) and a different group composition; *ii*) both mechanisms lead to an increase in the levels of contributions to the public good; *iii*) the tax-subsidy mechanism allows for surprisingly good point predictions of the average levels of contribution, and this is true

the compensation mechanism in a unilateral externality framework, which is different from ours.

[2]Experimental papers report disappointing performances for Groves-Clarke, Groves-Ledyard and the Walker mechanisms (see respectively Attiyeh et al (1999), Harstad and Marrese (1981, 1982), Chen and Plott (1996), and Chen and Tang (1998).

for two different parameter values; in contrast the compensation mechanism allows for less reliable predictions of the average levels of contributions.

The rest of the paper is organized as follows. Section 2 describes the public good economy we have reproduced in the laboratory; it also introduces the two mechanisms. The experimental design is discussed in Section 3 and empirical results are offered in Section 4. Section 5 discusses and summarizes these results.

2 A public good experiment with quadratic utility

The economic situation reproduced in the laboratory is as follows. Two agents $i = 1, 2$ are endowed with an exogenous income y_i, which they can divide between the consumption c_i of a composite private good and a contribution g_i to the production of a public good G. The production technology for the public good takes the simplest form : $G = g_1 + g_2$. While c_i is enjoyed by agent i only, the public good nature of G means that both agents benefit from it. Thus agent i's preferences are typically represented by utility functions of the form:

$$U^i (c_i, G) \ , \quad i = 1, 2. \tag{1}$$

In the laboratory, we have endowed the subjects with quasi-linear quadratic reward functions:

$$U^i (c_i, G) = M_i c_i - \frac{1}{2} N_i c_i^2 + G \ , \quad M_i, N_i > 0 \, , \quad i = 1, 2.$$

Note that those functions U^i are concave, increasing in public good consumption and, for $c_i \in [0, M_i/N_i]$, non decreasing in private good consumption. This functional form gives us the simplest framework consistent with the questions we want to challenge.

The reasons for this particular choice of utility functions are twofold. First of all there is a need to keep the framework as simple as possible to ensure that subjects will come to a good understanding of the link between the profile of decisions and their monetary earnings. Secondly the chosen framework has to be relevant to those theoretical properties of the two mechanisms we want to test experimentally, namely their ability to offer the subjects the correct incentives to take efficient decisions without requiring any knowledge of their preferences.

The compensation mechanism can be applied to a large set of situations, including those for which the tax-subsidy mechanism has been designed; logically we are therefore limited only by the requirements of the tax-subsidy mechanism. Most of the public good games used in experiments are games with linear payoffs, i.e. the simplest conceivable framework in which corner decisions are dominant strategies. Falkinger's mechanism can be applied to such games[3], but it then requires that the designer knows agents' preferences. If the ambition is to challenge asymmetric information issues where the designer does not know agents' preferences, then the mechanism does not have any advantage over more traditional pigovian tax / subsidy schemes, except in public good frameworks where agents undertake interior decisions, i.e. strictly positive contributions g_i. Only then the mechanism can Nash implement an allocation arbitrarily close to efficiency with the requirement that the regulator knows only the number of agents involved in the problem, and not their preferences. We shall come back to those subtleties in Section 2.2.1.

With quasi-linear quadratic utility functions, the Nash equilibrium consists of interior contributions. An additional advantage of the family chosen is that the marginal utility from public good consumption is constant: the Nash equilibrium involves dominant strategies, therefore subject shall be capable to deduce easily the equilibrium strategies (before the introduction of any mechanism)[4].

2.1 The free-rider problem

How much will agents contribute voluntarily to the public good? To answer this question, theorists suggest to focus on non cooperative decisions that form a Nash equilibrium, whereby each agent optimizes her utility, taking as given the other agents' decisions. Formally, under this behavioral assumption, each agent's problem reads as:

$$\max_{c_i \geq 0,\ g_i \geq 0} U^i(c_i, G)$$

$$s.t. \begin{cases} c_i + g_i = y_i \\ G = g_i + g_j\,, \quad g_j \text{ given}\,, \end{cases}$$

[3] Actually it has been applied to public good games with linear payoffs in Falkinger et al (2000), more precisely in the treatment experiment sessions which they called $M1, M2$ and $M3$.

[4] To our best knowledge, only four published articles have used this specific quadratic framework with interior dominant strategies for the purpose of experimentation. They are Sefton and Steinberg (1996), Keser (1996), Willinger and Ziegelmeyer (1999), and Falkinger et al (2000).

which, for an interior solution, leads to the first order condition: $MRS^i = \frac{U^i_G}{U^i_c} = 1$. For our quadratic example, the first order conditions of the two agents are:

$$\begin{cases} -M_1 + N_1 (y_1 - g_1) + 1 = 0, \\ -M_2 + N_2 (y_2 - g_2) + 1 = 0. \end{cases}$$

Solving those equations, the Nash equilibrium quantities are:

$$g_i^N = \frac{1 - M_i}{N_i} + y_i \ , \quad c_i^N = \frac{M_i - 1}{N_i}, \quad G^N = y_1 + y_2 + \frac{1 - M_1}{N_1} + \frac{1 - M_2}{N_2} \ .$$

For later use, the equilibrium utilities are:

$$U^i \left(c_i^N, G^N \right) = M_i \frac{M_i - 1}{N_i} - \frac{1}{2} N_i \left(\frac{M_i - 1}{N_i} \right)^2 + y_1 + y_2 + \frac{1 - M_1}{N_1} + \frac{1 - M_2}{N_2}$$

By contrast, Samuelson's condition for efficiency requires $MRS^1 + MRS^2 = 1$. Overall, this last condition and the feasibility condition form the system:

$$\begin{cases} \sum_i \frac{1}{M_i - N_i c_i} = 1 \ , \\ \sum_i c^i + G = \sum_i y^i \ , \end{cases}$$

or, equivalently,

$$\begin{cases} M_1 - N_1 c_1 + M_2 - N_2 c_2 = (M_1 - N_1 c_1)(M_2 - N_2 c_2) \ , \\ c_1 + c_2 + G = y_1 + y_2. \end{cases}$$

The Samuelson's condition differ from the Nash equilibrium conditions, meaning that the Nash equilibrium is generally not Pareto optimal.

2.2 Nash implementation with well-informed agents

This section describes two simple theoretical mechanisms designed to Nash-implement Pareto optimal decisions in informational frameworks where agents know each other preferences but, of course, the designer does not.

2.2.1 Tax-subsidy mechanism

This mechanism modifies each agent's budget constraint by rewarding (penalizing) contributions over (under) the mean of the other agents' contributions. To do so, the designer needs to choose a single parameter: a tax-subsidy rate β. Under this new institutional framework, the problem of agent i becomes:

$$\max_{c^i \geq 0 \ , \ g_i \geq 0} U^i (c_i, G)$$

$$s.t. \begin{cases} c_i + g_i = y_i + \beta\,(g_i - g_j)\ ,\quad \beta \geq 0 \\ G = g_i + g_j \\ g_j \text{ given.} \end{cases}$$

Note that this mechanism is balanced, whatever the contributions, since what an agent receives corresponds exactly to what the other agent pays. The first order conditions for interior solutions are:

$$\begin{cases} (\beta-1)M_1 - (\beta-1)N_1\,[y_1 - g_1 + \beta\,(g_1 - g_2)] + 1 = 0\ , \\ (\beta-1)M_2 - (\beta-1)N_2\,[y_2 - g_2 + \beta\,(g_2 - g_1)] + 1 = 0\ . \end{cases}$$

Solving this system one finds a solution for G and c_i configured by β. The nice feature of this mechanism is that, with a value for β arbitrarily close to $\frac{n-1}{n}$ (here $\beta \simeq 1/2$ since $n = 2$), the resulting equilibrium will be arbitrarily close to efficiency[5]. Indeed, letting $\beta \to 1/2$ those solutions tends to:

$$G^f = \frac{2 - M_1}{N_1} + \frac{2 - M_2}{N_2} + y_1 + y_2$$

$$c_i^f = \frac{M_i - 2}{N_i}\ ,\quad i = 1, 2.$$

The equilibrium quantities fulfill Samuelson's condition for efficiency. The resulting limit utilities are:

$$U^i\left(c_i^f, G^f\right) = M_i\frac{M_i - 2}{N_i} - \frac{1}{2}N_i\left(\frac{M_i - 2}{N_i}\right)^2 + y_1 + y_2 + \frac{2 - M_1}{N_1} + \frac{2 - M_2}{N_2}\ ,\quad i = 1, 2.$$

Clearly the only piece of information needed by the regulator to compute the critical β is the number of agents. No information as for the preferences is required. By contrast, consider what would be possible in a linear environment; payoffs would be:

$$U^i\,(c_i, G) = M_i c_i + G\ ,\quad M_i > 0\ ,\quad i = 1, 2.$$

Let the parameters M_i be such that $1 < M_i < 2,\ i = 1, 2$. In that configuration of parameters, the voluntary contribution equilibrium is characterized by a complete free-riding, i.e. $g_i^N = 0,\ i = 1, 2$, while Pareto optimality requires that both agents contribute their full endowment to the public good,

[5] When $\beta = 1/2$ the Nash equilibrium is not unique. In the symmetric case, where $M_i = M,\ N_i = N,\ y_i = y$, one can check that any pair (g_1, g_2) such that:

$$-\frac{1}{2}M + \frac{1}{2}N\left[y - \frac{1}{2}\,(g_1 + g_2)\right] + 1 = 0\ ,$$

is a Nash equilibrium.

so $g_i^P = y_i,\ i = 1, 2$. After introducing the tax-subisdy scheme, the first order conditions would drive the agents to efficiency if:

$$\begin{cases} (\beta - 1)M_1 + 1 > 0 \ , \\ (\beta - 1)M_2 + 1 > 0 \ , \end{cases}$$

or equivalently $\beta > (M_i - 1)/M_i,\ \ i = 1, 2$. In other words, the optimal β is given by any value such that

$$\beta > \max\left\{\frac{M_1 - 1}{M_1}, \frac{M_2 - 1}{M_2}\right\}.$$

It is readily deduced that the regulator now needs to know the preference parameters, and the mechanism is less attractive.

2.2.2 Compensation mechanism

Several versions of the compensation mechanism exist in the literature. We have chosen the simplest of those two-stage mechanisms[6].

First, agents voluntarily subsidize the other agents' contributions, i.e. agent i offers $s_i\, g_j$ to agent j (with $s_i \in [0, 1]$). Then, given this profile of subsidies, agents contribute to the public good.

It is very similar to the mechanism introduced by Danziger and Schnytzer (1991), albeit we have ignored a difficulty that may arise when, given the subsidy and the contribution chosen by the rival agent, agent i's own choice would entail that she spend more than her endowment on the public good. This happens for instance if agent 1 sets her subsidy rate close to one, and agent 2 choose a contribution larger than agent 1's income. To overcome this difficulty, Danziger and Schnytzer (1991) suggested to complete their mechanism with any rule that, in those situations generates a feasible profile of choices (see Danziger and Schnytzer (1991) on page 58). In the laboratory, two ways out could be contemplated: *i*) to make use of such a rule to avoid theoretical drawbacks, but at the risk of making the subjects even more confused with a mechanism that appeared already quite complicated; *ii*) ignoring this difficulty altogether and deleting afterwards those observations where one agent went bankrupt; this would make the subjects' decision context easier to understand, but at the risk of collecting many worthless observations. We preferred the second possibility. It turned out that, out of 600 observations under the compensation mechanism, only 7 incidences of bankruptcy have occurred.

[6] More sophisticated forms of this mechanism add penalization terms (see Varian (1994a)), which are ignored here.

Under this simple compensation mechanism, a subgame perfect Nash equilibrium is found by solving this game backward. Formally:

- each agent's maximization program in the second stage is:

$$\max_{c_i \geq 0\,,\, g_i \geq 0} U^i\left(c_i, G\right)$$

$$s.t. \begin{cases} c_i + \left(1 - s_j\right) g_i = y_i - s_i g_j \\ G = g_i + g_j \\ s_i, s_j, \text{ and } g_j \text{ given.} \end{cases}$$

  It is worth noting that adding up the two individual budget constraints, whatever the decisions as for the contributions and the subsidies, one has: $c_i + c_j + g_i + g_j = y_i + y_j$; this form of the mechanism meets the balanced aggregate budget requirement both at and off equilibrium. Let $\mathbf{s} = (s_1, s_2)$ stands for the profile of subsidies and let $c_i(\mathbf{s})$, $G(\mathbf{s})$ denotes the solutions to the above program; indirect utilities are

$$V^i(\mathbf{s}) = U^i\left(c_i(\mathbf{s}), G(\mathbf{s})\right).$$

- Moving backward to the first stage, the subsidy decisions solve for each agent

$$\max_{s_i \in [0,1]} V^i(\mathbf{s}) \,, \quad s_j \text{ given.}$$

Danziger and Schnytzer (1991) and Varian (1994) have established that, under reasonable assumptions, a subgame perfect Nash equilibrium for public good games with a subsidy stage exists and replicates a Lindahl equilibrium. In symmetric games, this means that $s_i = s_j = 1/2$. In our symmetric game, this would further imply that $c^i = c^j = 30$, $g^i = g^j = 20$.

However Varian's mechanism is slightly different from the one described above[7], and Danziger and Schnytzer' assumptions are not met by our quadratic economy: they require that $\lim_{c_i \to 0} \frac{\partial U^i / \partial G}{\partial U^i / \partial c_i} = 0\,, \forall i$, which does not hold here. Therefore, the existence of a SGPNE should be ascertained for the game we have reproduced in the laboratory. Even in this particular example, such an exercise is of some theoretical interest. Indeed it helps to clarify that Danziger and Schnytzer' s assumptions are sufficient but not necessary to guaranty their results[8].

[7] In Varian's mechanism there is an additional penalty term and both the subsidy and the tax that each agent faces are chosen by the other agent.

[8] For a related discussion concerning the existence of subgame perfect equilibria in this public good economy see also Althammer and Buchholz (1993).

Lemma 1 *Let* $M_i = 5$, $N_i = 1/10$ *and* $y^i = 50$. *With those numerical values, any subgame perfect equilibria of the compensation game involves a strictly positive level of public good* $G > 0$ *and* $s_i + s_j = 1$.

Proof 2 *See appendix A.*

Proposition 3 *For the above public good game with the compensation mechanism, the Lindahl allocation is implemented as a subgame perfect Nash equilibrium, where efficient equilibrium quantities are* $s_i = s_j = 1/2$, $G = 40$.

Proof 4 *See appendix B.*

An important remark is in order: this last result states that at a subgame perfect equilibrium, the level of public good is unique ($G = 40$). However it does not say that there is a unique subgame perfect equilibrium. Actually, the continuum of individual contributions such that $g_1 + g_2 = 40$, along with the pair of subsidies $s_i = 1/2, i = 1, 2$, are all subgame perfect equilibria. The intuition outlined in Danziger and Schnytzer (1991) is illuminating. When $1 - s_j = s_i$ the prices of direct contributions to the public good via g_i and indirect contributions via g_j are the same. Agent i is thus choosing effectively whether the aggregate level of public good to consume should be higher than, or equal to g_j. When the Lindahl level is affordable, it is optimal to choose it. Formally, agent i's reaction function is:

$$g_i = \max(40 - g_j, 0) \ . \tag{2}$$

The graphs of the two agents best response functions coincide for $g_1 + g_2 = 40$.

Clearly, the existence of a continuum of equilibria is a weakness of this implementation concept, for a coordination problem might prevent the agents to play their equilibrium strategies. To mitigate the extent of this issue, one may notice however that in this symmetric game the symmetric equilibrium where $g_1 = g_2 = 20$ is a focal point.

3 Experimental design

We ran two experiments, one for each mechanism. Thirty subjects took part in the experiment on the compensation mechanism and fifty-four subjects took part in the experiment on the tax-subsidy mechanism (thirty subjects with a parameter $\beta = \frac{9}{19}$ and twenty-four subjects with $\beta = \frac{1}{3}$). We conducted three sessions of each experiment. Subjects were recruited from Bristol

University students in Social Science and few from the English Department. In each experiment participants play two games: a control game, with no mechanism for twenty rounds, and a second game with either mechanism for other twenty rounds. Participants were divided into groups with two members each and were informed that they would play against the same opponent in the experiment but would never learn with whom they formed a group. The software z-tree has been used for programming and running the experiments. The instructions were read aloud to subjects who, before playing the game at the computer, had to fill in a questionnaire to check their understanding of the instructions. The total points scored at the end of each experiment were converted into pounds and added to the fee of £ 2.50 which each participant received for showing up. The average payment was £ 10.00. Each experiment lasted two hours.

In control games (without mechanism), each participant received 50 points as initial endowment and had to decide how to allocate this initial endowment between two activities, one beneficial to both players in the same group (activity A) and the other beneficial only to the donor (activity B). Their decision would have consequences in terms of income earned from both activities. Income from activity A would result from the sum of both group members' contributions to A, whereas income from activity B would result from the following formula:

$$\text{Income from B} = 5 \times \text{contribution to B} - \left(\frac{1}{20}\right) \times (\text{contribution to B})^2 \quad (3)$$

Subjects need not do any calculations as they were provided with a table listing the level of income from activity B corresponding to any possible value of the contribution to B that the subject could make, from zero to 50. The table also listed the income change if activity B increased by one unit and the income change if activity A increased by one unit, in order to make clear how income changed after an additional unit of contribution to either activity. Subjects were asked to decide how many points they wanted to contribute to activity A. Their choice would automatically determine the contribution to B, and income from both activities would be calculated by the software. Total income would result from the sum of income from activity B and income from activity A. This would give each subject a payoff which is derived from the utility function we used in the section 2:

$$U^i(c_i, G) = M_i c_i - \frac{1}{2} N_i c_i^2 + G \quad (4)$$

with $M_i = M = 5$, and $N_i = N = 0.1$. At the end of each period subjects were informed about the contribution of the other group member to activity

A, their income from activity A and their income from activity B. From a theoretical point of view, subjects participated to a 20-round repeated game of complete information, where the stage game had a unique Nash equilibrium $(g_i, c_i) = (10, 40)$, and where the subgame perfect Nash equilibrium of the total game was made of 20 repetitions of the static Nash equilibrium.

The only change between control games and mechanism games was the way we calculated the total contribution to activity B. In particular, the payoff function was the same for all the subjects in both mechanism games.

Both the tax-subsidy and the compensation mechanism act on the budget constraint. Remember that in the tax-subsidy game the budget constraint is:

$$c_i = y_i - g_i + \beta(g_i - g_j) \ , \tag{5}$$

and in the compensation game it is:

$$c_i = y_i - g_i + s_j g_i - s_i g_j \ . \tag{6}$$

This aspect made the design of the experiments we ran quite straightforward, as we could keep the same instructions, applying changes only for the way the contribution to the private good was calculated. This facilitated both communication to the subjects and their understanding of the consequences of their choice in the two treatments they played[9].

Falkinger et al. (2000) test the practical tractability and effectiveness of the Falkinger mechanism in quadratic games, as we did, but with different group size, different preferences and different tax-subsidy parameters. Table 1 describes both the experimental game used by Falkinger et al. (2000) - this is referred to as *C4* and *M4* in their paper - and our modification. We consider the same parameters for the payoff function, but we deal with only one group size, namely two players. We consider two different values for the parameter β. The values have to be below the optimal level $\beta^* = \frac{1}{2}$, that otherwise would have given multiple solutions to the game. We tried two different values, and first set β equal to $\frac{1}{3}$ and then to $\frac{9}{19}$, the later value being closer to the optimal one. Theoretically, we have again repeated games of complete information, where the subgame perfect Nash equilibrium was the repetition of the unique static Nash equilibrium, i.e. $(c_i, g_i) = (15, 35)$ for β equal to $\frac{1}{3}$ and $(c_i, g_i) = (19, 31)$ for β equal to $\frac{9}{19}$.

In the compensation mechanism treatment, subjects were first given the option to offer a rate at which to support the other group member in order to

[9]The set of instructions for the compensation mechanim is included in the Appendix. The full set of instructions is available from the authors upon request.

Table 1: Experimental design

	Falkinger 2000	BFR 2004	
Group size n	4	2	
Parameter M	5	5	
Parameter N	.1	.1	
Endowment y	50	50	
Benchmark: PG			
c^N	40	40	
g^N	10	10	
G^N	40	20	
U^N	160	140	
PG with Tax-Subsidy			
$\beta^* = (1 - \frac{1}{n})$	$\frac{3}{4}$	$\frac{1}{2}$	
β	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{9}{19}$
c^F	20	35	31
g^F	30	15	19
G^F	120	30	38
U^F	200	143.75	144.95
Efficiency ?	No		
PG with Compensation			
c^V	–	30	
g^V	–	20	
G^V	–	40	
U^V	–	145	
Efficiency ?	–	Yes	

encourage him to contribute to activity A. Once players had decided on the rate of support they had to choose how many points to allocate to activity A. A subgame perfect equilibrium of each two-stage game was a contribution of 20 to activity A and 30 to activity B, $(g_i, c_i) = (20, 30)$. And a subgame perfect Nash equilibrium over the 20 rounds was the repetition of this two-stage equilibrium.

4 Experimental results of the two experiments

This section examines the impact of the mechanisms by comparing the observed behavior over time in the control and in the treatment. It offers descriptive statistics of the contributions to the public good, results of tests on the effects of the treatments and it shows plots of time series of contributions. Throughout this section, the basic variable of interest is the "contribution to the public good by pairs of subjects".

4.1 Contributions to the public good

Table 2 reports on the descriptive statistic "average sum of contributions", for both parts of each experiment. Data from play of the game by the pairs of subjects are organized into blocks of five rounds. In each control of each experiment, subjects start at a very high level, close to the Pareto efficient level of 40, and although play moves towards Nash equilibrium, there is still substantial overcontribution after a few rounds of play. In the last five rounds, the observed average level is about 20% higher than predicted by theory.

As for the treatment sessions:

1. There is a rapid, large shift in the level of contributions after the mechanisms are introduced. The tax-subsidy mechanism with $\beta = \frac{1}{3}$ produces a jump from around 25 in the last five rounds of the control to around 38 in the first five rounds of the treatment. With $\beta = \frac{9}{19}$, the mechanism increases the level from 23 in the last five rounds of the control to around 52 of the first five rounds of the treatment. For the compensation mechanism, there is a jump from around 24 to around 37.

2. For all blocks, the contributions are higher under the tax-subsidy treatment with high value of β than under the compensation treatment.

3. For the last three blocks, the contributions are higher under the compensation treatment than under the tax-subsidy treatment with low

Table 2: Experimental results: Sum of contributions

	Experiment 1a		Experiment 1b		Experiment 2	
	Control	Tax-Subsidy $\beta = \frac{1}{3}$	Control	Tax-Subsidy $\beta = \frac{9}{19}$	Control	Compen-sation
Nash	20	30	20	38	20	40
Pareto	40					
Rounds 1-5	35.50	38.37	36.48	52.21	35.65	37.07
Rounds 6-10	29.58	30.88	27.99	47.05	30.51	34.13
Rounds 11-15	27.18	30.37	24.83	42.75	29.16	31.75
Rounds 16-20	25.13	29.00	23.03	38.97	24.36	31.65

value of β, which itself is characterized by higher contributions than under the control.

4. The sum of contributions under the tax-subsidy mechanism is predicted accurately, after some adjustment,

5. whereas the sum of contributions under the compensation mechanism is predicted inaccurately, even after some adjustment.

4.2 Average contribution and confidence bounds over time

Figures 1, 2 and 3 report the time series of the contributions. The figures show the sum of contributions to the public good averaged over either all control groups or all treatment groups. In addition to the average of the sum of contributions, the figures show the 95% confidence interval bounds for the average group sums. To calculate the confidence intervals, we used the method of bootstrapping (with $N = 1000$).

We observe movement towards Nash equilibrium under all controls, fast convergence to equilibrium under the treatment with tax-subsidy parameter $\beta = \frac{1}{3}$ and slower convergence to equilibrium under the treatment with $\beta = \frac{9}{19}$. In contrast, we do not observe movement towards the predicted value under the compensation mechanism.

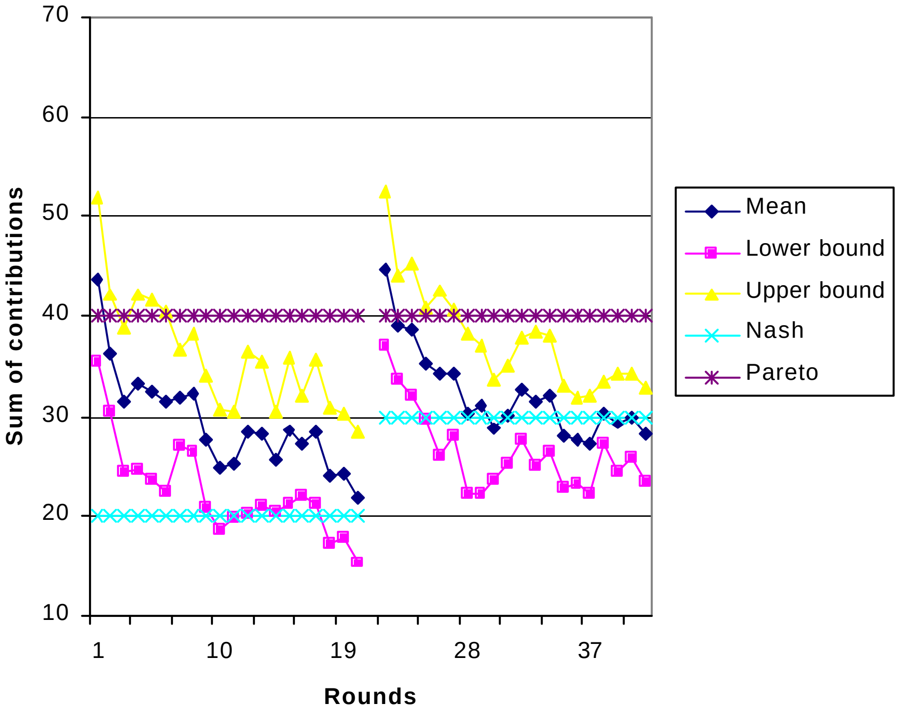

Figure 1: Sum of contributions and confidence bounds; Control and Tax Subsidy mechanism with beta = 1/3

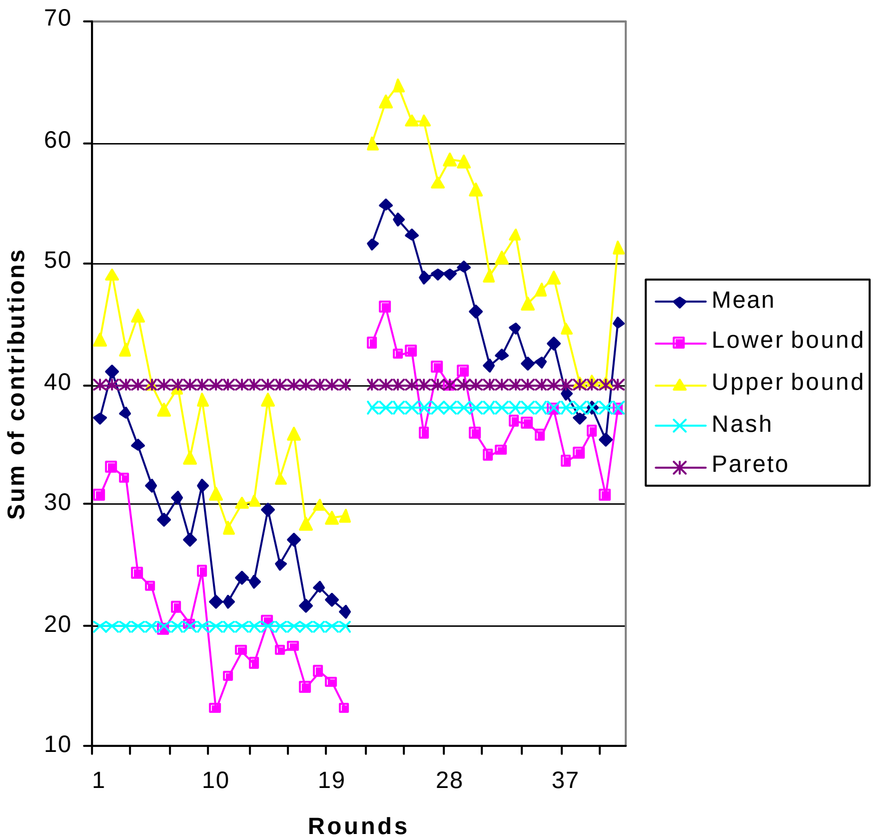

Figure 2: Sum of contributions and confidence bounds; Control and Tax-Subsidy mechanism with beta = 9/19

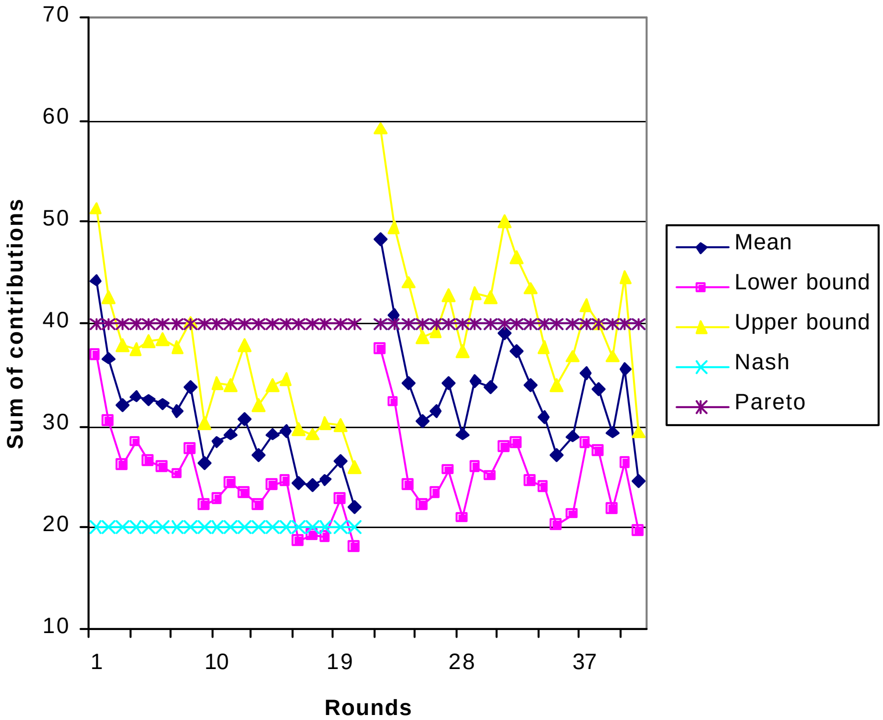

Figure 3: Sum of contributions and confidence bounds; Control and Compensation mechanism.

4.3 Parametric and nonparametric tests

In this subsection we shall repeatedly perform Mann-Whitney and bootstrap tests (i.e. parametric and non parametric tests). To perform the former is common practice in experimental economics, at least to allow for convenient comparisons with previous papers. However, some may prefer the later kind of test, for it does not rest on distributional assumptions.

The methodology followed here is first to compare the contributions under the controls in experiments 1a and 1b with the contributions under the control in experiment 2. This will lead to the conclusion that the contributions are similar in the experiments. Then, one can assess the effect of each treatment. That is, one can compare the contributions under the control with the contributions under the corresponding treatment. Lastly, one can compare the contributions under both treatments.

The average sum of contributions in the control sessions from experiments 1a and 1b are 29.35 and 28.08, respectively. The average sum of contributions in experiment 2 is 29.92. Can one consider that we observe similar levels of contributions in the controls? We use a two-sample Wilcoxon rank-sum (Mann-Whitney) test. Define $\bar{G}_1^N$ as the average sum of contributions under the control in experiments 1a and 1b and $\bar{G}_2^N$ as the average sum of contributions under the control in experiment 2. The null hypothesis is that the median of the difference $\bar{G}_1^N - \bar{G}_2^N$ is zero[10].

The results show that the medians are not statistically different at any level below .59 (with low value of β) and below 0.18 (with high value of β). We conclude that there is no difference between either control of experiment 1 and the control of experiment 2[11].

Above, the unit of observation was the average of the sum of contributions. That is, with twenty rounds, we calculated twenty statistics i.e. twenty average sums of contributions. Now, let the unit of observation be the sum of contributions of each pair. For instance, with twenty rounds and twelve pairs, we calculate two hundred forty statistics i.e. two hundred forty sums of contributions[12]. With the two-sample Wilcoxon rank-sum (Mann-Whitney) test, first, we find that $[Prob > |z|] = .28$ when we compare the control of experiment 1 with low value of β with the control of experiment 2; second, we find that $[Prob > |z|] = .05$ when we compare the control of experiment 1

[10] The test uses as inputs $\bar{G}_1^N$, the contributions summed over all pairs for each round $t = 1, 2, ..., 20$ and $\bar{G}_2^N$, the contributions summed over all pairs for each round $t = 1, 2, ..., 20$.

[11] The medians of the controls of experiment 1 are not statistically different at any level below 0.29. There is no difference between those controls, too.

[12] The test uses as inputs G_1^N of each pair for each round $t = 1, 2, ..., 20$ and G_2^N of each pair for each round $t = 1, 2, ..., 20$.

with high value of β with the control of experiment 2 and, finally, we find that $[Prob > |z|] = .24$ when we compare the controls of experiment 1. Overall we accept the conclusion that there is no difference between the controls.

4.3.1 Does the tax-subsidy mechanism change agents' behavior?

For the chosen value of the tax-subsidy parameter $\beta = \frac{1}{3}$, theory predicts a level of the public good equal to 30. The sum of contributions averaged over twelve pairs and twenty rounds is 31.63. In the last five rounds, the average sum of contributions is 29.00. The median average sum of contributions in the last five rounds is 29.33. We conclude that the observations are very close to equilibrium.

Does the treatment (with $\beta = \frac{1}{3}$) significantly increase the level of contributions? First, we use a Wilcoxon signed-ranks test. If $\bar{G}_a^N$ and $\bar{G}_a^F$ stand for the average sum of contributions under the control and the average sum of contributions under the treatment respectively, then the null hypothesis is that the median of the difference $\bar{G}_a^N - \bar{G}_a^F$ is zero. The null hypothesis can be rejected at any level below .09. We accept the conclusion that there is a significant difference between control and treatment over all rounds.

Second, we use a bootstrap test. The null hypothesis is that the level of the public good is identical in the control and the treatment sessions. Using a 5 % level of significance, the null hypothesis can be rejected on data from rounds 11-15 and data from rounds 16-20. More precisely, from the resulting data:

- In rounds $1-5$, the difference between the contributions in the treatment and the contribution in the control is 2.87, with a bias of .02, a standard error of 1.62 and a 95% confidence interval of $[-.31\ 6.04]$.
- In rounds $6-10$, the difference in contributions is 1.30, with a bias of $-.10$, a standard error of 1.58 and a 95% confidence interval of $[-1.80\ 4.40]$.
- In rounds $11-15$, the difference in contributions is 3.18, with a bias of .00, a standard error of 1.42 and a 95% confidence interval of $[.39\ 5.97]$.
- Finally, in rounds $16-20$, the difference in contributions is 3.87, with a bias of $-.02$, a standard error of 1.53 and a 95% confidence interval of $[.86\ 6.87]$.

We conclude that there is a significant difference between control and treatment in the last ten rounds.

When the tax-subsidy parameter β equals $\frac{9}{19}$, the predicted level of the public good is 38. The sum of contributions averaged over fifteen pairs and twenty rounds is 45.25. In the last five rounds, the average sum of contributions is 38.97. The median average sum of contributions in the last five rounds is 38.13. We conclude that the observations are close to equilibrium.

Does the treatment (with $\beta = \frac{9}{19}$) significantly increase the level of contributions? First, we use a Wilcoxon signed-ranks test. The null hypothesis is that the median of the difference of the average sum of contributions under the control, denoted $\bar{G}_b^N$, and the average sum of contributions under the treatment, denoted $\bar{G}_b^F$, is zero. The null hypothesis can be rejected at any reasonable level ($Prob > |z| = .00$). We conclude that there is a significant difference between control and treatment over all rounds.

Second, we use a bootstrap test. The null hypothesis is that the level of the public good is identical in the control and the treatment sessions.

- in rounds $1-5$, the difference between the contributions in the treatment and the contribution in the control is 15.73, with a bias of .21, a standard error of 2.86 and a 95% confidence interval of [10.12 21.35].

- In rounds $6-10$, the difference in contributions is 19.07, with a bias of .075, a standard error of 2.60 and a 95% confidence interval of [13.97 24.16].

- In rounds $11-15$, the difference in contributions is 17.92, with a bias of $-.03$, a standard error of 2.40 and a 95% confidence interval of [13.21 22.63].

- Lastly, in rounds $16-20$, the difference in contributions is 15.95, with a bias of .064, a standard error of 1.95 and a 95% confidence interval of [12.11 19.78].

Hence, using a 5% level of significance, we can reject the hypothesis on data from rounds $1-5$, $6-10$, $11-15$ and $16-20$.

We conclude that there is a significant difference between control and treatment.

4.3.2 Does the compensation mechanism change agents' behavior?

The predicted level of the public good is 40. The average sum of contributions of fifteen pairs and twenty rounds is 33.97. In the last five rounds, the average sum of contributions is 31.65 and the median average sum of contributions is 33.73.

Does the treatment has an effect? Again, using a Wilcoxon signed-ranks test, the null hypothesis is that the median of the difference of the average sum of contributions under the control ($\bar{G}_2^N$) and the average sum of contributions under the treatment ($\bar{G}_2^V$) is zero. The results indicate that the medians are not statistically different at any level below .22. On this basis, one may conclude that there is only an insignificant difference between control and treatment over all rounds.

However, a bootstrap test with a 5% level of significance rejects the null hypothesis that the level of the public good is the same under the control and the treatment on data from the last five rounds. Specifically, we find,

- in rounds $1-5$, the difference in contributions is 1.41, with a bias of $-.22$, a standard error of 2.36 and a 95% confidence interval of [-3.23 6.06],
- in rounds $6-10$, the difference in contributions is 3.63, with a bias of $-.06$, a standard error of 2.27 and a 95% confidence interval of [$-.85$ 8.10],
- in rounds $11-15$, the difference in contributions is 2.59, with a bias of $-.02$, a standard error of 2.26 and a 95% confidence interval of [-1.85 7.02]
- and in rounds $16-20$, the difference in contributions is 7.29, with a bias of $-.07$, a standard error of 2.11 and a 95% confidence interval of [3.16 11.42].

Hence, we conclude that there is a significant difference between control and treatment in the last five rounds.

4.3.3 Do the two mechanisms have different effects?

Does one observe significantly different levels of contributions in the two types of treatments? The average sum of contributions in the experiment with tax-subsidy parameters $\beta = \frac{1}{3}$ and $\beta = \frac{9}{19}$ are respectively 32.15 and 45.25. The average sum of contributions in the experiment with compensation option is 33.65.

Again, use has been made of a two-sample Wilcoxon rank-sum (Mann-Whitney) test. The null hypothesis is that the median of the difference of the average sum of contributions under the tax-subsidy mechanism, which we called earlier $\bar{G}_h^F$ ($h = a$ or b), and the average sum of contributions under compensation mechanism, denoted $\bar{G}_2^V$, is zero. The medians of the tax-subsidy mechanism with low value of β and the compensation mechanism are not statistically different at any level below .27. We conclude that there is only an insignificant difference between treatments.

The medians of the tax-subsidy mechanism with high value of β and the compensation mechanism are statistically different at any reasonable level ($[Prob > |z|] = .00$). We conclude that there is difference between treatments.

Some may worry that the average of the sum of contributions is quite a coarse information. Instead, let the unit of observation be the sum of contributions of each pair, so that with twenty rounds and twelve pairs, we have two hundred forty statistics i.e. two hundred forty sums of contributions. We use the two-sample Wilcoxon rank-sum (Mann-Whitney) test. Comparing the tax-subsidy mechanism with low value of β with the compensation mechanism, we find $[Prob > |z|] = .78$. Comparing the tax-subsidy mechanism with high value of β with the compensation mechanism, we find $[Prob > |z|] = .00$. From those findings, we conclude that there is no difference between the compensation mechanism and the mechanism with low value of the tax-subsidy parameter whereas there is a difference between the compensation mechanism and the mechanism with high value of the tax-subsidy parameter.

4.4 Tentative explanations

Two main findings about the performance of the incentive mechanisms in a controlled environment emerge from our analysis: *i*) the unequivocal superiority of the tax-subsidy mechanism in promoting efficiency and *ii*) the convergence of subjects' contributions to the equilibrium level in the tax-subsidy treatments and the lack of convergence to equilibrium in the compensation treatment.

How can we begin to explain the behavior of subjects?

Note that, under all three mechanisms, contributions were initially very high, before contributions decreased substantially. Initial play suggests that, at the outset, subjects may very well be motivated by, say, feelings of trust or altruistic tendencies; however those tendencies were clearly undermined in the course of play, though not completely eliminated. Elaborating on this remark, it is tempting to view the first finding as a confirmation of a "crowding out" effect. The compensation mechanism may be less efficient because it undermines intrinsic motivation for cooperation while, at the same time, substitutes real but weak incentives to depart from free-riding. Indeed, we find that the subsidy efforts were below the ones required by efficiency: in periods 21-30 with 300 observations we find an average subsidy rate of 0.185 (theory predicts 0.5) with a standard deviation of 19.37 and, in periods 31-40, with 300 observations, we find an average of 0.173 with a standard deviation of 20.04. The tax-subsidy mechanism may undermine altruistic behavior, too, however the mechanism provides strong penalties for subjects who contribute little. Note that we observed larger contributions under a high than under a low penalty regime[13].

We cannot look at the data from the tax-subsidy sessions without noticing that experience matters. Observed behavior changed as players acquire experience, in a way that suggests that learning is important[14]. Now, empirically, it has been found that modular or supermodular games like Falkinger's mechanism[15] do exhibit behavior of subjects that converge to equilibrium. Supermodularity is a technical property of games that ensures convergence to equilibrium under various learning dynamics (see Chen and Gazzale (2003)). Whether public goods with a compensation option are supermodular is an important and challenging yet still open question of ongoing theoretical research. However, if the lack of convergence to equilibrium that we observed in the laboratory is any guide, then one is lead to suspect that the supermodular property may very well be lacking in the compensation game.

We conclude by conjecturing that the superiority of the coercive mechanism might also be due to its simplicity. Subjects had to learn about the complex structure of the compensation game which is a challenging task. Furthermore, subjects had to overcome a coordination problem when devising the optimal strategy.

[13] See related evidence in a different context in Fehr and Rockenbach (2003).

[14] See the related survey by Ledyard (1995).

[15] Supermodularity would obtain for a value of $\beta \geq 1$. See Chen (2000).

5 Conclusion

The paper compares experimentally the performance of the tax-subsidy mechanism with the performance of the compensation mechanism in a public good problem. To summarize, it provides experimental evidence that: *i*) the two mechanisms change agents' behavior in the direction of increased contribution to the public good, *ii*) for a low value of the tax-subsidy parameter $\beta = 1/3$, the levels of public good resulting from the two mechanisms are not significantly different from one another, *iii*) however, with a higher value $\beta = 9/19$, closer to the one required for efficiency, the mechanisms do have different effects, *iv*) furthermore, with the tax-subsidy mechanism, theoretical predictions are more accurate and there is convergence whereas with the compensation mechanism variability is important and convergence is problematic. Points *ii*), *iii*) and *iv*) make a case in favour of the tax-subsidy mechanism: indeed, even with a value of β which is somehow below its "efficient" level, the performance of the mechanism in terms of promoting contribution to the public good is comparable with that of the compensation mechanism; besides the predictions of the tax-subsidy mechanism are more reliable.

The lack of convergence under the compensation treatment might be due to a learning problem. This mechanism is more complex than the tax-subsidy mechanism: there are two decision variables and backward induction is required within each stage game. For this reason the intuition on the benefits of the compensation mechanism may be more difficult to get within the time constraint of the experiment. In this respect, adding more rounds would be helpful.

Appendix

A Proof of Lemma 1

In the second period, agent i faces the following problem:

$$\max_{c_i \geq 0\,,\ g_i \geq 0} U^i\left(c_i, g_i + g_j\right) \tag{7}$$

subject to

$$c_i + \left(1 - s_j\right) g_i = y_i - s_i g_j \tag{8}$$

and s_i, s_j, g_j given. Or using the budget constraint to get rid of c_i, still considering s_i, s_j, g_j parametrically,

$$\max_{g_i} U^i \left(y_i - s_i g_j - (1 - s_j)\, g_i, g_i + g_j\right) \tag{9}$$

subject to

$$g_i \in \left[0, \frac{y_i - s_i g_j}{1 - s_j}\right] \ . \tag{10}$$

The optimal decision can be interior and verify:

$$\begin{aligned} \frac{U_G^i}{U_c^i} &= \frac{1}{M - N\left(y_i - s_i g_j - (1 - s_j)\, g_i\right)} \, , \\ &= MRS^i \left(y_i - s_i g_j - (1 - s_j)\, g_i\right) = 1 - s_j \, . \end{aligned}$$

Corner solutions are also a possibility; then if $g_i = 0$:

$$MRS^i \leq 1 - s_j$$

and if $g_i = \frac{y_i - s_i g_j}{1 - s_j}$:

$$MRS^i \geq 1 - s_j \, .$$

It is worth noting that strictly positive optimal decisions (interior or $g_i = \frac{y_i - s_i g_j}{1 - s_j}$) are increasing functions of s_j.

We shall first establish that $G = 0$ cannot be part of a perfect equilibrium. Indeed, $G \to 0$ means that $g_i \to 0 \, , \forall i$, therefore $c_i \to y_i \, , \forall i$. But, for any profile of subsidy that are chosen in the first stage:

$$\begin{aligned} \lim_{c_i \to y_i} MRS^i &= \lim_{c_i \to y^i} \frac{1}{M - N c_i} \, , & (11) \\ &= \frac{1}{5 - \frac{1}{10} \times 50} = +\infty > 1 - s_j \, , \quad \forall s_j \in [0, 1], & (12) \end{aligned}$$

so necessarily $G > 0$ in a perfect equilibrium.

Now, given that $G > 0$, assume that $s_1 + s_2 \neq 1$. We shall distinguish three cases.

Assume first that the two agents contribute an interior amount, therefore:

$$\begin{aligned} MRS^1 \left(y - (1 - s_2) g_1 - s_1 g_2\right) &= 1 - s_2 \, , & (13) \\ MRS^2 \left(y - (1 - s_1) g_2 - s_2 g_1\right) &= 1 - s_1 \, . & (14) \end{aligned}$$

Solving those equations, one obtains:

$$c_i = \frac{(1 - s_j) M - 1}{(1 - s_j) N} \ , \tag{15}$$

which is a decreasing function of s_j. Since by assumption $1-s_j \neq s_i$ for some i, assume without loss of generality that $1 - s_2 < s_1$. Then agent 1 could decrease her subsidy, increasing c_2 and thus decreasing g_2. Since from (15) her optimal c_1 is unaffected by this operation, but her total income $y_1 - s_1 g_2$ becomes higher, mechanically g_1 is increased. Clearly, at the unchanged own price $1-s_2$, g_1 can be increased to compensate for the decrease of g_2 so that G remains at the same level or higher but at a lower cost. Overall this agent would be better-off. The only possibility consistent with both contributions being interior is consequently $1 - s_2 = s_1$.

Second, assume that only one agent make an interior contribution, say agent 1 without loss of generality, while agent 2 contributes nothing. Then it follows from the first order conditions:

$$\begin{aligned} MRS^1 \left(y - (1 - s_2) g_1\right) &= 1 - s_2 \,, &(16)\\ MRS^2 \left(y - s_2 g_1\right) &\leq 1 - s_1 \,. &(17) \end{aligned}$$

If $1 - s_2 < s_1$, agent 2 can decrease s_2 to decrease g_1 until the point where it becomes optimal to contribute a positive g_2 such that $g_1 + g_2$ is equal to the initial G: the aggregate contributions are maintained at a lower cost. This would increase agent 2's utility. If on the contrary $1 - s_2 > s_1$, then there is an incentive for agent 1 to increase s_1 up to the point where agent 2 contribute. Concomitantly, g_1 will be reduced until the movements catch up with the previous G. This is yet another way to enjoy the same level of public good at a lower cost.

The third possibility is where one agent (or both) choose the maximum level of contribution consistent with her budget constraint, that is:

$$g_i = \frac{y - s_i g_j}{1 - s_j} \,, c_i = 0 \,.$$

For this agent

$$MRS^i \left(0\right) \geq 1 - s_j \,.$$

If $1 - s_j < s_i$, then agent i can cut s_i down to decrease g_j and increase g_i, keeping the consumption G unchanged at a lower cost. Again this is a beneficial deviation. QED.

B Proof of proposition 2

We shall follow the same lines as Danzinger and Schnitzer (1991). Let $q_i\left(g_i\right)$ stand for agent i's own price that makes g_i her most preferred individual purchase if she alone makes a contribution. In other words

$$q_i\left(g_i\right)\begin{cases} = & MRS^i\left(y-q_ig_i\right)=\frac{1}{M-N(y-q_ig_i)} & \text{for } g_i>0\,, \\ > & MRS^i\left(y\right)=\frac{1}{M-N(y)} & \text{for } g_i=0\,. \end{cases} \tag{18}$$

If $g_i > 0, q_i > 0$, then $q_i\left(g_i\right)$ is a continuous decreasing function of g_i; it can be expressed explicitly as the positive solution to the quadratic equation deduced from (18). From (18), and for our parameter values $M = 5, N = 1/10, y = 50$:

$$\frac{dq_i}{dg_i}=-\frac{q_i^2}{2g_iq_i}<0$$

which also follows from the fact that the public good is a normal good in our example. Consequently $q_1(G)+q_2(G)$ is a decreasing function for $G > 0$. Besides

$$\lim_{G\to 2y} q_1(G)+q_2(G)=\lim_{c_1\to 0, c_2\to 0} q_1(G)+q_2(G)=2/5\,,$$

and

$$\lim_{G\to 0} q_1(G)+q_2(G)=\lim_{c_1\to y, c_2\to y} q_1(G)+q_2(G)=+\infty\,.$$

From the intermediate value theorem, there exists a unique value G' such that

$$q_1(G')+q_2(G')=1\,. \tag{19}$$

In our numeric example, it is easy to check that $G' = 40$ solves (19). It follows that $q_1(40) = q_2(40) = 1/2$.

If agents choose $s_i = q_i(G')\ \forall i$, then agent i's budget constraint becomes:

$$\begin{aligned} c_i+\left(1-q_j\right)g_i &= y_i-q_ig_j\,, \\ \Rightarrow c_i+\left(1-q_j\right)g_i &= y_i-\left(1-q_j\right)g_j\,, \\ \Leftrightarrow c_i+\left(1-q_j\right)\left(g_i+g_j\right) &= y_i\,. \end{aligned}$$

where the second line is deduced from (19). Thus, for a given profile of individual prices $(1 - q_1, 1 - q_2)$, any combination of individual contributions such that $g_1 + g_2 = G'$ satisfies

$$1-q_j=MRS^i\left(y-\left(1-q_j\right)G'\right)$$

and is an equilibrium of the second stage of the game as each agent's contribution is optimal given the other agent's contribution.

It remains to show that setting $s_i = q_i\,,\ \forall i$, is also an equilibrium in the first stage of the game. Since $s_1+s_2 = 1$, from Lemma 1 it can be a candidate.

But it is not sufficient; it must be shown that no agent can increase her payoff by unilaterally deviating from $s_i = 1/2$.

Without loss of generality, suppose that agent 1 were to increase her subsidy. Since the other agent's contribution is a non decreasing function of s_1, agent 1 would find herself with a lower income (a higher g_2 to be paid for at a higher price s_1). Since both goods are normal, agent 1 would reduce her consumption and experience a loss of utility.

If instead agent 1 were to decrease her subsidy, then for agent 2 it becomes more expensive to maintain the same contribution as before, so she has an incentive to reduce it; as a result agent 1 benefits from a higher income and has an incentive to increase her own contribution, which increases further agent 2's incentives to cut down her contribution, and so on... So this deviation cannot result in a subgame perfect equilibrium where agent 2 makes a positive contribution. Agent 1 would find herself the sole contributor, at an unchanged own price for the public good $1 - s_2 = 1/2$, and where $g_1 = G' = 40$ as previously calculated. Overall she would enjoy the same level of utility and would be indifferent to the deviation. QED.

C Instructions for the compensation game

Instructions (Control)

You are now taking part in an experiment which has been financed by The Leverhulme Centre for Market and Public Organization at the Department of Economics, University of Bristol. If you read the following instructions carefully, you can, depending on your decisions, earn a considerable amount of money in addition to the £ 2.50 you will receive anyway. It is therefore very important that you read these instructions with care. At the end of the experiment all earnings will be added and paid to you in cash.

During the experiment your entire earnings will be calculated in "Points". At the end of the experiment the total amount of Points you have earned will be converted to Pounds at the following rate:

100 Points = £ 0.125

In this experiment there are 20 participants, which are divided into 10 groups with 2 members each. Except us, i.e., the experimenters, no one knows the group composition.

The experiment is divided into 20 periods. In each period you have to make a decision that you have to enter into the computer. During these 20 periods the group composition stays the same. You are, therefore, remaining with the same people in a group for 20 periods, but you will never learn with whom you formed a group.

The instructions which we have distributed to you, are solely for your private information. It is prohibited to communicate with the other participants during the experiment. Should you have any questions, please ask us. If you violate this rule, we will have to exclude you from the experiment and from all payments.

The following pages describe the course of the experiment in detail:

At the beginning of each period each participant gets **50 points**, which in the following we call **endowment**. Your task is to make a decision how to use your endowment. There are two activities for which you can use your endowment: **activity A** and **activity B**. These activities result in different incomes which we will explain in more detail below.

At the beginning of each period the following screen will appear:

[Insert screen shot of decision screen, Control]

You have to decide how many points you want to contribute to **activity A** by typing a number between 0 and 50 in the input field. This field can be reached by clicking it with the mouse. As soon as you have decided how many points to contribute to activity A, you have also decided how much your **contribution to activity B** is, namely, (**50 - your contribution to activity A**) points.

The current period appears in the top left corner of the screen. In the top right corner you can see how many more seconds remain for you to decide on the distribution of your points. Your decision must be made before the time displayed is 0 seconds. After you have inserted your contribution you have to press the OK-button (with the help of the mouse). As soon as you have pressed the OK-button you cannot revise your decision for the current period anymore.

After all members of your group have made their decision, the following income screen will appear.

[Insert screen shot of income screen 1, Control]

Besides the period and the remaining time for watching the income screen, the income screen shows the following entries, which we will explain in detail below.

1. Your contribution to activity A
2. Your income from activity A
3. Your contribution to activity B
4. Your income from activity B
5. Your total income in this period

In the following we will explain these items in detail.

1. Your contribution to activity A:

Here your contribution to activity A as you have inserted it previously is listed.

2. Your income from activity A:

Your income from activity A is your contribution to activity A plus the other group member's contribution to activity A.

Income from activity A = sum of contributions of both group members to activity A

The income of the other group member from activity A is calculated according to the same formula, i.e., each group member has the same income from activity A. If, for example, you contribute 20 points and the other group member contributes 40 points, the sum of the contributions is 60 points and you and the other group member receive an income of 60 points from activity A. If both group members together invest 10 points in activity A, you and the other group member each receive an income of 10 points.

Therefore, each point that you contribute to activity A, increases your income by 1 point. However, this also increases the income of the other group member by 1 point, such that the total income of the group increases by $2 \times 1 = 2$ points. **Hence, through your contribution to activity A the other group member earns something. In turn, it also holds that you earn something from the contribution of the other group member.**

3. Your contribution to activity B:

Your contribution to activity B is the difference between your endowment of 50 points and your contribution to activity A:

If, for example, you contribute 20 points to activity A, your contribution to activity B is 30 points.

Your contribution to activity B = 50 - contribution to activity A

4. Your income from activity B:

An important difference between the contributions to activity A and activity B is that from your contribution to activity A the other group member earns something in the same way, whereas from your contribution to activity B only you earn something. In turn, it also holds that you earn in the same way from the contribution of the other group member to activity A, while you earn nothing from the contribution of the other group members to activity B. The income of each group member from his contributions to activity B is calculated according to the following formula:

$$\begin{aligned}\textbf{Income from activity B} \quad &= 5\times\textbf{contribution to activity B} \\ &\quad \textbf{-}\ (\tfrac{1}{20})\times\textbf{(contribution to activity B)}^2\end{aligned}$$

The income from activity B will be calculated for both group members according to the same formula. In the Table **your income from activity B** is indicated for each level of your contribution to activity B. If your contribution to activity B is 0, for example, then your income from activity B is 0 (see Table). If your contribution is 20, then you will earn an income of 80.00 points from activity B (see Table). If your contribution is 50, then you will earn an income from activity B of 125.

Your income from activity B depends therefore on your contribution to activity B. The other group member earns - in contrast to activity A - nothing from your contribution to activity B.

In the enclosed **Table** not only your income from the contribution to activity B is listed, but also the **income change, if activity B increases by 1 unit**, as well as the **income change, if activity A increases by 1 unit**. As you see (and as already explained above under "Income from activity A") your income from **activity A** increases always exactly by 1 unit for each additional point you or the other group member contributes to activity A. The income change in activity B, however, is not constant. The income change is smaller, the larger your contribution to activity B already is. Therefore when your contribution to activity B is low (because your contribution to activity A is high) an additional contribution to activity B generates a relatively large additional income from activity B. If, in contrast, your contribution to activity B is high (because your contribution to activity A is low) an additional contribution to activity B generates a relatively low additional income from activity B.

5. Your total income in a period:

The total income (in points) in a period is

Income from activity A + income from activity B

At the end of the experiment the total incomes of all periods will be added up and exchanged into Pounds.

Control questionnaire:

Please answer all questions. The questions' purpose is only to enhance your understanding. Please always write down the whole calculation. If you have questions, please ask us!

Each group member is endowed with 50 points.

1. Assume that nobody (including you) contributes anything to activity A. What is
   Your contribution to activity B? ....
   Your income from activity A? ....
   Your income from activity B? ....
   Your total income in this period? ....
2. Assume that the other group member contributes nothing to activity A. You contribute 10 points. What is
   Your contribution to activity B? ....
   Your income from activity A? ....
   Your income from activity B? ....
   Your total income in this period? ....
3. Assume that each group member (including you) contributes 50 points to activity A. What is
   Your contribution to activity B? ....
   Your income from activity A? ....
   Your income from activity B? ....
   Your total income in this period? ....
4. Assume that the other group member contributes 50 points to activity A. You contribute 40 points to activity A. What is
   Your contribution to activity B? ....
   Your income from activity A? ....
   Your income from activity B? ....
   Your total income in this period? ....

Instructions (Varian)

We now repeat the experiment for further 20 periods. In this experiment, there are two stages.

You have a new option. You can offer a payment to the other group member to encourage him to contribute to activity A.

At the beginning of each period the following screen will appear:

[Insert screen shot of side payment screen]

The current period appears in the top left corner of the screen. In the top right corner you can see how many more **seconds** remain for you to decide on the payment to the other group member. Your decision must be made before the time displayed is 0 seconds.

On the screen you see an input field. This field can be reached by clicking it with the mouse. You decide how much you are willing to pay the other group member by typing a number between 0 and 100 in the input field. This number is the rate expressed in percentage terms at which you support the other group member's contribution to activity A. Suppose the other group member's contribution to activity A is 20. If you decide on a rate of 10% at which you support the other group member, you make a payment of $\frac{10}{100} \times 20 = \frac{200}{100} = 2$. If you decide on a rate of 30% at which you support the other group member, you make a payment of $\frac{30}{100} \times 20 = \frac{600}{100} = 6$. Suppose your contribution to activity A is 40. If the other group member decides on a rate of 10% at which he supports your contribution to activity A, he makes a payment of $\frac{10}{100} \times 40 = \frac{400}{100} = 4$. If he decides on a rate of 30% at which he supports your contribution to activity A, he makes a payment of $\frac{30}{100} \times 40 = \frac{1200}{100} = 12$. Neither group member will see how much the other group member is willing to pay until both group members have decided. As we will explain below, your payment will decrease your contribution to activity B. The other group member's payment will increase your contribution to activity B.

After you have inserted your rate you have to press the **OK-button** (with the help of the mouse). As soon as you have pressed the OK-button you cannot revise your decision for the current period anymore. After all members of your group have made their payment decision, you have to determine your contribution to activity A in the decision screen. The rate at which the other member supports your contribution to activity A appears in the center of the screen. Again you are endowed with 50 points per period. Again at the beginning of each period the following screen will appear:

[Insert screen shot of decision screen]

As previously your **income from activity A** is the *sum of contributions of both group members to activity A*. The income of both group members to activity A is calculated in the same way. In other words, the calculation of income from activity A is unchanged for both group members.

Hence, the change in this new experiment concerns the calculation of the contribution to activity B. Your direct contribution to activity B is the difference between your endowment, 50 points, and your contribution to activity A. Your total contribution to activity B is the sum of your direct contribution to activity B and the difference between your group member's contribution to your activity B and your contribution to your group member's activity B.

Your group member's contribution to your activity B depends on your contribution to activity A:

your group member's contribution to your activity B	= the other member's payment to you
	= (rate at which the other group member supports you)$\times$ (your contribution to activity A)

If, for example, the rate at which the other group member supports you is $\frac{1}{4}$ and your contribution to activity A is 20, the other member's payment to you is 5. The other member's payment is your group member's contribution to your activity B. Therefore, your group member's contribution to your activity B is 5.

Your contribution to the other group member's activity B depends on your group member's contribution to activity A:

your contribution to your group member's activity B	= your payment to the other group member
	= (rate at which you support the other group member)$\times$ (the other group member's contribution to activity A)

If, for example, the rate at which you support the other group member is $\frac{1}{5}$ and the other group member's contribution to activity A is 10, your payment to the other group member is 2. Your payment to the other group member is your contribution to your group member's activity B. Therefore, your contribution to your group member's activity B is 2.

It therefore holds:

Your total contribution to activity B = direct contribution to activity B + other member's contribution to your activity B - your contribution to the other member's activity B

Therefore, in the example, your direct contribution to activity B is 50 − your contribution to activity A : $50 - 20 = 30$, your group member's contribution to your activity B is 5 and your contribution to your group member's activity B is 2. Therefore your total contribution to activity B is: $30 + 5 - 2 = 33$.

The change has two consequences:

- In the previous experiment your total contribution to activity B decreased by 1-point if you have invested an additional point in activity A. Since in the new experiment the contribution to activity B depends on the other member's contribution to your activity B, the total contribution to activity B decreases now by 1−(rate at which the other group member supports you), if you contribute one more unit to activity A (see overview scheme).

If, for example, the rate at which the other group member supports you is $\frac{1}{4}$, and you contribute one more unit to activity A, your total contribution to activity B decreases by $1 - \frac{1}{4} = \frac{3}{4}$.

- Your contribution to activity B depends on the other group member's **level of contribution** to activity A. An increase of the other group member's contribution to activity A by one unit decreases your total contribution to activity B by the (rate at which you support the other group member)-point (see overview scheme).

If, for example, the rate at which you support the other group member is $\frac{1}{5}$, and the other group member contributes one more unit to activity A, your total contribution to activity B decreases by $\frac{1}{5}$.

Once the total contribution to activity B is determined, the calculation of the income from activity B is done according to exactly the same formula as in the previous experiment (see Table). The old income table is therefore still valid.

As previously, your total income in a period consists of the income from activity A plus the income from activity B.

After these 20 periods the experiments are definitively over and the payoffs of the two experiments will be added and paid to you in cash.

Control questionnaire:

Please answer all questions. The questions' purpose is only to enhance your understanding. Please always write down the whole calculation. If you have questions, please ask us!

Each group member is endowed with 50 points.

1. Assume that nobody (including you) contributes anything to activity A. The rate at which the other group member supports you is $\frac{1}{2}$. The rate at which you support the other group member is $\frac{1}{3}$.What is

Your direct contribution to activity B?	....
Your group member's contribution to your activity B?	....
Your contribution to your group member's activity B?	....
Your total contribution to activity B?	....
Your income from activity B?	....
Your income from activity A?	....
Your total income in this period?	....

2. Assume that the other group member contributes nothing to activity A. You contribute 10 points. The rate at which the other group member supports you is $\frac{1}{2}$. The rate at which you support the other group member is $\frac{1}{3}$. What is

Your direct contribution to activity B?	....
Your group member's contribution to your activity B?	....
Your contribution to your group member's activity B?	....
Your total contribution to activity B?	....
Your income from activity B?	....
Your income from activity A?	....
Your total income in this period?	....

3. The rate at which the other group member supports you is $\frac{1}{2}$. The rate at which you support the other group member is $\frac{1}{5}$. Assume that each group member (including you) contributes 50 points to activity A. What is

Your direct contribution to activity B?	....
Your group member's contribution to your activity B?	....
Your contribution to your group member's activity B?	....
Your total contribution to activity B?	....
Your income from activity B?	....
Your income from activity A?	....
Your total income in this period?	....

4. Assume that the other group member contributes 50 points to activity A. You contribute 40 points to activity A. The rate at which the other group member supports you is $\frac{1}{2}$. The rate at which you support the other group member is $\frac{1}{5}$ What is

Your direct contribution to activity B?	....
Your group member's contribution to your activity B?	....
Your contribution to your group member's activity B?	....
Your total contribution to activity B?	....
Your income from activity B?	....
Your income from activity A?	....
Your total income in this period?	....

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