Worker Substitutability and Bargaining Delays

Andreas Westermark*

Department of Economics

Uppsala University

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Abstract

We analyze a bargaining model with imperfect information where a firm bargains with more than one worker. We find that equilibrium delay decreases in the substitutability of workers. The reason is that a decrease in substitutability increases both wages and wage differentials between high and low productive firms. Then, to ensure separation, low productive firms must delay agreement longer to deter the high productive firm from mimicking. We also analyze how delay depend on the number of workers.

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*The author can be contacted at e-mail address Andreas.Westermark@nek.uu.se

1 Introduction

During the last two decades, noncooperative bargaining models have been studied extensively. One of the focuses of the analysis has been on delay in bargaining under imperfect information. The papers by Grossman (1986), Admati (1987), Gul (1988) and Cramton (1992) are all important contributions to the field. The three first papers analyze one sided uncertainty, while the fourth focuses on two sided uncertainty. Later contributions in the literature include Watson (1998) and Wang (1998). All these papers focus on the situation where two players bargain with each other. It is easy to believe that, in case of bargaining with more than two players, the results from the two player case generalizes as long as each bargaining pair is unrelated with the others. However, this is not such a reasonable assumption in for example firm worker negotiations. Then, one of the players (the firm) bargains with several partners (workers). Usually, the workers does not affect firm profits independently. The interrelationship among workers can occur in several different ways. The interrelationship can be purely technological - the degree of substitutability among workers - or it can work through the prior probability distribution - the correlation among worker productivity.

The purpose of this paper is to analyze how equilibrium bargaining delay is affected by changes in the degree of complementarity and correlation in productivity among workers in a firm which bargains with two workers. Also, the effect of firm size on delay, in terms of the number of workers employed by the firm, is analyzed.

The theoretical model builds primarily on Admati (1987) and Cramton (1992). We assume that the productivity of the workers is known only by the firm. A motivation of analyzing this type of asymmetric information is that it seems reasonable to assume that firms have an information advantage concerning future demand for the product that the firm produces and hence the profit level. [1] The bargaining takes place as follows. The workers alternate in bargaining with the firm. As in Ad-

[1] An alternative but perhaps less likely motivation is that, the firm is better in judging the productivity of the installed machines and equipment.

mati (1987) and Cramton (1992), when selected as proposer, any player can make a proposal immediately or delay and make a proposal later on. Following a proposal, the respondent accepts or rejects the proposal.

We first analyze a model where the productivity of workers is perfectly correlated. We find that a decrease in substitutability increases equilibrium delay. The reason for this is the following. As in models of perfect information, the worker payoff in equilibrium is half of the marginal return. This is a standard result in bargaining analyzing a firm bargaining with many workers, at least under binding contracts. If the degree of substitutability decreases, then the marginal return increases and hence the wage paid out to each worker increases. Second, the equilibrium has the property that it is fully separating, at least when the firm is proposer. As usual when analyzing separating equilibria, the type of the firm must be signalled credibly, i.e., such that no other type wants to mimic the firm. In this model delay is used to credibly signal the firm type. A firm with high productivity delays less than a firm with low, because a larger amount of the surplus disappears when delaying. Consider two firm types with productivities θ and θ' where $\theta > \theta'$ and suppose substitutability decreases, for some given equilibrium delay that separates θ' from θ. Since substitutability decreases, wages to the workers also increase. However, since productivity θ and the degree of substitutability[2] is multiplicatively separable, the wage paid when the firm has high productivity increases more than when productivity is low. Hence, the high productivity firm will find it profitable to mimic the low productivity firm, since the wage payments decrease more than before the decrease in substitutability. Thus, a decrease in substitutability lead to higher wages and more importantly to a higher wage difference between firm type θ and θ'. Hence the low productivity firm needs to delay a longer amount of time to deter the high productive firm from mimicking.

We also find that, an increase in the number of workers lead to a decrease in delay.

[2]This holds if the production function is a Cobb Douglas production function of the type $\theta (l_1)^a (l_2)^b$, which is a standard assumption in the macro literature.

The reason is that delay becomes more costly as the number of workers increase. If there number of workers increases, there is a larger surplus from agreement. Delaying then leads to a larger loss of surplus and hence, a shorter delay can credibly avoid mimicking.

The model is developed in the next section. Equilibrium under perfect correlation is analyzed in section 3, sections 4 and 5 extends the model by introducing imperfect correlation and more than two workers, respectively. Finally, section 6 concludes.

2 The Model

There are two workers and one firm. There are two goods, leisure and a consumption good. A worker either works some fixed amount of time or does not work at all. The utility of leisure for the workers is commonly known and for simplicity normalized to zero. As mentioned in the introduction, the value to the firm of hiring the workers is private information. The value of the production to the firm in state i when hiring only one worker is x_i and when hiring two workers is

$$x_i + y_i.$$

Thus, y_i is the marginal contribution of the second worker. The setup here follows the setup used in Horn and Wolinsky (1988).

At the beginning of the game none of the workers are employed by the firm. The firm bargains with one worker at a time. At the beginning of the bargaining game, the firm bargains with worker 1. The firm makes the first proposal and the worker responds yes or no. If an agreement is reached, the worker who signed the agreement leaves the game and bargaining continues with the remaining worker. If an agreement is reached the contract is observed by all players. In case of no agreement after the first round, the game continues to the second round, which is similar to the first, with the exception that worker 1 is replaced by worker 2. As long as no agreement has been reached with any of the workers, the game proceeds as above, with the firm bargaining with worker 1 in odd rounds and worker 2 in even rounds. The proposal rights alternate between the firm and a given worker, i.e., if the firm meets worker 1 at time period t and is proposer, then the next time the firm meets worker 1, the worker is proposer. When being proposer, the player that is proposer can either propose immediately or wait and delay the proposal. Thus, the offer can be made later than at the minimum time. A minimal amount of time passes between rounds, i.e., before another proposal can be made. If an agreement is reached between the firm and one worker, then the game proceeds as above, with the exception that the firm bargains with the other worker in all rounds.

Consider an outcome of the bargaining game. Suppose an agreement between worker i and the firm is reached at time t_i where $t_1 < t_2$. Let p_j denote the wage payment to worker j. Let r denote the rate of time preference of players. The payoff of the firm is,

$$V_i(t,p) = e^{-rt_1}(x_i - p_1) + e^{-rt_2}(y_i - p_2).$$

The payoff for worker j is

$$v_j(t,p) = e^{-rt_j} p_j.$$

As in Admati (1987) and Cramton (1992) there is a minimum amount of time that has to pass between rounds. This minimum amount is $-\frac{1}{r}\ln\delta$ where $\delta < 1$. If a player, say a worker, expects to receive p_j in the next round and if there is no delay beyond the minimum time the payoff is $e^{-r\left(-\frac{1}{r}\ln\delta\right)} p_j = \delta p_j$. As we analyze a model with imperfect information, the sequential equilibrium concept is used. We restrict attention to stationary strategies.

We restrict attention to one-sided asymmetric information. Specifically, we assume that the firm has full information, i.e., knows both x_i and y_i while the workers does not. The motivation for this assumption is that the firm has an information advantage about, e.g., the demand conditions for the product it sells, the prices of other inputs etc.

3 Equilibrium

We first focus on the case with two workers and perfect correlation. The production function of the firm is given by

$$(x + y)\,\theta$$

for any $\theta \in \Theta$. Thus, workers are symmetric and productivity is perfectly correlated. The production function above is consistent with the frequently used CES productivity function $A\left(a_1 (l_1)^{\rho} + a_2 (l_2)^{\rho}\right)^{\frac{1}{\rho}}$. To see this, set $A = \theta$, and $l_1 = l_2 = 1$, and, by symmetry, $a_1 = a_2 = 1$. Then we get $x = 1$ and $y = 2^{\frac{1}{\rho}} - 1$.

The productivity θ has probability distribution f and cumulative distribution F. We restrict attention to the same class of distributions as Cramton (1992), i.e., we have, when $b > s$,

$$F(b) - F(s) \leq f(b)(b - s).$$

3.1 Subgames with one worker remaining

When analyzing subgames where an agreement has been reached with one worker, the subgame is similar to the model in Cramton (1992), with the exception that there is only one-sided uncertainty. Let

$$p_f(\theta) = \frac{\delta y \theta}{1 + \delta} \text{ and } p_w(\theta) = \frac{y\theta}{1 + \delta}.$$

denote the perfect information prices when the firm and worker is proposer, i.e., the prices offered when both the firm and worker knows that the type of the firm is θ.[3]

We first start to analyze subgames where the firm is proposer.

3.1.1 Subgames where the firm is proposer

We first establish that equilibrium delay is nonincreasing in θ.

[3]When the productivity is revealed, bargaining takes place exactly as in a standard perfect information bargaining game, see e.g. Admati (1987) for an argument for why this holds.

Lemma 1 *Equilibrium delay is nonincreasing in θ.*

First, we claim that the set of types that delays exactly Δ must be an interval. Suppose type $\theta \in \Theta$ makes a proposal p_f that is accepted. Then, any $\theta' > \theta$ also make acceptable proposals. Too see this, let Δ' and p' denote the outcome for type θ' when not making an acceptable proposal. Then we must have

$$y\theta' - p_f < e^{-r\Delta'} \left(y\theta' - p'\right).$$

Since preferences satisfy the single crossing property, type θ has a profitable deviation by mimicking type θ'. A similar argument establishes that, if type θ does not make an acceptable proposal after delaying Δ, then any type $\theta' < \theta$ does not make an acceptable proposal. Hence, optimal delay is nonincreasing in θ.■

Note that the above lemma implies that, at any point in time in equilibrium, there is a cutoff level θ^0 such that all types with productivities above θ^0 have made acceptable proposals, while the others have not. Then we can define, given that workers believe that θ^0 is the cutoff value of the firm, $\theta\left(\Delta \mid \theta^0\right)$ as the (set of) productivity of the firm inferred by workers if the firm delays exactly Δ (from now on) before offering $p_f(\theta)$. Also, let $\Delta^*\left(\theta \mid \theta^0\right)$ denote the length of delay required to signal θ credibly. From the previous Lemma, we know that $\Delta^*\left(\theta \mid \theta^0\right)$ is nonincreasing. The following Lemma establishes some properties of $\theta\left(\Delta \mid \theta^0\right)$.

Lemma 2 $\theta\left(\Delta \mid \theta^0\right)$ *is a singleton and differentiable a.e.*

Proof: See the Appendix. ■

We restrict attention to functions $\theta\left(\Delta \mid \theta^0\right)$ that are differentiable everywhere. The lemma above indicates that this is a mild restriction. Below, it is shown that such a function exists.

Assumption 1. $\theta\left(\Delta \mid \theta^0\right)$ is differentiable.

Now let us find the functions $\Delta^*\left(\theta \mid \theta^0\right)$ and $\theta\left(\Delta \mid \theta^0\right)$. First, consider the equilibrium delay, for given worker beliefs $\theta\left(\Delta \mid \theta^0\right)$. From Lemma 2 we know that

$\theta(\Delta \mid \theta^0)$ is a singleton. Hence, the worker is in equilibrium able to infer the exact productivity of the firm, because each firm type delays a different amount of time. Also, following a proposal by the firm, the payoff offered to the worker is equal to the payoff the worker gets in case of rejection. However, in case of rejection we have a perfect information subgame where the worker can ensure a payoff of $p_w(\theta)$. Hence, by standard Rubinstein-Ståhl arguments, the firm must offer the worker $p_f(\theta)$. The payoff of the firm is then

$$e^{-r\Delta}\left(y\theta - p_f\left(\theta\left(\Delta \mid \theta^0\right)\right)\right).$$

Clearly, the expected utility of the firm from choosing $\Delta^*\left(\theta \mid \tilde{\theta}_w\right)$ must be larger than any other possible delay Δ, i.e., $\Delta^*\left(\theta \mid \tilde{\theta}_w\right)$ solves the first-order condition

$$-re^{-r\Delta}\left(y\theta - \frac{\delta y}{1+\delta}\theta\left(\Delta \mid \theta^0\right)\right) + e^{-r\Delta}\left(-\frac{\delta y}{1+\delta}\theta'\left(\Delta \mid \theta^0\right)\right) = 0$$

Simplifying and using truth-telling, i.e., $\theta(\Delta \mid \theta^0) = \theta$ gives $d\Delta = -\frac{\delta}{r\theta}d\theta$ Thus, we get a differential equation with initial condition $\theta(0 \mid \theta^0) = \theta^0$. Integrating gives

$$\Delta^*\left(\theta \mid \theta^0\right) = \frac{\delta}{r}\log\frac{\theta^0}{\theta}. \tag{1}$$

Note in particular that the delay of type $\theta = \theta^0$ is zero. Since equilibrium worker beliefs are correct in equilibrium, the worker infers that the firm has type θ, if the firm delays exactly $\Delta^*(\theta \mid \theta^0)$. Using the above expression then gives

$$\theta\left(\Delta \mid \theta^0\right) = \theta^0 e^{-\frac{r\Delta}{\delta}}.$$

3.1.2 Subgames when the worker is proposer

Suppose the worker has made a proposal p. Clearly, the firm accepts all offers that are better compared with rejecting and self becoming proposer. In equilibrium, the worker chooses the price to maximize payoffs. Also, for any given proposal p there is a critical value $\tilde{\theta}_w$ such that the firm type $\tilde{\theta}_w$ is indifferent between accepting the proposal p and rejecting and counteroffering $p_f(\tilde{\theta}_w)$ without additional delay,

$$y\tilde{\theta}_w - p = \delta\left(y\tilde{\theta}_w - p_f\left(\tilde{\theta}_w\right)\right).$$

For this given price, all firms with productivities higher (lower) than $\tilde{\theta}_w$ accepts (rejects) the proposal. Thus, by varying the price, the worker affects the critical value $\tilde{\theta}_w$. A lower price makes more firm types willing to accept and a higher price fewer firms accept. Thus, there is a direct relationship between p and the cutoff value θ. In particular, we have, using the definition of $p_f(\theta)$, solve for p;

$$p = \frac{y\tilde{\theta}_w}{1+\delta}.$$

Note that this is just $p_w(\tilde{\theta}_w)$.

Now consider the equilibrium cutoff level $\tilde{\theta}_w$. Note that, in equilibrium, the worker chooses the price to maximize payoffs. Since there is a one-to-one relationship between $p_w(\tilde{\theta}_w)$ and $\tilde{\theta}_w$ the worker can as well choose $\tilde{\theta}_w$ instead. In equilibrium, the firm type $\tilde{\theta}_w$ must be indifferent between accepting the workers proposal of $p_w(\tilde{\theta}_w)$ and rejecting and counteroffering $p_f(\tilde{\theta}_w)$ after the minimum amount of delay. This follows from expression (1). Also, all types $\theta > \tilde{\theta}_w$ strictly prefers to accept and all types $\theta < \tilde{\theta}_w$ strictly prefers to reject and counteroffering $p_f(\theta)$ after delaying $\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\delta}$. Hence the payoff of a worker is

$$p_w\left(\tilde{\theta}_w\right)\int_{\tilde{\theta}_w}^{\theta^0} f(\theta)\,d\theta + \int_{\underline{\theta}}^{\tilde{\theta}_w} \delta p_f(\theta)\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\delta} f(\theta)\,d\theta. \tag{2}$$

The worker chooses $\tilde{\theta}_w$ to maximize the expected payoff, subject to $\tilde{\theta}_w \in [0,\theta^0]$. We have the following result.

Lemma 3 *Suppose $\tilde{\theta}_w$ maximizes (2). Then $\tilde{\theta}_w < \theta^0$. For δ close to one, we have $\tilde{\theta}_w > \underline{\theta}$. In equilibrium, the worker proposes $p_w(\tilde{\theta}_w)$. Any firm with $\theta \geq \tilde{\theta}_w$ accepts the proposal. Any firm with $\theta < \tilde{\theta}_w$ rejects and counteroffers $p_f(\theta)$ after a delay of*

$\Delta^*(\theta \mid \tilde{\theta}_w)$.

Proof:

Step 1: Finding $\tilde{\theta}_w$.

The first-order condition of (2) is, using the definition of $p_f(\theta)$ and $p_w(\theta)$,

$$F\left(\theta^0\right) - F\left(\tilde{\theta}_w\right) - f\left(\tilde{\theta}_w\right)\tilde{\theta}_w + \delta^2\tilde{\theta}_w f\left(\tilde{\theta}_w\right) - \delta^3 \int_{\underline{\theta}}^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\delta+1} f(\theta)\, d\theta = 0. \quad (3)$$

Since the left-hand side is negative at $\tilde{\theta}_w = \theta^0$ we have $\tilde{\theta}_w < \theta^0$. Also, the left hand side evaluated at $\tilde{\theta}_w = \underline{\theta}$is

$$F\left(\theta^0\right) - F\left(\underline{\theta}\right) - \left(1 - \delta^2\right) f\left(\underline{\theta}\right) \underline{\theta}$$

For δ close to one, this expression is positive. Since (3) is continuous in $\tilde{\theta}_w$ there is a value of $\tilde{\theta}_w$ that solves (3).

Step 2: The equilibrium.

Suppose the worker has offered p. In response to rejection and a counteroffer after delay of Δ, the worker believes that the firm is of type $\theta\left(\Delta \mid \tilde{\theta}_w\right)$ with probability one. Equilibrium actions of the firms is as follows.

i). If $\theta \geq \tilde{\theta}_w$ and $\theta y - p \geq \delta\left(\theta y - p_f\left(\tilde{\theta}_w\right)\right)$ the firm accepts p.

ii). If $\theta \geq \tilde{\theta}_w$ and $\theta y - p < \delta\left(\theta y - p_f\left(\tilde{\theta}_w\right)\right)$, the firm rejects p and counteroffers $p_f\left(\tilde{\theta}_w\right)$ immediately.

iii). If $\theta < \tilde{\theta}_w$, the firm rejects p and counteroffers $p_f(\theta)$ after a delay of $\Delta^*\left(\theta \mid \tilde{\theta}_w\right)$.

Also, in equilibrium, the worker proposes $p = p_w(\tilde{\theta}_w)$.

Note that, since $p = p_w\left(\tilde{\theta}_w\right)$ we cannot have $\theta y - p < \delta\left(\theta y - p_f\left(\tilde{\theta}_w\right)\right)$. Hence, only i) and iii) are possible equilibrium actions of the firm. ■

3.1.3 The equilibrium in subgames with one worker remaining

The equilibrium outcome can be summarized as follows. In case the firm is recognized as proposer, firm type θ delays $\Delta^*(\theta \mid \theta^0)$ and then proposes $p_f(\theta)$. The worker accepts this proposal. If the worker is recognized as proposer, the worker proposes $p_w(\tilde{\theta}_w)$ All firm types with $\theta \geq \tilde{\theta}_w$ accepts while the other rejects. Following a rejected proposal firm type θ delays $\Delta^*(\theta \mid \tilde{\theta}_w)$ before proposing $p_f(\theta)$ which is accepted.

3.2 Subgames without any agreement

Consider a subgame without any agreement following a proposal by one of the workers. We first need to determine prices in a full information subgame.

3.2.1 Perfect information

With perfect information, the worker is in equilibrium able to infer the exact productivity of the firm, because each firm type delays a different amount of time. For such subgames, we have the following result. [4]

Lemma 4 *Suppose the workers know that the firm has productivity θ. For all x, y there exists a stationary equilibrium where the payoff as $\delta \to 1$ of the firm is $x\theta$ and the payoff of the workers are $\frac{y\theta}{2}$.*

Proof: See the appendix. ■

Note that the result above is similar to results in for example Horn and Wolinsky (1988) and Westermark (2003). The above equilibrium is the only symmetric stationary equilibrium. There are also some asymmetric stationary equilibria. These only exist when workers are complements. We have the following result.

[4] Note that an alternative setup that lead to a similar equilibrium is when the firm and one worker is selected to bargain at random. If a firm worker pair is selected, both the firm and the selected worker are allowed to make proposals. Following a rejection of a proposal by one of the players, the other can make a counteroffer. After another rejection, the firm and one of the workers is matched at random.

Lemma 5 *Suppose the workers know that the firm has productivity θ. For $x < y$ there exists a stationary equilibrium where the firm agrees first with worker* $1(2)$ *the payoff as $\delta \to 1$ of the firm is $\frac{1}{4}(3x + y)\theta$, the payoff for worker* $1(2)$ *is $\frac{1}{4}(x + y)\theta$ and the payoff of worker* $2(1)$ *is $\frac{y\theta}{2}$.*

Proof: See the appendix. ■

It can be shown that there are no other pure strategy equilibria. [5] Below, we restrict attention to the symmetric equilibrium in Lemma 4.

Now consider subgames where no information has been transmitted.

3.2.2 Subgames when the firm is proposer

We first show that equilibrium delay is nonincreasing in θ.

Lemma 6 *Equilibrium delay is nonincreasing in θ.*

Proof: First, we claim that the set of types that delays exactly Δ must be an interval. Suppose type $\theta \in \Theta$ makes a proposal p_f that is accepted. Then, any $\theta' > \theta$ also make acceptable proposals.

Step 1: Establishing that the set of types that delays exactly Δ is a singleton.

We prove this by contradiction. Let $\Delta' > \Delta$ and p' denote the outcome for type θ' when not making an acceptable proposal. For simplicity, let $p(i)$ denote the price offered when i workers remain. Also, let Θ and Θ' denote the posterior belief support following $p(2)$ and $p'(2)$, respectively. Note that, in the round following agreement with the first worker, there is some type that agrees immediately. This follows, since if there is no type that proposes immediately, then, by stationarity, no agreement is ever reached.

We first show that $\sup \Theta \geq \sup \Theta'$.

[5] A proof is available from the author on request. The proof goes through all possible cases and shows that all other possible equilibrium candidates besides those in Lemmata 4 and 5 can be ruled out.

Suppose by contradiction that $\sup\Theta < \sup\Theta'$. Then, for type $\sup\Theta'$ we have

$$\begin{aligned}&x\sup\Theta' - p_f(2) + \delta\left(y\sup\Theta' - p_w(1)\right)\\&\leq e^{-r\Delta'(2)}\left(x\sup\Theta' - p'(2) + \delta\left(y\sup\Theta' - p'_w(1)\right)\right)\end{aligned}$$

Then, since $e^{-r\Delta'(2)} < 1$ the same inequality holds strictly for type $\sup\Theta$. Hence type $\sup\Theta$ has a profitable deviation.

Case 1: For all $\varepsilon > 0$ such that $\varepsilon < \gamma$ for some $\gamma > 0$ there is some $\theta \in \Theta$ such that $\sup\Theta' - \varepsilon < \theta < \sup\Theta'$, i.e., there is no hole in Θ below $\sup\Theta'$.

In this case, delay in the subgame following agreement with the first worker is described by the differentiable method described in section 3.1. Then, since $\sup\Theta'$ does not want to deviate, we have

$$x\sup\Theta' - p_f(2) + \left(\frac{\sup\Theta'}{\tilde{\theta}_w}\right)^{\delta}\left(y\sup\Theta' - p_w(1)\right) \leq e^{-r\Delta'(2)}\left(x\sup\Theta' - p'(2) + \delta\left(y\sup\Theta' - p'(1)\right)\right)$$

In the limit, the derivative of the left hand side is, in the limit, using that $p_w(1) = \frac{y\theta}{2}$,

$$x + \frac{\theta}{\tilde{\theta}_w}y$$

if $\theta < \tilde{\theta}_w$ and

$$x + y$$

otherwise.

Subcase 1: Suppose for all $\varepsilon > 0$ such that $\varepsilon < \gamma'$ for some $\gamma' > 0$ there is some $\theta \in \Theta'$ such that $\sup\Theta' - \varepsilon < \theta < \sup\Theta'$, i.e., there is no hole in Θ' just below $\sup\Theta'$.

First, assume that $\theta < \tilde{\theta}_w$. The derivative of the right hand side is

$$e^{-r\Delta'(2)}\left(x + y\right)$$

for $\theta \geq \tilde{\theta}'_w$. Note that we must have indifference just below $\sup \Theta'$. Then

$$x\theta - p_f(2) + \left(\frac{\theta}{\tilde{\theta}_w}\right)\left(\frac{y\theta}{2}\right) = e^{-r\Delta'(2)}\left(x\theta - p'(2) + \frac{y\theta}{2}\right)$$

for $\tilde{\theta}'_w \leq \theta < \sup \Theta'$. Then the derivatives of the two expressions must be the same, i.e., we have

$$x + \frac{\theta}{\tilde{\theta}_w} y = e^{-r\Delta'(2)}(x+y).$$

However, this cannot hold for more than one θ and we have a contradiction.

Second, assume that $\theta > \tilde{\theta}_w$. Then, repeating the same argument establishes that

$$x + y = e^{-r\Delta'(2)}(x+y),$$

a contradiction.

Third, for $\theta < \tilde{\theta}'_w$ and $\theta < \tilde{\theta}_w$ we get, using a similar argument,

$$x + \frac{\theta}{\tilde{\theta}_w} y = e^{-r\Delta'(2)}\left(x + \frac{\theta}{\tilde{\theta}'_w} y\right).$$

Then

$$x\left(1 - e^{-r\Delta'(2)}\right) = e^{-r\Delta'(2)} \frac{\theta}{\tilde{\theta}'_w} y - \frac{\theta}{\tilde{\theta}_w} y$$

Since the left-hand side is constant we must have $e^{-r\Delta'(2)} \frac{y}{\tilde{\theta}'_w} = \frac{y}{\tilde{\theta}_w}$, implying that $e^{-r\Delta'(2)} = 1$, a contradiction.

Fourth, for $\theta < \tilde{\theta}'_w$ and $\theta > \tilde{\theta}_w$ we get, using a similar argument,

$$x + y = e^{-r\Delta'(2)}\left(x + \frac{\theta}{\tilde{\theta}'_w} y\right).$$

However, this cannot hold for more than one θ and we have a contradiction.

Subcase 2: Suppose there is no $\gamma' > 0$ such that for all $\varepsilon > 0$ such that $\varepsilon < \gamma'$ there is some $\theta \in \Theta'$, i.e., there is a hole in Θ' just below $\sup \Theta'$.

A. First, suppose $\sup \Theta' \geq \tilde{\theta}_w$. Also, suppose that for all $\varepsilon > 0$ such that $\varepsilon < \gamma$ for some $\gamma > 0$ there is some $\theta \in \Theta$ such that $\sup \Theta' + \varepsilon > \theta \geq \sup \Theta'$, i.e., there is no hole in Θ just above $\sup\Theta'$.

Note that type $\sup\Theta'$ is indifferent between following the equilibrium strategy and mimicking the strategy used by types in Θ. If type $\sup\Theta'$ were to strictly gain, then by continuity, types in Θ close to $\sup\Theta'$ also gain strictly by mimicking type $\sup\Theta'$. Since any $\theta\in\Theta$ is weakly better off by agreeing immediately, the derivative of the payoff is weakly smaller for type $\theta<\sup\Theta'$, when following the equilibrium strategy than when mimicking. However, the derivative of the equilibrium payoff is

$$x+y$$

and of the mimicking payoff

$$e^{-r\Delta'(2)}\left(x+y\right),$$

a contradiction.

B. Now, suppose $\sup\Theta'<\tilde{\theta}_w$.

Now, we claim that we cannot have an equilibrium such that types in Θ agree immediately, while the remaining types all delay at least $\Delta'\geq\gamma$ where $\gamma>0$ such that $\sup\Theta'<\tilde{\theta}_w$. Let θ be the maximum (supremum) in the set of types that delay at least γ. We have

$$x\theta-p_f\left(2\right)+\left(\frac{\theta}{\tilde{\theta}_w}\right)\left(\frac{y\theta}{2}\right)=e^{-r\gamma}\left(x\theta-p'\left(2\right)+\frac{y\theta}{2}\right)$$

Otherwise, either type θ mimics or types θ' just above θ mimics.

Consider out-of equilibrium beliefs and the delay of type $\hat{\theta}<\tilde{\theta}_w$. Let Γ denote the difference between the equilibrium payoff and the payoff when delaying $\Delta\left(2,\hat{\theta}\right)$. We get

$$\frac{d\Gamma}{d\theta}=\begin{cases} x+\left(\frac{\theta}{\tilde{\theta}_w}\right)y-e^{-r\Delta\left(2,\hat{\theta}\right)}\left(x+y\right) & \text{if } \theta>\tilde{\theta}'_w \\ x+\left(\frac{\theta}{\tilde{\theta}_w}\right)y-e^{-r\Delta\left(2,\hat{\theta}\right)}\left(x+\frac{\theta}{\tilde{\theta}'_w}y\right) & \text{if } \theta<\tilde{\theta}'_w. \end{cases}$$

Note that, if $\theta < \tilde{\theta}'_w$,

$$\frac{d\Gamma}{d\theta} = 0 \iff \theta = \frac{e^{-r\Delta\left(2,\hat{\theta}\right)} - 1}{\frac{1}{\tilde{\theta}_w} - \frac{e^{-r\Delta\left(2,\hat{\theta}\right)}}{\tilde{\theta}'_w}} \frac{x}{y}.$$

If $\frac{1}{\tilde{\theta}_w} > \frac{e^{-r\Delta(2,\hat{\theta})}}{\tilde{\theta}'_w}$ then $\frac{d\Gamma}{d\theta} > 0$ for all θ. If $\frac{1}{\tilde{\theta}_w} < \frac{e^{-r\Delta(2,\hat{\theta})}}{\tilde{\theta}'_w}$ there is some $\theta > 0$ such that $\frac{d\Gamma}{d\theta} = 0$. Then $\frac{d^2\Gamma}{d\theta^2} = \left(\frac{1}{\tilde{\theta}_w} - \frac{e^{-r\Delta(2,\hat{\theta})}}{\tilde{\theta}'_w}\right) y < 0$ and hence Γ attains a local (global, since Γ is quadratic) maximum at this value of θ.

Also, if $\theta > \tilde{\theta}'_w$ there is either some θ such that

$$\frac{d\Gamma}{d\theta} = 0 \iff \theta = \tilde{\theta}_w \frac{e^{-r\Delta\left(2,\hat{\theta}\right)} y - x\left(1 - e^{-r\Delta\left(2,\hat{\theta}\right)}\right)}{y}$$

or $\frac{d\Gamma}{d\theta} > 0$ for all $\theta > \tilde{\theta}'_w$. If $\frac{d\Gamma}{d\theta} = 0$ we have $\frac{d^2\Gamma}{d\theta^2} = \frac{1}{\tilde{\theta}_w} y > 0$ and hence Γ attains a local (global, since Γ is quadratic) minimum at this value of θ.

Since Γ attains a minimum at $\hat{\theta} = \tilde{\theta}_w \frac{e^{-r\Delta(2,\hat{\theta})} y - x\left(1-e^{-r\Delta(2,\hat{\theta})}\right)}{y}$, the worker puts probability one on this type after observing a delay of $\Delta\left(2,\hat{\theta}\right)$ .Note that this implies that the delay for type θ is exactly $\Delta(2,\theta)$. The delay cannot be lower, since then any type slightly above θ prefers to mimic type θ. If delay is higher, then if $p'_f(2) = \frac{y\theta}{2}$ type θ prefers to mimic types just below θ, since both delay and prices are lower for these types. If $p'_f(2) < \frac{y\theta}{2}$ then, since type θ prefers to delay $\Delta'(2) > \Delta(2,\theta)$, types slightly below $\theta \in \Theta$ prefers to mimic θ. (they are even more patient)

The payoff in equilibrium is, at $\hat{\theta}$,

$$x\theta - p_f(2) + \left(\frac{\theta}{\tilde{\theta}_w}\right)\left(\frac{y\theta}{2}\right)$$

Note that, for $\theta < \sup \Theta'$ close to $\sup \Theta'$ the payoff following a delay of $\Delta(2,\theta)$ is at least $e^{-r\Delta(2,\theta)}x\theta$, since the price paid is at most $\frac{\theta y}{2}$. Thus, delaying a little more for type $\theta < \hat{\theta}$ gives at least $e^{-r\Delta(2,\theta)}x\theta > x\theta - p_f(2) + \left(\frac{\theta}{\tilde{\theta}_w}\right)\left(\frac{y\theta}{2}\right)$, since the derivative of $e^{-r\Delta(2,\theta)}x\theta$ is smaller than the derivative of $x\theta - p_f(2) + \left(\frac{\theta}{\tilde{\theta}_w}\right)\left(\frac{y\theta}{2}\right)$. Hence, we have a contradiction.

Case 2: There is no $\gamma > 0$ such that for all $\varepsilon > 0$ with $\varepsilon < \gamma$ there is some $\theta \in \Theta$ such that $\sup \Theta' - \varepsilon < \theta < \sup \Theta'$, i.e., there is a hole in Θ just below $\sup \Theta'$.

Using case 1 implies that we can choose Θ' such that $\sup \Theta' - \tilde{\theta}_w < \epsilon$ for all $\epsilon > 0$.

Then there must be a hole in Θ such that there is an interval $\hat{\Theta}$ with $\max \Theta \geq \hat{\theta}$ for all $\hat{\theta} \in \hat{\Theta}$ and, for all $\theta \in \Theta \backslash \max \Theta$ we have $\theta \leq \hat{\theta}$ for all $\hat{\theta} \in \hat{\Theta}$. Also since $\theta' = \max\{\theta \in \Theta \backslash \max \Theta\} < \tilde{\theta}_w - \varepsilon$ for some $\varepsilon > 0$ there is positive delay for type θ' when bargaining with the second worker. Furthermore, we have

$$
\begin{aligned}
x \sup \Theta - p_f (2) + \delta (y \sup \Theta - p_w (1)) &= x \sup \Theta - p_f (2) + e^{-r\Delta'} (y \sup \Theta - p'_w (1)) \\
x\theta' - p_f (2) + \delta (y\theta' - p_w (1)) &= x\theta' - p_f (2) + e^{-r\Delta'} (y\theta' - p'_w (1))
\end{aligned}
$$

Rearranging gives

$$
\begin{aligned}
\delta (y \sup \Theta - p_w (1)) &= e^{-r\Delta'} (y \sup \Theta - p'_w (1)) \\
\delta (y\theta' - p_w (1)) &= e^{-r\Delta'} (y\theta' - p'_w (1))
\end{aligned}
$$

or

$$
\frac{y \sup \Theta - p_w (1)}{y\theta' - p_w (1)} = \frac{y \sup \Theta - p'_w (1)}{y\theta' - p'_w (1)}
$$

Then we must have $p_w (1) = p'_w (1)$. Then $e^{-r\Delta'} = \delta$, a contradiction.

Step 2: Establishing that delay is nonincreasing.

Step 1 establishes that $\theta (\Delta)$ is a singleton. All possible cases where Θ consists of more than one element leads to a contradiction. From step 1, we know that, the only possible case that did not lead to a contradiction is when there is a hole in Θ just above $\sup \Theta'$.

Suppose type $\hat{\theta} \in \Theta$ makes a proposal p_f that is accepted. Then, any $\theta' > \hat{\theta}$ also make acceptable proposals. Too see this, let Δ' and p' denote the outcome for type θ' when not making an acceptable proposal. Note that, since we know that the equilibrium is fully revealing, prices paid to both workers are the same in the limit.

Then we must have

$$(x+y)\,\hat{\theta} - 2p_f \geq e^{-r\Delta'}\left((x+y)\,\hat{\theta} - 2p'\right).$$

Since the left hand side increases faster than the right hand side when θ increases, the expression above holds for θ' also. Hence, type θ' has a profitable deviation by mimicking type $\hat{\theta}$. A similar argument establishes that, if type $\hat{\theta}$ does not make an acceptable proposal after delaying Δ, then any type $\theta' < \hat{\theta}$ does not make an acceptable proposal. ■

Note that the proof of this result is not as easily derived as in subgames with one worker remaining. In subgames with one player, it is enough to use the single crossing property of preferences to derive the above result. When we have two workers remaining, we could have the following equilibrium outcome. If we have two types θ and θ' where $\theta > \theta'$ we might have a longer delay in the two worker subgame for the high productivity case, compensated by a shorter delay in one worker subgames. The lemma above shows that this cannot occur.

As in subgames with one worker remaining, we know that the function $\theta\,(\Delta \mid \theta^0)$ is differentiable almost everywhere. Again we restrict attention to differentiable $\theta\,(\Delta \mid \theta^0)$ functions.

Now, let us find the functions $\Delta^*\,(\theta \mid \theta^0)$ and $\theta\,(\Delta \mid \theta^0)$. First, we need to introduce the following notation. Let $p_f\,(\theta, i)$ and $p_w\,(\theta, i)$ denote the price offered by the firm and worker in perfect information subgames when i workers remain. When there are two workers remaining the payoff of the firm following delay Δ and taking worker beliefs $\theta\,(\Delta \mid \theta^0)$ as given is

$$e^{-r\Delta}\left(x\theta - p_f\left(\theta\left(\Delta \mid \theta^0\right),2\right) + \delta\left(y\theta - p_w\left(\theta\left(\Delta \mid \theta^0\right),1\right)\right)\right)$$

The reason for this is that, when agreeing with the first worker the firm gets $x\theta$ in production and pays $p_f\,(\theta\,(\Delta \mid \theta^0)\,, 2)$ in wages and when agreeing with the second worker it gets $y\theta$ in production and pays $p_f\,(\theta\,(\Delta \mid \theta^0)\,, 1)$.

First, let us find optimal delay, for given worker beliefs $\theta\left(\Delta \mid \theta^0\right)$. The first-order condition is

$$-re^{-r\Delta}\left(x\theta - p_f\left(\theta\left(\Delta^* \mid \theta^0\right), 2\right) + \delta\left(y\theta - p_w\left(\theta\left(\Delta^* \mid \theta^0\right), 1\right)\right)\right)$$
$$+e^{-r\Delta}\left(-p_f'\left(\theta\left(\Delta^* \mid \theta^0\right), 2\right)\theta'\left(\Delta \mid \theta^0\right) - \delta p_w'\left(\theta\left(\Delta^* \mid \theta^0\right), 1\right)\theta'\left(\Delta \mid \theta^0\right)\right) = 0$$

Note that, from the proof of Lemma 4, p_f and p_w are linear in θ. Simplifying, setting $\delta = 1$ and using truth-telling, i.e., $\theta\left(\Delta \mid \theta^0\right) = \theta$ gives $d\Delta = -\frac{y}{r\theta x}d\theta$ Thus, we get a differential equation with initial condition $\theta\left(0 \mid \theta^0\right) = \theta^0$. Integrating and taking limits gives

$$\Delta^*\left(\theta \mid \theta^0\right) = \frac{y}{rx}\log\frac{\theta^0}{\theta}.$$

Note that delay here depends on the degree of substitutability, i.e., y. Since equilibrium worker beliefs are correct in equilibrium, the worker infers that the firm has type θ, if the firm delays exactly $\Delta^*\left(\theta \mid \theta^0\right)$. Using the above expression then gives

$$\theta\left(\Delta \mid \theta^0\right) = \theta^0 e^{-\frac{rx\Delta}{y}}.$$

Proposition 1 *Equilibrium delay is increasing in y.*

Proof: Using that we have $\theta^0 = 1$ gives equilibrium expected delay as

$$E\left(\Delta\right) = -\int_0^1 \frac{y}{rx}\left(\ln\theta\right) f\left(\theta\right) d\theta$$

Then the effect of a change in y is

$$\frac{dE\left(\Delta\right)}{dy} = -\int_0^1 \frac{1}{rx}\left(\ln\theta\right) f\left(\theta\right) d\theta = -\frac{1}{y}E\left(\Delta\right) > 0.$$

Thus, an increase in y leads to an increase in equilibrium delay.■

The intuition behind the result is the following. A firm with higher productivity delays less than a firm with low, because a larger amount of the surplus disappears when delaying. Consider two firm types with productivities θ and θ' where $\theta > \theta'$

and suppose substitutability decreases, for some given equilibrium delay that separates θ' from θ. Since substitutability decreases, wages to the workers also increase. However, since productivity θ and the degree of substitutability is multiplicatively separable, the wage paid when the firm has high productivity increases more than when productivity is low. Hence, the high productivity firm will find it profitable to mimic the low productivity firm, since the wage payments decrease more than before the decrease in substitutability. Thus, an decrease in substitutability lead to higher wages and more importantly to a larger wage difference between firm type θ and θ'. Hence the low productivity firm needs to delay a longer amount of time to deter the high productive firm from mimicking.

3.2.3 Subgames when a worker is proposer

Let us analyze the equilibrium cutoff level $\tilde{\theta}_w$. Since higher productivity types are more impatient than low productivity types, all types $\theta > \tilde{\theta}_w$ strictly prefers to accept and all types $\theta < \tilde{\theta}_w$ strictly prefers to reject and counteroffering $p_f(\theta)$ after delaying $\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{\delta y}{x}}$. Hence the payoff of a worker is, in the limit

$$p_w\left(\tilde{\theta}_w, 2\right) \int_{\tilde{\theta}_w}^{\theta^0} f(\theta)\, d\theta + \int_0^{\tilde{\theta}_w} p_f(\theta, 2) \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}} f(\theta)\, d\theta. \tag{4}$$

The worker chooses $\tilde{\theta}_w$ to maximize the expected payoff, subject to $\tilde{\theta}_w \in [0,\theta^0]$. The first-order condition is, in the limit,

$$F\left(\theta^0\right) - F\left(\tilde{\theta}_w\right) - \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta = 0$$

We have the following result in subgames where worker proposes.

Proposition 2 *We have* $\frac{dE(\Delta)}{dy} > 0$.

Proof: See the appendix. ■

4 Correlated signals

Now, let us analyze the case when productivities are only imperfectly correlated. For simplicity, we assume that the firm is initial proposer.

Above, the productivities of the workers are perfectly correlated. In this section we analyze the case when productivities are positively correlated with a correlation smaller than one. We assume that worker i has productivity θ_i. Let Θ_i denote the support of θ_i. We assume $\Theta_1 = \Theta_2$. Productivities follow the joint distribution $f : \Theta_i \times \Theta_j \to [0,1]$. We assume that, if worker i is hired first and worker j last, production is

$$x\theta_i + y\theta_j$$

where the relationship between x and y captures the degree of complementarity between workers.

4.1 Subgames where one worker remain

Note that, following an agreement with one of the workers, the analysis is analogous to the analysis in section 3.1, with the exception that an updated posterior distribution is used.

4.2 Subgames without any agreement

We focus on equilibria similar in style to the equilibria described in section 3.2.2. Again, let us analyze the equilibrium delay and worker beliefs $\Delta^* (\theta_1 \mid \theta^0)$ and $\theta (\Delta \mid \theta^0)$. First, let us find optimal delay, for given worker beliefs $\theta (\Delta \mid \theta^0)$. Again, optimal delay maximizes

$$e^{-r\Delta} \left(x\theta_1 - p_f \left(\theta_1 \left(\Delta^* \mid \theta^0 \right), 2 \right) + \delta E \left(y\theta_2 - p_w (\theta_2) \mid \theta_1 \left(\Delta^* \mid \theta^0 \right) \right) \right)$$

The following result shows that the effect of a change in complementarity on equilibrium delay under perfect correlation also holds when correlation is not perfect.

Proposition 3 *Equilibrium delay is increasing in y.*

Proof: See the Appendix. ■

Now, let us analyze effects of changes in correlation. Note that, if the productivities are uncorrelated then $\frac{\partial E\left(y\theta_2 - p_w(\theta_2) \mid \theta_1\left(\Delta^* \mid \theta^0\right)\right)}{\partial \theta_1} = 0$ while if they are perfectly correlated, we have, in the limit, since $\theta_2 = \theta_1$,

$$\frac{\partial E\left(y\theta_2 - p_w\left(\theta_2\right) \mid \theta_1\left(\Delta^* \mid \theta^0\right)\right)}{\partial \theta_1} = \frac{y}{2}.$$

Thus, the degree of correlation can be described in terms of how a change in one of the productivities affect the conditional expectation of the other productivity. We let ρ parametrize the degree of correlation. Hence, we say that the correlation of productivities increases if

$$\frac{\partial^2 E\left(\theta_2 \mid \theta_1\right)}{\partial \theta_1 \partial \rho} > 0.$$

Let $E\left(\theta_1\right)$ denote the unconditional mean of θ_1. We have (yet) the following limited result on the effects of changes in correlation on delay.

Proposition 4 $\Delta^*\left(\theta_1 \mid \theta^0\right)$ *is decreasing in ρ if $\theta_1 > E\left(\theta_1\right)$.*

Proof: The first-order condition is

$$\begin{aligned} &-r\left(x\theta_1 - p_f\left(\theta_1\left(\Delta^* \mid \theta^0\right), 2\right) + \delta E\left(y\theta_2 - p_w\left(\theta_2\right) \mid \theta_1\left(\Delta^* \mid \theta^0\right)\right)\right) \\ &+\left(-p_f'\left(\theta_1\left(\Delta^* \mid \theta^0\right), 2\right) + \delta \frac{\partial E\left(y\theta_2 - p_w\left(\theta_2\right) \mid \theta_1\left(\Delta^* \mid \theta^0\right)\right)}{\partial \theta_1}\right)\theta_1'\left(\Delta \mid \theta^0\right) = 0 \end{aligned}$$

The effect of a change in the degree of correlation on delay is then

$$\frac{d\Delta}{d\rho} = -\frac{-r\left(\frac{\partial E\left(y\theta_2 - p_w(\theta_2) \mid \theta_1\left(\Delta^* \mid \theta^0\right)\right)}{\partial \rho}\right) + \left(\frac{\partial^2 E\left(y\theta_2 - p_w(\theta_2) \mid \theta_1\left(\Delta^* \mid \theta^0\right)\right)}{\partial \theta_1 \partial \rho}\right)\theta_1'\left(\Delta \mid \theta^0\right)}{SOC}$$

The sign is indeterminate in general. Let $E(\theta_1)$ denote the unconditional mean of θ_1. Note that we have

$$\begin{array}{ll} \frac{\partial E(\theta_2|\theta_1)}{\partial \rho} > 0 & \text{if } \theta_1 > E(\theta_1) \\ \frac{\partial E(\theta_2|\theta_1)}{\partial \rho} < 0 & \text{if } \theta_1 < E(\theta_1) \end{array}$$

For $\theta_1 > E(\theta_1)$ we have $\frac{d\Delta}{d\rho} < 0$. ■

5 More than two workers

In this section, we assume that there are n workers. For simplicity, we focus on the case with perfect correlation and where the firm is the initial proposer. Let θx_i denote the i'th inframarginal contribution, i.e., the marginal contribution of a worker when $i-1$ workers already has been hired. Suppose an agreement between the j'th and the firm is reached at time t_j where $t_1 < \ldots < t_j < \ldots < t_n$. Let p_j denote the wage payment to worker j. The payoff of the firm is,

$$V_k(t,p) = \sum_{j=1}^{n} \delta^{t_j} (\theta x_j - p_j).$$

5.0.1 Equilibrium under perfect information

Again, we first need to find the equilibrium under perfect information. We focus on the case when workers are substitutes, i.e., the production function satisfies decreasing returns. Then, as the result below shows, there is a symmetric equilibrium with agreement in every negotiation. As is shown in Westermark (2003), symmetric need not exist when the production function satisfies increasing returns. We have the following result.

Lemma 7 *Suppose the workers know that the firm has productivity θ. Suppose the production function satisfies decreasing returns, i.e., $x_N > x_{N-1} > \ldots > x_2 > x_1$. For all x, y there exists a stationary equilibrium where the payoff as $\delta \to 1$ of the* *firm is*

$$\sum_{i=2}^{N} x_i \theta - N \frac{x_1 \theta}{2}$$

and the payoff of the workers are

$$\frac{x_1 \theta}{2}.$$

Proof: See the appendix. ■

5.0.2 Equilibrium under imperfect information

Recall that $p_f(\theta, i)$ and $p_w(\theta, i)$ denotes the price offered by the firm and worker in perfect information subgames when i workers remain. Let us find equilibrium delay and worker beliefs $\Delta^*(\theta \mid \theta^0)$ and $\theta(\Delta \mid \theta^0)$

First, let us find optimal delay, for given worker beliefs $\theta(\Delta \mid \theta^0)$. The firm chooses Δ to maximize

$$e^{-r\Delta} \left(\sum_{i=1, i \text{ odd}}^{n} \delta^{i-1} \left(x_i \theta - p_f \left(\theta \left(\Delta \mid \theta^0 \right), n - i - 1 \right) \right) \right.$$
$$\left. + \sum_{i=2, i \text{ even}}^{n} \delta \left(y\theta - p_w \left(\theta \left(\Delta \mid \theta^0 \right), n - i - 1 \right) \right) \right).$$

We have the following result.

Proposition 5 *Suppose the production function satisfies decreasing returns. Expected equilibrium delay decreases as the number of workers increase.*

Proof: The first-order condition is

$$-r \left(\sum_{\substack{i=1,\\ i \text{ odd}}}^{n} \delta^{i-1} \left(x_i \theta - p_f \left(\theta \left(\Delta \mid \theta^0 \right), n - i - 1 \right) \right) + \sum_{\substack{i=2,\\ i \text{ even}}}^{n} \delta^{i-1} \left(x_i \theta - p_w \left(\theta \left(\Delta \mid \theta^0 \right), n - i - 1 \right) \right) \right)$$
$$- \left(\sum_{\substack{i=1,\\ i \text{ odd}}}^{n} \delta^{i-1} p_f' \left(\theta \left(\Delta \mid \theta^0 \right), n - i - 1 \right) + \sum_{\substack{i=2,\\ i \text{ even}}}^{n} \delta^{i-1} p_w' \left(\theta \left(\Delta \mid \theta^0 \right), n - i - 1 \right) \right) \theta' \left(\Delta \mid \theta^0 \right) = 0$$

Since p_f is linear in θ we get, setting $\delta = 1$ and using truth-telling, i.e., $\theta\left(\Delta \mid \theta^0\right) = \theta$ gives

$$-r\left(\sum_{i=1}^{n} x_i - n\frac{x_1}{2}\right)\theta - n\frac{x_1}{2}\frac{d\Delta}{d\theta} = 0$$

The solution to this differential equation is

$$\Delta^*\left(\theta \mid \theta^0\right) = \frac{x_1}{r\left(\frac{2}{n}\sum_{i=1}^{n} x_i - x_1\right)} \log\frac{\theta^0}{\theta}.$$

Thus, equilibrium expected delay when there are n workers as

$$E_n\left(\Delta\right) = \int_0^1 \frac{x_1}{r\left(\frac{2}{n}\sum_{i=1}^{n} x_i - x_1\right)} \log\frac{1}{\theta} f\left(\theta\right) d\theta$$

We get

$$\begin{aligned}
E_{n+1}\left(\Delta\right) - E_n\left(\Delta\right) &= \int_0^1 \frac{x_1}{r\left(\frac{2}{n+1}\sum_{i=1}^{n+1} x_i - x_1\right)} \log\frac{1}{\theta} f\left(\theta\right) d\theta \\
&\quad - \int_0^1 \frac{x_1}{r\left(\frac{2}{n}\sum_{i=1}^{N} x_i - x_1\right)} f\left(\theta\right) \log\frac{1}{\theta} d\theta \\
&= \frac{2x_1}{r}\int_0^1 \frac{\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n+1}\sum_{i=1}^{n+1} x_i}{\left(\frac{2}{n+1}\sum_{i=1}^{n+1} x_i - x_1\right)\left(\frac{2}{n}\sum_{i=1}^{n} x_i - x_1\right)} \log\frac{1}{\theta} f\left(\theta\right) d\theta
\end{aligned}$$

If the production function satisfies decreasing returns, we have

$$\frac{1}{n+1}\sum_{i=1}^{n+1} x_i > \frac{1}{n}\sum_{i=1}^{n} x_i$$

Hence

$$E_{n+1}\left(\Delta\right) < E_n\left(\Delta\right)$$

and delay decreases as the number of workers increase. ■

The intuition is that delay becomes more costly as the number of workers increase. If there number of workers increases, there is a larger surplus from agreement. Then, delaying leads to a larger loss of surplus. Hence, a shorter delay can credibly avoid mimicking.

6 Conclusions

This paper analyzes how equilibrium bargaining delay is affected by changes in the degree of complementarity and correlation in productivity among workers in a firm which bargains with two workers. In addition, the effect of firm size on delay, in terms of the number of workers employed by the firm, is analyzed.

Bargaining takes place as follows. As is usual in bargaining models, workers alternate in bargaining with the firm. A proposer can either make a proposal immediately or delay and making a proposal later on. Following a proposal, the respondent accepts or rejects the proposal.

In equilibrium, the firm uses delay to credibly signal it's type. We find that equilibrium delay is increasing in the degree of complementarity. To see this, suppose we have two firms with productivities θ and θ' where $\theta > \theta'$. Suppose now that complementarity increases, for some given equilibrium delay that separates θ' from θ. When complementarity increases, since workers are paid half of the marginal contribution in wages, wages to the workers also increase. However, the high productivity wage increases more than the low productivity wage. If delay does not increase, the high productivity firm will find it profitable to mimic the low productivity firm. Hence the low productivity firm needs to delay longer to deter mimicking from the high productive firm.

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A Appendix

A.1 Proof of Lemma 2.

At some point in time $t+\Delta$, $\theta(\Delta \mid \theta^0)$ is an interval, followed by $\theta(\Delta' \mid \theta^0)$ being a singleton or empty for all higher $\Delta' > \Delta$.

Step 1. Showing that $\theta(\Delta \mid \theta^0)$ is a singleton for $\Delta > 0$.

First, suppose $\theta(\Delta' \mid \theta^0)$ is nonempty for some $\Delta' > \Delta$. Consider the maximal and minimal types $\theta^{\max}$ and $\theta^{\min}$ in $\theta(\Delta \mid \theta^0)$ after a delay of Δ. Note that the price $p_{t+\Delta}$ at time $t+\Delta$ satisfies $p_{t+\Delta} < p_f\left(\theta^{\min}\right) + \varepsilon$ for $\varepsilon > 0$. Otherwise, since $\theta^{\min} > \underline{\theta}$, then type $\theta^{\min}$ prefers to deviate and mimic type $\theta^{\min} - \gamma$ for γ small. Now, consider deviations by types that agree just before $t+\Delta$. Then, some set of types Θ' with $\inf \Theta' \geq \theta^{\max}$ accepts the proposal. Then, since all types in Θ' accept the price $p_f(\inf \Theta')$. To avoid a deviation by the types in Θ' we must have $p_{t+\Delta} > p_f(\inf \Theta') - \varepsilon \geq p_f(\theta^{\max}) - \varepsilon$ for $\varepsilon > 0$. Hence, we have a contradiction.

Second, suppose $\theta(\Delta \mid \theta^0)$ is empty for all $\Delta' > \Delta$. Then $\theta^{\min} = \underline{\theta}$. Note that, by the same argument as in the previous case, we must have $p_{t+\Delta} > p_f(\theta^{\max}) - \varepsilon$ for $\varepsilon > 0$. The payoff for type $\breve{\theta} \in (\theta^{\min}, \theta^{\max}]$ is then approximately

$$y\breve{\theta} - p_f(\theta^{\max}).$$

Consider the delay $\breve{\Delta}$ (in addition to Δ) and price $\breve{p}_f\left(\breve{\theta}\right)$ such that

$$y\breve{\theta} - p_f(\theta^{\max}) = e^{-r\breve{\Delta}}\left(y\breve{\theta} - p_f\left(\breve{\theta}\right)\right) \iff \breve{\Delta} = -\frac{1}{r}\log\left(\frac{y\breve{\theta} - p_f(\theta^{\max})}{y\breve{\theta} - \breve{p}_f}\right)$$

Note that, in case the worker should reject, it expects a higher price in the continuation game, leading to lower profits for the firm. Thus, all types $\theta < \breve{\theta}$ gains strictly by this deviation and all types $\theta > \breve{\theta}$ looses strictly. Hence, by the Intuitive Criterion, following this deviation the worker should put probability zero on the types $(\breve{\theta}, \theta^{\max}]$. Hence, if the worker rejects, it gets at most $p_f\left(\breve{\theta}\right)$. Thus, it is profitable for the worker to accept $p_f\left(\breve{\theta}\right)$. Then all firm types $\theta < \breve{\theta}$ has a profitable deviation. Thus, $\theta\left(\Delta \mid \theta^0\right)$ is a singleton.

Step 2. Showing that $\theta\left(\Delta \mid \theta^0\right)$ is a singleton for $\Delta = 0$.

Suppose not. Let $\tilde{\theta}_f$ denote the firm type that is indifferent between proposing immediately and delaying an infinitesimal amount of time. Let $p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right)$ denote the proposal that is made immediately by firm types $\theta \geq \tilde{\theta}_f$. Note that we either have $\tilde{\theta}_f > \tilde{\theta}_w$ or $\tilde{\theta}_f \leq \tilde{\theta}_w$.

Case 1. $\tilde{\theta}_f \geq \tilde{\theta}_w$.

If $\tilde{\theta}_f > \tilde{\theta}_w$ then all firm types that make acceptable offers also accept proposals made by the worker. Hence, in case the worker rejects, all such firm types pays $p_w\left(\tilde{\theta}_f\right)$ in the next period. Hence, the firm offers $\delta p_w\left(\tilde{\theta}_f\right)$. Since firm type $\tilde{\theta}_f$ must be indifferent between making an acceptable and an unacceptable offer, we must have $y\tilde{\theta}_f - \delta p_w\left(\tilde{\theta}_f\right) = \delta\left(y\tilde{\theta}_f - \delta p_w\left(\tilde{\theta}_f\right)\right)$. However, this cannot hold.

Case 2. $\tilde{\theta}_f \leq \tilde{\theta}_w$.

When the firm makes an acceptable proposal, the worker must be indifferent between accepting or rejecting. Hence, we have

$$p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right) = \frac{\delta}{F\left(\theta^0\right) - F\left(\tilde{\theta}_f\right)}\left(p_w\left(\tilde{\theta}_w\right)\int_{\tilde{\theta}_w}^{\theta^0} f(\theta)\, d\theta + \int_{\tilde{\theta}_f}^{\tilde{\theta}_w} \delta p_f(\theta)\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\delta} f(\theta)\, d\theta\right). \tag{5}$$

We restrict attention to the nontrivial case where some firms choose not to propose immediately. Then, the firm type $\tilde{\theta}_f$ that is indifferent between proposing $p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right)$ and delaying an infinitesimal amount of time is

$$y\tilde{\theta}_f - p_f\left(\tilde{\theta}_f\right) = y\tilde{\theta}_f - p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right) \tag{6}$$

Also, $\tilde{\theta}_w$ is determined by

$$F\left(\theta^0\right) - F\left(\tilde{\theta}_w\right) - f\left(\tilde{\theta}_w\right)\tilde{\theta}_w + \delta^2\tilde{\theta}_w f\left(\tilde{\theta}_w\right) - \delta^3\int_{\tilde{\theta}_f}^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\delta+1} f(\theta)\, d\theta = 0$$

Evaluating 5 and 6 in the limit gives, assuming $\tilde{\theta}_f < \theta^0$,

$$\tilde{\theta}_f = \frac{\tilde{\theta}_w}{F(\theta^0) - F\left(\tilde{\theta}_f\right)} \left(\int_{\tilde{\theta}_w}^{\theta^0} f(\theta)\, d\theta + \int_{\tilde{\theta}_f}^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^2 f(\theta)\, d\theta \right)$$

$$F\left(\theta^0\right) - F\left(\tilde{\theta}_w\right) = \int_{\tilde{\theta}_f}^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^2 f(\theta)\, d\theta$$

The second expression determines $\tilde{\theta}_w$ as a function of $\tilde{\theta}_f$, $\tilde{\theta}_w\left(\tilde{\theta}_f\right)$. If $\tilde{\theta}_f = \theta^0$ we have perfect information following a rejected offer we have, trivially, $\tilde{\theta}_w = \theta^0$ and hence $p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right) = \delta p_w\left(\theta^0\right)$.

Note that, for the case when $\tilde{\theta}_f < \theta^0$ we have, using the first-order condition,

$$\lim_{\tilde{\theta}_f \to \theta^0} p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right) = \frac{\lim_{\tilde{\theta}_f \to \theta^0} y\tilde{\theta}_w}{\lim_{\tilde{\theta}_f \to \theta^0}\left(F\left(\theta^0\right) - F\left(\tilde{\theta}_f\right)\right)} \lim_{\tilde{\theta}_f \to \theta^0} \left(\int_{\tilde{\theta}_w}^{\theta^0} f(\theta)\, d\theta + \int_{\tilde{\theta}_f}^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^2 f(\theta)\, d\theta \right)$$

Note that

$$\frac{d\tilde{\theta}_w}{d\tilde{\theta}_f} = -\frac{\left(\frac{\tilde{\theta}_f}{\tilde{\theta}_w}\right)^2 f\left(\tilde{\theta}_f\right)}{-2f\left(\tilde{\theta}_w\right) + \frac{2}{\tilde{\theta}_w}\int_{\tilde{\theta}_f}^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^2 f(\theta)\, d\theta}$$

Using L'Hospitals rule gives

$$\frac{\lim_{\tilde{\theta}_f \to \theta^0} y\tilde{\theta}_w}{\lim_{\tilde{\theta}_f \to \theta^0}\left(-f\left(\tilde{\theta}_f\right)\right)} \lim_{\tilde{\theta}_f \to \theta^0} \left(\frac{2}{\tilde{\theta}_w}\int_{\tilde{\theta}_f}^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^3 f(\theta)\, d\theta + \left(\frac{\tilde{\theta}_f}{\tilde{\theta}_w}\right)^2 f\left(\tilde{\theta}_f\right)\frac{d\tilde{\theta}_w}{d\tilde{\theta}_f} \right)$$

$$= \frac{y\theta^0}{\left(-f\left(\theta^0\right)\right)} \left(f\left(\theta^0\right) \lim_{\tilde{\theta}_f \to \theta^0} \frac{d\tilde{\theta}_w}{d\tilde{\theta}_f} \right) = \frac{y\theta^0}{2} = p_f\left(\theta^0\right)$$

Hence

$$\lim_{\tilde{\theta}_f \to \theta^0} p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right) = \lim_{\tilde{\theta}_f \to \theta^0} p_f\left(\tilde{\theta}_f\right)$$

Now consider the effects on $p_f\left(\tilde{\theta}_f\right)$ and $p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right)$ of changing $\tilde{\theta}_f$. We get

$$\frac{dp_f\left(\tilde{\theta}_f\right)}{d\tilde{\theta}_f} = \frac{y}{2}$$

and

$$\frac{dp_f\left(\theta^0,\tilde{\theta}_w,\tilde{\theta}_f\right)}{d\tilde{\theta}_f} = \frac{yf\left(\tilde{\theta}_f\right)\tilde{\theta}_w\left(\int_{\tilde{\theta}_w}^{\theta^0} f(\theta)\,d\theta + \int_{\tilde{\theta}_f}^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^2 f(\theta)\,d\theta - \left(F\left(\theta^0\right) - F\left(\tilde{\theta}_f\right)\right)\left(\frac{\tilde{\theta}_f}{\tilde{\theta}_w}\right)^2\right)}{2\left(F\left(\theta^0\right) - F\left(\tilde{\theta}_f\right)\right)^2}$$

$$= \frac{f\left(\tilde{\theta}_f\right)}{F\left(\theta^0\right) - F\left(\tilde{\theta}_f\right)}\left(p_f\left(\theta^0,\tilde{\theta}_w,\tilde{\theta}_f\right) - p_f\left(\tilde{\theta}_f\right)\frac{\tilde{\theta}_f}{\tilde{\theta}_w}\right)$$

The difference, evaluated at equilibrium, i.e., $p_f\left(\theta^0,\tilde{\theta}_w,\tilde{\theta}_f\right) = p_f\left(\tilde{\theta}_f\right)$ is

$$\frac{dp_f\left(\theta^0,\tilde{\theta}_w,\tilde{\theta}_f\right)}{d\tilde{\theta}_f} - \frac{dp_f\left(\tilde{\theta}_f\right)}{d\tilde{\theta}_f} = \frac{f\left(\tilde{\theta}_f\right)}{F\left(\theta^0\right) - F\left(\tilde{\theta}_f\right)}\left(p_f\left(\theta^0,\tilde{\theta}_w,\tilde{\theta}_f\right) - p_f\left(\tilde{\theta}_f\right)\frac{\tilde{\theta}_f}{\tilde{\theta}_w}\right) - \frac{p_f\left(\tilde{\theta}_f\right)}{\tilde{\theta}_f}$$

$$= \frac{p_f\left(\tilde{\theta}_f\right)}{\tilde{\theta}_f}\frac{1}{\tilde{\theta}_w}\left(\frac{\tilde{\theta}_f f\left(\tilde{\theta}_f\right)}{F\left(\theta^0\right) - F\left(\tilde{\theta}_f\right)}\left(\tilde{\theta}_w - \tilde{\theta}_f\right) - \tilde{\theta}_w\right)$$

At an equilibrium we need, in order to have $\frac{dp_f\left(\theta^0,\tilde{\theta}_w,\tilde{\theta}_f\right)}{d\tilde{\theta}_f} < \frac{dp_f\left(\tilde{\theta}_f\right)}{d\tilde{\theta}_f}$,

$$\tilde{\theta}_f f\left(\tilde{\theta}_f\right)\left(\tilde{\theta}_w - \tilde{\theta}_f\right) < \tilde{\theta}_w\left(1 - F\left(\tilde{\theta}_f\right)\right)$$

Assuming that

$$F(b) - F(s) \leq f(b)(b - s)$$

holds for all b, s (not just $b > s$) is sufficient. In the case when $s > b$ then the above expression cannot be reinterpreted as comparing F with a uniform distribution "from above" with pdf $f(b)$. However, rearranging gives

$$F(s) - F(b) \geq f(b)(s - b)$$

and hence, we can interpret the condition as comparing F with a uniform distribution "from below" with pdf $f(b)$

Setting $b = \tilde{\theta}_f$ and $s = \tilde{\theta}_w$ gives

$$\tilde{\theta}_f f\left(\tilde{\theta}_f\right)\left(\tilde{\theta}_w - \tilde{\theta}_f\right) \leq \tilde{\theta}_f \left(F\left(\tilde{\theta}_w\right) - F\left(\tilde{\theta}_f\right)\right) < \tilde{\theta}_f \left(1 - F\left(\tilde{\theta}_f\right)\right)$$

Hence, we have

$$\frac{dp_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right)}{d\tilde{\theta}_f} < \frac{dp_f\left(\tilde{\theta}_f\right)}{d\tilde{\theta}_f}$$

at any equilibrium. Note that the difference must alternate in sign between alternating equilibria. Hence, there cannot be any values of $\tilde{\theta}_f$ and $\tilde{\theta}_w$ such that $p_f\left(\theta^0, \tilde{\theta}_w, \tilde{\theta}_f\right) = p_f\left(\tilde{\theta}_f\right)$ below $\tilde{\theta}_f = \tilde{\theta}_w = \theta^0$ because we would require another point $\tilde{\theta}'_f$ and $\tilde{\theta}'_w$ where $p_f\left(\theta^0, \tilde{\theta}'_w, \tilde{\theta}'_f\right) = p_f\left(\tilde{\theta}'_f\right)$ and $\frac{dp_f\left(\theta^0, \tilde{\theta}'_w, \tilde{\theta}'_f\right)}{d\tilde{\theta}_f} > \frac{dp_f\left(\tilde{\theta}'_f\right)}{d\tilde{\theta}_f}$, a contradiction.

Step 3.

Since $\Delta^*\left(\theta \mid \theta^0\right)$ is nonincreasing in θ it is differentiable a.e.[6] Then, using Bayes rule, it follows that $\theta\left(\Delta \mid \theta^0\right)$ is differentiable a.e.■

A.2 Proof of Lemma 4

Suppose the firm and worker 1 has been selected to bargain. In the equilibrium any proposer makes an acceptable offer. Note that, following the selection of, say the firm and worker 1 to bargain, then there are two stages where the firm bargains exclusively with worker 1.

Let $V_F^{i,j}$ denote the continuation payoff at the start of the subgame for the firm when player i is proposer and j is respondent. Similarly, let $V_1^{i,j}$ and V_2^i denote the continuation payoff of workers 1 and 2 at the start of the subgame when player i is proposer and j is respondent. Note that, following an agreement with one of the workers, the firm offers $p_f(\theta)$ and the worker demands $p_w(\theta)$.

[6] See e.g., Royden (1988) 100f, Banks (1991) p. 18

The value functions when the firm meets worker 1 are

$$
\begin{aligned}
V_F^{F,1} &= x\theta + \delta\left(y\theta - p_w(\theta)\right) - \delta V_1^{2,F} \\
V_1^{F,1} &= \delta V_1^{2,F} \\
V_2^{F,1} &= \delta p_w(\theta) \\
V_F^{1,F} &= \delta V_F^{F,1} \\
V_1^{1,F} &= x\theta + \delta\left(y\theta - p_f(\theta)\right) - \delta V_F^{F,1} \\
V_2^{1,F} &= \delta p_f(\theta),
\end{aligned}
$$

and when the firm meets worker 2

$$
\begin{aligned}
V_F^{2,F} &= \delta V_F^{F,2} \\
V_1^{2,F} &= \delta p_f(\theta) \\
V_2^{2,F} &= x\theta + \delta\left(y\theta - p_f(\theta)\right) - \delta V_F^{F,2} \\
V_F^{F,2} &= x\theta + \delta\left(y\theta - p_w(\theta)\right) - \delta V_2^{1,F} \\
V_1^{F,2} &= \delta p_w(\theta) \\
V_2^{F,2} &= \delta V_2^{1,F}.
\end{aligned}
$$

The solution to the system of value equations is

$$
\begin{aligned}
V_F^{F,1} &= V_F^{F,2} &&= x\theta + \delta^2(1-\delta)\frac{y\theta}{1+\delta} \\
V_F^{1,F} &= V_F^{2,F} &&= \delta V_F^{F,1} \\
V_1^{F,1} &= V_2^{F,2} &&= \delta^3 \frac{y\theta}{1+\delta} \\
V_2^{F,1} &= V_1^{F,2} &&= \delta \frac{y\theta}{1+\delta} \\
V_1^{2,F} &= V_2^{1,F} &&= \delta^2 \frac{y\theta}{1+\delta} \\
V_1^{1,F} &= V_2^{2,F} &&= (x_2\theta + \delta y\theta)(1-\delta) + \delta^4 \frac{y\theta}{1+\delta}
\end{aligned}
$$

Note that, for it to be profitable to make acceptable offers, we need

$$\begin{aligned} V_F^{F,1} - \delta V_F^{2,F} &= V_2^{F,2} - \delta V_F^{1,F} > 0, \\ V_1^{1,F} - \delta V_1^{F,1} &= V_2^{2,F} - \delta V_2^{F,2} > 0. \end{aligned}$$

Since

$$\begin{aligned} V_F^{F,1} - \delta V_F^{2,F} &= (1-\delta)\, x\theta + \delta^2 (1-\delta) \frac{y\theta}{1+\delta}, \\ V_1^{1,F} - \delta V_1^{F,1} &= (x_2\theta + \delta y\theta)(1-\delta), \end{aligned}$$

it is profitable to make an acceptable proposal. Letting $\delta \to 1$ establishes the that equilibrium payoffs are as stated in the proposition.■

Proof of Lemma 5

The value functions when the firm meets worker 1 are

$$\begin{aligned} V_F^{F,1} &= x\theta + \delta\left(y\theta - p_w(\theta)\right) - \delta V_1^{2,F} \\ V_1^{F,1} &= \delta V_1^{2,F} \\ V_2^{F,1} &= \delta p_w(\theta) \\ V_F^{1,F} &= \delta V_F^{F,1} \\ V_1^{1,F} &= x\theta + \delta\left(y\theta - p_f(\theta)\right) - \delta V_F^{F,1} \\ V_2^{1,F} &= \delta p_f(\theta), \end{aligned}$$

and when the firm meets worker 2

$$\begin{aligned}
V_F^{2,F} &= \delta V_F^{F,2} \\
V_1^{2,F} &= \delta V_1^{F,2} \\
V_2^{2,F} &= \delta V_2^{F,2} \\
V_F^{F,2} &= \delta V_F^{1,F} \\
V_1^{F,2} &= \delta V_1^{1,F} \\
V_2^{F,2} &= \delta V_2^{1,F}.
\end{aligned}$$

The solution to the system of value equations is

$$\begin{aligned}
V_F^{F,1} &= x\theta \frac{1+\delta+\delta^2}{(1+\delta)(1+\delta^2)} + \delta^2 \frac{y\theta}{(1+\delta)(1+\delta^2)}, \\
V_F^{1,F} &= \delta V_F^{F,1}, V_F^{F,2} = \delta^2 V_F^{F,1}, V_F^{2,F} = \delta^3 V_F^{F,1} \\
V_1^{1,F} &= x\theta \frac{1}{(1+\delta)(1+\delta^2)} + \delta \frac{y\theta}{(1+\delta)(1+\delta^2)}, \\
V_1^{F,1} &= \delta^3 V_1^{1,F}, V_1^{F,2} = \delta V_1^{1,F}, V_1^{2,F} = \delta^2 V_1^{1,F} \\
V_2^{F,1} &= \delta \frac{y\theta}{1+\delta}, V_2^{2,F} = \delta^4 \frac{y\theta}{1+\delta}, V_2^{F,2} = \delta^3 \frac{y\theta}{1+\delta}, V_2^{1,F} = \delta^2 \frac{y\theta}{1+\delta}
\end{aligned}$$

We need to establish that it is profitable to make acceptable (unacceptable) offers when the firm and worker 1 (2) bargains. When the firm proposes to worker 1, we need

$$V_F^{F,1} - V_F^{2,F} = \left(1-\delta^3\right) V_F^{F,1} > 0.$$

Since $\delta < 1$ and $V_F^{F,1} > 0$, this holds for all $\delta \in [0,1)$. When worker 1 proposes to the firm, we need

$$V_1^{1,F} - V_1^{F,1} = \left(1-\delta^3\right) V_1^{1,F} > 0.$$

Since $\delta < 1$ and $V_1^{1,F} > 0$, this holds for all $\delta \in [0,1)$. When worker 2 proposes to the firm, we need

$$x\theta + \delta\left(y\theta - p_w\left(\theta\right)\right) - \delta V_F^{F,2} - \delta V_2^{F,2} = \frac{x\theta\left(1+\delta+\delta^2-\delta^4-\delta^5\right)+\delta^2\left(1-\delta^4-\delta^3\right)y\theta}{\left(1+\delta\right)\left(1+\delta^2\right)}$$
$$< 0.$$

In the limit, we get

$$\frac{x\theta - y\theta}{4} < 0 \iff x < y$$

When the firm proposes to worker 2, we need

$$x\theta + \delta\left(y\theta - p_w\left(\theta\right)\right) - \delta V_F^{1,F} - \delta V_2^{1,F} = \frac{x\theta\left(1+\delta-\delta^4\right)+\delta^2\left(1-\delta-\delta^3\right)y\theta}{\left(1+\delta\right)\left(1+\delta^2\right)}$$
$$< 0.$$

In the limit, we get

$$\frac{x\theta - y\theta}{4} < 0 \iff x < y$$

■

A.3 Proof of Proposition 2

General proof:

The first-order condition is

$$F\left(\theta^0\right) - F\left(\tilde{\theta}_w\right) - \frac{y}{x}\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f\left(\theta\right)d\theta = 0$$

Step 1. Showing that $\frac{d\tilde{\theta}_w}{dy} < 0$.

Then

$$\frac{d\tilde{\theta}_w}{dy} = -\frac{-\frac{1}{x}\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta + \frac{y}{x}\int_0^{\tilde{\theta}_w}\frac{1}{x}\ln\frac{\tilde{\theta}_w}{\theta}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta}{-f\left(\tilde{\theta}_w\right)\left(\frac{y}{x}+1\right) + \left(\frac{y}{x}+1\right)\frac{y}{x}\frac{1}{\tilde{\theta}_w}\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta}$$

$$= \frac{\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta - \frac{y}{x}\int_0^{\tilde{\theta}_w}\ln\frac{\tilde{\theta}_w}{\theta}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta}{(y+x)\left(-f\left(\tilde{\theta}_w\right) + \frac{y}{x}\frac{1}{\tilde{\theta}_w}\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta\right)}$$

$$= \frac{\int_0^{\tilde{\theta}_w}\left(1 - \frac{y}{x}\ln\frac{\tilde{\theta}_w}{\theta}\right)\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta}{(y+x)\left(-f\left(\tilde{\theta}_w\right) + \frac{y}{x}\frac{1}{\tilde{\theta}_w}\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta\right)}$$

The denominator is negative, from the second-order condition. The sign is thus determined by

$$\int_0^{\tilde{\theta}_w}\left(1 - \frac{y}{x}\ln\frac{\tilde{\theta}_w}{\theta}\right)\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta$$

Note that we can rewrite the above expression as

$$\int_0^{\tilde{\theta}_w e^{-\frac{x}{y}}}\left(1 - \frac{y}{x}\ln\frac{\tilde{\theta}_w}{\theta}\right)\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta + \int_{\tilde{\theta}_w e^{-\frac{x}{y}}}^{\tilde{\theta}_w}\left(1 - \frac{y}{x}\ln\frac{\tilde{\theta}_w}{\theta}\right)\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \tag{7}$$

Also, since we have, when $b > s$

$$F(b) - F(s) \leq f(b)(b - s)$$

we have $f(b) > f(s)$ except at a set of measure zero. Since $f(\theta) \geq f\left(\tilde{\theta}_w e^{-\frac{x}{y}}\right)$ for $\theta \geq \tilde{\theta}_w e^{-\frac{x}{y}}$ and $f(\theta) < f\left(\tilde{\theta}_w e^{-\frac{x}{y}}\right)$ for $\theta < \tilde{\theta}_w e^{-\frac{x}{y}}$, expression (7) is larger than, using partial integration in the last step

$$f\left(\tilde{\theta}_w e^{-\frac{x}{y}}\right)\int_0^{\tilde{\theta}_w}\left(1 - \frac{y}{x}\ln\frac{\tilde{\theta}_w}{\theta}\right)\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} d\theta = f\left(\tilde{\theta}_w e^{-\frac{x}{y}}\right)\frac{2\tilde{\theta}_w}{\left(\frac{y}{x}+2\right)^2},$$

which is positive. Hence, $\frac{d\tilde{\theta}_w}{dy} < 0$.

Step 2. Equilibrium expected delay is

$$E(\Delta) = \int_0^{\tilde{\theta}_w} \frac{y}{rx} \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) f(\theta)\, d\theta$$

The effect of a change in y is

$$\begin{aligned}
\frac{dE(\Delta)}{dy} &= \int_0^{\tilde{\theta}_w} \frac{1}{rx} \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) f(\theta)\, d\theta + \left(\frac{y}{rx} \left(\ln \frac{\tilde{\theta}_w}{\tilde{\theta}_w} \right) f\left(\tilde{\theta}_w\right) + \int_0^{\tilde{\theta}_w} \frac{y}{rx} \left(\frac{1}{\tilde{\theta}_w} \right) f(\theta)\, d\theta \right) \frac{d\tilde{\theta}_w}{dy} \\
&= \int_0^{\tilde{\theta}_w} \frac{1}{rx} \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) f(\theta)\, d\theta + \frac{y}{rx} \left(\frac{1}{\tilde{\theta}_w} \right) F\left(\tilde{\theta}_w\right) \frac{d\tilde{\theta}_w}{dy}
\end{aligned}$$

Using the expression for $\frac{d\tilde{\theta}_w}{dy}$ gives

$$\begin{aligned}
\frac{dE(\Delta)}{dy} &= \int_0^{\tilde{\theta}_w} \frac{1}{rx} \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) f(\theta)\, d\theta \\
&\quad + \frac{y}{rx} F\left(\tilde{\theta}_w\right) \frac{\int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta - \frac{y}{x} \int_0^{\tilde{\theta}_w} \ln \frac{\tilde{\theta}_w}{\theta} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta}{(y+x)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right) + \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \right)} \\
&= \int_0^{\tilde{\theta}_w} \frac{1}{rx} \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) f(\theta)\, d\theta + \frac{\frac{y}{rx} F\left(\tilde{\theta}_w\right) \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta}{(y+x)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right) + \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \right)} \\
&\quad - \frac{y}{rx} F\left(\tilde{\theta}_w\right) \frac{\frac{y}{x} \int_0^{\tilde{\theta}_w} \ln \frac{\tilde{\theta}_w}{\theta} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta}{(y+x)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right) + \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \right)}
\end{aligned}$$

Rewriting on a common denominator gives

$$\begin{aligned}
&\int_0^{\tilde{\theta}_w} \left(\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right) + \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \right) \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) + \frac{y}{y+x} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} \right. \\
&\left. - \frac{y}{y+x} F\left(\tilde{\theta}_w\right) \frac{y}{x} \ln \frac{\tilde{\theta}_w}{\theta} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} \right) f(\theta)\, d\theta
\end{aligned}$$

There is a critical value of θ where the above expression is zero, denoted θ^{crit}. For

$\theta \geq \theta^{crit}$ the expression is negative and for $\theta < \theta^{crit}$, the expression is positive. The above expression can be rewritten

$$\int_0^{\theta^{crit}} \left(\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right) + \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \right) \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) + F\left(\tilde{\theta}_w\right) \frac{y}{y+x} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} \right.$$
$$\left. - \frac{y}{y+x} F\left(\tilde{\theta}_w\right) \frac{y}{x} \ln \frac{\tilde{\theta}_w}{\theta} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} \right) f(\theta)\, d\theta$$
$$= \int_{\theta^{crit}}^{\tilde{\theta}_w} \left(\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right) + \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \right) \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) + F\left(\tilde{\theta}_w\right) \frac{y}{y+x} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} \right.$$
$$\left. - \frac{y}{y+x} F\left(\tilde{\theta}_w\right) \frac{y}{x} \ln \frac{\tilde{\theta}_w}{\theta} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} \right) f(\theta)\, d\theta$$

Since $f(\theta) \geq f(\theta^{crit})$ for $\theta \geq \theta^{crit}$ and $f(\theta) \leq f(\theta^{crit})$ for $\theta \leq \theta^{crit}$ the above expression is smaller than

$$f\left(\theta^{crit}\right) \int_0^{\tilde{\theta}_w} \left(\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right) + \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \right) \left(\ln \frac{\tilde{\theta}_w}{\theta} \right) + F\left(\tilde{\theta}_w\right) \frac{y}{y+x} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} \right.$$
$$\left. - \frac{y}{y+x} F\left(\tilde{\theta}_w\right) \frac{y}{x} \ln \frac{\tilde{\theta}_w}{\theta} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} \right) d\theta$$
$$= f\left(\theta^{crit}\right) \left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right) + \frac{y}{x} \int_0^{\tilde{\theta}_w} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\, d\theta \right) \left(\theta \ln \tilde{\theta}_w - (\theta \ln \theta - \theta) \right) \Big|_0^{\tilde{\theta}_w}$$
$$+ f\left(\theta^{crit}\right) \frac{F\left(\tilde{\theta}_w\right) \frac{y}{y+x} \tilde{\theta}_w}{\frac{y}{x}+2} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+2} \Bigg|_0^{\tilde{\theta}_w}$$
$$- f\left(\theta^{crit}\right) \frac{y}{y+x} F\left(\tilde{\theta}_w\right) \frac{y}{x} \left(\ln \frac{\tilde{\theta}_w}{\theta} \frac{\tilde{\theta}_w}{\frac{y}{x}+2} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+2} \Bigg|_0^{\tilde{\theta}_w} + \int_0^{\tilde{\theta}_w} \frac{1}{\frac{y}{x}+2} \left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} d\theta \right)$$

$$= f\left(\theta^{crit}\right)\left(\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+\frac{y}{x}\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\,d\theta\right)\tilde{\theta}_w+\frac{F\left(\tilde{\theta}_w\right)y}{y+x}\frac{\tilde{\theta}_w}{\frac{y}{x}+2}\right)$$

$$-f\left(\theta^{crit}\right)\frac{y}{y+x}F\left(\tilde{\theta}_w\right)\frac{y}{x}\frac{\tilde{\theta}_w}{\left(\frac{y}{x}+2\right)^2}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+2}\Bigg|_0^{\tilde{\theta}_w}$$

$$= f\left(\theta^{crit}\right)\left(\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+\frac{y}{x}\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\,d\theta\right)+\frac{y}{y+x}\frac{F\left(\tilde{\theta}_w\right)}{\frac{y}{x}+2}-\frac{y}{y+x}\frac{y}{x}\frac{F\left(\tilde{\theta}_w\right)}{\left(\frac{y}{x}+2\right)^2}\right)\tilde{\theta}_w$$

$$= f\left(\theta^{crit}\right)\left(\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+\frac{y}{x}\int_0^{\tilde{\theta}_w}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1} f(\theta)\,d\theta\right)+\frac{y}{y+x}\frac{2F\left(\tilde{\theta}_w\right)}{\left(\frac{y}{x}+2\right)^2}\right)\tilde{\theta}_w$$

The above is smaller than

$$f\left(\theta^{crit}\right)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+\left(\frac{y}{x}+\frac{y}{y+x}\frac{2}{\left(\frac{y}{x}+2\right)^2}\right)F\left(\tilde{\theta}_w\right)\right)\tilde{\theta}_w$$

$$= f\left(\theta^{crit}\right)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+\left(\frac{y\left(\frac{y}{x}+2\right)^2(y+x)+2xy}{x\left(\frac{y}{x}+2\right)^2(y+x)}\right)F\left(\tilde{\theta}_w\right)\right)\tilde{\theta}_w$$

$$f\left(\theta^{crit}\right)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+\int_0^{\tilde{\theta}_w}\left(\frac{y}{x}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1}+\frac{y}{y+x}\frac{2}{\left(\frac{y}{x}+2\right)^2}\right)f(\theta)\,d\theta\right)\tilde{\theta}_w$$

There is another critical value of θ where the above expression is zero, denoted θ^{crit*}. For $\theta \geq \theta^{crit*}$ the expression is negative and for $\theta < \theta^{crit*}$, the expression is positive. The above expression can be rewritten

$$f\left(\theta^{crit}\right)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+\int_0^{\theta^{crit*}}\left(\frac{y}{x}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1}+\frac{y}{y+x}\frac{2}{\left(\frac{y}{x}+2\right)^2}\right)f(\theta)\,d\theta\right)\tilde{\theta}_w$$

$$+f\left(\theta^{crit}\right)\left(\int_{\theta^{crit*}}^{\tilde{\theta}_w}\left(\frac{y}{x}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1}+\frac{y}{y+x}\frac{2}{\left(\frac{y}{x}+2\right)^2}\right)f(\theta)\,d\theta\right)\tilde{\theta}_w$$

Since $f(\theta) \geq f\left(\theta^{crit*}\right)$ for $\theta \geq \theta^{crit*}$ and $f(\theta) \leq f\left(\theta^{crit*}\right)$ for $\theta \leq \theta^{crit*}$ the above expression is smaller than

$$
\begin{aligned}
&f\left(\theta^{crit}\right)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+f\left(\theta^{crit*}\right)\int_0^{\tilde{\theta}_w}\left(\frac{y}{x}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+1}+\frac{y}{y+x}\frac{2}{\left(\frac{y}{x}+2\right)^2}\right)d\theta\right)\tilde{\theta}_w \\
&=f\left(\theta^{crit}\right)\left(-\tilde{\theta}_w f\left(\tilde{\theta}_w\right)+f\left(\theta^{crit*}\right)\frac{y}{x}\frac{\tilde{\theta}_w}{\frac{y}{x}+2}\left(\frac{\theta}{\tilde{\theta}_w}\right)^{\frac{y}{x}+2}+\frac{y}{y+x}\frac{2}{\left(\frac{y}{x}+2\right)^2}\theta\Bigg|_0^{\tilde{\theta}_w}\right) \\
&=f\left(\theta^{crit}\right)\tilde{\theta}_w\left(-f\left(\tilde{\theta}_w\right)+\frac{(y+x)\left(\frac{y}{x}+2\right)+2x}{\left(\frac{y}{x}+2\right)^2(y+x)}f\left(\theta^{crit*}\right)\frac{y}{x}\right)
\end{aligned}
$$

Note that

$$
\begin{aligned}
\frac{(y+x)\left(\frac{y}{x}+2\right)+2x}{\left(\frac{y}{x}+2\right)^2(y+x)}&=\frac{\frac{y}{x}(y+x)+2(y+2x)}{\left(\frac{y}{x}+2\right)^2(y+x)}=\frac{y\left(\frac{y}{x}+1\right)+2x\left(\frac{y}{x}+2\right)}{\left(\frac{y}{x}+2\right)^2(y+x)} \\
&<\frac{x}{y+x}
\end{aligned}
$$

Then

$$
\begin{aligned}
&f\left(\theta^{crit}\right)\tilde{\theta}_w\left(-f\left(\tilde{\theta}_w\right)+\frac{(y+x)\left(\frac{y}{x}+2\right)+2x}{\left(\frac{y}{x}+2\right)^2(y+x)}f\left(\theta^{crit*}\right)\frac{y}{x}\right) \\
&<f\left(\theta^{crit}\right)\tilde{\theta}_w\left(-f\left(\tilde{\theta}_w\right)+\frac{y}{y+x}f\left(\theta^{crit*}\right)\right)<0
\end{aligned}
$$

A.4 Proof of Proposition 3

The first-order condition is

$$
\begin{aligned}
&-r\left(x\theta_1-p_f\left(\theta_1\left(\Delta^*\mid\tilde{\theta}_w\right),2\right)+\delta E\left(y\theta_2-p_w(\theta_2)\mid\theta_1\left(\Delta^*\mid\tilde{\theta}_w\right)\right)\right) \\
&+\left(-p_f'\left(\theta_1\left(\Delta^*\mid\tilde{\theta}_w\right),2\right)+\delta\frac{\partial E\left(y\theta_2-p_w(\theta_2)\mid\theta_1\left(\Delta^*\mid\tilde{\theta}_w\right)\right)}{\partial\theta_1}\right)\theta_1'\left(\Delta\mid\tilde{\theta}_w\right)=0
\end{aligned}
$$

Note that there is no explicit solution for delay in this setup. However, we get, using that prices in the limit are $\frac{y\theta_1(\Delta^*|\tilde{\theta}_w)}{2}$, $\frac{y\theta_2}{2}$ and the first-order condition;

$$\begin{aligned}\frac{d\Delta}{dy} &= -\frac{\frac{r\theta_1(\Delta^*|\tilde{\theta}_w)}{2} + E\left(\frac{\theta_2}{2} \mid \theta_1\left(\Delta^* \mid \tilde{\theta}_w\right)\right) + \left(-\frac{1}{2} + \frac{1}{y}\frac{\partial E\left(y\theta_2 - p_w(\theta_2)|\theta_1\left(\Delta^*|\tilde{\theta}_w\right)\right)}{\partial\theta_1}\right)\theta_1'\left(\Delta \mid \tilde{\theta}_w\right)}{SOC} \\ &= -\frac{1}{y}\frac{xr\theta_1}{SOC} > 0\end{aligned}$$

Thus, delay is again increasing in y.■

A.5 Proof of Lemma 7

Let $V_F^{i,j}(m)$ denote the continuation payoff at the start of the subgame for the firm when player i is proposer and j is respondent when m workers remain. Similarly, let $V_k^{i,j}(m)$ denote the continuation payoff of workers k at the start of the subgame when player i is proposer and j is respondent when m workers remain.

The value functions when the firm meets worker 1 are

$$\begin{aligned} V_F^{F,1}(N) &= x_N\theta - \delta V_1^{2,F}(N) + \delta V_F^{2,F}(N-1) \qquad (8) \\ V_1^{F,1}(N) &= \delta V_1^{2,F}(N) \\ V_2^{F,1}(N) &= \delta V_2^{2,F}(N-1) \\ &\vdots \\ V_N^{F,1}(N) &= \delta V_N^{2,F}(N-1) \\ V_F^{2,F}(N) &= \delta V_F^{F,3}(N) \\ V_1^{2,F}(N) &= \delta V_1^{F,3}(N-1) \\ V_2^{2,F}(N) &= x_N\theta - \delta V_F^{F,3}(N) + \delta V_F^{F,3}(N-1) \\ V_3^{2,F}(N) &= \delta V_3^{F,3}(N-1) \\ &\vdots \\ V_N^{2,F}(N) &= \delta V_N^{F,3}(N-1) \end{aligned}$$

Since $V_1^{2,F}(N) = \delta V_1^{F,3}(N-1)$ we get

$$V_F^{F,1}(N) = x_N\theta - \delta^2 V_1^{F,3}(N-1) + \delta V_F^{2,F}(N-1)$$

Step 1: Equilibrium candidate payoffs.

In step 1 , we compute the candidate equilibrium payoffs from the above system of value equations using induction.

Suppose equilibrium payoffs in subgames with $N-1$ workers are as follows;.

$$\lim_{\delta\to 1} V_F^{F,1}(N-1) = \lim_{\delta\to 1} V_F^{2F}(N-1) = \sum_{i=2}^{N-1} x_i\theta - (N-1)\frac{x_1\theta}{2} \tag{9}$$

and

$$\lim_{\delta\to 1} V_k^{F,1}(N-1) = \lim_{\delta\to 1} V_k^{2F}(N-1) = \frac{x_1\theta}{2} \text{ for all } k = 1, \ldots, N-1. \tag{10}$$

Then, using the value functions when N workers remain, we get

$$\lim_{\delta\to 1} V_k^{F,1}(N-1) = \frac{x_1\theta}{2} \text{ for } k \neq 1 \text{ and } \lim_{\delta\to 1} V_k^{2F}(N-1) = \frac{x_1\theta}{2} \text{ for } k \neq 1.$$

Also,

$$\lim_{\delta\to 1} V_1^{F,1}(N) = \lim_{\delta\to 1} V_1^{2,F}(N) = \frac{x_1\theta}{2}$$

and

$$\begin{aligned}\lim_{\delta\to 1} V_F^{F,1}(N) = \lim_{\delta\to 1} V_F^{2,F}(N) &= x_N\theta - \lim_{\delta\to 1} V_1^{2,F}(N) + \lim_{\delta\to 1} V_F^{2,F}(N-1) \\ &= x_N\theta - \frac{x_1\theta}{2} + \sum_{i=2}^{N-1} x_i\theta - (N-1)\frac{x_1\theta}{2} = \sum_{i=2}^{N} x_i\theta - N\frac{x_1\theta}{2}.\end{aligned}$$

Then

$$\lim_{\delta\to 1} V_2^{2,F}(N) = x_N\theta - \lim_{\delta\to 1} V_F^{F,3}(N) + \lim_{\delta\to 1} V_F^{F,3}(N-1)$$
$$= x_N\theta - \left(\sum_{i=2}^{N} x_i\theta - N\frac{x_1\theta}{2}\right) + \left(\sum_{i=2}^{N-1} x_i\theta - (N-1)\frac{x_1\theta}{2}\right) = \frac{x_1\theta}{2}$$

In subgames where two workers remain, i.e., $N-1=2$, we know that payoffs are gives by expressions (9) and (10). Using induction then establishes the result.

Step 2: Establishing that it is profitable to make acceptable proposals.

Step 1 only established that, if there is an equilibrium characterized by the system of value equations in (8), then equilibrium payoffs are as stated in the Proposition. Now we establish existence. Consider whether it is profitable to make acceptable offers for the proposers.

First, the gain from making an acceptable offer for the firm is, using that $V_F^{F,3}(N) = V_F^{F,1}(N)$

$$V_F^{F,1}(N) - \delta V_F^{2,F}(N) = \left(1-\delta^2\right) V_F^{F,1}(N)$$

This expression is positive as long as $V_F^{F,1}(N) > 0$. If the production function satisfies decreasing returns, then we know that $\lim_{\delta\to 1} V_F^{F,1}(N) = \sum_{i=2}^{N} x_i\theta - N\frac{x_1\theta}{2}$. Decreasing returns implies that $\lim_{\delta\to 1} V_F^{F,1}(N) > 0$, since $x_i > x_1$ for all $i = 2,\ldots,N$.

Now consider worker 2. The gain from making an acceptable offer for the worker is, using that $V_2^{F,3}(N) = V_N^{F,1}(N) = \delta V_N^{2,F}(N-1)$ and $V_F^{F,3}(N) = V_F^{F,1}(N)$

$$V_2^{2,F}(N) - \delta V_2^{F,3}(N) = V_2^{2,F}(N) = x_N\theta - \delta V_F^{F,3}(N) + \delta V_F^{F,3}(N-1) - \delta^2 V_N^{2,F}(N-1)$$
$$= x_N\theta(1-\delta) - \delta^2\left(V_N^{2,F}(N-1) - \delta V_1^{F,3}(N-1)\right)$$
$$+\,\delta\left(V_F^{F,1}(N-1) - \delta V_F^{2,F}(N-1)\right)$$

Note that, by induction, the last term is positive. Also, $V_N^{2,F}(N-1)$ is defined following an agreement with worker 1 and $V_1^{F,3}(N-1)$ following an agreement with worker 2. Hence, in equilibrium, $V_N^{2,F}(N-1)$ and $V_1^{F,3}(N-1)$ are the payoff for the last worker to agree. Then $V_N^{2,F}(N-1)$ is at most $\delta^{N-2}\frac{x_1\theta}{1+\delta}$ and $V_1^{F,3}(N-1)$ at least $\delta^{N-2}\delta\frac{x_1\theta}{1+\delta}$. Hence, by decreasing returns, i.e., $x_N > x_1$, we have

$$V_2^{2,F}(N) - \delta V_2^{F,3}(N) > x_N\theta(1-\delta) - \delta^N\left(\left(1-\delta^2\right)\frac{x_1\theta}{1+\delta}\right) = (1-\delta)\left(x_N - \delta^N x_1\right)\theta > 0.$$

Thus, it is profitable for worker 2 to make an acceptable proposal. By symmetry, it is profitable for all workers to make acceptable proposals.

Step 3: Equilibrium.

Step 1 established equilibrium payoffs and Step 2 that it is profitable to make acceptable proposals. Since by construction respondents are indifferent between accepting and rejecting, existence of equilibrium is established. ■