Political Alternation : a suggested interpretation*

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Abstract

This paper proposes an explanation for the instability of parties in power in democratic regimes. We consider a dynamic political game, a multidimensional public consumption model in which public goods are durable and parties/candidates are opportunistic and heterogeneous. In this setting, we first present an intuitive manner to endogeneize "issue voting". Then, we describe the conditions under which political alternation occurs in studying the influence of (i) ideology, (ii) polarization in political competition (iii) candidates' competencies.

keywords : political alternation, issue voting, valence, durable public goods.

JEL classification : H41, D72, C61

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1 Introduction

Political alternation, that is the change of party[1] in office after an election, has been a very frequent phenomenon in most of western democracies[2] since 1945. As one can observe on Table 1, alternation has several salient features (Quermione [03]) :

(i) It occurs in countries with different political institutions (presidential, parliamentary or mixed regimes) and systems (bipartisan or multi partisan systems).

(ii) Its frequency (incumbent is beaten in about 50% of elections) is compatible either with "regular" political histories (partisan cycles are quite similar in their frequency and/or length : France, United States...) or with more chaotic ones (England, New Zealand...).

COUNTRY	**Alternations since 1945**
Germany	1966, 1969, 1982, 1998
Australia	1946, 1972, 1975, 1983, 1996
Austria	1970, 1990, 1999
Belgium	1954
Canada	1957, 1963, 1979, 1980, 1993
Denmark	1950, 1953, 1968, 1971, 1973, 1975, 1982, 1993, 1901
Spain	1982, 1996, 2004
USA	1952, 1960, 1968, 1976, 1980, 1992, 2000
France	1981, 1986, 1988, 1993, 1995, 1997, 2002
UK	1945, 1951, 1964, 1970, 1974, 1979, 1997
Greece	1981, 1989, 1993
Ireland	1948, 1951, 1954, 1973, 1977, 1981, 1982, 1986, 1993, 1994
Italy	1994, 1996, 2001
Luxembourg	1974, 1979
Norway	1963, 1965, 1971, 1972, 1973, 1981, 1990, 1997, 2001
NewZeeland	1949, 1957, 1960, 1972, 1975, 1990
Portugal	1987, 1995, 2002
Sweden	1976, 1982, 1991, 1994

source: J.L Quermione (2003)

[1]In the rest of the paper we will use alternatively the terms party and candidate. *In our setting* they are going to be synonyms

[2]One may retort that it is probably the *essence* of Democracy to allow such changes...but this remark does not spare us of trying to understand why they occur !

Surprisingly, despite its long history since Downs [1957] seminal work, political economics has largely neglected this topic and, as far as we know, our paper is the first attempt to explicitly consider alternation in a formal model of electoral competition.

Wise readers may notice, however, that standard spatial model a la Hotelling-Downs allows to obtain alternation, and, then, they may wonder about the interest of this work. Indeed, it is true that if one repeats canonical Downsian game a given number of times, one will observe changes in office. The reason is clear : both parties perfectly converge toward median ideal policy[3], so voters are indifferent between them. Each candidate then has a probability of victory equal to 1/2, which is also the probability to have alternation.

This approach, beyond its extreme simplicity, has the attractive feature to fit with several stylized facts : first, it is able to exhibit any kind of political histories one can meet in reality[4] (regular or chaotic); second, for a sufficiently large number of elections, alternation will occur in 50% of cases, which is also what we observe. But, in the one dimension setting with perfect information, this possibility of alternation fails to exist if a candidates have heterogeneous valences or competencies. Indeed, Ansolabere and Snyder [2000] have studied the case of certainty and have found that the advantaged candidate chooses the central position and the disadvantaged candidate locates anywhere on the political interval. Furthermore, the advantaged candidate wins the election with probability 1, then no alternation can occur. On the contrary, our model permits alternation when candidates are heterogeneous. Furthermore, in our view, explaining alternation with the Downsian model is not satisfying for three reasons:
(i) Despite the scope of elements influencing democratic life (some of them being random) we don't consider alternation as a totally stochastic process. Rather, we believe that there exist predictable forces that (at least partially) drive this phenomenon. Furthermore several empirical studies (...) exhibit that the probability for an incumbent to be replaced is increasing with the number of periods he has spent in office consecutively, which is not the case in a repeated spatial competition game[5]

[3]Which is median voter's ideal policy only when individual heterogeneity is unidimensional.

[4]By construction, since electoral results are random !

[5]Probability of defeat is always 1/2, whatever the past political history.

(ii) One never observes a total convergence of platforms[6] and/or voters are generally not indifferent between candidates or programs[7].

(iii) Political parties are not identical in their ability to offer platforms, or, at least, voters believe they are not.

The ambition of our paper is to propose a stylized model of electoral competition which takes into account these elements, overtakes quoted limits of standard spatial model, and offers a rich and consistent description of political alternation.

First of all, it is crucial to observe that a priori ideological preferences (partisanship), relying on party identity rather than on programs, don't explain alternation very well. If one considers the electoral competition as an opposition between "left" and "right" and asks voters how they feel about this cleavage, a large majority of them seems to be ideologically attached at one of the two sides[8], this attachment being quite stable in time. So one may expect that the party with the largest support in a given population always wins the election. This is not the case, even if one sometimes observes long periods of political stability. A first plausible explanation for alternation, then, is turnout : what is crucial, the day of the election is not the ideological structure of the citizenship, but the composition of the effective voting population (electorate). In the past few decades, abstention has became a very strong phenomenon, and, if we assume that from an election to the following those who abstain are not systematically the same, these modifications in electorate might explain changes in the identity of victorious candidate. But alternation occurs even in countries where voting is obligatory, which militates for other kinds of explanations.

Our view is that one can approach alternation in a quite general framework, without regard of any change in electorate structure and/or size, from an election to the following. The starting point of this work has to be found in a very simple observation : beyond the fact (already noticed) that partisanship does not explain alternation, one can observes that many individuals sometimes vote for a party that is not the one they announced as their "favorite". In a single issue election, this could hardly happen[9], because the ideological dimension translates into the (unique) political dimension. But when political competition is multi-dimensional, there is no reason to believe that a unique candidate is preferred to any other on every dimensions by a given citizen. This might be a reason for "unfaithful" voting behaviors.

Two related questions arise at this point : What makes a given individual consider one party as his favorite? Why, then, does this individual sometimes vote for an other candidate?

Our answer to the first question is very simple, and (we believe) quite intuitive : among the different policies that enter parties platforms, any citizen considers one as being more important than the others. The party sharing the same opinion and/or more likely to implement this policy will then be his favorite. This, in a very large sense, can be assimilated to *ideology*.

Our answer to the second question is connected to what political scientists call *"issue voting"*, namely the fact that, during electoral campaigns, one aspect of platforms emerges as a priority in a large part of society. Such an issue has two main features : first, the distribution of voters who consider it as a priority is largely independent of ideological affiliations[10]. Second, it is not systematically the same from an election to the following. Here, it is crucial to distinguish between voters *preferences*, and voters *priorities*. We claim that preferences are, for a large part, independent of the place and the moment, while priorities depend upon circumstances. For instance, a voter may usually consider public research as more important than military power, but, because of an international crisis, may (punctually) want his government to invest in army as a matter of priority. This distinction will become clear when going deeper in the model.

The model is inspired from the Multidimensional Public consumption model introduced by Tabellini and Alesina [1990], which has the nice characteristic not to generate cycles. Furthermore, we add two crucial assumptions to this model. First, we suppose that *policies are durable*, that is what has been implemented during a legislature still has an impact (though sometimes limited) on the following ones. One can think about any kind of public equipment, investments in research or education, etc. Our distinction between preference and priority will rely on this assumption. Second we consider *parties as different*. Recently, many papers have emphasized "valence" aspects in electoral competition, then introducing asymmetries among candidates. Basically, citizens enjoy a politician's valence, whatever his ideology or the policies he implements are. Two families of such asymmetries have been explored : candidates' personal features (charisma, reputation,...), and greater ability/competence in implementing policies. When assuming differences between parties, we follow the second stream of literature[11]. The novelty of our approach, however, is to consider that, in a multidimensional setting, there is no reason to believe that one candidate is systematically more competent than the other when implementing policies[12]. On the contrary, we assume that each candidate is better than his competitor on one dimension, that is, in usual political economics terminology, we consider a *policy-based valence*. This assumption is consistent with empirical electoral studies : voters believe that parties[13] are not identical in their ability to implement policies. For instance, right-wing parties are often considered[14] as being better than left-wing parties in decreasing taxes or for security programs. In the same time, left-wing parties are viewed as more efficient in building welfare state programs[15].

As already noticed, there is very few literature studying political alternation. Many papers study the links between changes in power (or proximity of elections) and changes in policies implemented[16], but they don't try to explain alternation itself. Conversely, several empirical studies exhibit correlation between incumbent defeat and inflation or unemployment. But to our knowledge, there is no attempt to "rationalize" these points. The closest work to ours is probably the one by Roemer [1995]. Roemer considers a model of electoral competition a la Wittman [73] (i.e. with partisan politicians) in which individual preferences for public and private goods depend upon a random variable whose distribution is influenced by past political history (i.e. policies implemented during previous legislatures). Although it is not the subject of the paper, this model exhibits some alternation. We share with this work the very general idea that *political history has a great role* on electoral results. Yet, we show that one does not need any form of uncertainty to describe it.

Another related stream of literature is the one devoted to "divided government" (Alesina and Rosenthal [96]). As in this family of models, we assume that voters want a relative mix of several policies, no party being able to offer them an optimal combination. Then, facing two elections (presidential and congress, usually), voters elect candidates from different parties in order to get a less extreme average policy, closer to their bliss point. The contribution of our work is to consider this idea in a multi period perspective, with only one election per period. This dynamic setting has the advantage of being applicable to parliamentary regimes, where political power is unique.

The rest of the paper is organized as follows. Section 2 presents the main features and assumptions of our model. Section 3 considers a one-period game and the nature of its equilibria. Sections 4 and 5 propose two refinements in parties goals and strategies and study the dynamics of political equilibria and alternation in office. Finally, section 6 concludes and propose several directions for future research.

2 Presentation of the model

2.1 Durable Public Goods

We consider a bi-dimensional dynamic electoral competition over *two durable public goods* (policies), X and Y. At election t, party p proposes a program (x_t^p, y_t^p). Between two elections, quantities depreciate linearly at exogenous rate $\delta \in [0,1]$.

Let Y_t, X_t be the total available quantities (stocks) of X and Y during a legislature t and x_t^I, y_t^I the quantities provided by incumbent during the same legislature[17]. Then :

$$Y_t = (1-\delta)Y_{t-1} + y_t^I$$
$$X_t = (1-\delta)X_{t-1} + x_t^I$$

2.2 The Political Parties

We consider a two-parties (L and R) electoral competition. They only care about being in office and don't have any preferences on policies ("opportunist politicians"). Once elected, they implement their campaign promises ("preelection politics").

Each candidate, when proposing its platform, must respect a budget constraint (fiscal resources, for instance[18]), normalized to 1.

The crucial assumption concerns candidates' valence. More precisely, each of them has a greater ability to implement one of the policies than its competitor. Formally :

$$c_L(x,y) = y + \lambda_L x \leq 1$$
$$c_R(x,y) = x + \lambda_R y \leq 1$$

Where $c_L(.)$ and $c_R(.)$ are respectively the cost function for party L and for party R, $\lambda_L > 1$ and $\lambda_R > 1$ measure each party relative inefficiency in providing public goods : X for L and Y for R).

As a direct consequence of previous equations, candidates have different sets of possible programs, i.e. some programs can be proposed both by Left-wing and Right-wing candidate, and some others by one candidate only. We assume that voters are perfectly aware of such constraints[19] and any non credible platform would be punished in ballot box.

Once in power, an incumbent has the possibility to "sell" part of the stocks of public goods remaining from previous periods, in order to relax its budget constraint. Reminding that X_t and Y_t are the effective total quantities of X and Y at period t (that is the sum of remaining stocks and new productions), one must respect :

$$X_t \geq 0$$
$$Y_t \geq 0$$

Note that these constraints are compatible with negative new provision of public goods ($x_t^I \leq O$ or $y_t^I \leq O$), which would simply mean that incumbent has destroyed part of the stock of one public good in order to have the possibility to produce more of the other one.

Furthermore, one can notice that :

$$x_t^I = X_t - (1-\delta)X_{t-1}$$
$$y_t^I = Y_t - (1-\delta)Y_{t-1}$$

This leads to new constraints, in terms of total available quantities of public goods[20] :

$$C_L(X_t, Y_t) = Y_t + \lambda_L X_t \leq B_{L,t} = 1 + (1-\delta)(\lambda_L X_{t-1} + Y_{t-1})$$
$$C_R(X_t, Y_t) = X_t + \lambda_R Y_t \leq B_{R,t} = 1 + (1-\delta)(X_{t-1} + \lambda_R Y_{t-1})$$

For the clearness of the presentation we will then mainly consider total quantities X_t and Y_t in the rest of the paper.

Graph 1 : budget constraints and sets of feasible platforms.

2.3 The Electorate

As in the Multidimensional Public Consumption model introduced by Tabellini and Alesina [1990], individual consumption of private goods is homogeneously fixed (then can be excluded from the model) and preferences of the electorate are defined, at any period t, over couples of available public goods quantities $(X_t, Y_t) = (x_t^s + x_t^p, y_t^s + y_t^p)$. That is not only voters consider candidates' platform, but also they evaluate what the consequences of past political history are (in terms of levels of public goods).

We assume a continuum of citizens and no abstention. Any individual prefers one of the two goods, but there exist a certain substitutability between them[21]. A natural way to modelize these assumptions is to consider standard Cobb-Douglas utility functions :

$$U^i(X_t, Y_t) = (x_t^s + x_t^p)^{\alpha_i} . (y_t^s + y_t^p)^{1-\alpha_i} = X_t^{\alpha_i} Y_t^{1-\alpha_i}$$

We will say that if $\alpha_i > \frac{1}{2}$ then citizen i prefers X to Y, if $\alpha_i < \frac{1}{2}$ then i prefers Y to X, and if $\alpha_i = \frac{1}{2}$ then i likes X and Y equally. In very weak sense, one can consider that parameter α_i captures "ideology".

Obviously, because of the durable nature of public goods and the existence of stocks at the beginning of every period, voters' preferred policies will be different from an election to the following. The main intuition of the model then appears clearly : *political history has a role* because it influences economic or social environment, which, in turn, influences the electorate wishes. Furthermore, because of parties' specificities, there is some space for possible alternation in office.

Note that even if bliss points are not the same from an election to the following, preferences do not change across time, each voter i being characterized by a constant parameter α_i. But it would be formally equivalent to consider that preferences evolve endogenously across time, if we define $U_e^i(.)$ as an endogenous utility function such that:

$$\forall x_t^s, y_t^s, x_t, y_t, \; U_e^i(\alpha_i^e, x_t^s, y_t^s, x_t, y_t) = U^i(X_t, Y_t)$$

This equation defines implicitly α_i^e as a function of x_t^s, y_t^s, x_t, and y_t. Yet, we believe that interpreting our model via the impact of political history is more relevant than via some changes in preferences[22].

A useful property of this model is that voters have "intermediate preferences" (Grandmont [1978]), which implies that a Condorcet winner exist. Intermediate preferences are defined as follows:

Definition 1 *Voters in the set [0,1] have intermediate preferences, if their indirect utility function U(X,Y,α^i) can be written as U(X,Y,α^i) = $J(X,Y) + K(\alpha^i)H(X,Y)$*

Let (X_t^m, Y_t^m) be the median voter's preferred program at election t.

Lemma 1 *In the model, at each election t, a Condorcet winner exists and is given by* (X_t^m, Y_t^m).

3 One period game and Political Equilibria

In this section, we only consider the one-election game. Because of his pivotal role, we first describe median voter preferred platform (Condorcet Winner) and then specify which are the equilibria of the electoral competition. The main point is that, because of differences in abilities, only one candidate has (in general) the possibility to propose a winning program. Furthermore, the Condorcet Winner is not the unique winning program.

3.1 Median voter's choice

Median voter chooses (X_t, Y_t) in order to maximized utility. Her optimization program (M) is then :

$$MAX[U^m = (X_t)^{\alpha_m}(Y_t)^{1-\alpha_m}]$$
$$\text{r.t. } (X_t, Y_t) \in \Re^+ \times \Re^+$$
$$\text{s.t. (BL): } C_L(X_t, Y_t) \leq B_{L,t} \text{ or (BR): } C_R(X_t, Y_t) \leq B_{R,t}$$

Let $(X_{R,t}^m, Y_{R,t}^m)$ be the solution of program (M) without constraint (BL) and $(X_{L,t}^m, Y_{L,t}^m)$ be the solution of program (M) without constraint (BR). Then, straightforward calculations lead to the following results :

$$X_{R,t}^m = \alpha_m B_{R,t}$$
$$Y_{R,t}^m = \left(\frac{1-\alpha_m}{\lambda_R}\right) B_{R,t}$$
$$X_{L,t}^m = \frac{\alpha_m}{\lambda_L} B_{L,t}$$
$$Y_{L,t}^m = (1-\alpha_m) B_{L,t}$$

The mainline of the reasoning is to compare maximal utility obtained under left-wing and right-wing candidates victory. It will be useful to define the function $\Delta U_t^m(.)$ as, for median voter, utility difference between best program R can offer to her $((X_{R,t}^m, Y_{R,t}^m))$ and best program L can offer to her $((X_{L,t}^m, Y_{L,t}^m))$ at election t.

$$\Delta U_t^m \equiv U^m\left(X_{R,t}^m, Y_{R,t}^m\right) - U^m\left(X_{L,t}^m, Y_{L,t}^m\right)$$

Let (X_t^m, Y_t^m) be the best platform for median voter at election t, then :

$$(X_t^m, Y_t^m) = (X_{R,t}^m, Y_{R,t}^m) \text{ if } \Delta U_t^m(.) > 0$$
$$\text{and}$$
$$(X_t^m, Y_t^m) = (X_{L,t}^m, Y_{L,t}^m) \text{ if } \Delta U_t^m(.) < 0.$$

Finally, let E_t be the set of winning programs at period t.

Then the following proposition summarizes which are the winning platforms in a one-election setting. Of course, any couple of strategies such that one of them belong to E_t, is a political equilibrium, because the candidate who is sure to loose can propose any (feasible) program.

3.2 Political equilibria

The game is sequenced as follow. First, citizens and parties observe available stocks of public goods. Second, parties simultaneously choose their platforms. Then elections are hold and, finally, new incumbent implements its program.

By the Single Crossing Property, there exists a Condorcet-Winner, namely the platform preferred by voter with median heterogeneity parameter (median voter). However, because of differences in parties' competencies, one candidate only can propose this platform in general. This appears clearly on the following graph.

GRAPH 2 : Sets of credible platforms, indifference curves and median voter's preferred program.

What does determine which candidate will win the election ? One can answer in considering the sign of $\Delta U_t^m(.)$:

$$\Delta U_t^m(.) \propto [\lambda_R^{\alpha_m} - \lambda_L^{-\alpha_m}]y_t^s - [\lambda_L^{1-\alpha_m} - \lambda_R^{\alpha_m - 1}]x_t^s - [\lambda_L^{-\alpha_m} - \lambda_R^{\alpha_m - 1}]$$

Recalling that winning party is R if $\Delta U_t^m(.) > 0$, and L if $\Delta U_t^m(.) < 0$, it is clear that the more important is the stock of public good Y, and the smallest is the stock of X when the election takes place, the more likely is R to win. The reason is obvious : given such an economic environment, median voter[23] is likely to require modest amount of Y and a lot of X. R, which is more competent candidate in providing X, then has a comparative advantage. Of course, this advantage is not systematically sufficient to ensure the victory, especially if $\lambda_R > \lambda_L$: in such a case, candidate L's structural advantage may counterbalance R's conjunctural advantage (economic situation). A symmetric reasoning applies for the circumstances under which L is more likely to be elected[24].

It is important to notice that "winning candidate" (i.e. the candidate which is certain to win the election) has a certain latitude when choosing its platform. Indeed there exists a full set of winning programs. More precisely, any feasible couple (X_t, Y_t) such that the other party can't offer a better platform to median voter, is a winning program.

Let us then define $\Phi_{L,t}^m$ (resp. $\Phi_{R,t}^m$) as the set of programs considered by median voter as better than best program L (resp. R) can offer to her :

$$\Phi_{L,t}^m = \left\{(X,Y) \in \Re^+ \times \Re^+ / U^m(X,Y) > U^m(X_{L,t}^m, Y_{L,t}^m)\right\}$$
$$\Phi_{R,t}^m = \left\{(X,Y) \in \Re^+ \times \Re^+ / U^m(X,Y) > U^m(X_{R,t}^m, Y_{R,t}^m)\right\}$$

Let $W(t)$ be a map valuated in {R,L,RL} such that :
$W(t) = R$ means that R wins the election at time t,
$W(t) = L$ means that L wins the election at time t,
$W(t) = RL$ means that R and L win the election at time t with proba $\frac{1}{2}$.

Proposition 1 (Winning strategies) .
If $\Delta U_t^m(.) > 0$*, then* $W(t) = R$*,* $E_t = \Phi_{L,t}^m \cap \boldsymbol{B}_{R,t} \neq \oslash$
If $\Delta U_t^m(.) = 0$ *, then* $W_t^m = RL$ *and* $E_t \equiv \left\{ (X_{L,t}^m, Y_L^m(t)), (X_{R,t}^m, Y_R^m(t)) \right\}$
If $\Delta U_t^m(.) < 0$ *, then* $W_t^m = L$ *and* $E_t = \Phi_{R,t}^m \cap \boldsymbol{B}_{L,t} \neq \oslash$

These results lead to several observations. First, because our model is deterministic, one candidate is in general certain to be elected. Furthermore, this candidate can propose many winning programs (graphs 3 and 4), each of them leading to different dynamics of the model. That is why, in the rest of the paper, it will be necessary to define more precisely how candidates choose their announced programs.

GRAPH 3 Set of winning strategies for candidate L.

GRAPH 4 Set of winning strategies for candidate R.

Second, in very specific circumstances, median voter is indifferent between candidates. This is not a surprise, public goods being substitutable for voters. As it is usually done in political economics models, we assume that each candidate then wins with proba 1/2. However, such an event, which would be of modest importance in a static model, becomes crucial in a dynamic one. Indeed, because of the role of political history (via stocks of public goods), the identity of elected candidate will dramatically affect future priorities of voters, and then their political choices !

GRAPH 5 Indifferent median voter.

4 Dynamic electoral competition and political alternation

4.1 When parties are secondary Democratic

We first consider "democratic" parties, in the sense that, among all the possible winning platforms, they propose the one which is preferred by median voter. Such a definition of democracy may seem somehow surprising, but it only says that victorious candidate implements the Condorcet Winner of the game.

4.1.1 The one period Game

Let (X_t^*, Y_t^*) be the platform chosen among all the winning strategies by victorious candidate. Then :

If $\Delta U_t^m > 0$, then $W(t) = R$ and $(X_t^*, Y_t^*) = (X_{R,t}^m, Y_{R,t}^m)$.

If $\Delta U_t^m = 0$, then $W(t) = R, L$ and :

-$(X_t^*, Y_t^*) = (X_{R,t}^m, Y_{R,t}^m)$ with probability 1/2.
-$(X_t^*, Y_t^*) = (X_{L,t}^m, Y_{L,t}^m)$ with probability 1/2.

if $\Delta U_t^m < 0$, then $W(t) = L$ and $(X_t^*, Y_t^*) = (X_{L,t}^m, Y_{L,t}^m)$.

4.1.2 Infinite horizon Game

We now consider our political game repeated over an infinite number of legislatures. The one period game is played at each period and stocks result from the whole political history. We first show that infinite stability in power is impossible for some values of ideological parameter in median voter's utility function. Secondly, we study how the different elements of the model influence the position of these values.

Proposition 2 (No Power stability) .
If parties are democratic and if $0 < \delta < 1$, *there exist* $\alpha_1 < \alpha_2$ *belonging to* $]0, 1[$, *such that if* $\alpha_1 < \alpha_m < \alpha_2$ *then no party can win consecutively the elections an infinite number of times.*

Basically, with democratic candidates, there exist a subset of ideological parameters such that, if median voter belongs to this subset, no party can stay in office for ever. We can rewrite this proposition in the following way :

Proposition 3 (Alternation) .
If parties are democratic and if $0 < \delta < 1$, *there exist* $\alpha_1 < \alpha_2$ *belonging to* $]0, 1[$, *such that if* $\alpha_1 < \alpha_m < \alpha_2$, *then each party wins consecutively the elections a finite number of times.*

Proposition 3 emphasizes that "no power stability" does not say alternation is systematic. On the contrary, our model allows very long periods of political stability as well as more chaotic political histories.

The following natural step consist in studying the features of such an "alternation interval". More precisely, it is important to know on which part of the ideological spectrum it is, and how it is influenced by parameters $(\delta, \lambda_R, \lambda_L)$.

Corrolary 1 *If* $\lambda_R = \lambda_L$, *then* $\alpha_1 < \frac{1}{2} < \alpha_2$

The interpretation of this result is easy. When electoral competition is "symmetric", that is when candidates' relative advantages in providing public goods are similar then alternation occurs when median voter is "moderate" in her preferences.

4.1.3 Simulations

We first consider the case where parties have similar comparative advantages (i.e. $\lambda_L = \lambda_R$). Then, these simulations emphasize that the more specialized are the candidates (that is the higher are λ_L and λ_R), the largest is alternation interval.

α_m	x_0^s	y_0^s	λ_R	λ_L	δ	interval
0.5	0	0	1.2	1.2	0,5	[0.455,0.529]

α_m	x_0^s	y_0^s	λ_R	λ_L	δ	interval
0.5	0	0	2	2	0,5	[0.35,0.65]

α_m	x_0^s	y_0^s	λ_R	λ_L	δ	interval
0.5	0	0	10	10	0,5	[X,0.899]

The intuition behind the influence of specialization on alternation intervals is the following. Let's assume highly specialized candidates (λ_P high). If, for instance R is elected (a similar reasoning would apply for L), it is extremely likely that its platforms includes big quantities of X and very few of Y. The reason is clear : it would not be efficient to make R provide too much Y. Imperfect substitutability between public goods and high specialization put together drive to such asymmetric platforms. Now, given the stock of X at the end of the legislature, even voters with great taste for this public good (*preference*) may require bigger quantity of Y (*priority*), and then vote for L at the next election. This explain why alternation interval increases with λ_P.

asymmetry in technologies...permanent advantage...

asymmetry in stocks...temporary advantage...

various depreciation rates...

4.2 When parties maximize their possibilities of reelections: the incumbency advantage

We now assume that, when choosing their platforms, parties also care about what will happen in the future periods. In other words, when certain of its victory, a candidate proposes the winning platform which is the more favorable to its reelection. We will call such politicians "Long Term Opportunists" (LTO).

4.2.1 Political equilibria

Best strategies, given this long-term objective[25], are very intuitive : candidates will offer the winning program including the smallest quantity of public good he is more efficient to provide, and as much as possible of the other good. Then, the priority for the electorate to elect the other party at next election is limited. Formally:

If $W(t) = R$, party R's program is:

$$MAX[\Delta U_{t+1}^m(X_{R,t}^*, Y_{R,t}^*)]$$
$$\text{r.t. } (X_{R,t}^*, Y_{R,t}^*) \in E_t$$

If $W(t) = L$, party L's program is:

$$MIN[\Delta U_{t+1}^m(X_{L,t}^*, Y_{L,t}^*)]$$
$$\text{r.t. } (X_{L,t}^*, Y_{L,t}^*) \in E_t$$

ΔU_{t+1}^m is affine, increasing with $Y_{P,t}^*$ and decreasing with $X_{P,t}^*$ ($P = R, L$. Let respectively $(X_{R,t}^{**}, Y_{R,t}^{**})$ and $(X_{L,t}^{**}, Y_{L,t}^{**})$ be solutions of these programs. Next proposition then describes the winning strategies in the LTO game.

Proposition 4 (LTO game winning strategies) .

If $\Delta U_t^m > 0$, *then* $W(t) = R$ *and*
$Y_{R,t}^{**} = Argmax[Y \in \Re^+ / X + \lambda_R Y = B_{R,t} \text{ and } U_t^m(X,Y) = U_t^m(X_{L,t}^m, Y_{L,t}^m)]$
$X_{R,t}^{**} = B_{R,t} - \lambda_R Y_{R,t}^{**}$

If $\Delta U_t^m = 0$, *then* $W(t) = L, R$ *and winning strategies are the same as in the case of parties having just first objective.*

If $\Delta U_t^m < 0$, *then* $W(t) = L$ *and*
$X_{L,t}^{**} = Argmax[X \in \Re^+ / \lambda_L X + Y = B_{L,t} \text{ and } U_t^m(X,Y) = U_t^m(X_{R,t}^m, Y_{R,t}^m)]$
$Y_{L,t}^{**} = B_{L,t} - \lambda_L X_{L,t}^{**}$

[25] "Long term" may seem an unappropriate term, but one has to remember that legislatures are usually 4 or 5 years long.

First and third cases describes the situation in which one candidate is certain to be elected. Then, as expected, L (resp. R) will propose, among his winning platforms, the one which includes the minimal amount of Y (resp. X). As one can see on the following graphs, this LTO strategy is the south-est (resp north-west) extremity in the set of winning platforms[26].

GRAPH 6 LTO strategy for candidate L.

GRAPH 7 LTO strategy for candidate L.

In the second case, median voter is indifferent between candidates, that is best program proposed by L and by R gives her the same utility level. In such a situation, no party can afford to propose any other platform (and notably a LTO platform) at the risk of being beaten.

GRAPH 8 LTO strategies when median voter is indifferent between L and R.

4.2.2 Infinite horizon Game

Given strategies presented in proposition 4, the following result emphasizes, for some values of heterogeneity parameters, the impossibility of infinite stability in power.

Proposition 5 (Alternation) .
If parties are LTO and if $0 < \delta < 1$ *and , there exist* $\alpha_1 < \alpha_2$*, such that if* $\alpha_m \in]\alpha_1, \alpha_2[$*, then each party wins consecutively the election a finite number of times.*
Furthermore $\frac{Ln[\lambda_R]}{Ln[\lambda_R \lambda_L]} \in]\alpha_1, \alpha_2[$

This result is very similar of the one we got in the democratic context. It emphasizes the fact that, even when politicians systematically propose the program which is the more favorable to their reelection, there exists a range of ideological parameter for which stability in power is impossible.

It is now important to discuss the influence of parameters on this "alternation space"...still has to be done...

[26]It is interesting to notice that winning candidates platforms converge toward the center. Usually, the main motivation for such a convergence is to increase vote share and/or probability of victory. Here it is not the case, rather, parties try to create a "need" in the electorate.

5 Conclusion

Political alternation is one of the main features of electoral competition. The main ambition of this paper has been to propose a theory which shed some light on this phenomenon. We suggest a very intuitive link between alternation and issue voting, through a model incorporating two ingredients of democratic life : the durable nature of public policies and policy-based asymmetries in parties' competencies. In this setting, we show that political alternation has to be viewed as a natural "breathing" of democracy, in the sense it directly results from changes in priorities of the electorate. We have also discussed the role of citizen's ideology and polarization in electoral competition.

In the same time, we have proposed a very simple and consistent way of modeling "issue voting". Basically, the salience of specific issues results from evaluation of economic and/or social environments by voters at the time elections take place ("priorities"), rather than from any shock on preferences. More generally, we emphasized the crucial role of History on voting behaviors.

We believe that this work is of some interest for two kind of reasons. First, it constitutes a first step in a more systematic study of political alternation. Second it proposes a quite intuitive and tractable setting in which many aspects of democratic life can be analyzed.

References

[1] Alesina, A. and H.Rosenthal, 1996, "A Theory of Divided Government", *Econometrica*, V.64, N°6, 1311-1341.

[2] Ansolabere, S. and J.M. Snyder, 2000, ”Valence Politics and Equilibrium in Spatial Election Models”, *Public Choice* 103: 327-336.

[3] Aragones, E. and T. Palfrey, 2002, "Mixed Equilibrium in a Downsian Model with a Favored Candidate", *Journal of Economic Theory*, N°103, 131-161.

[4] Aragones, E. and T. Palfrey, 2004, "Electoral Competition between Two Candidates of Different Quality", Mimeo.

[5] Barnes, S.H. and R.Pierce, 1971, "Public Opinion and Political Preferences in France and Italy", *Midwest Journal of Political Science*, V.15, N°4, 643-660.

[9] Drazen, A., 2000, *Political Economy in Macroeconomics*, Princeton University Press.

[10] Grandmont J.M., 1978, ”Intermediate preferences and the majority rule”, *Econometrica* 46: 317-330.

[11] Mayer, N. and P. Perrineau, 1990 and 1997, ..., Fondation Nationale des Sciences Politiques.

[12] Persson, T. and G. Tabellini, 2002, *Political Economics, Explaining Economic Policy*, MIT Press.

[13] Petrocik, J.R., 1996, "Issue Ownership in Presidential Elections, with a 1980 Case Study", *American Journal Of Political Science*, V.40, N°3, 825-850.

[14] Quermione, J.L., 2003, *L'alternance au pouvoir*, Montchretien.

[15] Roemer, J.E., 1995, "Political cycles", *Economics and Politics* 7,1-20.

[16] Roemer, J.E., 2001, *Political Competition*, Harvard University Press.

[17] Tabellini G. and A. Alesina, 1990, ”Voting on the budget deficit”, *American Economic Review* 80: 37-39

[18] Wittman, D., 1973, "Parties as Utility Maximizers", *American Political Science Review* 67, 490-498.

[19] Wittman, D., 1977, "Candidate motivation : A synthesis of alternative theories", *American Political Science Review* 77, 142-157.

6 ANNEX

6.1 proofs

Proof of lemma 1: It is straightforward that voters preferences are "intermediate", and Grandmont [1978] has proved that the Condorcet winner is the median voter's preferred program.

Proof of proposition 1:
We make the proof of (i) and (ii). Proof of (iii) is the same as proof of (i). Let firstly prove part (i):
If $\Delta U_t^m > 0$ and R plays $(X_{R,t}^m, Y_{R,t}^m)$, then $W(t) = R$ and $E_t \neq \oslash$. Now, suppose R choose a program which does not belong to E_t. By definition, this program does not belong to $\mathbf{B}_{R,t}$. Then the only possible case is that it does not belong to $\Phi_{L,t}$. But, since U is strictly increasing with respect to X and Y, there exists a program such that is feasible by party L that defeats party's R program. This contradicts $W(t) = R$. Now prove part (ii):
If $\Delta U_t^m = 0$. If R and L respectively plays $(X_{L,t}^m, Y_{L,t}^m)$ and $(X_{R,t}^m, Y_{R,t}^m)$ they give the same utility to median voter. If one party do not, then he gives less utility to median voter and lose the election.

Proof of proposition 2:
The mainline of this proof is the following : we first assume that party R wins the election from a certain date to infinity and then show that if α_m is below a certain value (α_2), this assumption is violated. With the same reasoning in the case of party L, we show that L can not win the elections to infinity if α_m is above an other value (α_1). Finally, we show that α_1 is smaller than α_2.

Let h_∞^P be an history infinite dimension vector of the game, such that, for all $t \geq d_P$, W(t)=P. If R wins the elections an infinity of times, then :

$$X_{R,\infty}^m(h_\infty^R, \alpha_m) = \frac{\alpha_m}{\delta}$$
$$Y_{R,\infty}^m(h_\infty^R \alpha_m) = \frac{1-\alpha_m}{\delta\lambda_R}$$

Then utility difference will be :

$$\Delta U_\infty^m(h_\infty^R, \alpha_m) = \left(\lambda_R^{\alpha_m} - \lambda_L^{-\alpha_m}\right)\left[\tfrac{1-\alpha_m}{\delta\lambda_R}\right] - \left(\lambda_L^{1-\alpha_m} - \lambda_R^{\alpha_m-1}\right)\tfrac{\alpha_m}{\delta} - \left(\lambda_L^{-\alpha_m} - \lambda_R^{\alpha_m-1}\right)$$

With the same reasoning, we obtain:

$$\Delta U_\infty^m(h_\infty^L, \alpha_m) = \left(\lambda_R^{\alpha_m} - \lambda_L^{-\alpha_m}\right)\left[\tfrac{1-\alpha_m}{\delta}\right] - \left(\lambda_L^{1-\alpha_m} - \lambda_R^{\alpha_m-1}\right)\tfrac{\alpha_m}{\delta\lambda_L} - \left(\lambda_L^{-\alpha_m} - \lambda_R^{\alpha_m-1}\right)$$

Let us write the difference between these two last expressions:

$\Delta U^m_\infty(h^L_\infty, \alpha_m) - \Delta U^m_\infty(h^R_\infty, \alpha_m)$
$= \left(\lambda_R^{\alpha_m} - \lambda_L^{-\alpha_m}\right) \frac{1}{\delta} \left[1 - \alpha_m - \frac{1-\alpha_m}{\lambda_R}\right] + \left(\lambda_L^{1-\alpha_m} - \lambda_R^{\alpha_m - 1}\right) \frac{1}{\delta} \left[\alpha_m - \frac{\alpha_m}{\delta \lambda_L}\right]$

This difference is strictly positive if $\alpha_m \in]0, 1[$.
Furthermore:

$$\Delta U^m_\infty(h^R_\infty, 0) = \Delta U^m_\infty(h^L_\infty, 0) = \frac{1}{\lambda_R} - 1 < 0$$

and,

$$\Delta U^m_\infty(h^R_\infty, 1) = \Delta U^m_\infty(h^L_\infty, 1) = 1 - \frac{1}{\lambda_L} > 0$$

Since $\Delta U^m_\infty(h^L_\infty, \alpha_m)$ and $\Delta U^m_\infty(h^R_\infty, \alpha_m)$ are continuous with respect to α_m, they are null for specific values of α_m. Then, there exist $\alpha_2 \in]0, 1[$ such that $\Delta U^m_\infty(h^R_\infty, \alpha_m)$ is zero. Then, $\Delta U^m_\infty(h^L_\infty, \alpha_2) > 0$, since $\Delta U^m_\infty(h^L_\infty, 0) < 0$ and with continuity, there exist $\alpha_1 < \alpha_2 \in]0, 1[$ such that $\Delta U^m_\infty(h^L_\infty, \alpha_1) = 0$.

Furthermore, if $\alpha_m \in]\alpha_1, \alpha_2[$, $\Delta U^m_\infty(h^R_\infty, \alpha_m) < 0 < \Delta U^m_\infty(h^L_\infty, \alpha_m)$.
This concludes the proof.

Proof of proposition 3:

The proof of this proposition is straightforward. Suppose party R wins the elections from a date t to infinity. By proposition 3, that is false, then there exists a date T>t, such that L wins the election. Now, suppose that party L wins the elections from T to unity, proposition 3 says that is false, then there exists a date $d > T$, such that party R win the election, and so on... Repeating this reasoning to infinity ends the proof.

Proof of proposition 5:

We have to show that the following statements can be false for the same values of α_m:
(H1) There exist d_R such that for all $t \geq d_R$, $W(t) = R$
(H2) There exist d_L such that for all $t \geq d_L$, $W(t) = L$
Suppose (H1) is true. Then:

$$\forall t \geq d_R + 1,\ X^{**}_{R,t} + \lambda_R Y^{**}_{R,t} = 1 + (1-\delta)[X^{**}_{R,t-1} + \lambda_R Y^{**}_{R,t-1}]$$

Then, this budget converges (because $\delta < 1$) toward $X^{**}_{R,\infty} + \lambda_R Y^{**}_{R,\infty} = 1/\delta$ which is finite (because $0 < \delta$).
Furthermore, $X^{**}_{R,t} \geq \frac{\lambda_R - 1}{\delta[\lambda_R \lambda_L - 1]} + \epsilon$

where the right hand term is the "abcisse" of the intersection point of possibility production frontiers and ϵ is an infinitesimal strictly positive number. Let write maximum level of utility party L can offer to the median voter:

$$U^m(X^m_{L,\infty}, Y^m_{L,\infty}) = \left(\frac{\alpha_m}{\lambda_L}\right)^{\alpha_m} (1-\alpha_m)^{1-\alpha_m} [\lambda_L X^{**}_{R,\infty} + Y^{**}_{R,\infty}]$$

$$= \frac{\left(\frac{\alpha_m}{\lambda_L}\right)^{\alpha_m} (1-\alpha_m)^{1-\alpha_m}}{\lambda_R}[\frac{1}{\delta} + (\lambda_L\lambda_R - 1)X^{**}_{R,\infty}]$$

$$\geq \left(\frac{\alpha_m}{\lambda_L}\right)^{\alpha_m} (1-\alpha_m)^{1-\alpha_m} [\frac{1}{\delta} + \widetilde{\epsilon}]$$

Furthermore, $U^m(X^m_{R,\infty}, Y^m_{R,\infty}) = \alpha_m^{\alpha_m} \left(\frac{1-\alpha_m}{\lambda_R}\right)^{1-\alpha_m} \frac{1}{\delta}$

It follows:
$\Delta U^m_\infty \leq \alpha_m^{\alpha_m} \frac{(1-\alpha_m)^{1-\alpha_m}}{\delta}[\lambda_R^{\alpha_m - 1} - \lambda_L^{-\alpha_m}(1+\widetilde{\widetilde{\epsilon}})]$

Then it suffices that $\alpha_m < \frac{Ln[\lambda_R(1+\widetilde{\widetilde{\epsilon}})]}{Ln[\lambda_R\lambda_L]}$ for (H1) to be contradicted. Now, suppose (H2) is true.Then:

$\forall t \geq d_L + 1,\ \lambda_L X^{**}_{L,t} + Y^{**}_{R,t} = 1 + (1-\delta)[\lambda_L X^{**}_{L,t-1} + Y^{**}_{R,t-1}]$

Then, this budget converge (because $\delta < 1$) toward $\lambda_L X^{**}_{L,\infty} + Y^{**}_{L,\infty} = 1/\delta$ which is finite (because $0 < \delta$).
Furthermore, $Y^{**}_{L,t} \geq \frac{\lambda_L - 1}{\delta[\lambda_R\lambda_L - 1]} + \epsilon'$

where the right hand term is the "ordonnée" of the intersection point of possibility production frontiers and ϵ' is an infinitesimal strictly positive number. Let write maximum level of utility party R can offer to the median voter:

$$U^m(X^m_{R,\infty}, Y^m_{R,\infty}) = \alpha_m^{\alpha_m} \left(\frac{1-\alpha_m}{\lambda_R}\right)^{1-\alpha_m} [X^{**}_{L,\infty} + \lambda_R Y^{**}_{L,\infty}]$$

$$= \alpha_m^{\alpha_m} \left(\frac{1-\alpha_m}{\lambda_R}\right)^{1-\alpha_m} \frac{1}{\lambda_L}[\frac{1}{\delta} + (\lambda_L\lambda_R - 1)Y^{**}_{L,\infty}]$$

$$\geq \alpha_m^{\alpha_m} \left(\frac{1-\alpha_m}{\lambda_R}\right)^{1-\alpha_m} [\frac{1}{\delta} + \widetilde{\epsilon}']$$

Furthermore, under (H2), $U^m(X^m_{L,\infty}, Y^m_{L,\infty}) = (\frac{\alpha_m}{\lambda_L})^{\alpha_m} (1-\alpha_m)^{1-\alpha_m} \frac{1}{\delta}$

It follows:
$\Delta U^m_\infty \leq \alpha_m^{\alpha_m} \frac{(1-\alpha_m)^{1-\alpha_m}}{\delta}[\lambda_R^{\alpha_m - 1}(1+\widetilde{\widetilde{\epsilon}}') - \lambda_L^{-\alpha_m}]$

Then it suffices that $\alpha_m > \frac{Ln\left[\frac{\lambda_R}{(1+\widetilde{\widetilde{\epsilon}})}\right]}{Ln[\lambda_R \lambda_L]}$ for (H2) to be contradicted. Finally, it suffices that

$$\frac{Ln\left[\frac{\lambda_R}{(1+\widetilde{\widetilde{\epsilon}})}\right]}{Ln[\lambda_R \lambda_L]} < \alpha_m < \frac{Ln[\lambda_R(1+\widetilde{\widetilde{\epsilon}})]}{Ln[\lambda_R \lambda_L]}$$

for (H1) and (H2) to be contradicted.