Inducing Efficiency in Oligopolistic Markets with Increasing Returns to Scale [*]

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Abstract

We consider a Cournot Oligopoly market of firms possessing increasing returns to scale technologies (which may not be identical). It is shown that an external regulating agency can increase total social welfare without running a deficit by offering to subsidize one firm an amount which depends on the output level of that firm and the market price. The firms bid for this contract, the regulator collects the highest bid upfront and subsidizes the highest bidding firm. It is shown that there exists a subsidy schedule such that (i) The regulator breaks even (ii) The subsidized firm obtains zero net profit and charges a price equal to its average cost (iii) Every other firm willingly exit the market and (iv) Market price decreases, consumers are better off and total welfare improves.

Keywords: Regulation, Oligopoly, Increasing Returns.

JEL Classification: D43, H21, L11, L13, L51.

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1 Introduction

The central motivation of this paper is to come up with a mechanism which promotes efficiency in an oligopolistic industry which is characterized by increasing returns to scale technologies (not necessarily identical). We show that there exists a mechanism which provides incentives to existing firms in an industry to implement a strictly welfare improving outcome. Specifically this mechanism makes a *command* structure, where production and pricing decisions of firms are controlled by a regulatory authority, redundant.

We arrive at incentive mechanisms by which a benevolent regulator is able to implement, without running a deficit, a welfare improving outcome by making every Cournot oligopolistic firm but one, exit the market. The regulator offers a contract to exclusively subsidize one firm – the one who is willing to pay him the highest upfront fee for that contract. The subsidy offered by the regulator is of the form of a non-linear scheme which depends on the output of the winning firm and market price. It is shown that the winning firm sets a price equal to average cost and obtains zero net profit. All other firms voluntarily exit the industry and the regulator breaks even. The total welfare of the society is improved.

In a classic paper, Demsetz (1968) points out that regulation of a monopolistic firm might be carried out in the *entry* stage itself. In an industry which is a natural monopoly, suppose the regulator wants to grant a license which gives the right to produce to a single firm. Each firm quotes a bid for the price at which it will sell if granted the license. The firm with the lowest bid would ultimately be awarded the contract to be the monopoly producer in the market. With a large number of firms "competing for the field", the level of competitiveness among potential entrants would ensure that the price charged by the winning monopolist would be close to per-unit cost of production. Demsetz contends that the winner is committed to charge no more than what it bids as it enters a binding legal contract with the regulator.

Baron and Myerson (1982) and Berg and Tschirhart (1988) deal with ways to control a natural monopoly under asymmetric information, where the firm

has more information about its technology than the regulator. Sappington and Sibley (1988) provide a mechanism where a regulator without any information about the firm's cost structure can implement marginal cost pricing and zero rents over multiple periods. In the same vein, Vogelsang and Finsinger (1979) describe an incentive mechanism which induces the management of a multi-product monopoly to adjust the price structure step-by-step in the direction of the optimum. Guesnerie and Laffont (1978) examine the likelihood of the government ensuring a first best outcome in a general equilibrium framework where one of the firms is a monopolist in a particular commodity. They show that when non-convexities are present in the profit function of the monopolist, there exists optimal taxation schemes which ensure optimal pricing by the monopolist. The most comprehensive treatment of a large number of issues related to regulation of a natural monopoly is by Laffont and Tirole (1993).

There have been a few papers dealing with industry structure *per se*. Auriol and Laffont (1992) discuss costs and benefits of a duopoly structure when marginal costs are private information and fixed costs are common knowledge. A duopoly is preferred by the government if the market structure is to be chosen before marginal costs have been revealed to the firms themselves, whereas a monopoly is socially better if the government needs to fix the market structure after the firms come to know their private marginal costs.

Grimm et al. (2003) examine under incomplete information, the design of mechanisms to implement the optimal market structure assuming that a regulator is unable to control firm behavior once firms have entered the market.

The paper closest in spirit to this one is by Liao and Tauman (2002). It considers a Cournot oligopoly industry with constant marginal costs. It is shown that a certain linear per unit subsidy schedule can induce the socially best outcome while the regulator breaks even in the process. The result is quite straightforward. The regulator offers a linear subsidy to just one firm. If the per unit subsidy is chosen to be the ratio between marginal cost and the price elasticity of demand (at marginal cost), then the subsidized firm will produce the efficient outcome. Firms therefore would bid up to the subsidy cost for the

right to be subsidized and the regulator will not run any deficit. The current paper generalizes this to the case of a Cournot industry with increasing returns to scale technology.

This paper focusses on the case of complete information. The cost functions and the demand functions are all commonly known by the firms in the market as well as by the regulator. In Section 2 we set up the basic model with symmetric firms and state our main result. In Section 3, we generalize our model to incorporate asymmetry in the firms' technologies.

2 The Model

We first examine the situation when there are $N \geq 2$ identical firms producing a single commodity in the market. Market inverse demand is $P(Q)$ and each firm's production cost is given by $C(q_i)$. We consider the case where the technology exhibits increasing returns to scale. The firms are engaged in Cournot competition. The demand and cost functions are common knowledge to all agents.

We postulate the following assumptions.

Assumption 1 $P(Q)$ *is twice differentiable, strictly decreasing and concave, i.e. $P' < 0$ and $P'' \leq 0$ for all $Q > 0$.*

Assumption 2 $C(q)$ *is twice differentiable for all $q > 0$. $C'(q) > 0$ and $C''(q) < 0$ for all $q > 0$. $C(0) = 0$ but $lim_{q \to 0} C(q) \geq 0$.[1] Namely, $C(\cdot)$ may include a fixed cost component.*

The requirement that $P'' < 0$ is not standard in the literature. However, it guarantees the concavity of the profit functions (see Lemma 1, below) and hence the existence of the Cournot equilibrium. Alternatively, we could drop the concavity assumption of demand and require instead that

$$2P'(Q) + QP''(Q) - C''(q) < 0$$

for all $Q > 0$ and all $0 < q \leq Q$.

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[1]In particular, we assume that this limit exists.

Assumption 3 $P(q) - C'(q)$ *is strictly decreasing for all* $q > 0$ *and there exists* $\bar{q} > 0$, *such that* $P(\bar{q}) = C'(\bar{q})$.

For every $q > 0$, let $AC(q) = \dfrac{C(q)}{q}$. By Assumption 2, $AC'(q) < 0$ and consequently $AC(q) > C'(q)$ for all $q > 0$.[2] Assumptions 3 implies that $P(\cdot)$ and $C'(\cdot)$ intersect exactly once. Let,

$$\pi_i(q_1, \ldots, q_N) \quad = \quad q_i P(Q) - C(q_i) \tag{1}$$

be the profit function of firm i, where $Q = \sum_{j=1}^{N} q_j$. Then,

$$\frac{\partial \pi_i}{\partial q_i} \quad = \quad P(Q) + q_i P'(Q) - C'(q_i)$$

and

$$\frac{\partial^2 \pi_i}{\partial q_i^2} \quad = \quad 2P'(Q) + q_i P''(Q) - C''(q_i)$$

Lemma 1 $\dfrac{\partial^2 \pi_i}{\partial q_i^2} < 0.$

Proof: $\dfrac{\partial^2 \pi_i}{\partial q_i^2} = P'(Q) + q_i P''(Q) + P'(Q) - C''(q_i).$

By Assumption 1, $P'(Q) + q_i P''(Q) < 0$ and $P'(Q) \leq P'(q_i)$. Consequently,

$$\frac{\partial^2 \pi_i}{\partial q_i^2} < P'(q_i) - C''(q_i) < 0$$

since $P(q_i) - C'(q_i)$ is decreasing (Assumption 3). ∎

Lemma 1 implies the existence of a Cournot equilibrium.

In order to simplify the analysis we focus on a duopoly. The analysis of the general case is a straightforward extension of the duopoly case. Suppose that the total Cournot duopoly output levels of the firms are Q_1^d and Q_2^d, and let $Q_1^d + Q_2^d = Q^d$, $Q_i^d > 0$ for $i = 1, 2$.

Lemma 2 *There exists* $q > 0$ *such that* $P(q) = AC(q)$.

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[2]Clearly, $AC(q)$ is decreasing in q if $\frac{C(q)-C(0)}{q}$ is decreasing. Let $q_2 > q_1$. Then $\frac{C(q_2)-C(0)}{q_2} < \frac{C(q_1)-C(0)}{q_1}$ if and only if, $\frac{C(q_2)-C(q_1)}{q_2-q_1} < \frac{C(q_1)-C(0)}{q_1}$. But $\frac{C(q_2)-C(q_1)}{q_2-q_1} = C'(\xi_2)$ and $\frac{C(q_1)-C(0)}{q_1} = C'(\xi_1)$ where $q_1 < \xi_2 < q_2$ and $0 < \xi_1 < q_1$. Since $C''(\cdot) < 0$, $C'(\xi_2) < C'(\xi_1)$ and hence $AC(q_2) < AC(q_1)$.

Proof: $\pi_i(Q_1^d, Q_2^d) = Q_i^d P(Q^d) - C(Q_i^d) \geq 0$. Hence, by Assumption 2 $P(Q^d) \geq AC(Q_i^d) > AC(Q^d)$. On the other hand, for each $q > \bar{q}$, $P(q) < C'(q) < AC(q)$ (see Assumption 3). Since $P(q)$ and $AC(q)$ are continuous for $q > 0$, by the Mean Value Theorem there exists q such that $P(q) = AC(q)$. ∎

Let,

$$A = \{q | P(q) = AC(q)\}. \tag{2}$$

Lemma 3 *The set A is nonempty and compact.*

Proof: See Appendix.

Let,

$$\widetilde{Q} = Max_{q \in A} \; q. \tag{3}$$

Lemma 4 $\widetilde{Q} < \bar{q}$.

Proof: Suppose to the contrary that $\widetilde{Q} \geq \bar{q}$. Then by Assumption 3, $P(\widetilde{Q}) \leq C'(\widetilde{Q}) < AC(\widetilde{Q})$, a contradiction. ∎

This situation is depicted in Figure 1 of Appendix C. $AC(q)$ is the downward sloping average cost function and $\widetilde{Q}$ is the maximal element of A (set of all q, such that $P(q) = AC(q)$).

2.1   The Role of the Regulator

Consider next a regulator which desires to increase the total welfare of the society. It could be the government or a government run regulatory authority. It is assumed that the regulator has full information about market demand and cost structures of the firms. The interaction between the regulator and the firms is described as follows.

In the first stage, the regulator announces a subsidy schedule $f(q, Q)$ to be awarded exclusively to one firm, where Q is the total industry output. The firm producing q would be awarded a lump-sum amount of $f(q, Q)$. In the second stage, the firms announce simultaneously (or through sealed tenders) their own willingness to pay (bids) for this contract. The firm with the highest willingness to pay is awarded the exclusive contract. In case of a tie, the regulator chooses

one of the highest bidders and awards it the subsidy. The winning firm pays its bid to the regulator upfront. In the last stage, the firms compete *a lá* Cournot, that is in quantities.

Consider the following subsidy schedule,

$$f(q, Q) \;=\; -qP(Q) + \int_0^q P(x)dx - \widetilde{t} \cdot q + \delta \tag{4}$$

where $\widetilde{t} = P(\widetilde{Q}) - C'(\widetilde{Q})$, Q is the total industry output and $\delta > 0$. The payoff function of the subsidized firm i is,

$$\pi_i^S(q_i, q_j) = q_i P(Q) - C(q_i) + f(q_i, Q) - \alpha_i$$

where α_i is the bid of i and $Q = q_i + q_j$. By (4),

$$\pi_i^S(q_i, q_j) \;\equiv\; \pi_i^S(q_i) = \int_0^{q_i} P(x)dx - C(q_i) - \widetilde{t} \cdot q_i + \delta - \alpha_i \tag{5}$$

and as we see, π_i^S depends only on the quantity q_i produced by the subsidized firm only, and not on q_j, the quantity produced by the non-subsidized firm.

The payoff function of the other firm j, $j \neq i$, is

$$\pi_j(q_i, q_j) \;=\; q_j P(Q) - C(q_j)$$

and the payoff of the regulator is the total welfare of the society.[3]

The above describes a game G between the regulator and the firms of the industry.

Let the parameter $\delta > 0$ be chosen to be sufficiently large to guarantee the inequality,

$$\pi_i^S(\widetilde{Q}) > \pi_i(Q_1^d, Q_2^d) \qquad i = 1, 2.$$

This eliminates an equilibrium where both firms do not bid for the contract.

We are ready to state and prove our first main result.

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[3]The regulator takes an action only in the event of a tie between the highest bidders. Since each of them would produce the same quantity if subsidized (the solution of $\frac{\partial \pi_i^S}{\partial q_i} = 0$), the choice of one would have the same impact on welfare as any other. Hence the regulator can choose any one at random without affecting the outcome.

Theorem 1 *The game G has a unique subgame perfect equilibrium. The subsidized firm produces the output level $\widetilde{Q}$ and earns a net profit of zero. The other firms exit the market and the regulator breaks even. Finally, the total welfare of the society increases as compared with the no subsidy case.*

Proof: By (5), for the subsidized firm i,

$$\frac{\partial \pi_i^S}{\partial q_i} = P(q_i) - C'(q_i) - \widetilde{t} = P(q_i) - C'(q_i) - [P(\widetilde{Q}) - C'(\widetilde{Q})]$$

By Assumption 3, $P(q) - C'(q)$ is strictly decreasing. Thus $\dfrac{\partial \pi_i^S}{\partial q_i} = 0$, iff $q_i = \widetilde{Q}$. Namely, independent of the output level q_j of j, firm i is best off producing $\widetilde{Q}$ units, and $\pi_i^S(\widetilde{Q}) = f(\widetilde{Q}) - \alpha_i$.

We now prove that the other firm j is best off exiting the market given that i produces $\widetilde{Q}$. Firm j will stay in the market only if $\pi_j(q_j, \widetilde{Q}) \geq 0$, that is if, $P(\widetilde{Q} + q_j) \geq AC(q_j)$. Since $AC(\cdot)$ is decreasing $AC(q_j) > AC(\widetilde{Q} + q_j)$. Consequently, j may stay in the market only if $P(\widetilde{Q} + q_j) > AC(\widetilde{Q} + q_j)$. Suppose that the last inequality holds and $q_j > 0$. By Assumption 3, $P(q) < C'(q) < AC(q)$ for all $q > \overline{q}$. Hence by continuity of $P(q)$ and $AC(q)$ there exists q such that $\widetilde{Q} < q < \overline{q}$ and $P(q) = AC(q)$, a contradiction to the definition of $\widetilde{Q}$. Consequently, $q_j = 0$ must hold and j exits the market. We can now write the subsidy schedule as $f(q)$ instead of $f(q, Q)$, since $Q = q$.

Lemma 5 $f(\widetilde{Q}) > 0$.

Proof: See Appendix B.

Thus in order to win the subsidy, each firm would be willing to pay its *entire* gross profit, that is $\alpha_i = f(\widetilde{Q})$. Hence, the subsidized firm makes zero *net* profit in equilibrium and the regulator breaks even. As we have already see, the subsidized firm i produces $\widetilde{Q}$ irrespective of what j produces. Firm j in turn has the unique best response of $q_j = 0$. Hence this equilibrium is unique.

We next show that the subsidy schedule of the regulator improves the total welfare of society. The total welfare under the Cournot duopoly is,

$$TW_1 = \pi_i^d(Q_i^d, Q_j^d) + \pi_j^d(Q_i^d, Q_j^d) + CS(Q^d)$$

where $CS(Q^d)$ is the consumer surplus. Hence,

$$
\begin{aligned}
TW_1 &= \pi_i^d(Q_i^d, Q_j^d) + \pi_j^d(Q_i^d, Q_j^d) + \int_0^{Q^d} P(x)dx - Q^d P(Q^d) \\
&= \int_0^{Q^d} P(x)dx - C(Q_i^d) - C(Q_j^d)
\end{aligned} \tag{6}
$$

On the other hand, the total welfare with the subsidy schedule $f(q)$ is,

$$
TW_2 = CS(\widetilde{Q}) = \int_0^{\widetilde{Q}} P(x)dx - \widetilde{Q}P(\widetilde{Q}) = \int_0^{\widetilde{Q}} P(x)dx - C(\widetilde{Q})
$$

The net gain in welfare,

$$
\begin{aligned}
\Delta \equiv W(\widetilde{Q}) &= TW_2 - TW_1 \\
&= \int_{Q^d}^{\widetilde{Q}} P(x)dx - C(\widetilde{Q}) + C(Q_i^d) + C(Q_j^d)
\end{aligned} \tag{7}
$$

Denote

$$
g(q) = \int_{Q^d}^{q} P(x)dx - C(q) + C(Q_i^d) + C(Q_j^d).
$$

By the concavity of $C(\cdot)$,

$$
g(Q^d) = C(Q_i^d) + C(Q_j^d) - C(Q_i^d + Q_j^d) > 0.
$$

Again,

$$
g'(q) = P(q) - C'(q).
$$

By Assumption 3, $g'(q) > 0$ iff, $q < \overline{q}$. By Lemma 4, $\widetilde{Q} < \overline{q}$ and thus if $Q^d < \widetilde{Q}$ then $\Delta = g(\widetilde{Q}) > g(Q^d) > 0$, as claimed. We are left to prove that $Q^d < \widetilde{Q}$.

Since $\pi_i^d \geq 0$, we have by Assumption 2,

$$
P(Q^d) \geq AC(Q_i^d) > AC(Q^d)
$$

Also, $\widetilde{Q}$ is the maximal number such that $P(\widetilde{Q}) = AC(\widetilde{Q})$. Namely, for any $q > \widetilde{Q}$, $P(q) < AC(q)$. Hence, $Q^d < \widetilde{Q}$. As a result, total welfare of the society improves. *blacksquare*

Note that the lowest price the regulator can induce without running a deficit is $P(\widetilde{Q}) = AC(\widetilde{Q})$. The reason is that for any $q > \widetilde{Q}$, $P(q) < AC(q)$, which

means that the gross profit of the subsidized firm is less than f(q). Hence, the regulator will not be able to recover the subsidy.

Remark The following is a simple alternative mechanism which yields the same equilibrium outcome as the one described in Theorem 1.[4] Consider the subsidy scheme,

$$\hat{f}(q) = \begin{cases} \delta & \text{if } q \geq \widetilde{Q} \\ -\delta & \text{if } q < \widetilde{Q} \end{cases}$$

where $\delta > 0$ is determined below. The idea is to induce the subsidized firm to produce the output level $\widetilde{Q}$. If it produces q, such that $q < \widetilde{Q}$, it will have to pay the regulator a fine of δ. As before, the regulator offers an exclusive contract to the highest bidder. Let $\pi_1^0, \ldots, \pi_N^0$ be the Cournot oligopoly profit levels of the firms in the standard non-subsidy case. To eliminate the equilibrium outcome where no firm is bidding for the contract $\hat{f}$, we require $\delta > \pi_i^0 - [\widetilde{Q}P(\widetilde{Q}) - C(\widetilde{Q})]$. Otherwise, no firm has an incentive to win the contract even for a zero bid. In addition, δ should be sufficiently large to guarantee $\delta > \pi_M$ where π_M is the monopoly profit. Otherwise, the subsidized firm is best off deviating from $\widetilde{Q}$ to the monopoly output. It is easy to verify that in every subgame perfect equilibrium at least two firms will bid δ and the other firms will either bid below δ or will not bid at all. The winning produces $\widetilde{Q}$ and all the other firms exit the market.

The problem with this mechanism is that the equilibrium outcome is based on weakly dominated strategies. The subsidized firm obtains in equilibrium a zero net payoff but if some firms do not exit the market and produces positive output levels, then the subsidized firm is guaranteed to make a loss. If it does adjust its output level and produce below $\widetilde{Q}$ it will have to pay the fine δ (in addition to the bid δ). If it continues to produce $\widetilde{Q}$, its gross profit is $(\widetilde{Q} + q)P(\widetilde{Q} + q) - C(\widetilde{Q} + q)$ which by definition of $\widetilde{Q}$ (and Assumption 3) is negative.

With the subsidy scheme $f(q, Q)$ given by (4), the regulator takes the risk of over production. The subsidized firm will end up with zero profit on or off

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[4]This mechanism was suggested to us by Elchanan Ben-Porath during a seminar given by one of the authors, held in the Center for Rationality in the Hebrew University of Jerusalem.

the equilibrium path, since its net profit does not depend on the output levels of the other firms.

2.2 An Example

Let $P(q) = a - rq$ and $C(q) = -q^2 + 2bq$ where $b > 0$, $0 \le q \le b$, $2b < a < (r+1)b$ and $r > 2$. Then $P(q) = C'(q)$ implies $\overline{q} = \dfrac{a - 2b}{r - 2}$ and $P(q) = AC(q)$ implies $\widetilde{Q} = \dfrac{a - 2b}{r - 1}$. Note that $\widetilde{Q} < b$ since $a < (r+1)b$. The total welfare when the regulator offers the subsidy is,

$$TW_1 = CS(\widetilde{Q}) = \frac{r}{2}\widetilde{Q}^2 = \frac{r}{2}\left(\frac{a - 2b}{r - 1}\right)^2$$

Consider now the case where the regulator offers no subsidy. There are N firms competing *a lá* Cournot. The profit of every firm is then,

$$\pi_i = q_i\left(a - r\sum_{j=1}^{N} q_j\right) + q_i^2 - 2bq_i, \qquad \forall\, i = 1, \dots, N.$$

Setting $\dfrac{\partial \pi_i}{\partial q_i} = 0$, we find that the Cournot output of every firm is

$$q_i^* = \frac{a - 2b}{r(N + 1) - 2}$$

and the total output is,

$$Q^* = \frac{N(a - 2b)}{r(N + 1) - 2} < \frac{a - 2b}{r - 1} < \widetilde{Q}.$$

The total industry profit is,

$$\Pi^* = \frac{(r - 1)N(a - 2b)^2}{[r(N + 1) - 2]^2}$$

and the consumer surplus is,

$$CS^* = \frac{r}{2}Q^* = \frac{rN^2}{2}\left[\frac{a - 2b}{r(N + 1) - 2}\right]^2$$

Hence the total welfare of the society is,

$$TW_2 = \Pi^* + CS^* = \frac{[\frac{rN^2}{2} + N(r - 1)](a - 2b)^2}{[r(N + 1) - 2]^2}.$$

Also,

$$\frac{\partial TW_2}{\partial N} = \frac{r(r - 2)(a - 2b)^2}{[r(N + 1) - 2]^2} > 0.$$

Consequently, the total welfare is maximized in a competitive industry (i.e. when $n \to \infty$). Now,

$$lim_{n \to \infty} TW_2 = \frac{1}{2r}(a - 2b)^2 \quad \text{and} \quad lim_{n \to \infty} TW_2 < TW_1$$

iff $r^2 > (r - 1)^2$, which always holds.

3   The Case of Non Symmetric Firms

Consider the case where the N firms are not necessarily symmetric. Their cost functions are $C_i(q_i)$, where $i = 1, \ldots, N$. Assumptions 2 and 3 are modified accordingly. Namely,

Assumption 2a *For every i, $C_i(q_i)$ is differentiable for all $q_i > 0$, $C_i'(q_i) > 0$ and $C_i''(q_i) < 0$ for all $q_i > 0$. $C(0) = 0$ but $lim_{q_i \to 0} C(q_i) \geq 0$.*

Assumption 3a *For every i, $P(q) - C_i'(q)$ is strictly decreasing for all $q > 0$ and there exists $\overline{q}_i > 0$, such that $P(\overline{q}_i) = C_i'(\overline{q}_i)$.*

Lemma 2a *For every i, $1 \leq i \leq N$, there exists q_i such that $P(q_i) = AC_i(q_i)$.*

The proof of Lemma 2a is the same as proof of Lemma 2. Define,

$$A_i = \{q_i | P(q_i) = AC_i(q_i)\}$$

and let, $A = \bigcup_{i=1}^{N} A_i$.

Lemma 3a *The set A is non-empty and compact.*

The proof follows from Lemma 3 and from the fact that a finite union of compact sets is compact.

Let,

$$\widetilde{Q} = Max_{q \in A} \ q$$

Let I be the set of all firms i such that $\widetilde{Q} \in A_i$. Namely, I is the set of all firms such that $P(\widetilde{Q}) = AC_i(\widetilde{Q})$. Such a situation is depicted in Figure 2 of Appendix C where there are three asymmetric firms, each represented by its own average cost curve, $AC_i(\cdot)$, $i = 1, 2, 3$. As per our definition, $I = \{1\}$ and $P(\widetilde{Q}) = AC_1(\widetilde{Q})$.

For every i, $1 \leq i \leq N$, define

$$f_i(q_i, Q) = -q_i P(Q) + \int_0^{q_i} P(x)dx - [P(\widetilde{Q}) - C_i'(\widetilde{Q})]q_i \tag{8}$$

where Q is the total industry output. Suppose that a firm i is subsidized according to $f_i(q_i, Q)$. Then by (8) its net profit is,

$$\pi_i^S(q_i) = -C_i(q_i) + \int_0^{q_i} P(x)dx - q_i[P(\widetilde{Q}) - C_i'(\widetilde{Q})] - \alpha_i \tag{9}$$

where α_i is the fee it pays to the regulator. As before, that π_i^S depends only on q_i and not on Q. The first order condition of this subsidized firm then is,

$$\frac{\partial \pi_i^S}{\partial q_i} = P(q_i) - C'(q_i) - [P(\widetilde{Q}) - C'(\widetilde{Q})] = 0$$

By Assumption 3a, the unique solution is $q_i = \widetilde{Q}$. Namely, irrespective of the cost function of the subsidized firm, it will produce $\widetilde{Q}$ units as output.

We distinguish two cases.

Case 1: $|I| \geq 2$. This case is very similar to the symmetric case (where all firms are identical). The regulator offers an exclusive subsidy schedule to the highest bidder. The schedule is $f_i(q_i, Q)$ given by (8) for some $i \in I$. The competition between the firms in I induce them to bid $\alpha_k = f_i(\widetilde{Q}, \widetilde{Q})$, $k \in I$. A firm j not in I, will not be able to afford a bid of $f_i(\widetilde{Q}, \widetilde{Q})$ since by definition of $\widetilde{Q}$, $P(\widetilde{Q}) - AC_j(\widetilde{Q}) < 0$. Hence, the subsidized firm is one from I.

Case 2: $|I| = 1$. And, suppose that $I = \{1\}$. This case is more complicated.

Consider the following sequential interaction between the regulator and the firms.

Stage 1. The regulator orders the N firms in a sequence where firm 1 (the only firm in I) precedes any other firm. Without loss of generality, assume that the sequence is $(1,2,\ldots,N)$. This sequence is publicly announced.

Stage 2. The regulator makes a "take-it-or-leave-it" offer to firm 1 to subsidize it according to $f_1(q_1, Q)$ (given by (8)), in return for an upfront fee $\alpha_1 = f_1(\widetilde{Q}, \widetilde{Q})$. If 1 accepts the offer, it will become the unique subsidized firm. Otherwise the regulator makes a similar offer to firm 2, namely he offers firm 2 the subsidy schedule $f_2(q_2, Q)$ in return for the fee $\alpha_2 = f_2(\widetilde{Q}, \widetilde{Q})$ and so on. If the first $N - 1$ firms reject the offer then the regulator will offer the last firm N, the subsidy schedule $f_N(q_N, Q)$ but for a reduced fee of $\alpha_N = f_N(\widetilde{Q}, \widetilde{Q}) - \pi_N^0$ where π_N^0 is the Cournot oligopoly profit of N in case no firm is subsidized.

Stage 3. The firms compete *a lá* Cournot.

The payoff function of the subsidized firm (if there is one) is given by (9). The payoff function of any other firm k is given by,

$$\pi_k(q_k) = q_k P(Q) - C_k(q_k),$$

and the regulator's payoff is the total welfare of the society.

The above mechanism describes a game G_a between the regulator and the firms in the industry.

Theorem 1a *Suppose that $I = \{1\}$. Then in every subgame perfect equilibrium of G_a, firm 1 is subsidized, it produces $\widetilde{Q}$ and earns a net profit of zero. All other firms exit the market, the regulator breaks even and the total welfare of the society is improved.*

Proof: As mentioned above, every subsidized firm will produce the output $\widetilde{Q}$. In this case it is a best reply for any other firm to exit the market (see proof of Theorem 1). Also, similar to the symmetric case, the total welfare under $\widetilde{Q}$ is higher than that of the Cournot oligopoly when no firm is subsidized.

Suppose next that the regulator's offer is rejected by the first $N-1$ firms. Then we argue that in equilibrium the last firm N, will accept the offer. Actually, this firm is indifferent between accepting and rejecting the offer $f_N(q_N, Q)$ in exchange for $\alpha_N = f_N(\widetilde{Q}, \widetilde{Q}) - \pi_N^0$, since its net payoff will be π_N^0 in both cases. But if N rejects the offer, the regulator has an incentive to slightly reduce his fee α_N in order to induce N to accept the offer. The regulator himself benefits from reducing the fee as the total welfare under $\widetilde{Q}$ is strictly larger than that of the Cournot oligopoly when no firm was subsidized. Thus the firm $N-1$ knows that if it does not accept the offer it would ultimately be driven out by N and hence accepts. Working backwards, firm 1 knows that if it rejects the offer, some other firm will be subsidized and will produce $\widetilde{Q}$. Thus firm 1 is indifferent between accepting and rejecting its offer. On the other hand the regulator strictly prefers to subsidize 1 rather than any other firm. Hence, in equilibrium 1 will be subsidized and the regulator himself breaks even. ∎

Appendix

A   Proof of Lemma 3

First note that there exists $\delta > 0$ such that for $0 \leq q < \delta$, $AC(q)$ and $P(q)$ do not intersect. Indeed, if $C(0) = 0$ then $lim_{q \to 0} AC(q) = C'(0)$ and by Assumption 3, $lim_{q \to 0} AC(q) < P(0)$. By the continuity of $AC(q)$ and $P(q)$, there exists $\delta > 0$ such that $AC(q) < P(q)$ for the $q < \delta$. Consequently, $A \subseteq \{q | q \geq \delta\}$. By Lemma 2, $A \neq \phi$. Again by Assumption 3, for $q > \overline{q}$, $P(q) < C'(q) < AC(q)$. Hence, $A \subseteq \{q | \delta \leq q \leq \overline{q}\}$, and thus is bounded. By the continuity of $AC(\cdot)$ and $P(\cdot)$ for $q > 0$ A is closed, implying A is nonempty and bounded. ∎

B   Proof of Lemma 5

We have,

$$
\begin{aligned}
f(\widetilde{Q}) &= -\widetilde{Q}P(\widetilde{Q}) + \int_0^{\widetilde{Q}} P(x)dx - \widetilde{t} \cdot \widetilde{Q} \\
&= \int_0^{\widetilde{Q}} P(x)dx - \widetilde{t} \cdot \widetilde{Q} - C(\widetilde{Q})
\end{aligned}
$$

as $P(\widetilde{Q}) = AC(\widetilde{Q})$. Let us show that

$$
\int_0^{\widetilde{Q}} P(x)dx - \widetilde{t}\widetilde{Q} > C(\widetilde{Q}).
$$

Define $S(q) = \int_0^q P(x)dx - \widetilde{t}q$. Then

$$
S'(q) = P(q) - [AC(\widetilde{Q}) - C'(\widetilde{Q})].
$$

Thus $S'(q) \geq C'(q)$ iff $P(q) - C'(q) \geq AC(\widetilde{Q}) - C'(\widetilde{Q}) = P(\widetilde{Q}) - C'(\widetilde{Q})$.

But the last inequality holds by Assumption 3 for any $0 < q \leq \widetilde{Q}$ (since $\widetilde{Q} < \overline{q}$). Also $S'(0) = P(0) - \widetilde{t} > P(\widetilde{Q}) - \widetilde{t} = C'(\widetilde{Q}) > 0$ and $S(0) = 0 = C(0)$. Consequently $S(q) > C(q)$ for all $0 < q \leq \widetilde{Q}$. In particular, $C(\widetilde{Q}) < S(\widetilde{Q}) = \int_0^{\widetilde{Q}} P(x)dx - \widetilde{t}$ and $\pi_M(\widetilde{Q}) > 0$. ∎

C   Figures

Figure 1: Efficient level of output with Symmetric firms.
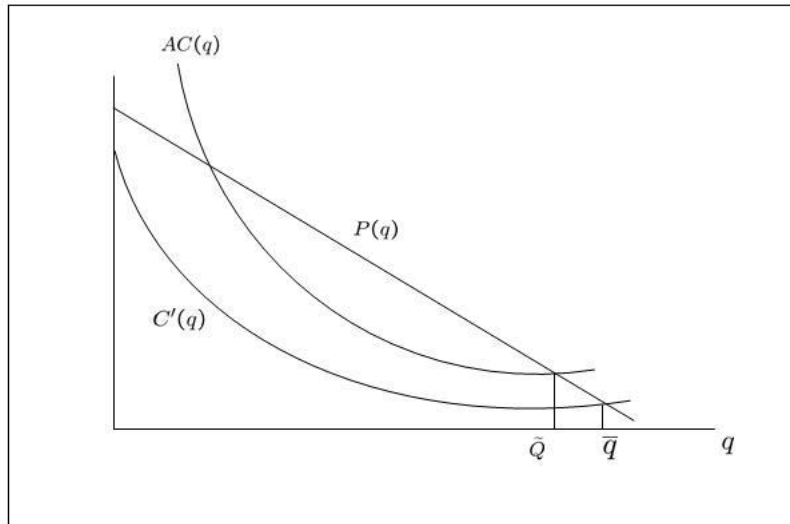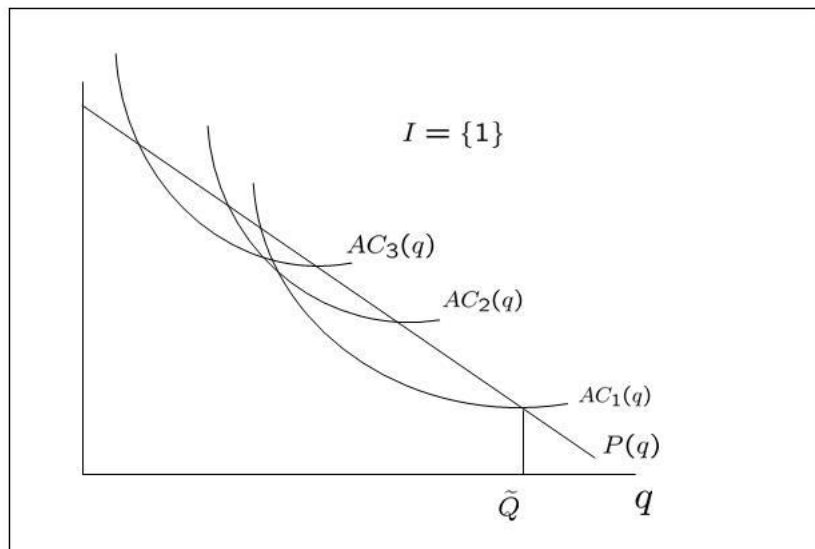

Figure 2: Efficient level of output with Asymmetric firms.

References

Auriol, E. and J.-J. Laffont (1992). "Regulation by Duopoly". *Journal of Economics and Management Strategy 1*(3), 507–533.

Baron, D. and R. Myerson (1982). "Regulating a Monopolist with Unknown Costs". *Econometrica 50*, 911–930.

Berg, S. and T. Tschirhart (1988). *"Natural Monopoly Regulation"*. Cambridge: Cambridge University Press.

Demsetz, H. (1968). "Why Regulate Utilities?". *Journal of Law and Economics 11*, 55–66.

Grimm, V., F. Riedel, and E. Wolfstetter (2003). "Implementing Efficient Market Structure. Optimal Licensing in Natural Oligopoly when Tax Revenue Matters". *Review of Economic Design 7*, 443–463.

Guesnerie, R. and J.-J. Laffont (1978). "Taxing Price Makers". *Journal of Economic Theory 19*, 423–455.

Laffont, J.-J. and J. Tirole (1993). *A Theory of Incentives in Procurement and Regulation.* (Cambridge: MIT Press).

Liao, C.-H. and Y. Tauman (2002). "Implementation of the Socially Best Outcome". Memo: http://ms.cc.sunysb.edu/∼ytauman/research.html.

Sappington, D. E. and D. S. Sibley (1988). "Regulating without Cost Information: The Incremental Surplus Subsidy Scheme". *International Economic Review 29*(2), 297–306.

Vogelsang, I. and J. Finsinger (1979). "A regulatory adjustment process for optimal pricing by multiproduct monopoly firms". *The Bell Journal of Economics 10*(1), 157–171.