All-Pay Auctions with Endogenous Asymmetries

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Abstract

We study N-bidders, asymmetric all-pay auctions under incomplete information. First, we solve for the equilibrium of a parametric model. Each bidder's valuation is independently drawn from an uniform $[0, \alpha_i]$ where the parameter α_i may vary across bidders. In this game, asymmetries are exogenously given. Next, a two-stage game where asymmetries are endogenously generated is studied. At the first stage, each bidder chooses the level of an observable, costly, value-enhancing action. The second stage is the bidding sub-game, whose equilibrium is simply the equilibrium of the, previously analyzed, game with exogenous asymmetries. Finally, natural applications of the all pay-auction in the context of political lobbying are considered: the effects of excluding bidders, as well as, the impact of caps on bids.

1 Introduction

All-pay auctions under perfect information are well understood, see for instance, Baye *et. al.* (1993, 1996) and Che and Gale (1998). The main results are: Only high valuation bidders, those bidders who have the highest or the second-highest valuation, participate. Low valuation bidders do not participate; in equilibrium, they bid zero. The equilibrium is unique provided there are only two high valuation bidders. Otherwise, there is a continuum of equilibria, which are not revenue equivalent.

Amann and Leininger (1996) prove existence and uniqueness of the equilibrium for two-bidders, all-pay auctions under incomplete, independent information.

All-pay auctions under incomplete information, with many bidders having symmetric, affiliated information were studied by Krishna and

Morgan (1997). The all-pay auction revenue dominates the first-price auction but, it is revenue dominated by the War of Attrition.

The novelty of this paper is that it presents a model of incomplete information, N-bidders, asymmetric all-pay auction. Although, the model is parametric, valuations are uniformly distributed, the first section presents a general model; useful to compute the equilibrium for other parameterizations.

The papers is organized as follows. Section 2 lays down the necessary conditions for the existence of an equilibrium in pure increasing strategies. Section 3 solves the model when the buyers' valuations are uniformly distributed and, it obtains the equilibrium payoffs and revenue. Section 4 performs comparative statics. Section 5 solves the model when bidders' are allowed to perform observable, value-enhancing investments previously to the auction. All proofs are collected into the Appendix.

2 A General Model

There are $j = 1, \ldots, N$ buyers whose valuations are independent draws from F_j, which support is $[\underline{v}, \overline{v}]$. We restrict attention to equilibria in strategies b_j that are constant in $[\underline{v}, v_j^*]$ and strictly increasing in $(v_j^*, \overline{v}]$. The inverse bid functions are denoted by ϕ_j. Let $\underline{b}$ be the lowest bid placed in equilibrium. The payoff of bidding $b > \underline{b}$ for a buyer i who has valuation ϕ is

$$\Pi_i(b|\phi_i) = \phi_i \prod_{j \neq i} F_j(\phi_j(b)) - b \,, \tag{1}$$

and the corresponding first-order condition for buyer i is,

$$\phi_i \sum_{j \neq i} \prod_{k \neq i,j} F_k(\phi_k(b)) f_j(\phi_j(b)) \phi_j'(b) - 1 = 0 \tag{2}$$

Let $Q = (id, Q_2, \ldots, Q_N)$ be the *tying mapping*: $Q_i(\phi)$ is the type of buyer i that in equilibrium bids the same amount as the type ϕ of buyer 1. By combining the first-order conditions of buyer i and buyer 1, we obtain the following differential equations,

$$\begin{aligned} &\sum_{j \neq 1,i} \frac{f_j(Q_j(\phi_1))}{F_j(Q_j(\phi_1))} Q_j'(\phi_1) - \frac{\phi_1 f_i(Q_i(\phi_1))}{Q_i(\phi_1) F_1(\phi_1) - \phi_1 F_i(Q_i(\phi_1))} Q_i'(\phi_1) = \\ &= \frac{Q_i(\phi_1) f_1(\phi_1)}{\phi_1 F_i(Q_i(\phi_1)) - Q_i(\phi_1) F_1(\phi_1)} \qquad \text{for} \quad i = 2, \ldots, N. \end{aligned} \tag{3}$$

Notice that the above system of $N - 1$ differential equations is invertible. More exactly, one has:

Proposition 1 *If bidders' equilibrium strategy is strictly increasing at the bid $b = b_1(\phi)$ then the tying function of bidder $i = 1, ..., N$ must satisfy:*

$$\frac{\partial}{\partial \phi} Q_i = \frac{\sum_{j \neq i} F_j(Q_j(\phi)) \prod_{k \neq j} Q_k(\phi) - (N-2) F_i(Q_i(\phi)) \prod_{k \neq i} Q_k(\phi)}{\sum_{j \neq 1} F_j(Q_j(\phi)) \prod_{k \neq j} Q_k(\phi) - (N-2)\ F_1(\phi)\ \prod_{k \neq 1} Q_k(\phi)} \frac{F_i(Q_i(\phi))}{f_i(Q_i(\phi))} \frac{f_1(\phi)}{F_1(\phi)}$$

In sum, as in Griesmer, Levitan, and Shubik (1967), see also Amann and Leininger (1996) and Parreiras (2002), one can solve for the inverse bid functions recursively: first, the tying map Q is obtained by solving the system of differential equations given by Proposition 1. Secondly, the solution is then used to eliminate the ϕ_j for $j \neq 1$, from the first-order condition (2). The resulting equation is just another differential equation, which can be solved by direct methods.

3 The Parametric Model

We specialize the model to uniform distributions, $F_i \sim \mathrm{U}[0, \alpha_i]$ for $i = 1, \ldots, N$ and, without any loss of generality, we assume that $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_N$. In the uniform model, the system of differential equations for the inverse bids is,

$$\frac{\partial Q_i}{\partial \phi_1}(\phi_1) = \frac{\sum_{j \neq i} \alpha_j^{-1} - (N-2)\alpha_i^{-1}}{\sum_{j \neq 1} \alpha_j^{-1} - (N-2)\alpha_1^{-1}} \frac{Q_i(\phi_1)}{\phi_1} \text{ for all } i.$$

Thus, the tying function is given by,

$$Q_i(\phi_1) = \alpha_i \left[\frac{\phi_1}{\alpha_1}\right]^{\kappa_i} \qquad \text{where } \kappa_i = \frac{\sum_{j \neq i} \alpha_j^{-1} - (N-2)\alpha_i^{-1}}{\sum_{j \neq 1} \alpha_j^{-1} - (N-2)\alpha_1^{-1}}.$$

Therefore, a necessary and sufficient condition for the above characterization of the tying function is $\kappa_i > 0$ for all i or, equivalently,

$$\frac{\sum_{j \neq i} \alpha_j^{-1}}{N-2} > \alpha_i^{-1} \text{ for all } i. \tag{4}$$

Notice that $\kappa_i > 0$ holds whenever buyers are not too asymmetric. Later on this section, we characterize equilibrium when the asymmetry is high and so, the above condition fails to hold.

The auxiliary identities: $\sum_{i \neq 1} \kappa_i = \frac{(N-1)\alpha_1^{-1}}{\sum_{i \neq 1} \alpha_i^{-1} - (N-2)\alpha_1^{-1}}$, $1 + \sum_{i \neq 1} \kappa_i = \frac{\sum_i \alpha_i^{-1}}{\sum_{i \neq 1} \alpha_i^{-1} - (N-2)\alpha_1^{-1}}$, and $\frac{\sum_{i \neq 1} \kappa_i}{1 + \sum_{i \neq 1} \kappa_i} = \frac{(N-1)\alpha_1^{-1}}{\sum_i \alpha_i^{-1}}$ shall be used in the following computations.

First, notice we can re-write equation 2 as,

$$\phi_1(b) \sum_{i \neq 1} \prod_{j \neq 1, i} \left[\frac{\phi_1(b)}{\alpha_1}\right]^{\kappa_j} \frac{\kappa_i \phi_1^{\kappa_i - 1}(b)}{\alpha_i^{\kappa_i}} \phi_1'(b) - 1 = 0 \Longrightarrow$$

$$\left(\sum_{i \neq 1} \kappa_i\right) \left[\frac{\phi_1(b)}{\alpha_1}\right]^{\sum_{i \neq 1} \kappa_j} \phi_1'(b) - 1 = 0$$

For simplicity, let $\kappa = \sum_{i \neq 1} \kappa_i$, then the inverse bid and the equilibrium bid functions are respectively,

$$\phi_1(b) = \alpha_1 \left[\frac{1+\kappa}{\kappa} \frac{b}{\alpha_1} \right]^{\frac{1}{1+\kappa}} = \alpha_1 \left[\frac{\sum_i \alpha_i^{-1}}{N-1} b \right]^{\frac{1}{1+\kappa}} \text{ and}$$

$$b_1(\phi) = \alpha_1 \frac{\kappa}{1+\kappa} \left[\frac{\phi}{\alpha_1} \right]^{1+\kappa} = \frac{N-1}{\sum_i \alpha_i^{-1}} \left[\frac{\phi}{\alpha_1} \right]^{1+\kappa}.$$

The lowest and highest equilibrium bids are respectively,

$$\underline{b} = 0 \text{ and } \overline{b} = \alpha_1 \frac{\kappa}{1+\kappa} = \frac{N-1}{\sum_i \alpha_i^{-1}}$$

The interim revenue is simply,

$$R(\phi_1, ..., \phi_N) = \sum_{i=1}^{N} b_i(\phi_i) = \frac{N-1}{\sum \alpha_j^{-1}} \sum_{i=1}^{N} \left[\frac{\phi_i}{\alpha_i} \right]^{\frac{1+\kappa}{\kappa_i}}$$

and so, the expected revenue is:

$$R(\alpha_1, ..., \alpha_N) = \frac{N-1}{\sum \alpha_j^{-1}} \sum_{i=1}^{N} \frac{\sum \alpha_j^{-1} - (N-1)\alpha_i^{-1}}{2\sum \alpha_j^{-1} - (N-1)\alpha_i^{-1}} \tag{5}$$

Buyer i's interim payoff is,

$$\Pi_i(\phi_i) = \phi_i \prod_{j \neq i} F_j(Q_j(Q_i^{-1}(\phi_i))) - b(\phi_1) = \left[\alpha_i - \frac{N-1}{\sum \alpha_j^{-1}} \right] \left[\frac{\phi_i}{\alpha_i} \right]^{\frac{1+\kappa}{\kappa_i}}, \tag{6}$$

and consequently, i's expected payoff is

$$\Pi_i = \left[\alpha_i - \frac{N-1}{\sum \alpha_j^{-1}} \right] \frac{\sum \alpha_j^{-1} - (N-1)\alpha_i^{-1}}{2\sum \alpha_j^{-1} - (N-1)\alpha_i^{-1}}. \tag{7}$$

All previous computations were derived under the provisory assumption, the level of asymmetries among bidders is not too high or, in other words, that condition (4) holds. Also, observe that condition (4) is likely to hold when the number of bidders is small. In particular, it always holds for $N = 2$, the two bidder case. When the condition fails, the 'weakest' bidders, bidders likely to have low valuations, will not participate. That is, in equilibrium, they will always bid zero. The following proposition characterizes precisely the set of non-participating bidders. First, recall that bidders were ordered in such way that, bidder i likely (first-order stochastic dominance) has 'higher' valuations than bidders $j > i$: $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_N$. Secondly, define κ^K by:

$$\kappa^K = \frac{\sum_{j=1}^{K-1} \alpha_j^{-1} - (K-2)\alpha_K^{-1}}{\sum_{j=2}^{K} \alpha_j^{-1} - (K-2)\alpha_1^{-1}}.$$

Proposition 2 *Bidders* 1 *and* 2 *always participate. Moreover, whenever bidder i participates, bidder $i-1$ also participates. Exactly K bidders participate, if and only if, $\kappa^K > 0$ and $\kappa^{K+1} < 0$.*

4 Comparative Statics

Assume that N bidders are participating. If i's valuation is stochastically raised (put simply, α_i increases) and, all the other bidders remain active, it's straightforward to show that i's payoff is increasing in α_i and decreasing in α_j.

Proposition 3 *If* $\kappa_i > 0$ *then* $\dfrac{\partial}{\partial \alpha_i}\Pi_i > 0$ *and* $\dfrac{\partial}{\partial \alpha_j}\Pi_i < 0$.

But, it may happen that, as i's valuation stochastically increases, 'weak' bidders drop out. In this instance, i's payoff increases continuously; however, it is not differentiable. More exactly,

Proposition 4 *At the values of* α_i *where 'weak' bidders drop out, the marginal return of increasing* α_i *decreases discontinuously.*

Nevertheless, for the range of α_i where competitors do not drop out, one has that:

Proposition 5 *At the values of* α_i *where* Π_i *is differentiable, the marginal return of* α_i *is increasing.*

Figure 1 below illustrates the content of the previous propositions; it depicts the bidders' payoffs (for $N = 3$, $\alpha_1 = 6$ and $\alpha_2 = 3$) as α_3 increases from 1 to 8.

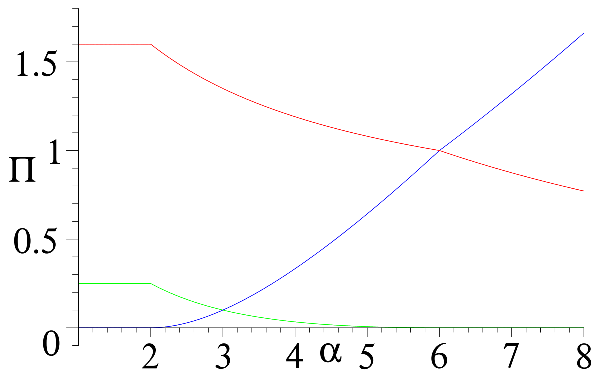

Figure 1: The bidders' payoffs as functions of α_3

5 Raising the Stakes

We now look at the effects of endogenous changes in the distribution of the buyers' valuations. Previous to the auction, each buyer i can perform an investment that stochastically increase his/her own valuation. More specifically, if bidder i takes the action $a_i \geq \alpha_i$, his/her valuation are drawn from $\mathrm{U}[0, a_i]$ and, the cost of action a_i is given by $c(a_i - \alpha_i)$, where c satisfies the following properties:

1. $a_i \leq \alpha_i \Rightarrow c(a_i - \alpha_i) = 0$;
2. $c' > 0$ and $c'' > 0$;
3. $c'(0) < \dfrac{1}{2} < c'(+\infty)$.

Property 1 says, disinvestments do not yield positive returns. That is, we interpret α_i as being the *status-quo*. If $a_i = \alpha_i$, no investments are made and the bidder does not incurs any cost. Property 2 is a standard assumption; the cost is increasing and convex. Property 3 rules out uninteresting cases where: the optimal action is always to not invest ($c'(0) > \frac{1}{2}$); the optimal action is to invest infinity ($\frac{1}{2} > c'(+\infty)$).

Moreover, Property 3, the convexity of the payoff equilibrium of the bidding sub-game, $\Pi_i'' > 0$, and the fact that $\Pi_i'' < \frac{1}{2}$, altogether, imply the existence of an unique optimal level of investment.

6 The Exclusion Principle

7 Caps on Political Contributions

8 Conclusion

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9 Appendix

9.1 Proof of Lemma 2

To prove this lemma a few auxiliary results are needed first:

Remember that when bidder N does not participate, we define κ by $\kappa = \sum_{i \neq 1,N} \kappa_i$ and $\kappa_1 = 1$. So, the inverse bid functions when bidder N does not participate are:

$$\phi_i(b) = \alpha_i \left[\frac{\sum_{j=1}^{N-1} \alpha_j^{-1}}{N-2} b \right]^{\frac{\kappa_i}{1+\kappa}}$$

Consequently, the first-order condition of bidder N is simply:

$$\phi_N \sum_{i \neq N} \prod_{j \neq i,N} F_j(\phi_j(b)) f_i(\phi_i(b)) \phi_i'(b) - 1 =$$

$$\phi_N \sum_{i \neq N} \prod_{j \neq i,N} \frac{\phi_j(b)}{\alpha_j} \frac{1}{\alpha_i} \phi_i'(b) - 1 =$$

$$\phi_N \sum_{i \neq N} \prod_{j \neq i,N} \frac{\alpha_j \left[\frac{\sum_{k=1}^{N-1} \alpha_k^{-1}}{N-2} b \right]^{\frac{\kappa_j}{1+\kappa}}}{\alpha_j} \frac{1}{\alpha_i} \frac{\kappa_i}{1+\kappa} \alpha_i \left[\frac{\sum_{k=1}^{N-1} \alpha_k^{-1}}{N-2} \right]^{\frac{\kappa_i}{1+\kappa}} b^{\frac{\kappa_i}{1+\kappa}-1} - 1 =$$

$$\phi_N \frac{1+\kappa}{1+\kappa} \frac{\sum_{k=1}^{N-1} \alpha_k^{-1}}{N-2} b^{\frac{1+\kappa}{1+\kappa}-1} - 1 < 0 \Longleftrightarrow \frac{\sum_{k \neq N} \alpha_k^{-1}}{N-2} < \phi_N^{-1} \Longleftarrow \frac{\sum_{k \neq N} \alpha_k^{-1}}{N-2} < \alpha_N^{-1} \quad Q.E.D.$$

In the case that N participates, $\kappa = \sum_{i \neq 1} \kappa_i$ and $\kappa_1 = 1$, and the corresponding inverse bids are $\phi_i(b) = \alpha_i \left[\frac{\sum_{j=1}^{N} \alpha_j^{-1}}{N-1} b \right]^{\frac{\kappa_i}{1+\kappa}}$

first-order condition is:

$$\phi_N \sum_{i\neq N}\prod_{j\neq i,N} F_j(\phi_j(b))f_i(\phi_i(b))\phi_i'(b) - 1 =$$

$$\phi_N \sum_{i\neq N}\prod_{j\neq i,N} \frac{\phi_j(b)}{\alpha_j}\frac{1}{\alpha_i}\phi_i'(b) - 1 =$$

$$\phi_N \sum_{i\neq N}\prod_{j\neq i,N} \frac{\alpha_j\left[\frac{\sum_{k=1}^N \alpha_k^{-1}}{N-1}b\right]^{\frac{\kappa_j}{1+\kappa}}}{\alpha_j}\frac{1}{\alpha_i}\frac{\kappa_i}{1+\kappa}\alpha_i\left[\frac{\sum_{k=1}^N \alpha_k^{-1}}{N-1}b\right]^{\frac{\kappa_i}{1+\kappa}-1} - 1 =$$

$$\phi_N \frac{1+\kappa-\kappa_N}{1+\kappa}\left[\frac{\sum \alpha_k^{-1}}{N-1}b\right]^{\frac{1+\kappa-\kappa_N}{1+\kappa}-1} - 1 =$$

$$\phi_N \left(1-\frac{\kappa_N}{1+\kappa}\right)\left[\frac{\sum \alpha_k^{-1}}{N-1}b\right]^{-\frac{\kappa_N}{1+\kappa}} - 1 =$$

$$\phi_N \frac{(N-1)\alpha_i^{-1}}{\sum \alpha_k^{-1}}\left[\frac{\sum \alpha_k^{-1}}{N-1}b\right]^{-\frac{\kappa_N}{1+\kappa}} - 1 = 0 \Longleftrightarrow$$

$$\phi_N = \frac{\alpha_i\sum \alpha_k^{-1}}{N-1}\left[\frac{\sum \alpha_k^{-1}}{N-1}b\right]^{\frac{\kappa_N}{1+\kappa}}$$

9.2 Revenue and Payoffs

The revenue is simply,

$$R(\phi_1,...,\phi_N) = \sum_{i=1}^N b_i(\phi_i) = \sum_{i=1}^N b_1(Q_i^{-1}(\phi_i)) = \frac{N-1}{\sum \alpha_j^{-1}}\sum_{i=1}^N \left[\frac{\phi_i}{\alpha_i}\right]^{\frac{1+\kappa}{\kappa_i}}$$

and so, the expected revenue is:

$$R(\alpha_1,...,\alpha_N) = \frac{N-1}{\sum \alpha_j^{-1}}\sum_{i=1}^N \frac{\kappa_i}{\kappa_i+\kappa+1} = \frac{N-1}{\sum \alpha_j^{-1}}\sum_{i=1}^N \frac{\sum_{j\neq i}\alpha_j^{-1}-(N-2)\alpha_i^{-1}}{2\sum_{j\neq i}\alpha_j^{-1}-(N-3)\alpha_i^{-1}} \tag{8}$$

Buyer i's interim payoff is,

$$\Pi_i(\phi_i) = \phi_i\prod_{j\neq i}F_j(Q_j(Q_i^{-1}(\phi_i))) - b(\phi_1) = \phi_i\prod_{j\neq i}\left[\frac{\phi_i}{\alpha_i}\right]^{\frac{\kappa_j}{\kappa_i}} - \frac{N-1}{\sum \alpha_j^{-1}}\left[\frac{\phi_i}{\alpha_i}\right]^{\frac{1+\kappa}{\kappa_i}} =$$

$$= \left[\alpha_i - \frac{N-1}{\sum \alpha_j^{-1}}\right]\left[\frac{\phi_i}{\alpha_i}\right]^{\frac{1+\kappa}{\kappa_i}},$$

and consequently, i's expected payoff is

$$\Pi_i = \left[\alpha_i - \frac{N-1}{\sum \alpha_j^{-1}}\right]\frac{\kappa_i}{\kappa_i+\kappa+1} = \left[\alpha_i - \frac{N-1}{\sum \alpha_j^{-1}}\right]\frac{\sum \alpha_j^{-1}-(N-1)\alpha_i^{-1}}{2\sum \alpha_j^{-1}-(N-1)\alpha_i^{-1}}.$$

9.3 Comparative Statics

9.3.1 The payoff of bidder i is increasing in α_i

$$\frac{\partial}{\partial \alpha_i} \log\left(\Pi_i(\alpha)\right) = \frac{\alpha_i \sum_{j\neq i} \alpha_j^{-1} \left[4 + 5\alpha_i \sum_{j\neq i} \alpha_j^{-1} - N + 2\alpha_i^2 (\sum_{j\neq i} \alpha_j^{-1})^2\right] + \left(\alpha_i \sum_{j\neq i} \alpha_j^{-1} - N + 2\right)^2 \left(2\alpha_i\right.}{\alpha_i \left(\alpha_i \sum_{j\neq i} \alpha_j^{-1} - N + 2\right) \left(2\alpha_i \sum_{j\neq i} \alpha_j^{-1} - N + 3\right) \left(1 + \alpha_i \sum_{j\neq i} \alpha_j^{-1}\right)} \tag{9}$$

As long as $\kappa_i > 0$, both, the denominator and the numerator of (9) are positive.

$$\frac{\partial}{\partial \alpha_j} \log\left(\Pi_i(\alpha)\right) = \frac{\partial}{\partial \alpha_j} \sum_{j\neq i} \alpha_j^{-1} \times$$
$$\times \frac{\alpha_i \left[N - (N-2)^2 + 3(N-1)\alpha_i \sum_{j\neq i} \alpha_j^{-1}\right]}{\left(\alpha_i \sum_{j\neq i} \alpha_j^{-1} - N + 2\right) \left(2\alpha_i \sum_{j\neq i} \alpha_j^{-1} - N + 3\right) \left(1 + \alpha_i \sum_{j\neq i} \alpha_j^{-1}\right)}$$

9.3.2 The payoff Π_i is not differentiable in α_i at the points where other bidders drop out

Let $\alpha = (\alpha_j)_{j=1}^N$ be such that all bidders are participating and consider the critical $\alpha_i^* = \dfrac{1}{(N-2)\alpha_N^{-1} - \sum_{j\neq i,N} \alpha_j^{-1}}$. If α_i is increased but kept below α_i^*, all bidders still participate. But if α_i increases above α_i^*, bidder N drops out. Bidder N bids zero in equilibrium. The payoff of bidder i is not differentiable at α_i^*. More exactly,

$$0 < \frac{\partial_+}{\partial \alpha_j} \log\left(\Pi_i(\alpha)\right) - \frac{\partial_-}{\partial \alpha_j} \log\left(\Pi_i(\alpha)\right) =$$
$$= \frac{(N-2)\left[3\alpha_i \sum_{j\neq i} \alpha_j^{-1} - (N-5)\right]}{\alpha_i (N-1) \left[\alpha_i \sum_{j\neq i} \alpha_j^{-1} - (N-3)\right] \left[2\alpha_i \sum_{j\neq i} \alpha_j^{-1} - (N-4)\right] \left(\alpha_i \sum_{j\neq i} \alpha_j^{-1} + 1\right)}.$$

Using the fact that $\alpha_i \sum_{j\neq i} \alpha_j^{-1} > N - 2$ when N bidders participate, it is possible to prove that Π_i is locally convex.

$$0 < \frac{\partial^2}{\partial\alpha_j\partial\alpha_j}\Pi_i(\alpha) \times \frac{\left\{\left[\alpha_i\sum_{j\neq i}\alpha_j^{-1}-(N-3)\right]\left(\alpha_i\sum_{j\neq i}\alpha_j^{-1}+1\right)\right\}^3}{2\sum_{j\neq i}\alpha_j^{-1}(N-1)} =$$

$$= - N^3 + 9N^2 + 6\alpha_i\sum_{j\neq i}\alpha_j^{-1}N^2 - 26N - 33\alpha_i\sum_{j\neq i}\alpha_j^{-1}N - 9\left[\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right]^2 N+$$

$$+ \left[\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right]^3 N + 27\left[\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right]^2 + 45\alpha_i\sum_{j\neq i}\alpha_j^{-1} + 5\left[\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right]^3 + 24 =$$

$$= \left[\left(\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right)^3 - (N-3)^3\right] + \left(\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right)^2\left[\alpha_i\sum_{j\neq i}\alpha_j^{-1} - (N-3)\right] +$$

$$+ (N+3)\left(\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right)^3 - 8(N-3)\left(\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right)^2 + (6N^2 - 33N + 45)\left(\alpha_i\sum_{j\neq i}\alpha_j^{-1}\right) +$$

$$+ N - 3$$