NON-GENERICITY OF STRICT CORRELATED EQUILIBRIUM

EXTENDED ABSTRACT

A correlated equilibrium of a strategic form *n*-person game is called *strict* if all the "associated inequalities" (those appearing in the definition of "correlated equilibrium," other than that the probabilities are non-negative and sum to 1) are strict. The game itself is called *strict* if it possesses at least one strict correlated equilibrium.

In as yet unpublished work, Aumann and Dreze characterize all possible expected payoffs of a player in a game G, when common priors and common knowledge of rationality are assumed. The characterization has a particularly simple form when G is strict.

The question thus arises how general strictness is. Not all games are strict. For example, "matching pennies" is not, and neither is any game in a sufficiently small neighborhood U of "matching pennies," in the 8-dimensional space of payoffs in 2-person 2x2 games. Thus for 2-person 2x2 games, strictness is not generic. But this could be an artifact of the small size of these games; for larger games, one might conjecture that strictness *is* generic.

The main objective of this study is to show that it is not, even for arbitrarily large games. Specifically, we prove

Theorem. Let G be an n-person game with a unique correlated equilibrium. Suppose that this equilibrium is also a completely mixed Nash equilibrium. Then there exists a neighborhood of G in which every game has a unique correlated equilibrium, and all the associated inequalities for this equilibrium are equalities.

The proof uses properties of the orthogonal space, uppersemicontinuity of the Nash correspondence, and theorems of the alternative.

It remains to show that there are arbitrarily large games satisfying the condition of the theorem. For each m, no matter how large, we construct two-person $m \times m$ games of this kind. An example in the 3×3 case is "Rock, Scissors, Paper" where the unique Nash equilibrium is $((\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}))$, and this is also the unique correlated equilibrium.

Finally, it is known that the set of correlated equilibria is a nonempty, compact, and convex polytope containing the convex-hull of the Nash equilibria. However, the exact relationship between the set of correlated equilibria and the set of Nash equilibria has not yet been completely clarified. This work sheds light on this relationship by providing sufficient conditions for the two sets to coincide in a whole neighborhood of games.