Abstract

This paper considers the formation of political parties and their choice of platform. In a representative democracy, individuals vote for the party that represents their preferred position on the issue and the party with the most votes wins the election. The question raised here is whether parties align themselves so that the winner has chosen a platform that is preferred by only a minority of the population.

I show in a model with binary issues that the minority platform can win the election. There exist equilibria where a third party enters with the platform preferred by the majority of the population splitting the votes. This allows the party with the minority platform to win. Furthermore, under certain environments, it is the only outcome that exists. As a consequence, restricting entry to the political process is beneficial when it stops entrants from causing the minority position to win.

Party Formation and Platform Selection with Binary Issues: Can the Minority Win the Election?[1]

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1 Introduction

Individuals differ in their views about political issues. Consequently, political parties differ in their stances regarding these issues. Individuals have the opportunity to cast a vote for a candidate of a party in an election. The winner of the election, in a majority winning environment, reacts to the issue implementing a stance. Can political parties align themselves in a way so that the winning party chooses a platform that is preferred by only a minority of the population? In a representative democracy people vote for representatives to decide political issues rather than directly vote of these issues themselves. One may believe that choosing the representatives by a majority winning vote guarantees that the preferences of a minority of the population are not implemented. Here, I challenge this belief. By not voting directly on the issues the platform of the elected officials may well be misaligned with the population.

There are few examples of this outcome in the literature. The standard voting model assumes that voters have an ideal point on a line segment and they prefer the stance implemented by the victor to be as close to their ideal point as possible.[1] Myerson and Weber (1997) consider such a model with three parties and show, under plurality voting, that there exists equilibria where the party preferred by only a minority of the population can win the election. To do so, they must exogenously fix the positions and number of the parties. Once they allow candidates to position themselves the third party is ignored by the voters and the remaining party that represents the most voters wins. Myerson (1993) considers minority representation but considers

[1] This is known as single-peaked preferences.

only elections where there are multiple seats. Therefore, I develop a model to explain winning by minority candidates when there is only one seat to win and parties can select their platforms.

To address this question I develop a model of party formation with a new twist. A stance on an issue takes the form of a yes or no. Should abortions be legal? Should we cut taxes on dividends? Should we go to war with Iraq? Parties choose platforms consisting of a yes or no for each issue. The standard model, with utility increasing as the stance moves closer to a voter's ideal point, fails to address the question. In these models the existence of a party representing a minority of the population can, at best, only influence the policy of the winning party. Myerson (1993) briefly discusses a similar setup but, as stated, addresses minority representation in multiseat elections.

The parties are assumed to be Downsian in that they do not have a preference for the stance implemented but only care about obtaining votes to win the election. Most authors have, as a result of Anthony Downs' work, assumed that parties receive their utility from winning the election. Downs repeatedly refers to his assumption on the preferences of parties as "vote maximization".[2] (Downs, 1957) In a two party system maximizing votes and winning the election and identical goals - one cannot be achieved without the other. Rather than exogenously fix the number of active parties at two I allow multiple party to enter the race. I assume that the political party's utility increases both with the number of votes obtained and with winning

[2] Anthony Downs lays out his assumption on pages 30-31. A description of his idea of the fundamentals of government decision making are covered on pages 52-53. In these sections Downs expresses his behavioral assumption as "vote maximization" and even states, "we do not distort the motives of party members by saying that their primary objective is to be elected...each party seeks to receive more votes".

the election. Both components are necessary to study multiple Downsian political parties competing in a representative democracy. This distinctive setup leads to new interesting results.

I consider the model with one single issue. Interestingly, I find that the minority can win and implement their stance on the issue. Two parties, attempting to gain votes, may choose the platform that is preferred by the majority. They split the votes and the party representing the minority can win. This outcome seems quite realistic. For example, in the 2000 Presidential election many believe that Ralph Nader took enough votes from Al Gore to allow George W. Bush to win.[3] Another example, discussed by Fey (1997) and Riker (1982), is the 1970 New York senatorial election. The majority where unable to coordinate between two liberal candidates allowing the conservative to win with 39% of the vote. Therefore, in equilibrium, the party with the platform preferred by the minority of the population can win if multiple parties split the votes of the majority of the population.

The result of minority winning rests on the existence of "non-Duvergerian" equilibria. This refers to equilibria that violate Duverger's Law. (Duverger, 1954) Duverger's Law states that in a plurality voting system there should be no more than two political parties. Many authors have debated the validity of Duverger's Law. Morelli (2002) argues that, with certain distributions of preferences, multiple district elections can give rise to many po-

[3] A CNN exit poll found that 47% of people that voted for Nader would have voted for Gore as compared to 21% that would have voted for Bush. In the state of Florida, Nader received approximately 97,000 votes and Bush had a lead less than 2000. Thus, if Nader had not been on the ballot Gore would have presumably gained over 25,000 votes, won Florida, and won the Presidency. Furthermore, one can argue that Bush won with a platform preferred only by a minority of the population since he lost the popular vote. (Wise, 2000)

litical parties. Myerson and Weber (1997) argue that plurality voting gives rise to non-Duvergerian outcomes which are robust to changes in the distribution of preferences. In recent history, one need only look at the United States Presidential elections of 1992 and 2000 to find examples of races where third parties had dramatic effects on the outcomes of the elections. Hence, non-Duvergerian outcomes have the potential problem of misaligned political parties and issues, preferred by only a minority of the population, being implemented.

This raises an additional provoking question. Can barriers in the formation of political parties be a good thing? Or stated another way, can a two party system be more beneficial than democracy with free entry? If the cost to organizing a political party is relatively cheap then a third party will enter with the majority and potentially cause the minority platform to win the election. As entry becomes more expensive the third party may be deterred and the outcome of minority winning is averted. These results are new to the literature and are due to the new environment.

The paper is organized as follows. Section 2 presents the model while Section 3 demonstrates an example useful to highlight the outcomes of the model. The general solution is presented in Section 4. Section 5 concludes and discusses future work.

2 The Model

Suppose there is a continuum of voters with a mass normalized to unity and an unlimited supply of potential political parties.[4] Consider, first, the case where there is a single issue for which voters have a preference. A political party can choose either yes, y, or no, n, on the issue. Denote the fraction of the population of voters that prefer y as μ^Y and the fraction of the population that prefers n as μ^N. Assume that $\mu^Y + \mu^N = 1$ and, without loss of generality, $\mu^Y \geq \mu^N$ so that a majority of the population prefers y.

Consider the following game. Each party simultaneously selects a platform announcing y or n. A party may choose not to form by announcing $\emptyset$. Because there is a continuum of voters, no one voter can ever influence the outcome of the election. Thus, voters are assumed to not be strategic and vote for the party that has chosen the platform that he prefers. If more than one party chooses the platform then the votes are shared equally.[5] The party with the most votes wins the election. If multiple parties receive the same number of votes each wins with equal likelihood.

The parties are assumed to be Downsian, or rather, they only care about the number of votes they receive and winning the election and do not have a preference for the issue. (Downs, 1957) The payoffs to the political parties are comprised of three components. First, a party receives a payoff equal to the fraction of the population that voted for it, η^i. For example, if party A is the only party with a platform of y then A receives a benefit equal to μ^Y.

[4] All that is required is that the number of potential political parties exceed $\frac{1}{c}$ where c will be defined as the cost to forming a party.

[5] If no party chooses a particular platform then the voters that prefer the platform do not vote.

Secondly, the party that wins the election receives a benefit of $w > 0$. It is assumed that the party with the greatest number of votes wins. Finally, there is a cost $c > 0$ to organizing a party ($w \geq c$). Therefore, the payoff function for party i is

$$\pi^i = \begin{cases} \eta^i + w - c & if\ i\ wins \\ \eta^i - c & if\ i\ loses \\ 0 & otherwise \end{cases} \tag{1}$$

where η^i represents the number of votes received by party i.[6] Each component in the payoff function is necessary. Without w the model is of a proportional representation system. Without η^i there would be no gain to representing anyone but the majority and the outcome would be either only one political party (dictatorship) or many parties with the exact same platform. Finally, the cost is necessary to determine the number of active parties, rather than set it exogenously. The functional form of the payoff function will be generalized. The solution concept is Nash equilibrium.

3 An Illustrative Example

Consider the following example. Let the fraction of the voters who prefer y be 0.6. Furthermore, suppose that the cost to organizing a political party is 0.2 and the benefit to winning the election is 1. Consider the outcome of two parties, A and B, with a platform of y and one party, C, with a platform of n. Parties A and B split the μ^Y votes each receiving 30% while C receives 40%. Thus, C wins the election earning a payoff of $\pi^C = 0.4 + 1 - 0.2 = 1.2$.

[6] One may think that η^i represents the number of Congressional seats a platform generates, w represents the payoff (in terms of number of Congressional seats) winning the Presidency earns, and c represents the number of seats required to make running a race profitable.

Parties A and B each earn $\pi^A = \pi^B = 0.3 - 0.2 = 0.1$. To show that this is indeed an equilibrium it must be that no party wants to change it's decision. Consider first A and B. By switching to n they again lose the election and now split 0.4 rather than 0.6. If C switches to y the votes and gains from winning are split three ways. Party C's payoff when deviating to y would be $\frac{1.6}{3} - 0.2$ which is strictly less. All three parties are earning a positive payoff as well. Thus, none of the organized parties has a profitable deviation. Finally, consider the parties that did not organize. If they enter with y they split the losing vote receiving $0.2 - 0.2 = 0$. If they enter with n then the make n the losing platform and again split the losing vote earning $0.2 - 0.2 = 0$. Therefore, no political party wants to enter making this a Nash equilibrium. As a consequence, the minority wins the election and implements their desired stance.

It turns out that there are, in fact, four pure strategy Nash equilibria in this example. In one, two parties choose y and two choose n resulting in the majority winning. In a second, three parties choose y and two choose n. This results in a tie. The final two equilibria have the feature of the minority platform winning the election. One is the outcome previously described and the other Nash is where a third party enter with a y platform while only one has a platform of n.

Additionally, let the cost to organizing a political party increase, in this example, from 0.2 to 0.21. With this increase it turns out that the unique equilibrium is the one initially described where two parties choose the y platform and one selects n. Thus, minority winning is the only outcome that exists! Once the cost to organizing exceeds 0.3 it is no longer profitable for the third party to enter. At these high costs the platform that is preferred by the majority wins the election.

Finally, suppose there existed many issues instead of just the one. For simplicity let the stance of y on an issue be the majority position. For an example, consider another issue where 0.55 of the population prefers y and the population is skewed on all other issues so that many more people prefer y than n on each. Denote this new issue as Issue 1 with $\mu_1^Y = 0.55$ and the other as Issue 2 with $\mu_2^Y = 0.6$. Suppose that Party A announces a platform of y on every issue, $(yy...y)$, Party B announces a n on the new issue, $(ny...y)$, and Party C announces n only on the second issue, $(yny...y)$. Since every party agrees on all issues except the first and second voters select based only on these two issues. In this scenario all voters who prefer y on both issues vote for A ($\eta^A = 0.33$), all voters who prefer y on only one issue vote for the party that has selected their platform, and the voters who prefer n on both issues split between B and C ($\eta^B = 0.36$ and $\eta^C = 0.31$). Thus, Party B wins the election.

Again, it can be shown that this is indeed a Nash equilibrium of the game. To be a Nash no organized party desires to exit, no unorganized party wants to enter, and no party wants to switch to a new platform. Party C receives the smallest payoff of $0.31 - 0.2 = 0.11 > 0$. Thus, no want wants to exit. A new party, entering with any previously selected platform earns less than half of what the party already at that platform earns. Since each platform receives less that 0.4 of the vote entering with any platform is not profitable. Finally, no party wants to switch. Party B is winning the election. Any deviation results in fewer votes and is not preferrable. Parties A and C could select n on the skewed issues, match B's platform, or match the others. In each case less votes is received than by continuing with their original platform. Thus, this is a Nash equilibrium of the game and the minority stance on an issue is implemented even when extending the model to many issues.

The model lends itself to a simple geometrical representation of the Nash equilibria. Consider, the example with only the original issue. For any outcome it must be that

(1) There are more votes for the winning platform.
(2) Winning is profitable.
(3) Losing is profitable.
(4) Entering with the losing platform is not profitable.
(5) Entering with the winning platform is not profitable.
(6) Winning parties do not switch platforms.
(7) Losing parties do not switch platforms.

Consider, first, an outcome of majority winning. For this example the seven requirements are

(1) $\frac{0.6}{Y} \geq \frac{0.4}{N}$

(2) $\frac{1.6}{Y} - 0.2 \geq 0$

(3) $\frac{0.4}{N} - 0.2 \geq 0$

(4) $\frac{0.4}{N+1} - 0.2 \leq 0$

(5) If $\frac{0.6}{Y+1} \geq \frac{0.4}{N}$, then $\frac{1.6}{Y+1} - 0.2 \leq 0$; otherwise $\frac{0.6}{Y+1} - 0.2 \leq 0$.

(6) $\frac{0.4}{N+1} - 0.2 \leq \frac{1.6}{Y} - 0.2$

(7) If $\frac{0.6}{Y+1} \geq \frac{0.4}{N-1}$, then $\frac{1.6}{Y+1} - 0.2 \leq \frac{0.4}{N} - 0.2$; otherwise $\frac{0.6}{Y+1} - 0.2 \leq \frac{0.4}{N} - 0.2$.

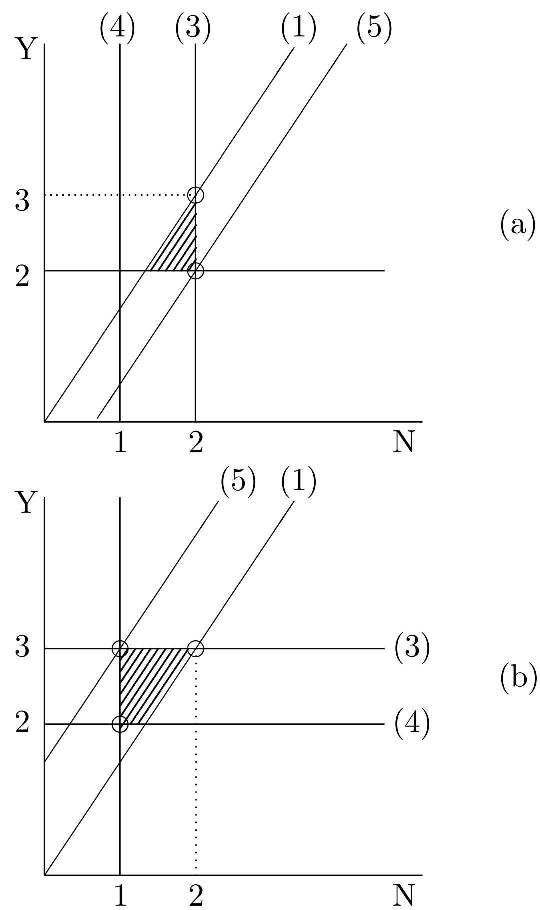
where Y and N represent the number of firms with a platform of y and n respectively. Figure 1(a) illustrates the set of Nash equilibria outcomes where the majority wins the election.[7] The striped area represents the set of feasible points. The outcome where the minority platform wins follows analogously. Figure 1(b) depicts the set of Nash equilibria outcomes that arise from these seven conditions adjusting each for the minority winning. Pure strategy Nash equilibria result in integer points within these striped areas and are depicted with the circles.

4 The General Solution

The general solution can be presented geometrically as well. For convenience, let W (L) denote the number of parties who take the platform that wins (loses) the election. I will first consider the pure strategy equilibria. The goal of this paper is to identify whether the implementation of the platform preferred by only a minority of the population is possible. The confirmation of such a result is strengthened if it can be shown to occur without the use of randomization by political parties.

[7]Conditions (6) and (7) assume that $N \geq 1$ and $Y > 1$. For this example, there are 0.4 voters preferring n. Since the cost is only 0.2, at least one party will prefer to announce a platform of n. Also, there are 0.6 voters that prefer y. With a cost of 0.2 even if the parties lose the election, it is worthwhile for at least two firms to announce y. In the next section it will be shown that conditions (2) and (4) imply (6) holds, when (5) is satisfied so to is (7); and (2) is never a binding constraint. Therefore, for clarity, conditions (2), (6), and (7) are omitted in Figure 1.

Figure 1: Nash Equilibrium in the Example



4.1 The General Solution with One Issue

4.1.1 Pure Strategy Equilibria

Initially assume that $\mu^N \geq c + \frac{cw}{\mu^Y}$ so that the size of the minority of the population is sufficiently larger than the cost. This guarantees that it is worthwhile for at least one party to represent each of the platforms. As a consequence, W and L are greater than or equal to 1. This assumption will later be relaxed. The same seven conditions given in the previous section describe the pure strategy Nash equilibrium in the general case. A pure strategy Nash equilibrium outcome is an integer point that satisfies

(1) $\frac{\mu^W}{W} \geq \frac{\mu^L}{L}$

(2) $\frac{\mu^W+w}{W} - c \geq 0$

(3) $\frac{\mu^L}{L} - c \geq 0$

(4) $\frac{\mu^L}{L+1} - c \leq 0$

(5) If $\frac{\mu^W}{W+1} \geq \frac{\mu^L}{L}$, then $\frac{\mu^W+w}{W+1} - c \leq 0$; otherwise $\frac{\mu^W}{W+1} - c \leq 0$.

(6) When $W = 1$, $\frac{\mu^W+w}{W} - c \geq \frac{\mu^L+w}{L+1} - c$.
When $W > 1$, $\frac{\mu^W+w}{W} - c \geq \frac{\mu^L}{L+1} - c$.

(7) When $L = 1$, $\mu^L - c \geq \frac{\mu^W+w}{W+1} - c$.
When $L > 1$, if $\frac{\mu^W}{W+1} \geq \frac{\mu^L}{L-1}$ then $\frac{\mu^L}{L} - c \geq \frac{\mu^W+w}{W+1} - c$; otherwise $\frac{\mu^L}{L} - c \geq \frac{\mu^W}{W+1} - c$.

There are two simplifying results that can be shown. First, conditions (2) and (6) are never binding constraints. If losing is profitable and each of the parties with the winning platform receives more votes, then winning must also be profitable. Thus, (2) does not constrain the set of equilibria outcomes. Also, if winning is profitable and entering with the losing platform is not then switching platforms from the winning one to the losing one is not profitable either. Thus, (6) does not constrain the set of equilibria outcomes.

Lemma 1 *Conditions (2) and (6) are never binding constraints.*

Proof. From (1), $\frac{\mu^W}{W} \geq \frac{\mu^L}{L}$. Subtracting c from both sides, $\frac{\mu^W}{W} - c \geq \frac{\mu^L}{L} - c$. Condition (3) states that $\frac{\mu^L}{L} - c \geq 0$, which implies $\frac{\mu^W}{W} - c \geq 0$. As a consequence, $\frac{\mu^W+w}{W} - c \geq 0$. Therefore, when (1) and (3) hold (2) always holds as well. Furthermore, (1) states that $\frac{\mu^W}{W} \geq \frac{\mu^L}{L}$ so that $\frac{\mu^W+w}{W} - c \geq \frac{\mu^L+w}{L} - c$. It follows that, since $\frac{\mu^L+w}{L} - c \geq \frac{\mu^L+w}{L+1} - c$, $\frac{\mu^W+w}{W} - c \geq \frac{\mu^L+w}{L+1} - c$. Therefore, (6) always holds when (1) does. ■

Second, Conditions (5) and (7) simply require that $W \geq \frac{\mu^W - c}{c}$. This condition requires that a nonpositive payoff must be earned by a party that enters with the winning platform.

Lemma 2 *Conditions (5) and (7) only require that* $W \geq \frac{\mu^W - c}{c}$.

Proof. First, consider Condition (5) which states that if entering with the winning platform maintains that platform's victory, then the entry should not be profitable. If entry with the winning platform causes that platform to lose, then entry again should not be profitable. Consider the first statement. If losing is profitable (Condition (3)) and entering with the winners results in more votes, then it too must be profitable. Therefore, (5) requires that both entry causes the platform to lose and this entry is not profitable. Notice that if entry is not profitable, but losing was, then it trivially holds that entry causes the platform to lose. Therefore, (5) simply requires that $W \geq \frac{\mu^W - c}{c}$.

Now consider the first half of Condition (7) when $L > 1$. It requires that if switching from the losing to winning platform maintains that platform's victory then switching should not be profitable. Otherwise, if switching causes the platform to lose this again should not be profitable. Consider the first statement. If switching maintains the victory then the platform must still receive more votes. Since (3) requires that losing is profitable then so too must switching. Therefore, (7) requires both that switching causes the platform to lose and switching is not profitable. If switching allows the party to win then it must always be profitable. As a result, (7) requires that $\frac{\mu^L}{L-1} - c \geq \frac{\mu^W}{W+1} - c$. Since (5) states that the right hand side is negative and (3) implies the left hand side is positive, (7) holds when (3) and (5) do.

Finally, consider Condition (7) when $L = 1$. This requires that $\mu^L - c \geq \frac{\mu^W + w}{W+1} - c$. This simplifies to requiring that $W \geq \frac{\mu^W - \mu^L + w}{\mu^L}$. Since it was assumed that $\mu^N \geq c + \frac{cw}{\mu^Y}$ it follows that $cw \leq \mu^W \mu^L - c\mu^W$, or rather, $cw + c\mu^W - c\mu^L \leq \mu^W \mu^L - c\mu^L$ so that $\frac{\mu^W - \mu^L + w}{\mu^L} \leq \frac{\mu^W - c}{c}$. Therefore, any outcome that satisfies $W \geq \frac{\mu^W - c}{c}$ satisfies (7). ■

As a result of Lemmas 1 and 2, any number of winners and losers that satisfies the following conditions is a Nash equilibrium outcome.

I. $W \leq \left(\frac{\mu^W}{\mu^L}\right) L$

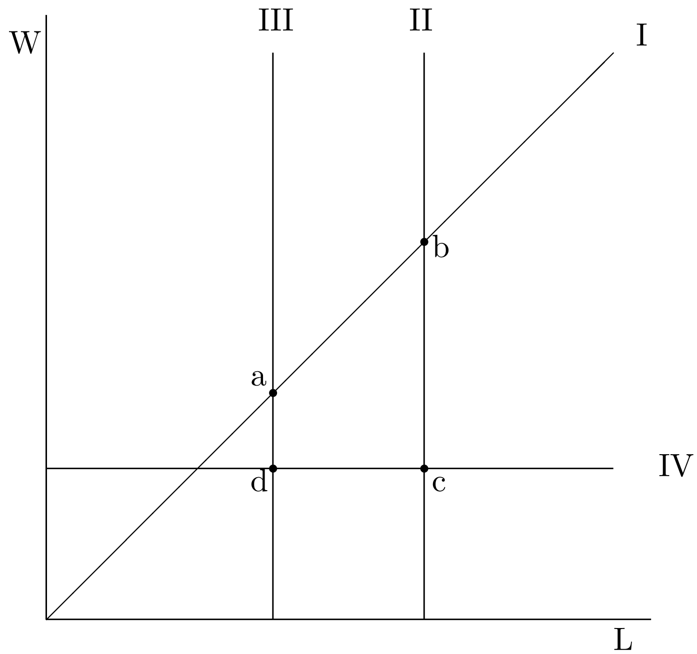
II. $L \leq \frac{\mu^L}{c}$

III. $L \geq \frac{\mu^L - c}{c}$

IV. $W \geq \frac{\mu^W - c}{c}$

Figure 2 depicts the set of Nash equilibrium outcomes described by these

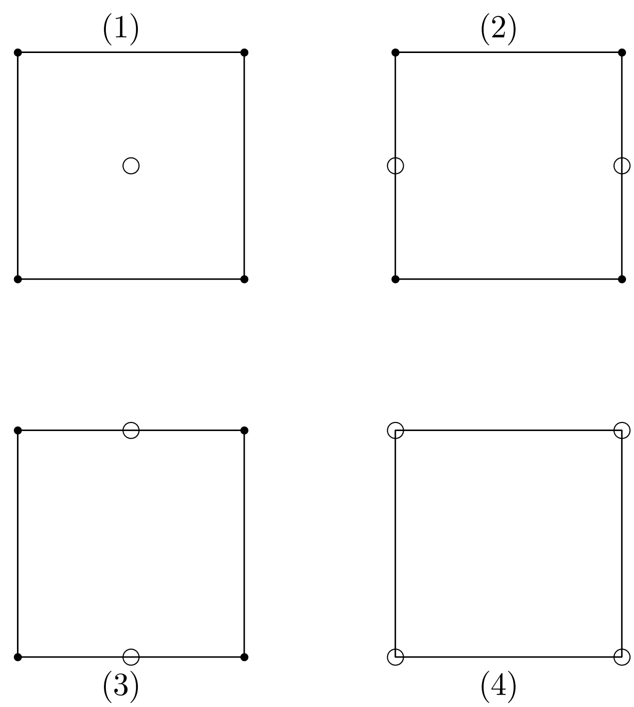
Figure 2: Nash Equilibrium In The General Solution



four conditions. Specifically, any integer pair (L, W) that satisfies I – IV is a pure strategy Nash equilibrium outcome. The set of feasible equilibria outcomes is represented by the quadrilateral *abcd*.

What points lie within this region? Notice that I and IV require that $W \in \left[\frac{\mu^W - c}{c}, \left(\frac{\mu^W}{\mu^L}\right) L\right]$ and II and III imply that $L \in \left[\frac{\mu^L - c}{c}, \frac{\mu^L}{c}\right]$. Since $L \leq \frac{\mu^L}{c}$ it is necessary that $W \in \left[\frac{\mu^W - c}{c}, \frac{\mu^W}{c}\right]$. As a result, it is necessary that both Y is in the interval $\left[\frac{\mu^Y - c}{c}, \frac{\mu^Y}{c}\right]$ and that N is in the interval $\left[\frac{\mu^N - c}{c}, \frac{\mu^N}{c}\right]$. Consider the square created by the intervals of Y and N. There are four possible scenarios. Since the square has sides of one unit in length, if neither $\frac{\mu^Y}{c}$ nor $\frac{\mu^N}{c}$ are integers then there is one integer point in the square. If both $\frac{\mu^Y}{c}$ and $\frac{\mu^N}{c}$ are integers then there are four point's in the square. The remaining two scenarios are when only one is an integer. Figure 3 depicts

Figure 3: Four Scenarios



the scenarios where the circles represent the integer points.

It turns out that Figure 3 depicts the pure strategy Nash equilibria outcomes of the game. The squares were created by using the constraint $W \leq \frac{\mu^W}{c}$ but all that was required, from Condition I, is that $W \leq \left(\frac{\mu^W}{\mu^L}\right) L$. Notice that an integer point in the square that does not satisfy Condition I for platform y winning must be satisfied for platform n winning.[8] Thus, Figure 3 depicts all Nash outcomes of the game. Proposition 1 formally states the equilibrium. For notational purposes, let the point (N, Y) describe the num-

[8]Suppose that, in a proposed equilibrium point in the square, $\frac{\mu^Y}{Y} > \frac{\mu^N}{N}$. It follows that $Y < \frac{\mu^Y}{\mu^N} N$. Therefore, Condition I is satisfied for a party with the platform y to win the election.

ber of political parties with platforms of n and y that organize and let Y' and N' denote the integers strictly within the intervals described.

Proposition 1 *All pure strategy Nash equilibria are described for the four scenarios.*

(1) If $\frac{\mu^Y}{c}, \frac{\mu^N}{c} \notin \mathbb{Z}$ *then* (N', Y')

(2) If $\frac{\mu^Y}{c} \notin \mathbb{Z}$, $\frac{\mu^N}{c} \in \mathbb{Z}$ *then* $\left(\frac{\mu^N}{c}, Y'\right)$ *&* $\left(\frac{\mu^N - c}{c}, Y'\right)$

(3) If $\frac{\mu^Y}{c} \in \mathbb{Z}$, $\frac{\mu^N}{c} \notin \mathbb{Z}$ *then* $\left(N', \frac{\mu^Y}{c}\right)$ *&* $\left(N', \frac{\mu^Y - c}{c}\right)$

(4) If $\frac{\mu^Y}{c}, \frac{\mu^N}{c} \in \mathbb{Z}$ *then* $\left(\frac{\mu^N}{c}, \frac{\mu^Y}{c}\right)$, $\left(\frac{\mu^N}{c}, \frac{\mu^Y - c}{c}\right)$, $\left(\frac{\mu^N - c}{c}, \frac{\mu^Y}{c}\right)$ *&* $\left(\frac{\mu^N - c}{c}, \frac{\mu^Y - c}{c}\right)$.

Proof. Since $\mu^Y \geq \mu^N > c + \frac{cw}{\mu^Y}$ both $\frac{\mu^N}{c}$ and $\frac{\mu^Y}{c}$ are greater than one so that these intervals are in the positive quadrant. Consider any potential Nash equilibrium described. By construction the platform that wins the election does not want to deviate to a new platform or exit (Lemma 1). A party that chooses a platform that does not win the election does not want to deviate to a new platform (Lemma 2) or exit (Condition II). Finally, a party that did not organize does not want to join in with the winning platform (Condition IV) nor the losing platform (Condition III). Therefore, these are each Nash equilibria outcomes. ■

Proposition 1 describes all pure strategy Nash equilibria that exist. Two consequences of this proposition can be highlighted. First, the question posed was whether there existed an equilibrium where a party, choosing the platform preferred by a minority of the population, won the election. Consider first the outcome $\left(\frac{\mu^N}{c}, \frac{\mu^Y}{c}\right)$. In this outcome the number of parties are $Y = \frac{\mu^Y}{c}$ and $N = \frac{\mu^N}{c}$. This implies that $\frac{\mu^Y}{Y} = \frac{\mu^N}{N}$ and there is a tie. Thus, the outcomes $\left(\frac{\mu^N - c}{c}, \frac{\mu^Y}{c}\right)$ and $\left(N', \frac{\mu^Y}{c}\right)$ in scenarios (3) and (4) have the majority platform winning and the outcomes $\left(\frac{\mu^N}{c}, \frac{\mu^Y - c}{c}\right)$ and $\left(\frac{\mu^N}{c}, Y'\right)$ in scenarios (2) and (4) have the party, representing the minority of the population, winning the election. Finally, in scenario (1) there is a unique equilibrium outcome. Thus, if $\frac{\mu^Y}{Y'} > \frac{\mu^N}{N'}$ then minority winning is the only outcome.

Second, a common result in two party models is that both parties more to the "center". (Cox, 1990) For example, suppose voter's ideal points are distributed uniformly along the $[0, 1]$ line and each party chooses a point on the line to implement. If each individual votes for the party closest to their own ideal and the parties only care about acquiring votes then the equilibrium is that both parties choose to locate at $\frac{1}{2}$. This common result seems rather unrealistic if one considers how different the two major parties stances actually are. If, for illustration, $\frac{\mu^Y}{c}$ and $\frac{\mu^N}{c}$ are in the interval $(1, 2)$ the unique pure strategy equilibrium has one party choosing the y platform and one party choosing the n platform. Thus, the "centrist" result needs not hold in this model.[9]

Finally, notice that the number of active political parties is independent of the size of the payoff to winning the election. The number of parties is solely determined by their ability to obtain enough votes to cover the fixed cost of organizing. This outcome does not support Duverger's Hypothesis. Duverger's Hypothesis states that there should be more parties under proportional representation than with plurality voting. (Duverger, 1954) Here, the number of parties is the same regardless of which way the translation

[9]For example, keep $\mu^Y = 0.6$ and $\mu^N = 0.4$. Let $c = \frac{1}{3}$ and $w = 0.1$. Thus, $N \in \left[\frac{1}{5}, \frac{6}{5}\right]$ and $Y \in \left[\frac{4}{5}, \frac{9}{5}\right]$ so that there is one party with each platform and the assumption made, $\mu^N > c + \frac{cw}{\mu^Y}$ $\left(\frac{2}{5} > \frac{35}{90}\right)$, holds.

from votes to power occurs.

Proposition 1 describes all pure strategy Nash equilibria when $\mu^N \geq c + \frac{cw}{\mu^Y}$. Let us consider relaxing this assumption. So long as $\mu^N \geq c$ at least one party forms at each of the two platforms. As shown in Lemma 2, constraints (5) and (7) can be reduced to $W \geq \frac{\mu^W - c}{c}$ when the assumption holds. Suppose that $\mu^N \in \left(c, c + \frac{cw}{\mu^Y}\right)$. It follows from the proof of Lemma 2 that $W \geq \frac{\mu^W - \mu^L + w}{\mu^L}$ must also hold for a potential outcome to be a pure strategy equilibrium. This additional constraint may eliminate some equilibria and if $\mu^N < c + \frac{cw}{\mu^Y} - \frac{\mu^N}{\mu^Y}$ then no pure strategy equilibria survive the additional requirement.[10]

4.1.2 Mixed Strategy Equilibria

I turn attention to the derivation of the mixed strategy equilibria. Suppose there are two parties that organize. Let p^A and p^B denote the probability that parties A and B select the y platform. The following proposition describes the mixed strategy equilibrium.

Proposition 2 *With two parties, the unique mixed strategy equilibrium is for both to select the y platform with probability $2\mu^Y - \mu^N + w$.*

Proof. If A selects y it's expected payoff is

$$p^B \left(\frac{\mu^Y + w}{2}\right) + \left(1 - p^B\right)\left(\mu^Y + w\right) - c.$$

If A selects n it's expected payoff is

$$p^B \mu^N + \left(1 - p^B\right) \frac{\mu^N + w}{2} - c.$$

[10] Condition I requires that $W \leq \frac{\mu^W}{c}$. Thus, when $\frac{\mu^W - \mu^L + w}{\mu^L} > \frac{\mu^W}{c}$ no pure strategy equilibria exist. This simplifies to $\mu^N < c + \frac{cw}{\mu^Y} - \frac{\mu^N}{\mu^Y}$.

Setting the two equal and simplifying

$$p^B = 2\mu^Y - \mu^N + w.$$

From the symmetry of the payoff functions it follows that $p^A = p^B$. $\blacksquare$

Since $\mu^Y \geq \mu^N$ this probability is always positive. If $2\mu^Y - \mu^N + w \geq 1$ (or rather, $\mu^Y \geq 1 - \frac{w}{3}$) then both parties choose the pure strategy of representing the majority platform. The expected payoff to a firm playing this mixed strategy equilibrium is

$$\frac{\left(3\mu^N - w\right)\left(\mu^Y + w\right)}{2} - c \tag{2}$$

and neither party wants to exit if this payoff is nonnegative. The mixed strategy was derived guaranteeing that neither A nor B preferred to switch strategies. Therefore, to guarantee that this is an equilibrium, we need only consider a party, C, choosing to enter the race. If C selects a platform of y it earns

$$p^2\left(\frac{\mu^Y + w}{3}\right) + p\left(1-p\right)\mu^Y + \left(1-p\right)^2\left(\mu^Y + w\right) - c.^{11} \tag{3}$$

If C selects the platform n then it earns

$$p^2\left(\mu^N + w\right) + p\left(1-p\right)\mu^N + \left(1-p\right)^2\left(\frac{\mu^N + w}{3}\right) - c. \tag{4}$$

As a consequence, no party enters if both are nonpositive. Notice that the payoff, w, determines the platforms selected in a two-party system. As w increases the probability that a party announces the majority platform increases and when the payoff is sufficiently large both parties announce the

[11] This payoff assumes that $\mu^Y \leq \frac{2}{3}$.

majority platform. Thus, the payoff to winning the election, which had no influence over the number of political parties that organized or over the platforms selected in the pure strategy equilibria, now determines the frequency at which the majority is represented and whether new parties enter the race.

A similar analysis can be done with more than two parties to determine the mixed strategy equilibria. The goal of this paper is to address the phenomenon of the platform, preferred by only a minority of the population, being implemented. When (2) is nonnegative and equations (3) and (4) are nonpositive the minority platform is implemented with probability $(1-p)^2$. Therefore, the minority platform is implemented with a positive probability even in a two-party system!

4.1.3 The Generalized Payoff Function

The payoff function of the political parties has a rather specific form. Consider a generalized function with the same two arguments, the number of votes and the surplus from winning, determing the payoff, $u(\eta^i, \varpi) - c$, where $\varpi = w$ if the party wins and zero otherwise. I will assume that payoff is increasing in the political support (the number of votes received), increasing in the payoff to winning (when the party collects the most votes), and that $u(\eta^i, \varpi)$ and $u(\eta^i, 0)$ are H^1. Given this generalized payoff function the following proposition shows that the solution previously described continues to hold.

Proposition 3 *With the generalized payoff function in all pure strategy Nash* *equilibria the number of parties that announce each platform is*

$$Y \in \left[\frac{u\left(\mu^{Y},0\right)}{c} - 1, \frac{u\left(\mu^{Y},0\right)}{c} \right]$$

and

$$N \in \left[\frac{u\left(\mu^{N},0\right)}{c} - 1, \frac{u\left(\mu^{N},0\right)}{c} \right].$$

Proof. To show that any outcome that satisfies the above conditions is a pure strategy Nash equilibrium requires that (1) no organized party wants to exit the race, (2) moving to the losing platform is not profitable, and (3) moving to the winning platform is not profitable. I assume that $u\left(\mu^{N},0\right) > c$ so that these intervals include a positive integer.

Consider the first. It is required that $Y \leq \frac{u\left(\mu^{Y},0\right)}{c}$. Thus, $c \leq \frac{1}{Y} u\left(\mu^{Y},0\right)$. Since $u\left(\mu^{Y},0\right)$ is H^1, it follows that $u\left(\frac{\mu^{Y}}{Y},0\right) - c \geq 0$. Analogously, $u\left(\frac{\mu^{N}}{N},0\right) - c \geq 0$ and no party wants to exit. Now consider deviating to the losing platform. This can be done by either a party with the winning platform or by an unorganized party entering the race. Notice that $Y \geq \frac{u\left(\mu^{Y},0\right)}{c} - 1$. This implies that $c \geq \frac{1}{Y+1} u\left(\mu^{Y},0\right)$. Since $u\left(\mu^{Y},0\right)$ is H^1, it follows that $u\left(\frac{\mu^{Y}}{Y+1},0\right) - c \leq 0$. Analogously, $u\left(\frac{\mu^{N}}{N+1},0\right) - c \leq 0$. Therefore, regardless of which platform is the losing platform, a party deviating to the losing one results in a nonpositive payoff. Since every party previously earned a nonnegative payoff such a deviation is not profitable.

Finally consider deviating to the winning platform. If moving to the losing platform is not profitable then it suffices to show that deviating to the winning platform causes it to lose when there is at least two parties with the losing platform. First, it follows from the required condition that $W \geq \frac{u\left(\mu^{W},0\right)}{c} - 1$ where W denotes the winning platform (either Y or N).

This implies that $\frac{W+1}{\mu^W} \geq \frac{u(1,0)}{c}$. Secondly, the conditions require that $L \leq \frac{u(\mu^L,0)}{c}$ where L is the number of parties that select the losing platform. This implies that $\frac{L}{\mu^L} \leq \frac{u(1,0)}{c}$. Combining the two iuequalities it follows that $\frac{W+1}{\mu^W} \geq \frac{L}{\mu^L}$, or rather, $\frac{\mu^L}{L} \geq \frac{\mu^W}{W+1}$. Thus, moving to the winning platform causes it to lose and losing with an additional party is not profitable. This is true for either an organized party switching platforms or an unorganized party entering the race. Therefore, all pure strategy Nash equilibria have Y (N) parties announce a platform of y (n) where $Y \in \left[\frac{u(\mu^Y,0)}{c} - 1, \frac{u(\mu^Y,0)}{c}\right]$ and $N \in \left[\frac{u(\mu^N,0)}{c} - 1, \frac{u(\mu^N,0)}{c}\right]$. $\blacksquare$

This result requires that at least one party prefers to announce each platform, $u\left(\mu^N,0\right) > c$ and that there is only one party with the losing platform. If there is only one party that announces the losing platform then deviating to the winning platform maintains that platforms victory. Hence, a Nash equilibrium requires that the payoff of deviating and maintaining the win $\frac{u(\mu^W,w)}{W+1} - c$ is less than the payoff to losing, $u\left(\frac{\mu^L}{L},0\right) - c$. This extra condition must hold when the number of losing parties equals one. If this additional condition fails then only a mixed strategy equilibrium exists.

As can be seen, the previous model was simply the case where $u\left(\mu^i,0\right) = \mu^i$ and $u(\eta^i,w) = \mu^i + w$. Notice that the number of parties that compete in the race is independent of the size of the payoff to winning. The reason for this is that parties differ in the platforms the announce and only one platform can win the election. For a party that has chosen a platform that does not win the decision to be in the race depends only on its ability to gain enough support to make running the race worthwhile. For the winning platform the surplus from winning makes the platform very profitable. Entry will occur up to the point that one additional party causes that platform to lose. An unorganized party deciding whether or not to enter then only cares whether it is able to collect enough votes to cover the cost to running.

4.2 Minority Winning with Many Issues

The example illustrates a case where there are an arbitrarily many issues a minority stance can be implemented. What the example required was that the two main parties (those that collected the most votes) differ on an issue that is quite split in the population at large (55% majority). A third party entered with a platform that was appealing to many of the supporters of the majority but differed on an issue that was more unevenly distributed in the population (only 40% preferred n on the new issue). The third party was able to pull enough voters away from the majority candidate to allow that minority candidate to win. Thus, the results from the one issue model hold when there are many issues. Also, this new outcome gives possibly a more realistic portrayal since the two parties no longer announce exactly the same platform but rather similar platforms and the less popular third party attracts a minority of the voters - enough to influence the outcome of the election!

5 Conclusion

In a representative democracy can stances on issues preferred by only a minority of the population be implemented even when political parties credibly announce that they will do so before the election? Yes. The example illustrates that it is quite possible. If more than two parties form then the competition for the majority's votes allow for the minority platform to win. In fact, in one scenario it was the only outcome! Can barriers to entry in the political process be a good thing? In the example, the unique equilibrium with three parties is that the platform preferred by a minority of the voters won. If the cost is sufficiently great the stances preferred by the majority are always implemented. Hence, it may be socially better to restrict entry.

Also, consider the role the environment plays to the outcome. It is important that parties organize to gain votes. Without this there would be little incentive to compete in the political process. If we assumed that political parties have preferences for the issues, so long as there is a benefit to running a race and obtaining support, we should still expect the outcome to persist. It is also important the parties receive an additional benefit from winning. Without this, the model would be a democracy with proportional representation. The use of binary issues is not only realistic but also necessary for the minority winning outcome. In this environment there is no way for a losing party to alter it's platform to entice voters away from the winner and there is no way for similar parties to differentiate themselves.

A critique that can be made is that in a circumstance where multiple parties split the votes we should expect coordination to occur either between the political parties or between voters to eliminate this outcome. The study of coalitions in systems with proportional representation is common. (see Bandopadhyay and Oak (2002) for a discussion of the literature) With plurality voting parties often are unable to coordinate. So long as they are not perfectly altruistic, Downsian parties, gaining from the support of voters, compete in elections even when it would be better for their constituents if they exited the race. With regards to the coordination of votes, I assume that everyone votes sincerely. Instead, if they are allowed to vote strategically there would exist equilibria where the majority coordinate on a candidate. Sincere voting remains an equilibrium though. Some propose that the role of political parties is to coordinate voters but, so long as their are sufficient gains to forming a party, the inability of political parties to coordinate results in the minority winning outcome to exist.

The model presented is novel in its formation and has many extensions to be considered. Assumptions were made to simplify the analysis to answer the main question posed. Relaxing these assumptions may lead to either new, interesting results or may discredit the results argued here. Specifically, future work should consider the impact of multiple issues. The example showed that if parties were to choose platforms that consisted of a list of yes or no responses, third parties would no longer need to match another's platform exactly but may instead deviate to the minority position on some other issue. Also, with the single issue studied, strategic voting has no role but with multiple issues it would be important. Expanding the results to that the result holds with many issues is necessary. Finally, I assume here that parties can commit to implement their announced platform and parties cannot form coalitions sharing the gains from victory. Casual observation of politics shows that assumptions of the commitment by candidates and the formation of coalitions are debatable. Future work should therefore address party formation and platform selection in a broader model relaxing these assumptions.

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