Jobless Recovering and Equilibrium Involuntary Unemployment with a Simple Efficiency Wage Model*

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Abstract

The U.S. economy had experienced the "jobless recovering" after the 1990-1991 and 2001 recessions, which has been constantly puzzling the economists, market analysts, and policymakers. This paper uses a simple hiring game in an efficiency wage model framework to resolve that puzzle. Our efficiency wage model emphasizes the importance of the local unemployment rate, which is endogenously determined by firms' hiring decision at a symmetric Nash equilibrium. Our model has a new feature such that nonzero steady involuntary unemployment at equilibrium may coexist with an efficiency wage that stays below the market-clearing wage. Moreover, we show how it is possible to use our model to study income inequality as a result of skill-biased technical change, inter-industry wage differentials, and skill wage premiums. We also demonstrate how it is possible to derive the wage curve (Blanchflower and Oswald (1994)) as an equilibrium locus of our model. *Journal of Economic Literature Classification Numbers: D0, D24, J41*

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1 Introduction

According to the Bureau of Economic Analysis, real gross domestic product (GDP)-the output of goods and services produced within the U.S. grew at an annual rate of 8.2 percent in the third quarter 2003. The Labor Department reported that in the same quarter nonfarm productivity of American workers-output per hour increased at an annual rate of 9.4 percent. The strong growth in GDP and productivity in the quarter lead economists and market analysts to expect the labor market to create 100,000 to 150,000 new nonfarm jobs in the December of 2003. In contrast, the Labor Department reported on January 9, 2004 that only 1,000 nonfarm jobs were created in the month[1] and there were 309,000 discouraged workers who dropped out of the workforce.

The weak job market situation in December 2003 was by no means alone. It had been a characteristic of the U.S. economy during the latest two recessions and their later recoverings. The payroll survey from the Bureau of Labor Statistics showed that there were more than one and half million job losses during the 2001 recession from March 2001 to November 2001, largely due to the massive layoffs exercised simultaneously by the U.S. firms, even though real GDP fell just mildly. During the recovering of the 2001 recession in years 2002 and 2003, real GDP grew at annual rates of 2.2 and 3.1 percent respectively. But there were still 726,000 nonfarm jobs that had been lost from November 2001 to November 2003 (Gould (2003)). The 1990-1991 recession had experienced a similar pattern in creation of new jobs during recovering, though less severe than the 2001 recession. Now economists call such a character of a weak job market during a recovering the "jobless recovering". It has been extensively discussed in the media and puzzling the economists, market analysts, and policymakers. Indeed, why did the U.S. labor market not create jobs during an economic recovering?

Bernanke (2003) and Schreft and Singh (2003), among others, had provided some detailed discussions of the "jobless recovering" of the 2001 recession. Bernanke (2003) emphasized that the increase in "the long run productivity growth of the U.S. economy", "the unmeasured increases in the work effort of employees", and "the delayed result of firms' heavy investment in high-technology equipment in the later part of the 1990s" might be the key factors that contribute to the 2001 "jobless recovering". Schreft and Singh (2003) observed that "productivity gains in the current recovering [after the 2001 recession were] determined to a relatively greater extent by firms' efforts to lower costs by shrinking payrolls and working existing employees more intensively." Thus, factors such as effort of work, productivity, technological progress, and firms' investment in capital before the recession appear to be important in explaining the "jobless recovering". An interesting question is why it is a recession during a business cycle that triggers these factors to work together against the creation of new jobs. Indeed, why?

This paper contributes to the literature with a simple hiring game, which is able to integrate those factors discussed above into a simple efficiency wage model, to provide some insights in answering these two questions.

It is not new, at least in theory, that (involuntary) unemployment can persist with sticky wage in the short run with an efficiency wage model. The challenge part in answering the questions is how it is possible to have a persistent involuntary unemployment with flexible wage. This is relevant because empirical data (Table 1) showed that both nominal and, in particular, real wages were not sticky as expected during the 2001 recession and its later recovering. For example, as output fell by an annual rate of 2.7 percent in the second quarter 2001, real compensation per hour had fallen by 3 percent. Sticky efficiency wage theory can not explain the "jobless recovering" for one single reason. Sticky wage can only imply that a strong growth in GDP should have generated

[1] The revised number reported on February 8 was 16,000, majority of them are the labor-intensive construction and retail trade service jobs. The capital-intensive manufacturing labor market is still quite weak.

a strong labor market because strong demand in goods and services must shift the demand curve for labor upward. Such a shift, in conjunction with sticky wage, must imply a strong demand for labor.

In this paper we try to find a missing piece with the existing efficiency wage models. It turns out that workers' risk aversion over unemployment matters very much. By incorporating the risky part of unemployment into a simple efficiency wage model, we show that there is a sticky involuntary unemployment at equilibrium for a wide class of effort functions of work even if real wage is perfectly flexible, a conclusion against the conventional view in the neoclassic theory. During an economic boom with a tight labor market, the unemployment risk may not matter much. But this is no longer true if the economy is in recession during which job security becomes a foremost factor for a worker. Firms know it and can take advantage of it by squeezing the existing workers without much fear of job quitting.

Akerlof (2002), Akerlof and Yellen (1986), and Stiglitz (2002) have provided an account of the literature of the existing efficiency wage models. The central idea behind an existing efficiency wage model to explain involuntary unemployment was well stated in Akerlof (1984): "If there is involuntary unemployment in an equilibrium situation, it must be that firms, for some reason or other, wish to pay more than the market-clearing wage. And that is the heart of any efficiency-wage theory." Therefore, nonzero involuntary unemployment is a result of supra-competitive wage that firms are willing to pay. This idea has been used by Krueger and Summers (1988) to explain the inter-industrial wage differentials of identical workers. A departure of our simple efficiency wage model from the existing ones is that it is possible that involuntary unemployment at equilibrium coexists with an efficiency wage that is lower than the competitive wage.[2] The intuition of this result is simple: The demand curve for labor shifts down when unemployment matters in the effort of work. It may shift down to such a degree that the efficiency wage of the shifted demand curve of labor is below the competitive wage of the original demand curve. Nevertheless, we will show how it is possible to use our model to explain the inter-industrial wage differentials.

The overall unemployment rate in an economy is an important indicator. But it alone is probably not a good way to study how a labor market operates. It is known that different worker groups across age, sex, race, skills, education, industries, states, counties, and so forth have very different unemployment rates. For example, unemployment rate for workers with greater education and high-skill is much lower than that for workers with less education and low-skill. The unemployment rates for those with Bachelor's degree and higher varied in a range between 1.7% and 3.1% during the 1990-2001 business cycle. In contrast, the unemployment rates for workers with less than a high school diploma varied in a range between 6.3% and 10.8% during the same period. Such a pattern of unemployment is quite steady across time and spaces (Table 2). The pattern of unemployment across skills is quite similar. Unemployment rates for workers with high-skill remain quite steady across time and are much lower than those for workers with low-skill. For example, the unemployment rates for managerial and professional specialty workers were 2.1% in 1987 and 3.1% in 2002. But the unemployment rates for operators and laborers were 9.8% in 1987 and 8.9% in 2002 (Table 3). Similar patterns of unemployment across industries, age, sex, races, and so forth can be obtained through payroll or household survey data (see Table 4 for unemployment rates across industry and sex in years 2001 and 2002).

The steady pattern of unemployment across time and different work groups reveals the importance of "local" unemployment rates. Here locality has a broader meaning beyond geographic locations or areas alone. Phelps (1970) initiated the idea of island economies to study labor mar-

[2] Based on this result, one may argue that our model is no longer an efficiency wage model. But such an argument misses a key point of the original logic behind the efficiency wage: A worker's productivity is affected, at least in part, by the real wage he receives.

kets. Island economies are perfect in the study of local labor markets by treating each work group as an island economy. The whole economy can be seen as a network of island economies. Such an idea has been embodied in the Macroeconomics Textbook and the empirical research. For example, Dornbusch, Fischer, and Startz (2003) defined the overall unemployment rate as the weighted average of the unemployment rates of the groups. Dickens and Katz (1987) and Krueger and Summers (1988) provided studies about wage differentials across skills and industries. Their empirical studies show that each industry or skill group deserves a closer look in terms of unemployment rate and wage determination. The wage curve in Blanchflower and Oswald (1994) that relates the wage to the local (area or industrial) unemployment rates is clearly consistent with this island framework.

Our model uses this idea of island economies. But a major building block of our model, the effort function of work, has been taken from the Akerlof (1982) "partial-gift exchange" efficiency wage model. We illustrate our approach as follows. Consider an island economy with two identical firms and a finite number of identical workers N, as in the shirking model of Shapiro and Stiglitz (1984). Each firm has a production function $F(el)$ of its employment level l and the effort of work e of an individual worker. The two firms sell all their products in a competitive commodity market outside the island. Workers receive real wage and purchase consumption goods from the markets outside the island. The two firms behave in a non-cooperative manner and there is no auctioneer on the island. Thus, each firm has to decide a wage offer w and an employment level l to maximize its profit. The effort e is a function of real wage w and the local unemployment rate u (Akerlof (1982, 1984)). Since the local unemployment rate u depends on both firms' employment levels, the hiring issue faced by the two firms becomes a strategic game.[3] Naturally, we consider the symmetric Nash equilibrium since the two firms are identical. With identical workers and firms, the definition of involuntary unemployment is unambiguous: An unemployed worker is willing to work at the wage offered in the market but there is no additional position available for him.

A job is a means to generate income. It is also a risky means. Unemployment rate provides a measure how much risk a worker may have in his job. We assume that workers are risk averse with respect to the unemployment. This is in particular true when job security matters during a recession and its recovering phase. Thus, risk aversion means that the effort function of work $e(w, u)$ is concave in u.

Suppose that the effort function of work is as follows:

$$(*) \qquad e(w, u) = aw + bu^\beta, \ a > 0, b > 0, \beta > 0.$$

What is the unemployment rate u^Δ at equilibrium for this island economy? Surprisingly, u^Δ equals $\frac{\beta}{1+\beta}$. Moreover, there exists a positive constant c such that the effort of work e^Δ and efficiency wage w^Δ at equilibrium are given respectively by

$$e^\Delta = \frac{c}{1 - u^\Delta}$$

and

$$w^\Delta = \frac{e^\Delta}{a} - \frac{b}{a}(u^\Delta)^\beta.$$

[3]It should be aware that our use of an island economy to study involuntary unemployment is a major departure from the use of overall unemployment rate in Akerlof (1982, 1984). It is reasonable to consider the overall unemployment rate as an exogenous variable as in Akerlof (1982) since each firm's hiring or firing can be seen as a drop of water in the ocean. But if local unemployment matters in the determination of wage and unemployment rate, then a firm's decision about hiring or firing will matter not only for its own workforce but also for those of the "adjacent" firms on the same island. Under such a situation, one can no longer consider the local unemployment rate as an exogenous variable. It should be endogenously determined. By doing so, we find that our simple efficiency wage model owns many novel features not shared by the existing efficiency wage models (see discussions in Section 5).

Thus, involuntary unemployment at equilibrium does not depend on the production function $F(el)$ and the coefficients a and b. It depends on only the parameter β, a parameter that is related to workers' attitudes for being unemployed. Wage and productivity at equilibrium depend on the production function and unemployment rate at equilibrium. Moreover, wage at equilibrium grows in the trend of productivity. To see the implication of it, consider two different economies in two different time periods (say, 2002 and 2003) for the same work group. Then the two economies may have quite different production functions, coefficients a and b. But if the two economies had effort functions with a steady β,[4] then our result shows that the two economies had in fact the same involuntary unemployment rates at equilibrium. Thus, our efficiency wage model shows that two different economies with different levels in real wage and productivity can have the same or close nonzero involuntary unemployment rate at equilibrium. Since our supply of labor is fixed, this implies that the total number of workers unemployed across the two years remains constant. Such a feature does show that there is no new job created in the economy even if GDP and productivity both grow. Thus, we do have the "jobless recovering".

Let $F(el) = A(el)^{\alpha}$, where α is the labor share. How fast can this economy grow without creating jobs? We provide an interesting answer: With a labor share $\alpha = 0.70$, it can grow 3.33 percent without creating new jobs for each percent increase in the multifactor productivity A.

What matters here is the upper bound of the effort function. An upper bound over the effort function of work represents the productivity potential of an individual worker. The unbounded effort function of work can happen under different scenarios. First, firms may have the ability to work the existing workers more and more intensively for a given level of capital stock and technology or they may enhance the productivity potential through internal improvement in organization or reallocation of the existing resources (Greenspan (2002)). Second, a firm can grow individual worker's productivity potential (i.e., the upper bound) with new investment in capital stock or technology for its long run growth. Third, such a bound, though existing, may not be reached during a different phase in a business cycle. In a recession, it is known that the utilization of capacity in an economy dropped substantially (Corrado (2002)). This implies that workers and capital have not operated up to their potential during a recession. This "idle" potential in productivity is the key to understand the "jobless recovering".

The explanation of "jobless recovering" above is incomplete without answering the question when an economy does start to create new jobs in recovering or expansion. As the economy grows for a substantial period, workers and capital start to operate at their potential. Thus, there is an issue what may happen if the upper bound has been reached. Things change dramatically. With such a reached bound, we show that additional growth in GDP and productivity is tied up with new hiring at equilibrium together with firms' investment in capital goods. With a reached bound, we also obtain the wage curve in Blanchflower and Oswald (1994) as an equilibrium locus of our efficiency wage model.

The story delivered by our model about the "jobless recovering" may be summarized as follows: Suppose the economy expands in an equilibrium path such that both labor and capital operate at their productivity potential. Under such an environment, our model shows that any additional increase in demand must be met by adding new labor and capital stock (Theorem 3). Suppose this process continues up to a point when an adverse demand shock hits the economy into a recession. Suppose that investment is irreversible or it is very costly to liquidate the extra capital in the short run. So it is in the firms' interest to layoff workers but not to liquidate the extra capital stock. This exercise raises the ratio of capital per existing worker. Thus, the productivity potential of the existing workers is higher. With higher productivity potential and a weaker demand, it is

[4]It should be aware that we do not claim that the β must be steady year after year.

plausible to expect that workers and capital operate under their potential productivity. Then our model shows that the economy will jump into an equilibrium path with a sticky unemployment rate (Theorem 2). The economy jumps because the sticky unemployment rate is much higher than that before the recession. Since there is some 'idle' productivity at equilibrium, any increase in demand during the recovering will be met by the 'idled' productivity of the existing workers. There is no creation of new jobs no matter how strong the economy may grow as long as such idle productivity exists. The higher the idle productivity, the longer the "jobless recovering". The weaker the demand during the covering, the longer the "jobless recovering". After the 'idled' productivity potential has been used up, the economy will enter another equilibrium path with new hiring and investment. Along the new equilibrium path of another expansion, the level of productivity is permanently higher than the previous expansion, and the unemployment rate also becomes lower gradually.

Our bounded and unbounded effort functions of work also provide a useful framework to study involuntary unemployment in the long run and short run. In the long run, real wage and productivity in the U.S. economy have formed an upward trend without a bound in sight while unemployment rates have been fluctuated in a range. Blanchard and Katz (1997) pointed out that any model on productivity and the average rate of unemployment should satisfy the condition that there is no long run effect of the level of productivity on the average rate of unemployment. Stiglitz (1997) and Trehan (2001) both insisted that the tradeoff between level or rate of change of productivity and the unemployment rate, if any, must be transient not permanent. Summer (1988) made a similar remark on the tradeoff between real wages and average unemployment rates in the long run. He wrote that "[i]t is striking that real wages have doubled several times over the last century without having a large impact on average unemployment rates". Our unbounded effort function of work captures the idea that the growth in productivity is in a trend in the long run without a bound. But the bounded effort function captures the idea that the productivity potential of an individual worker for time being can be constrained by the current technology, the existing accumulated capital stock, and the current economic status.

One may have noted that our island economy is quite similar to the simple efficiency wage model studied in Solow (1979) in which a representative neoclassical firm with production function $F(e(w)l)$ chooses an efficiency wage such that the elasticity of effort with respect to the real wage is unity[5]. The equilibrium employment level is chosen such that the marginal product of efficient labor equals the efficiency wage. Involuntary unemployment is nonzero as long as the inelastic labor supply surpasses the equilibrium employment level. The Solow model differs from ours in the effort function of work. Effort function of work in Solow (1979) depends on the real wage only so that efficiency wage at equilibrium is sticky and depends on the effort function alone. In our model, the effort function of work depends on not only the real wage but also the local unemployment rate. By doing so, we have a situation with a "sticky" involuntary unemployment rate at equilibrium.

The shirking model of Shapiro and Stiglitz (1984) had used the "local" unemployment rate as a variable endogenously determined by the no shirking condition. The shirking model is based on the idea that the costs to detect a worker who shirks may be so high that a firm may benefit in saving in the monitoring costs by offering a higher wage for the worker so that he has less incentive to shirk due to a tougher labor market created by the higher wage. Thus, nonzero involuntary unemployment is needed in order to prevent workers from shirking. Unemployment, as "a worker discipline device", must exist in order for it to be effective. In our model, we also have effort functions of work such that full employment equilibrium is possible (Subsection 5.2). That is, in certain circumstance, we need the "worker discipline device" as an ax but we may never use it at

[5]Here we follow the reformulated version of the Solow (1979) model well presented in Yellen (1984) and Akerlof and Yellen (1986).

equilibrium.

The nonzero involuntary unemployment at equilibrium in our model is due to a very different reason: the risk aversion. If a worker is risk averse toward unemployment, then he has to work harder for a tougher labor market in order to be employed. With risk aversion workers, firms get benefits from higher productive workers by creating certain number of unemployed workers in the marketplace. That is, involuntary unemployment persists in our model not due to the sticky wage or monitoring costs but the risk aversion workers.

Acemoglu (2002) provided a survey and a detailed discussion on technical change and income inequality of the U.S. labor markets during the last several decades. One important conclusion he reached is that "technical change has been skill-biased during the past sixty years, and probably for most of the twentieth century". Such a skill-biased technical change has been recognized to be the main cause for the sharp rising since 1970s in the income inequality between high-skill and low-skill workers. In fact, real income for high-skill workers rises sharply during 1980s while real wages for low-skill workers have fallen since 1970. These facts in conjunction with the steady pattern of unemployment call up a question: How is it possible that a steady pattern of unemployment coexists with deteriorating income inequality? We will use our model to provide an answer to the question.

The rest paper is organized as follows. Section 2 provides the general model of a hiring game. We provide an example to illustrate why divisible labors and the general form of effort functions of work are important to us. Section 3 presents the general results derived from our hiring game. Section 4 provides a detailed discussion how our model may resolve the "jobless recovering" puzzle. Section 5 presents some novel features of our model that are not shared by the existing efficiency wage models. Section 6 presents some analyses and evidences. In particular, we show how it is possible to use our model to study the issue of wage premium across industries and skills and the issue of income inequality due to skill-biased technical change. Moreover, we show how it is possible to obtain the wage curve. Further, we also present one of the examples used by Akerlof (1982) to illustrate how endogeneity of the unemployment rate may alter the equilibrium efficiency wage, the unemployment rate, and the effort of work. A few remarks in Section 7 conclude our paper.

2 The Hiring Game

There are a finite number of workers, $i = 1, 2, \cdots, N$, and a finite number of firms, $j = 1, 2, \cdots, m$. A wage offer w_j by firm j is defined by $0 \leq w_j = \{w_{ij}\}_{i=1,2,\cdots,N}$. A worker i who is hired by firm j and paid with wage offer w_{ij} chooses an effort e of work to maximize her utility:

$$\max_{e \geq e_{min}(w_{ij}, u)} U_i(e_n, w_{ij}, e, \bar{w}, u),$$

where $e_{min}(w_{ij}, u)$ is the minimum effort required by firm j for worker i, which depends on the real wage w_{ij} that worker i receives from firm j and the (local) unemployment rate u. Therefore, a wage offer w_{ij} for a worker contains not only the real wage the firm will pay the worker, if employed, but also the job descriptions and the working rule. If a worker receives more than two offers, she chooses the offer that is the best for her. This maximization problem for the worker is quite similar to that in Akerlof (1982) in which the minimum effort e_{min} is constant not a function. It is plausible to assume that firms may impose a different working rule along business cycles as observed by Schreft and Singh (2003). The wage $\bar{w}$ is either the reference wage in Akerlof (1982) or the fair-wage in Akerlof and Yellen (1990). Thus, worker i, if hired by firm j, chooses an effort function of work

$$e_{ij} = e(e_n, w_{ij}, \bar{w}, u)$$

of the norms of work e_n, his real wage offer w_{ij}, the fair-wage $\bar{w}$, and the unemployment rate u. Akerlof (1984) pointed out that such effort function of work is central to any efficiency wage model. Unemployment rate u affects the effort of work in Akerlof (1982) through the norms of effort and the reference wage. It had been considered as an exogenous variable. With a finite number of firms considered in this paper and the island economy, it is natural to consider it as an endogenous variable.

Given an effort function of work $e(e_n, w, \bar{w}, u)$, the Arrow-Pratt coefficient of absolute risk aversion is given by

$$r_e(u) = -\frac{e''_u(e_n, w, \bar{w}, u)}{e'_u(e_n, w, \bar{w}, u)};$$

see Mas-Colell, Whinston, and Green (1995). We assume that the effort function of work is concave with respect to the unemployment rate u. This measure $r_e(u)$ is relevant because unemployment provides an uncertainty for a worker in his income flow. The Arrow-Pratt coefficient of absolute risk aversion provides a measure for the attitude that a worker or a work group has toward this risk. We should expect that the duration and magnitude of unemployment benefits have impacts on the measure. Longer duration and better benefits should be associated with effort function of work that is less risk averse. Other institutional factors like job protection and regulation may also affect it.

Since the norms of effort e_n depend on the unemployment rate u too, one can define such a measure on the norms of effort. It is plausible to assume that a higher pay work group has norms of effort that is more risk averse. For example, if e_n^1 and e_n^2 are the norms of effort for high and low pay work groups respectively, then

$$r_{e_n^1}(u) \geq r_{e_n^2}(u).$$

Since they are the norms, they may well be "steady" across time for the same work group in the long run.

A firm j has a production function $F_j : 2^{\{e_{1j}, e_{2j}, \cdots, e_{Nj}\}} \to R_+$, where e_{ij} is the effort of work of worker i in firm j if worker i is hired by firm j. Since a worker's effort of work depends on unemployment rate u and many other factors, firm j's profit depends on not only those hired by firm j but also those hired by other firms. So the choices of what wage offers may be provided and who will be hired become a strategic game.

Let w_j denote the wage offer by firm j and $S_j \subset \{1, 2, \cdots, N\}$ denote the group of workers hired by firm j, $j = 1, 2, \cdots, m$. Let $W = (w_1, w_2, \cdots, w_m)$ denote a strategy profile of wage offers and $S = (S_1, S_2, \cdots, S_m)$ denote a strategy profile of workers hired. Given a strategy profile (W, S), a firm j's profit function is defined by

$$\pi_j(W; S) = F_j(\{e_{ij:i \in S_j}\}) - \sum_{i \in S_j} w_{ij}.$$

A strategy profile (W, S) is a (pure) Nash equilibrium if for every j, $j = 1, 2, \cdots, m$,

$$\pi_j((w_j, w_{-j}); (S_j, S_{-j})) \geq \pi_j((w'_j, w_{-j}); (S'_j, S_{-j}))$$

for all $w'_j \geq 0$ and all $S'_j \subset \{1, 2, \cdots, N\}$.

A Nash equilibrium should satisfy the feasibility condition. If labor is assumed to be indivisible, then a strategy profile (W, S) is feasible if for all i, worker i is in some S_j, then worker i is not in $S_{j'}$ for all other firms $j' \neq j$. That is, a worker can only work at most for one firm. If labor is assumed to be divisible, then feasibility implies that the unemployment rate u, which will be defined later, satisfies the condition such that $0 \leq u \leq 1$.

2.1 An Example

This subsection presents an example with indivisible labors to illustrate the importance of divisible labors and the general structure in the effort function of work. The example presented below is similar to the job matching market of Kelso and Crawford (1982), with the exception that the presence of unemployed workers may affect the employed workers' productivity.

Suppose there are three identical workers, $W = \{1, 2, 3\}$, and one firm j. Firm's production function is as follows, where unemployment does not affect the productivity of hired workers:

$$\begin{aligned} &F_j(W) = 3.5 \\ &F_j(S) = 2.75, \quad \forall S \subset W \text{ such that } \mid S \mid = 2 \\ &F_j(S) = 1.5, \quad \forall S \subset W \text{ such that } \mid S \mid = 1 \\ &F_j(\emptyset) = 0. \end{aligned}$$

The demand curve $D_j(w)$ is defined by

$$D_j(w) = \begin{cases} W, & 0 \leq w \leq 0.75 \\ \{S \subset W :\mid S \mid = 2\}, & 0.75 \leq w \leq 1.25 \\ \{S \subset W :\mid S \mid = 1\}, & 1.25 \leq w \leq 1.5 \end{cases}$$

Thus any wage in the interval $[0, 0.75]$ clears the market.

Now suppose that unemployment affects the productivity of hired workers as follows:

$$\begin{aligned} &\tilde{F}_j(W) = 3.5 \\ &\tilde{F}_j(S, W - S) = 3, \quad \forall S \subset W \text{ such that } \mid S \mid = 2 \\ &\tilde{F}_j(S, W - S) = 2, \quad \forall S \subset W \text{ such that } \mid S \mid = 1 \\ &\tilde{F}_j(\emptyset, S) = 0, \quad \forall S \subset W. \end{aligned}$$

The new demand curve $\tilde{D}_j(w)$ is given by

$$\tilde{D}_j(w) = \begin{cases} W, & 0 \leq w \leq 0.50 \\ \{S \subset W :\mid S \mid = 2\}, & 0.50 \leq w \leq 1 \\ \{S \subset W :\mid S \mid = 1\}, & 1 \leq w \leq 2 \end{cases}$$

Note that the new demand curve shifts downward due to the externality created by unemployment on hired workers' productivity. The new market-clearing wages are belong to the interval $[0, 0.50]$, which is a proper subset of $[0, 0.75]$.

Now suppose there are two identical firms, say, k and j. Both firms have the same production functions given by F and $\tilde{F}$ above. Let us assume that firm j is the row player and firm k is the column player. So if firms j and k each hire one single worker, then this leaves one worker unemployed and each firm enjoys profit $2 - w$ if the wage is w. But if firm j hires two workers and firm k hires one, then there is no unemployment in the market so firm j has profit $2.75 - 2w$ and firm k has profit $1.5 - w$ if the wage is w. The hiring game is given by

	0	1	2	3
0	0,0	$0, 2 - w$	$0, 3 - 2w$	$0, 3.5 - 3w$
1	2-w,0	$2 - w, 2 - w$	$1.5 - w, 2.75 - 2w$	$1.5 - w, 3.5 - 3w$
2	3-2w,0	$2.75 - 2w, 1.5 - w$	$2.75 - 2w, 2.75 - 2w$	$2.75 - 2w, 3.5 - 3w$
3	3.5-3w,0	$3.5 - 3w, 1.5 - w$	$3.5 - 3w, 2.75 - 2w$	$3.5 - 3w, 3.5 - 3w$

Note that this hiring game is different from our general structure given in the above since the wage w is seen as a given variable not a strategy. But this will not affect our purpose to show that there is no Nash equilibrium in pure strategy in this game.

Note that, unlike an ordinary game, not all pairs of strategies in our hiring game are feasible, since each worker can only work for at most one firm.

Now we show that the pair of strategies $(3, 0)$ or $(0, 3)$ can't be a Nash equilibrium. By symmetry, it is enough to show that there is no wage w such that the pair of strategies $(3, 0)$ is a Nash equilibrium. In order for $(3, 0)$ to be a Nash equilibrium, we have that $w \leq 0.5$ and $w \geq 1.5$, a contradiction.

Next, we show that there is no wage w such that $(2, 1)$ is a Nash equilibrium. In order for $(2, 1)$ to be a Nash equilibrium, we have that $w = 0.75$ for firm j. But $w \geq 1$ in order for firm k to hire one worker without incentive to deviate. This is a contradiction. Similarly, one can show that there is no wage such that $(1, 2)$, $(2, 0)$ or $(0, 2)$ is a Nash equilibrium.

The pair of strategies $(1, 0)$ or $(0, 1)$ is a Nash equilibrium only if the wage $w = 2$, with zero profit for each firm. But wage $w = 2$ can't be a Nash equilibrium because if a firm, with no hiring, proposes a wage lower than 2 and hires the unemployed worker, then such an offer will be accepted and the firm makes a positive profit.

The pair of strategies $(1, 1)$ is a Nash equilibrium only if the wage w is belong to $[0.75, 2]$. But no wage offer in $[0.75, 2]$ can be a Nash equilibrium, because one firm can always make a wage offer w' that is lower than 0.75 but higher than 0 to the unemployed worker. Such an offer will be accepted and the firm gets strictly better off.

Note that when unemployment does not matter in the productivity of hired workers, the neoclassical market-clearing wage at equilibrium with two firms is 1.25, with one firm hiring two workers and the other hiring one. When unemployment matters, the neoclassical market-clearing wage w at equilibrium with two firms is 1, which is smaller than 1.25. That is, the demand curve for labor when unemployment matters shifts downward.

This example demonstrates some difficulty to obtain a symmetric Nash equilibrium with our hiring game when labors are indivisible and wage does not matter in workers' productivity in production. This helps illustrate some important assumptions behind our general results presented next about the effort function of work and divisible labors, even though they are quite standard in the literature of efficiency wage models.

3 General Results

Section 2 has established a general hiring game whose main structure is quite similar to the "partial-gift exchange" model initiated by Akerlof (1982) and the job matching market of Kelso and Crawford (1982). In this section we present an application of it to study the issue of involuntary unemployment in a labor market with identical firms and identical workers with divisible labors. Since we have identical workers and firms, the wage offer at equilibrium is the same for all workers. Moreover, the norms of effort e_n equals the effort of work e (Akerlof (1982)). So the effort of work for a worker

$$e = e(w, u)$$

is a function of his wage w and the unemployment rate u such that $e'_w(w, u) > 0$ and $e'_u(w, u) > 0$ (the exogenous variables are left out in the expression which can be embodied without changing the results in this paper). Let $F(el)$ be the production function for a firm, with $F(0) = 0$, $F' > 0$, and $F'' < 0$. Since all firms are identical, it is natural to consider a symmetric Nash equilibrium (in pure strategy) under which all firms choose the same wage offer w and the same employment level

$0 \leq l \leq N$ at equilibrium. Thus, the unemployment rate u in the economy is defined by $u = 1 - \frac{m}{N}l$ or equivalently $\frac{m}{N}l = 1 - u$.

At a symmetric Nash equilibrium, firms choose wage offer w and employment level l to maximize their profits, subject to the constraints of $1 \geq u \geq 0$ and nonnegative wage $w \geq 0$,

$$\Pi(w, l) = F(e(w, u)l) - wl,$$

and the first order conditions are respectively given by

$$F'(e(w, u)(1 - u)\frac{N}{m})e'_w(w, u) - 1 = 0, \tag{1}$$

$$F'(e(w, u)(1 - u)\frac{N}{m})(-(1 - u)e'_u(w, u) + e(w, u)) - w = 0, \tag{2}$$

after replacing l with $(1 - u)\frac{N}{m}$ and assuming that there is an interior solution at equilibrium.

Thus, one can solve equations (1) and (2) to obtain the optimal wage offer w^Δ and the unemployment rate u^Δ at a symmetric Nash equilibrium. The optimal employment level is defined by $l^\Delta = (1 - u^\Delta)\frac{N}{m}$.

It should be aware that the price level p for each unit output has been normalized to be unity. As a result, w is the real wage. Eq.(2) is the demand curve for labor which should be nonlinear in general. It says that at a symmetric Nash equilibrium, the marginal product of an efficiency labor equals the real wage w. Note that real wage at equilibrium when unemployment matters is lower than that when it does not matter. That is, our demand curve for labor shifts down when unemployment matters.

Substitute (1) into (2) by eliminating the term $F'(e(w, u)l)$, we have

$$\lambda_{e,w} = \frac{e'_w(w, u)}{-(1 - u)e'_u(w, u) + e(w, u)}w = 1, \tag{3}$$

which is similar to the Solow equilibrium condition (Subsection 5.1) but with very different consequences as shown in Theorems 1 and 2.

Theorem 1. *Full employment at a symmetric Nash equilibrium is achieved only if the following holds:*

$$e(w, 0) = e'_w(w, u)w + e'_u(w, u) \mid_{u=0} .$$

Conversely, if $e'_u(w, u) \neq 0$ *and the following holds for all* w, u*:*

$$e(w, u) = e'_w(w, u)w + e'_u(w, u), \tag{4}$$

then there is a full employment at every symmetric Nash equilibrium.

Proof. A full employment at a symmetric Nash equilibrium means that $u = 0$. It follows from Eq.(3) that

$$e(w, 0) = e'_w(w, u)w + e'_u(w, u) \mid_{u=0} .$$

If Eq.(4) holds, then Eq.(3) implies that $ue'_u(w, u) = 0$. Thus, we have $u = 0$. □

A surprising fact about Theorem 1 is that even if the demand curve for labor depends on the production function, the sufficient condition to achieve full employment at a symmetric Nash equilibrium does not depend on it. One can tell from effort function of work alone if there is nonzero involuntary unemployment at equilibrium, no matter what the production function may be. The necessary condition for full employment at a symmetric Nash equilibrium shows that nonzero involuntary unemployment is by no means a specific phenomenon but a general one. Theorem 1 shows that it is possible to find a family of effort functions of work to achieve full employment at equilibrium. But any economy with a general form of effort function that does not satisfy the necessary condition in Theorem 1 has nonzero involuntary unemployment at equilibrium (if feasible), no matter what a production function the economy may have. Because of the restrictive manner in the necessary condition for full employment, full employment at equilibrium is quite special in nature.

The natural rate of unemployment, defined as "the average level around which the unemployment rate fluctuates" (Mankiw (1997, p.124)), is important for both theory and economic policy. The conventional view is that the natural rate of unemployment generally consists of frictional and structural unemployment rates but not the involuntary unemployment rate (Rogerson (1997)). Our result above shows that this is possible only for a particular family of effort functions. In general cases, nonzero involuntary unemployment persists in the average level of unemployment rates.

Next we consider a class of effort functions with nonzero involuntary unemployment at equilibrium such that the underlying production function has impacts on the individual worker's productivity and equilibrium wage but not on the involuntary unemployment rate. This result makes our study useful to analyze involuntary unemployment in the long run and the "jobless recovering". It provides an additional evidence that involuntary unemployment may present in the average or natural rate of unemployment.

Theorem 2. *Assume the effort function of work is as follows:*

$$e(w, u) = aw + g(u),$$

where $g(u)$ is any nonnegative continuous function that is bounded on $[0, 1]$, satisfying $g'(0) \to \infty$.[6] *Then there exists a unique symmetric Nash equilibrium that is also feasible. Moreover, the involuntary unemployment rate u^{Δ} at equilibrium satisfies the following:*

$$(1 - u)g'(u) = g(u). \qquad (5)$$

Furthermore, there exists a constant $c > 0$ such that the effort of work e^{Δ} at equilibrium is given by

$$e^{\Delta} = \frac{c}{1 - u^{\Delta}} \qquad (6)$$

and the efficiency wage w^{Δ} at equilibrium is given by

$$w^{\Delta} = \frac{c}{a(1 - u^{\Delta})} - \frac{1}{a}g(u^{\Delta}). \qquad (7)$$

Proof. Eq.(5) follows from Eq.(3). Note that the function $f(u) = (1 - u)g'(u) - g(u)$ is monotone and continuous on $(0, 1)$. Moreover, $f(0) \to \infty$ and $f(1) \leq 0$. Thus, the existence and uniqueness follow. It follows from Eqs.(2) and (5) that

$$aF'(e^{\Delta}(1 - u^{\Delta})\frac{N}{m}) = 1. \qquad (8)$$

[6]This condition means that, with or without the probability to be unemployed, it matters very much for workers in their productivity. Our economists are not lack of the same experience: It matters very much whether an academic position is tenured or not tenured.

Thus, there is a constant $c > 0$ such that

$$e^{\Delta}(1 - u^{\Delta}) = c.$$

This shows eq.(6). Note that the efficiency wage w^{Δ} at equilibrium is given by

$$w^{\Delta} = \frac{e^{\Delta}}{a} - \frac{1}{a}g(u^{\Delta}).$$

Thus, eq.(7) follows. □

Eq.(5) shows that not all effort functions of work can have a symmetric Nash equilibrium that is also feasible since it may happen that a solution u of Eq.(5) is out of the interval $[0, 1]$.

Henceforth, the effort function of work is assumed to have the separable form given in Theorem 2. Moreover, we assume that the function $g(u)$, if without further specification, is concave.

To see how technical progress and demand or supply shock may affect the economy described in Theorem 2, we write the production function F as $A\theta F$, where A and θ represent the technical progress and the demand shock respectively. Then the effort of work e^{Δ} at equilibrium is explicitly given by

$$e^{\Delta} = \frac{m}{N}\frac{1}{1 - u^{\Delta}}(F')^{-1}(\frac{1}{aA\theta}).$$

Since $F'' < 0$, it follows that

$$\frac{\partial e^{\Delta}}{\partial A} > 0, \qquad \frac{\partial e^{\Delta}}{\partial \theta} > 0$$

and

$$\frac{\partial w^{\Delta}}{\partial A} > 0, \qquad \frac{\partial w^{\Delta}}{\partial \theta} > 0.$$

The relationship between the effort of work and the unemployment rate at equilibrium is defined as the effort curve, which is upward-sloping. So higher unemployment rate at equilibrium implies higher effort of work at equilibrium. The effort curve shifts if there is a technical change or demand shock (Fig.1).

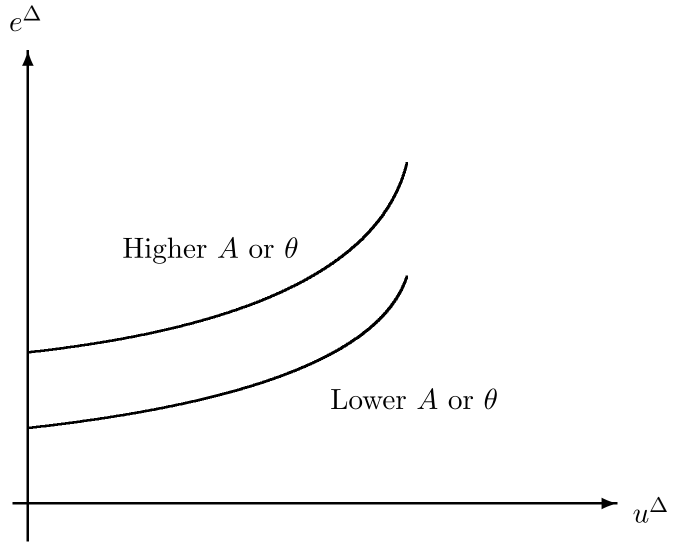

Fig.1 Effort Curve, i.e., Relationship Between e^{Δ} and u^{Δ}

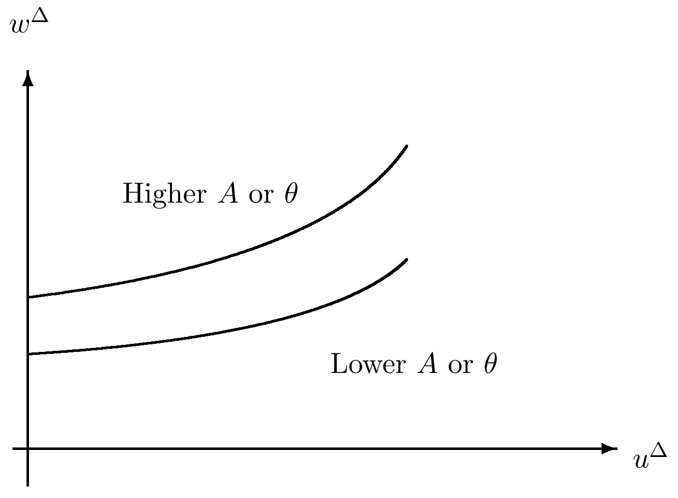

Fig.2. Wage Curve, i.e., Relationship Between w^{Δ} and u^{Δ}

The relationship between the efficiency wage and unemployment rate at equilibrium is defined as the wage curve (Blanchflower and Oswald (1994)). Depending on $g(u)$ and the production function F, the wage curve can be upward or downward sloping[7].

[7]Precisely, one can show that the wage curve is upward-sloping if $c > (1-u)g(u)$. It is downward-sloping if $c < (1-u)g(u)$.

An upward-sloping wage curve is given in Fig. 2. Note that it shifts upward if there is a technical progress or a positive demand shock.

Theorem 2 can be used to analyze involuntary unemployment and the efficiency wage or productivity at equilibrium in the long run. For example, consider a sequence of economies with effort function $e_t(w, u) = a_t w_t + b_t g(u)$ and the production function $F_t(e_t l)$ with N_t number of workers and m_t number of firms. Then such a sequence of economies has the same involuntary unemployment rates at equilibrium. But equilibrium wage and productivity may well be different for different time periods. In particular, it is possible that real wage and productivity form an upward trend in the long run while involuntary unemployment rate at equilibrium remains steady, a theoretic result that is consistent with the view expressed by Blanchard and Katz (1997), Stiglitz (1997), and Trehan (2001), among others. Therefore, Theorems 1 and 2 provide some interesting insights on the involuntary unemployment rate at equilibrium and its relationships with the equilibrium effort of work and the equilibrium efficiency wage. More importantly, Theorem 2 can be used in part to resolve the "jobless recovering" puzzle experienced by the U.S. economy in the 1990-1991 and 2001 recessions.

4 "Jobless Recovering"

A key feature in the effort function of work given in Theorem 2 is that it does not have an upper bound. An unbounded effort function of work is consistent with the upward trend in the level of productivity in the U.S. economy. In particular, it has been the characteristic of the U.S. economy in recession. For example, Corrado (2002) reported that, during the 1990-1991 recession, utilization of capacity in the manufacturing industries had dropped from its peak 84.1 percent in 1988 to the trough 78.3 percent in 1991; During the 2001 recession, utilization of capacity had fallen from 81.4 percent in 2000 to 73.8 percent in 2002. The total industrial capacity utilization during the latest two recessions had dropped in a similar pattern. These data provided strong evidence to support our view that workers and capital stock have not operated up to their potential during a recession, which is right the situation described by the unbounded effort function of work.

When an expansion or recovering lasts for a substantial period, an upper bounded effort function of work is probably more realistic in the economic analysis, at least in the short run. Such a bound mainly depends on the capital stock per labor and the technical progress. Let K denote the total capital stock in a firm and l denote the number of hired workers. Thus, the productivity potential of an individual worker $\bar{e}(A, k)$ is defined as a function of the multifactor productivity or technical progress A and the ratio $k = \frac{K}{l}$ of capital stock per labor. At steady state of the economy, k is constant. This implies that $\bar{e}(A, k)$ must be constant for a given production technology A. Note that managerial skills, organization forms in production and job market status may also affect $\bar{e}$ through A.[8]

We want to see how the existing of such a constant bound may imply for our model. If such a bound is not reached at equilibrium, then our analysis in Theorem 2 still holds without any change. Next we analyze what may happen if such a bound is always reached in equilibrium.

Let $\bar{e}$ be the fixed upper bound. Then a firm's maximization problem at a symmetric Nash equilibrium is to maximize its profit

$$A\theta F(\bar{e}l) - \frac{1}{a}(\bar{e} - g(u))l$$

[8]Greenspan (2002) explicitly stated that the multifactor productivity A "includes technical progress, organizational improvements, cyclical factors, and myriad other influences on output per hour."

with respect to l. Note that we explicitly introduce the technical progress A and the demand shock θ into our production function for the convenience of analysis below.

The first order condition is given by

$$A\theta F'(\bar{e}l)\bar{e} - \frac{1}{a}[g'(u)\frac{m}{N}l + \bar{e} - g(u)] = 0.$$

That is,

$$A\theta F'(\bar{e}(1-u)\frac{N}{m})\bar{e} - \frac{1}{a}[g'(u)(1-u) + \bar{e} - g(u)] = 0.$$

Let

$$\begin{aligned} L(u) &= A\theta F'(\bar{e}(1-u)\frac{N}{m})\bar{e} - \frac{1}{a}[g'(u)(1-u) + \bar{e} - g(u)] \\ &= \frac{\bar{e}}{a}[aA\theta F'(\bar{e}(1-u)\frac{N}{m}) - 1] - \frac{1}{a}[(1-u)g'(u) - g(u)]. \end{aligned}$$

By the assumption that the bound $\bar{e}$ is reached at equilibrium, it follows that $e^{\Delta} \geq \bar{e}$. Since, by eq. (8),

$$aA\theta F'(e^{\Delta}(1-u^{\Delta})\frac{N}{m}) = 1,$$

it follows that

$$aA\theta F'(\bar{e}(1-u^{\Delta})\frac{N}{m}) \geq 1$$

since $A\theta F'' < 0$. This means that $L(u^{\Delta}) \geq 0$ since, by eq. (5),

$$(1-u^{\Delta})g'(u^{\Delta}) - g(u^{\Delta}) = 0.$$

Theorem 3 *There exists a unique symmetric Nash equilibrium such that the equilibrium unemployment rate u^{Γ} satisfies that*

$$0 < u^{\Gamma} \leq u^{\Delta}.$$

Proof. Recall that $g'(0) \to \infty$ (Theorem 2). Thus, we have that $L(0) \to -\infty$. Since $L(u)$ is continuous in $(0, u^{\Delta}]$, there exists $u^{\Gamma} \in (0, u^{\Delta}]$ such that $L(u^{\Gamma}) = 0$. Note that $L'(u) > 0$. Thus, uniqueness follows from the fact that $L(u)$ is monotonely increasing. □

The efficiency wage at equilibrium w^{Γ} is given by

$$w^{\Gamma} = \frac{1}{a}(\bar{e} - g(u^{\Gamma})),$$

which is an equilibrium locus that has a negative slope. Moreover, it is a convex function as shown by the two wage curves of $\bar{e}$ and $\bar{e}_1$ in Fig.3. Thus, the wage curve in Blanchflower and Oswald (1994) can be obtained as an equilibrium locus of our efficiency wage model, with a bounded effort function of work.

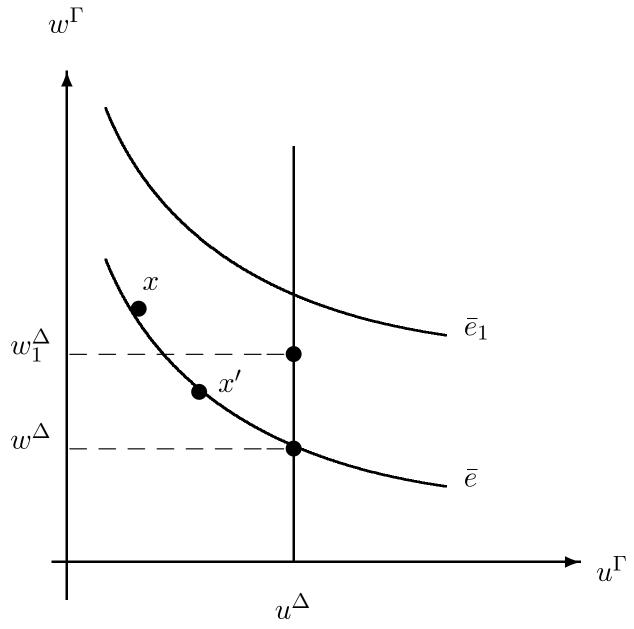

Fig.3. The gap between (u^Δ, w_1^Δ) and the wage curve of $\bar{e}_1$ contributes to the "jobless recovering".

4.1 "Jobless Recovering"

Theorems 2 and 3 provide the tools to resolve the "jobless recovering" puzzle experienced by the U.S. economy in the latest two recessions. Recall that θ is the demand shock and note that the function $L(u)$ shifts to the left with higher θ. Suppose the economy starts with the equilibrium (u^Δ, w^Δ), where $e^\Delta = \bar{e}$. Any further growth in demand θ results in an equilibrium with lower unemployment rate u^Γ and higher equilibrium wage w^Γ, by following the wage curve $\bar{e}$ in Fig.3.[9] Since u^Γ is smaller than u^Δ and $\bar{e}$ is constant, firms also accumulate capital stock along the equilibrium path of $\bar{e}$. Suppose the expansion of the economy continues up to the equilibrium point x, say. After equilibrium x, suppose that there exists an adverse demand shock in the economy resulting in a recession. A decline in θ from demand level θ_x at equilibrium x creates several possible adjustments in equilibrium. The ideal case is that each firm layoffs its workers, reduces the wage, and cuts capital stock along the equilibrium path of $\bar{e}$. But investment in capital stock is typically irreversible, at least in the short run. Moreover, a substantial reduction in real wage may cause morale problems in the working place. Thus, in response to an adverse demand shock, a firm most likely shrinks its labor force first, a scenario that had been observed during the latest two recessions of the U.S. economy. A reduction in the labor force without reduction in capital stock for a firm raises the ratio of capital per labor. This capital deepening process raises the productivity potential for the existing hired workers. This means that the equilibrium path described by $\bar{e}$ shifts to the new equilibrium path described by $\bar{e}_1$ in Fig.3. With the adverse shock in demand and the increase

[9] Note that the economy is at steady state with a constant ratio of capital per labor. The steady state can be relaxed by working out the equilibrium level of capital stock per labor. Such an equilibrium level may be higher or lower than that at steady state. A higher ratio of capital per labor at equilibrium will shift up the equilibrium path of $\bar{e}$ and a lower ratio at equilibrium will shift it down. Such an exercise, though a little more complicated, will not change the insights obtained from our steady state. Bernanke (2003) pointed out that the heavy investment in high-technology equipment by the U.S. firms in the late 1990s is important in understanding the 2001 "jobless recovering". Our model is useful in understanding why this is indeed the case.

in the bound of effort of work, the new equilibrium now should be at $(u^{\Delta}, w_1^{\Delta})$, which is given by Theorem 2, since it is likely during a recession that the new upper bound $\bar{e}_1$ is not binding.[10] Note that the equilibrium unemployment rate goes back to u^{Δ}. Thus, the unemployment rate jumps from u_x^{Γ} to u^{Δ} in a recession. The equilibrium wage w_1^{Δ} may be lower than the equilibrium wage at point x. But the reduction in the equilibrium wage is much smaller than that by following the equilibrium path $\bar{e}$ to an equilibrium point x', because a gradual adjustment along the equilibrium path of $\bar{e}$ results in at most an increase in unemployment rates from u_x^{Γ} to u^{Δ}.

Observe that there is a gap between $e_1^{\Delta} = aw_1^{\Delta} + g(u^{\Delta})$ and $\bar{e}_1$. This gap plays a very important role in the job creation during recovering. After the adverse shock in demand for the recession, let us assume that demand gradually picks up so that the economy enters a recovering phase. Due to the existing gap, an increase in demand will first result in an increase in equilibrium effort (thus productivity) and wage. The unemployment rate at equilibrium remains at u^{Δ}. That is, the economy enters a recovering phase without creating any new jobs. This "jobless recovering" process can continue until the gap is closed. The wider the gap, the longer the "jobless recovering". After the gap has been closed, the economy will enter a new expansion phase by creating new jobs and investing in additional capital stock.

How does the technical progress A affect the creation of jobs? A technical progress that only affects the productivity potential $\bar{e}$ but not the production function F can slow the creation process of new jobs during the recovering, since the gap must be widen for higher A. But if technical progress affects production function F but not $\bar{e}$, then technical progress can speed up the creation of new jobs. If it affects both, then either case can happen, depending on which force dominates.

During the expansion with equilibrium path $\bar{e}$ or $\bar{e}_1$, it is also uncertain whether the growth in the technical progress A will definitely create jobs. This can be made precisely by taking derivative of $L(u)$ over A:

$$\frac{dL(u)}{dA} \mid_{u=u^{\Gamma}} = \frac{\bar{e}'_A}{a}(aA\theta F'(\bar{e}(1-u^{\Gamma})\frac{N}{m}) - 1) + \bar{e}\theta F' + \bar{e}\theta\bar{e}'_A F'',$$

where $\bar{e}'_A$ is the partial derivative of the function $\bar{e}(A,k)$ over A. By Theorem 3, $u^{\Gamma} \leq u^{\Delta}$. This implies that $\frac{\bar{e}}{a}(aA\theta F'(\bar{e}(1-u)\frac{N}{m}) - 1) \mid_{u=u^{\Gamma}}$ is nonnegative since

$$\frac{\bar{e}}{a}(aA\theta F'(\bar{e}(1-u^{\Gamma})\frac{N}{m}) - 1) = (1-u^{\Gamma})g'(u^{\Gamma}) - g(u^{\Gamma}) \geq (1-u^{\Delta})g'(u^{\Delta}) - g(u^{\Delta}) = 0.$$

The first two terms in $\frac{dL(u)}{dA} \mid_{u=u^{\Gamma}}$ are positive. But the third term is negative. So, if the sum is positive, the equilibrium unemployment rate u^{Γ} will be lower since $L(u)$ shifts to the left and the equilibrium is unique. But if the sum is negative, the unemployment rate at equilibrium will be higher.

Our model shows that the equilibrium adjustment in the creation of new jobs in the expansion phase is a gradual process along the wage curve. The creation of new jobs has been slowed by the accumulation in capital stock at steady state where unemployment rate at equilibrium is gradually lower. But the destroy of jobs during a recession can be a very rapid process. This implication of our model is quite consistent with the empirical evidence (see Figures 7 and 8 in Appendix). For example, it took the California State seven years to lower its unemployment rates from 10 percent in 1983 to 5 percent 1990. After the 1990-1991 recession, in less than three years, the CA unemployment rates increased from 5 percent in 1990 to 9.6 percent in 1993. The creation and

[10] Under-utilization of capacity after a recession provides a strong evidence that this may indeed be the case; see Corrado (2002).

destroy of jobs before and after the 2001 recession in CA had the same pattern that had also been observed in all other states in the U.S. during the 1990-1991 and 2001 recessions.

One important implication from our analysis is that, everything else constant, economic recovering in labor-intensive sectors should create new jobs faster than the capital-intensive sectors. This is because the shift from equilibrium path $\bar{e}$ to $\bar{e}_1$ for the labor-intensive sectors should be less than that for the capital-intensive sectors. A narrower gap implies a faster pace of new job creation along the equilibrium path $\bar{e}_1$. Since capitals are expected to play more and more important role in the productivity in the U.S. economy in the future, it is quite safe to expect that "jobless recovering" won't go away in a future recession.

5 Discussions

In the above we have explored the possible cause for "jobless recovering" by using a simple efficiency wage model. In what follows, we will demonstrate some novel aspects of our model that have not been observed in the existing efficiency wage models. They also challenge certain conventional views in the neoclassic theory.

5.1 The Solow Equilibrium Condition

One may have observed that our model is quite similar to the Solow (1979) simple efficiency wage model in its reformulated version presented by Yellen (1984). The difference lies in the function form of effort of work. In our model effort of work depends on the real wage and the endogenously determined "local" unemployment rate. Such a change makes our model very different from the original Solow model.

Suppose that the effort function of work does not depend on the unemployment rate u or the unemployment rate u is considered to be exogenous, then the first order condition in Eq.(2) should be given by

$$F'(e(w,u)l)e(w,u) - w = 0, \tag{9}$$

which does not have the first term $-(1-u)e'_u(w,u)$ in the bracket as in Eq.(2). The elasticity of effort

$$\eta_{e,w} = \frac{e'_w(w,u)}{e(w,u)}w = 1$$

must be unity. This is the known Solow equilibrium condition. Thus, the equilibrium wage must be sticky and depends on the effort function of work alone. In contrast, Theorem 2 shows that our model has sticky unemployment rate at equilibrium when the effort function of work does not have a bound.

5.2 Full Employment

Theorem 1 provides a general condition on the effort function of work for an economy to achieve full employment. Note that our equilibrium notion is the Nash equilibrium. That is, given every other stays with the recommended equilibrium wage and employment level, a firm does not have any incentive to deviate unilaterally from the equilibrium wage offer and employment level. This makes our equilibrium notion different from the notion of full employment in competitive equilibrium in the neoclassic theory, in which an auctioneer is needed to determine the equilibrium wage and make necessary adjustments if the economy is at disequilibrium.

Let $e(w,u) = aw + bexp(u)$ and $F(el) = A(el)^{\alpha}$. The fair-wage $\bar{w}$ has been embodied in the coefficient a. Then, we know from Theorem 1 that the equilibrium unemployment rate u^{Δ} equals zero. Therefore, the equilibrium employment level $l^{\Delta} = \frac{N}{m}$. Then the equilibrium wage and effort of work are given, respectively, by

$$w^{\Delta} = \frac{m}{aN}(A a\alpha)^{\frac{1}{1-\alpha}} - \frac{b}{a}$$

and

$$e^{\Delta} = \frac{m}{N}(A a\alpha)^{\frac{1}{1-\alpha}}.$$

Note that the neoclassical market-clearing wage w_0^{**} is determined by

$$ln(A\alpha) + (\alpha - 1)ln\frac{N}{m} = lnw_0^{**} - \alpha ln(aw_0^{**} + b).$$

An increase in the supply of labor N lowers the equilibrium wage w^{Δ} and effort of work $e^{\Delta} = aw^{\Delta} + b$. Thus, productivity is lower as N increases. The aggregate output of this island economy is given by

$$\bar{Y} = mF(e^{\Delta}l^{\Delta}) = mA^{\frac{1}{1-\alpha}}(a\alpha)^{\frac{\alpha}{1-\alpha}},$$

which depends on the number of firms m but not the number of workers N. This means that an increase in supply of labor alone, i.e., without an increase in the number of firms m, leaves the aggregate output unchanged. This is because the increase in the number of workers depresses the efficiency wage and thus lowers the productivity. This is in the contrast to the neoclassical theory with competitive equilibrium in which an increase in the supply of labor results in an increase in the aggregate output. Such a conclusion is based on the assumption that higher supply of labor does not create any adverse effect on the productivity of the existing hiring workers, which is of course not necessarily true, especially for risk aversion workers.

An increase in the number of firms m increases the equilibrium efficiency wage w^{Δ} and effort of work e^{Δ}, generating higher productivity and higher aggregate output.

Let $\lambda_{w,A}$, $\lambda_{e,A}$, and $\lambda_{\bar{Y},A}$ denote the elasticity of the equilibrium efficiency wage, effort, and aggregate output with respect to the coefficient A, respectively. Then

$$\lambda_{w,A} = \frac{1}{1-\alpha}(1 + \frac{b}{aw^{\Delta}}),$$

$$\lambda_{e,A} = \frac{1}{1-\alpha},$$

and

$$\lambda_{\bar{Y},A} = \frac{1}{1-\alpha}.$$

For example, if $\alpha = 0.7$, then one per cent increase in technical progress results in 3.33% increase in the aggregate output and effort of work. The real wage increases a little more than 3.33%. Therefore, our economy can grow at 3.33% without creating new jobs. This conclusion depends on the ability that firms can work existing workers more and more intensively or invest in capital stock so that our effort function does not surpass the productivity potential $\bar{e}$.

5.3 Persistence Involuntary Unemployment

The above Subsection provides an economy with full employment at symmetric Nash equilibrium. The effort function of work is convex, which means that workers are risk-takers toward unemployment. This subsection presents a class of effort function such that workers are risk aversion toward unemployment. We show that our economy has a constant nonzero involuntary unemployment rate at equilibrium, even if wage is perfectly flexible. This is in contrast to the neoclassic theory under which full employment is implied by flexible wage.

Assume that the effort function of work is defined as follows:

$$e(w,u) = aw + bu^{\beta},\ \ a, b, \beta > 0$$

and the production function $F(el) = A(el)^{\alpha}$. The Arrow-Pratt coefficient of absolute risk aversion is given by

$$r_e(u) = \frac{1-\beta}{u}.$$

By Theorem 2, the equilibrium unemployment rate u^{Δ}, the equilibrium efficiency wage w^{Δ} and effort of work e^{Δ} are respectively given by

$$u^{\Delta} = \frac{\beta}{1+\beta},$$

$$w^{\Delta} = \frac{m}{aN}(1+\beta)(Aa\alpha)^{\frac{1}{1-\alpha}} - \frac{b}{a}(\frac{\beta}{1+\beta})^{\beta},$$

and

$$e^{\Delta} = \frac{m}{N}(1+\beta)(Aa\alpha)^{\frac{1}{1-\alpha}}.$$

Let $c = \frac{m}{N}(Aa\alpha)^{\frac{1}{1-\alpha}}$. Then

$$e^{\Delta} = \frac{c}{1-u^{\Delta}}.$$

Thus, the effort and wage curves are both upward sloping since

$$\frac{de^{\Delta}}{du^{\Delta}} = c(1+\beta)^2 > 0$$

and

$$\frac{dw^{\Delta}}{du^{\Delta}} = \frac{(1+\beta)^2}{a}[c - b(ln\beta - ln(1+\beta))\frac{\beta^{\beta}}{(1+\beta)^{1+\beta}}] > 0.$$

Therefore, we provide a family of effort functions of work such that involuntary unemployment at equilibrium is nonzero and does not depend on the production function. Unlike the existing efficiency wage model, such a conclusion does not necessarily require firms to pay supra competitive wage.

The neoclassical market-clearing wage w^{**} is given by

$$w^{**} = \frac{m}{aN}(Aa\alpha)^{\frac{1}{1-\alpha}}.$$

Let $d = \frac{b}{a}(\frac{\beta}{1+\beta})^{\beta}$. Then the equilibrium wage w^{Δ} can be written as follows

$$w^{\Delta} = -d + (1+\beta)w^{**}.$$

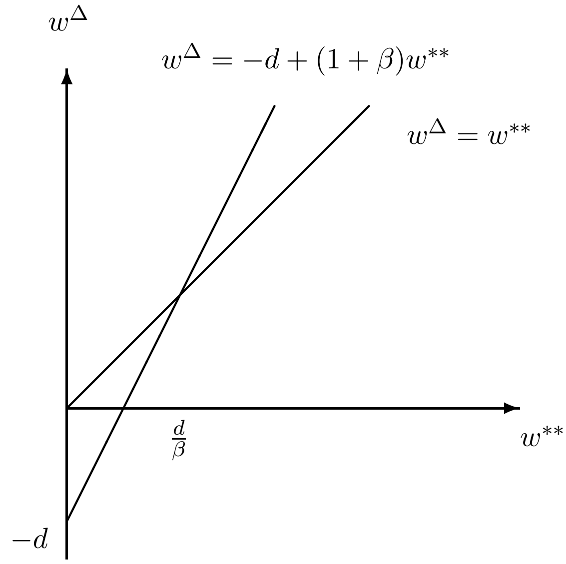

Fig.4 Relationship Between w^{Δ} and w^{**}

Theorem 4. *If* $w^{**} \leq (\geq) \frac{d}{\beta}$, *then* $w^{\Delta} \leq (\geq) w^{**}$.

Proof. See Fig.4. □

The intuition why w^{Δ} may be below w^{**} in Theorem 4 is quite simple and has been given in the example in Subsection 2.1. When unemployment affects the effort of work, the demand curve of labor shifts down, see eq. (2). The demand curve can shift down so much that lower than the market-clearing efficiency wage and nonzero involuntary unemployment coexist at equilibrium.

Note that the aggregate output at equilibrium $\bar{Y}$ is given by

$$\bar{Y} = mA^{\frac{1}{1-\alpha}}(a\alpha)^{\frac{\alpha}{1-\alpha}},$$

which equals the aggregate output at full employment provided in the last subsection. Even if there is involuntary unemployment, the aggregate output equals the full employment level! This is due to higher effort of work at equilibrium that has compensated the loss in the output of the unemployed workers.

Again, elasticities of the equilibrium efficiency wage, effort, and aggregate output with respect to the coefficient A are given, respectively, by

$$\lambda_{w,A} = \frac{1}{1-\alpha}(1 + \frac{b}{aw^{\Delta}}),$$

$$\lambda_{e,A} = \frac{1}{1-\alpha},$$

and

$$\lambda_{\bar{Y},A} = \frac{1}{1-\alpha}.$$

Once again, with $\alpha = 0.7$, our economy can grow at 3.33% without creating new jobs for each percent increase in the technical progress.

6 Analysis and Evidence

Our simple model has many interesting implications for the questions related to the wage curve (Blanchflower and Oswald (1994)), the unemployment insurance, the inter-industry wage differentials (Krueger and Summers (1988)), technical change and income inequality (Acemoglu (2002)).

6.1 Wage Curve

Blanchflower and Oswald (1994) used the cross-sections data to study the relationship between (annual) wages and contemporaneous local unemployment rates. They found that the elasticity of the real wage with respect to the local unemployment rate is about -10% across countries. Such a relationship can be expressed by

$$lnw = -0.1 lnu + \text{ other terms.}$$

Blanchflower and Oswald (1994) called it the wage curve. Blanchard and Katz (1997) obtained the "wage curve" with the hourly earnings and the state unemployment rates from 1980 to 1991 for the U.S. economy and found that their results are more in favor of the classic Phillips curve rather than the wage curve. Nevertheless, they still asked how to reconcile the wage curve. Card (1995) did an excellent review of the wage curve. He also obtained several wage curves with both hourly wage and annual earnings, and the state unemployment rates for quite different groups of workers for the U.S. economy in selected years. By the end of his review, Card (1995) concluded that

> "There is a 'wage curve.' The wages of individuals who work in labor markets with higher unemployment are lower than those of similar workers in markets with lower unemployment. Furthermore, the tendency for the wage curve to show up for different kinds of workers, in different economies, and at different times, suggests that the wage curve may be close to an 'empirical law of economics.'"

The efficiency wage model of Shapiro and Stiglitz (1984) is one of the three leading theories that Blanchflower and Oswald (1994) had developed to explain the wage curve. They showed that the region with higher nonpecuniary benefits (i.e., the 'nice' region) offers lower wage and higher unemployment and the region with lower nonpecuniary benefits (i.e., the 'nasty' region) offers higher wage and lower unemployment. Such a conclusion is based on the idea that different regions must offer the same expected utility at a zero-migration equilibrium under the assumption that there are zero-migration costs. If a 'nice' region is defined by the one with a lower crime rate, a better school district, higher real estate prices, and less congestion of traffics etc, then the reality seems to be the opposite of what has been described in their equilibrium.

Our efficiency wage model follows the spirit of Shapiro and Stiglitz (1984) and is built on the idea of the norms of effort in Akerlof (1982). Yet our "wage curve" with unbounded effort function of work can be upward sloping instead downward sloping required by Blanchflower and Oswald (1994). It should be aware that a more compatible wage curve with our study here is the wage curve across different groups of workers as suggested by Card (1995):

> "A fundamental property of the efficiency wage model is that wages of a given group of workers are related to the *group-specific* unemployment rate. Higher unemployment

for unskilled workers, for example, should have no effect on the wages of skilled workers, once their own unemployment is taken into account. The absence of any empirical evidence on such questions, noted earlier, is disappointing. It is also somewhat disappointing that Blanchflower and Oswald did not give more thought to developing the implications of an efficiency wage model for differences in the slope of the wage curve across different groups of workers."

One important feature in the shirking model discussed in Blanchflower and Oswald (1994) is that the effort of work is fixed at certain level by "technology" (see their Assumption A.2). We show that if the effort of work is fixed at certain level and it has been reached, then the equilibrium locus of our model is indeed a downward sloping convex curve (Fig.3 and 5), as long as workers' attitude toward unemployment is risk aversion.

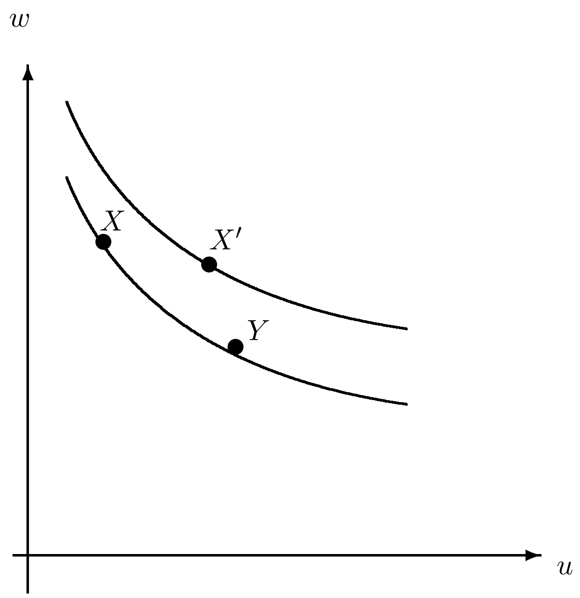

Fig.5. The Wage Curve. X with higher A versus Y with lower A.

Thus, we show that the condition whether the effort of work is bounded or not is critical for the wage curve.

Suppose that the bound is not affected by A. Then higher technical progress A, everything else constant, implies lower unemployment rate u at equilibrium since the function $L(u)$, defined Section 4, shifts to the left and the equilibrium is unique. This means that technology advance without impact on the bound helps reduce the equilibrium unemployment rate but increase the efficiency wage at equilibrium. X and Y in Fig.5 are the points on the wage curve corresponding to higher and lower A respectively. Thus, a favorable shock in A makes the economy boom (i.e., higher real wage, higher employment level, and higher aggregate output) and an adverse shock in A makes the economy sluggish (i.e., lower real wage, lower employment level, and lower aggregate output). Consequently, a business cycle emerges along with technical progress and regress. Therefore, the wage curve derived from our model may be useful in the study of real business cycle.

The assumption that the effort of work is fixed by "technology" in Blanchflower and Oswald (1994) should be taken carefully. If technology does not change $\bar{e}$, then a change in technology provides a movement along the wage curve from points Y to X analyzed above. If technology advance also increases the upper bound $\bar{e}$, then the wage curve shifts upward, together with a movement along the new wage curve. Point X' in Fig.5 is a situation where an increase in A due to technology advance shifts the wage curve and causes a movement along the new curve. The impact of technology advance on the creation of jobs has been studied in Section 3.

6.2 Unemployment Insurance

The economy presented in Subsection 5.3 may be used to study unemployment insurance. An increase in the parameter β implies a decrease in risk aversion. This can be accomplished by unemployment insurance which helps reduce the fear of a worker for being unemployed. A consequence of such a policy is an increase in the voluntary unemployment rate at equilibrium. The reduction in employment level at equilibrium has been compensated by an increase in the effort of work. So the aggregate output at equilibrium remains unchanged. Everything else constant, an increase in risk aversion decreases real wage at equilibrium.

The impact of unemployment benefits and job protection or regulation can be analyzed with respect to their risk aspects. Longer duration and better unemployment benefits are expected to reduce risk aversion. Such a policy, if implemented permanently, helps the unemployed but has potential to increase involuntary unemployment rate at equilibrium in the long run. Thus, unemployment insurance with better benefits makes unemployed better off in the short run but worse off in the long run. Thus, unemployment insurance and the related policy may increase the overall average unemployment rate.

One key difference between the labor markets in the U.S. and OECD Europe is that the existing workers in the U.S. labor market are less secure in their jobs. It is well known that it is harder for a firm to fire a worker in the OECD. Therefore, it is plausible to expect that workers in OECD Europe are less risk aversion than in the United States. Our model then shows that the labor market in OECD Europe should have higher unemployment rates at equilibrium than the U.S.

6.3 Inter-industry Wage Differentials

Krueger and Summers (1988) found that inter-industry wage differentials for equally skilled workers are substantial and stable across time and spaces. They showed that these wage differences are hardly explained by the unobserved quality of workers, the union threats, and various working conditions. They concluded that their findings support the efficiency wage theory in the sense that the high pay industries paid supra-competitive wage. But we show that when unemployment rate matters, efficiency wage at equilibrium is not necessarily higher than the competitive wage. There is a question how to use our model to explain their empirical findings.

Their studies suggested that each industry should be examined in detail for its wage determination. Note that different industries also have quite different unemployment rates (Table 4). This implies that our stylized island economy is probably the right one in the study of both wage determination and unemployment rates for each industry. We now provide an explanation why inter-industry wage differentials for equally skilled workers can remain stable across time and spaces, by using the economy given in subsection 5.3.

Consider two island economies $\imath$ and $\jmath$ with different coefficients A. All else are equal. Suppose that island $\imath$'s $A(\imath)$ is greater than island $\jmath$'s $A(\jmath)$. Then the wage premium at equilibrium is given by

$$\frac{w^{\Delta}(\imath)+d}{w^{\Delta}(\jmath)+d} = (\frac{A(\imath)}{A(\jmath)})^{\frac{1}{1-\alpha}},$$

which is larger than 1. Note also that the efforts of work at equilibrium in the two islands also satisfy that

$$\frac{e^{\Delta}(\imath)}{e^{\Delta}(\jmath)} = (\frac{A(\imath)}{A(\jmath)})^{\frac{1}{1-\alpha}}.$$

Therefore, our results show that the island that is initially endowed with more "profitable" production function pays higher equilibrium wage than the island endowed with less "profitable" production function. Note that higher wage needs also higher effort of work at equilibrium. The next question is how it is possible such a wage difference can sustain across time, with a possible mobility of labors between the two islands.

Consider two periods 0 and 1 island economies. Let $x_\imath$ and $y_\jmath$ denote the growth rates of $A(\imath)$ and $A(\jmath)$ respectively. Let z denote the net mobility rate of labors from island $\jmath$ to island $\imath$ at the beginning of period one (there is no mobility after that). That is, the ratio of number of firms over number of workers in island $\imath$ becomes $\frac{m}{N}(1-z)$ at period one. It is $\frac{m}{N}(1+z)$ in island $\jmath$ at period one. Then the wage premium at equilibrium in period one is given by

$$\frac{w_1^{\Delta}(\imath)+d}{w_1^{\Delta}(\jmath)+d} = \frac{w^{\Delta}(\imath)+d}{w^{\Delta}(\jmath)+d}\frac{1-z}{1+z}(\frac{1+x}{1+y})^{\frac{1}{1-\alpha}} \approx \frac{w^{\Delta}(\imath)+d}{w^{\Delta}(\jmath)+d} > 1,$$

as long as x, y and z are small and $x = y + 2(1-\alpha)z$. The mobility rate z can be small due to mobility barriers or costs. Once again, higher wage at equilibrium in period one in island $\imath$ is not for free. The higher equilibrium wage in island $\imath$ in period one corresponds to a higher effort of work, since the efforts of work in period one also satisfy

$$\frac{e_1^{\Delta}(\imath)}{e_1^{\Delta}(\jmath)} = \frac{e^{\Delta}(\imath)}{e^{\Delta}(\jmath)}\frac{1-z}{1+z}(\frac{1+x}{1+y})^{\frac{1}{1-\alpha}} \approx \frac{e^{\Delta}(\imath)}{e^{\Delta}(\jmath)}.$$

It should be noted that the unemployment rates at equilibrium for the two islands do not change in period one even though there is a mobility of labors between the two islands. Therefore, our results show that both inter-industry wage differentials and inter-industry involuntary unemployment rates can be steady across time and spaces.

There is another reason why inter-industry wage differentials persist across time and spaces. Suppose that islands $\imath$ and $\jmath$ have workers with different norms of work

$$e_\imath(w, u) = a(\imath)w + b(\imath)u^{\beta}$$

and

$$e_\jmath(w, u) = a(\jmath)w + b(\jmath)u^{\beta}.$$

Assume that $a(\imath) > a(\jmath)$ and $b(\imath) > b(\jmath)$, with constant $\frac{a(\imath)}{b(\imath)}$ and $\frac{a(\jmath)}{b(\jmath)}$, everything else being equal. Then both islands have the same unemployment rates at equilibrium. But workers in island $\imath$ enjoy higher wage with higher effort of work at equilibrium than workers in island $\jmath$. Such a wage difference can persist across time and spaces, as shown in Fig.6. The two curves in Fig.6 are the two indifference curves of utility for the two islands. The line follows from the wage equation

$$w^{\Delta} = \frac{e^{\Delta}}{a} - \frac{b}{a}(u^{\Delta})^{\beta}.$$

Since the elasticity of equilibrium wage and effort of work with respect to a is given respectively by

$$\lambda_{w,a} = \frac{\alpha}{1-\alpha}$$

and

$$\lambda_{e,a} = \frac{1}{1-\alpha},$$

it follows that with higher a, the equilibrium wage and effort of work are both higher.

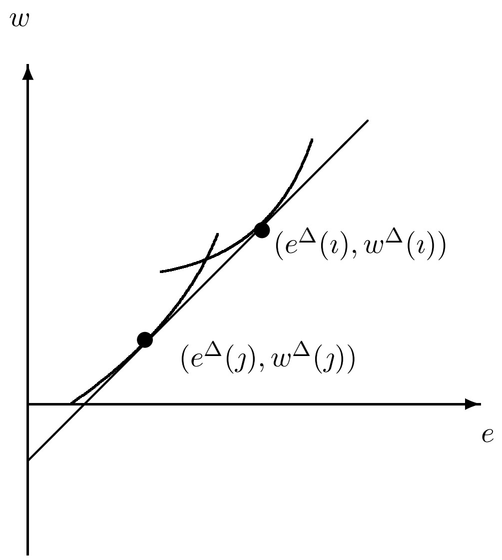

Fig.6 Persistent Wage Differentials

6.4 Technical Change and Income Inequality

Acemoglu (2002) provided a survey and a detailed discussion on technical change and income inequality of the U.S. labor markets during the last several decades. He concluded that "technical change has been skill-biased during the past sixty years, and probably for most of the twentieth century". Such a skill-biased technical change has been recognized to be the main cause for the sharp rising since 1970s in the income inequality between high-skill and low-skill workers. In fact, real income for high-skill workers rises sharply during 1980s while real wages for low-skill workers have fallen since 1970. These facts together with the pattern of unemployment invite an interesting question: How is it possible that a steady pattern of unemployment coexists with deteriorating income inequality?

We now provide an answer to the question with our simple island economy in Subsection 5.3. Imagine that island $\imath$ is for the high-skill workers and island $\jmath$ is for the low-skill workers. Assume that high-skill workers as a work group are more risk averse than low-skill workers. Then island $\imath$ has lower involuntary unemployment rate than island $\jmath$ at equilibrium. The involuntary unemployment rates at equilibrium remain steady unless workers have changes in their attitudes toward unemployment risk.

Now, the skill (wage) premium (Acemoglu (2002)) at equilibrium is given by

$$\frac{w^{\Delta}(\imath) + d(\imath)}{w^{\Delta}(\jmath) + d(\jmath)} = (\frac{A_1}{A_2})^{\frac{1}{1-\alpha}}.$$

The income inequality can be widened at period one as long as $x - y > 2(1-\alpha)z$, because the ratio of equilibrium wages at period one becomes

$$\frac{w^{\Delta}(\imath) + d(\imath)}{w^{\Delta}(\jmath) + d(\jmath)} \frac{1-z}{1+z} (\frac{1+x}{1+y})^{\frac{1}{1-\alpha}} > \frac{w^{\Delta}(\imath) + d(\imath)}{w^{\Delta}(\jmath) + d(\jmath)},$$

where x, y, and z are defined in Subsection 6.3.

Technical change that is skill-biased means that $x > y$. If the net mobility from low-skill labor market to high-skill one is small, then the condition that $x - y > 2(1-\alpha)z$ can be easily satisfied.

If technical change generates an "erosion effect" (i.e., $y < 0$) on the productivity of low-skill workers (Acemoglu (2002)), then equilibrium wage for low-skill workers can decline in trend if $(1-\alpha)z + y < 0$. Acemoglu (2002) pointed out that the decline in real wages for the low-skill workers since 1970 is one of the puzzles associated with technical progress. Our model can have a reasonable explanation of that fact. It does one more thing because it also shows why unemployment rate can remain high for low-skill workers, while income inequality deteriorates.

6.5 Akerlof's Example

Akerlof (1982) used two examples to illustrate how it is possible to have involuntary unemployment at equilibrium in his partial-gift-exchange model by treating the unemployment rate u as an exogenous variable. He used the Solow equilibrium condition to find out the equilibrium employment level in his first example. We now used his example to show how an endogenous unemployment rate may change some of the computation.

Let $e = -a + b(\frac{w}{b_u})^{\gamma u}, b > 0, 0 < \gamma < 1$, where b_u denotes the unemployment benefit. The production function is given by $F(el) = (el)^{\alpha}$, $0 < \alpha < 1$. Akerlof (1982) assumed that $a > 0$. In our study below, we assume that $a = 0$ in order to simplify the computation. It is of interest to note that the efficiency wage w^{Δ} is not zero at equilibrium even if $a = 0$.

It follows from Theorem 1 that such an economy does not have a symmetric Nash equilibrium with full employment. So if there is any symmetric Nash equilibrium that is feasible, there must exist nonzero involuntary unemployment rate at equilibrium.

It follows from the effort function of work e that

$$e'_w(w,u) = \frac{b\gamma u}{b_u}(\frac{w}{b_u})^{ru-1} = \frac{\gamma u b^{\frac{1}{ru}}(e+a)^{\frac{ru-1}{ru}}}{b_u},$$

$$e'_u(w,u) = \gamma e(lnw - lnb_u),$$

and

$$e'_l(w,u) = e'_u(w,u)(-\frac{m}{N}) = -\frac{m}{N}\gamma e(lnw - lnb_u).$$

Thus, at the symmetric Nash equilibrium, by eq.(3), we have that

$$(10) \qquad e^{\Delta} = bexp(\frac{u^{\Delta}}{1-u^{\Delta}}(1-\gamma u^{\Delta})),$$

$$(11) \qquad w^{\Delta} = b_u exp(\frac{1-\gamma u^{\Delta}}{\gamma(1-u^{\Delta})}),$$

and

$$(12) \qquad l^{\Delta} e_l(w,u) + e^{\Delta} = e^{\Delta}(1-(1-u^{\Delta})\gamma(lnw^{\Delta} - lnb_u)) = e^{\Delta}\gamma u^{\Delta}.$$

By eq.(2), the unemployment rate u^{Δ} at the symmetric Nash equilibrium satisfies the following:

$$(13) \qquad \alpha b^{\alpha}(\frac{N}{m}(1-u^{\Delta}))^{\alpha-1}\gamma u^{\Delta} = b_u exp(\frac{(1-\gamma u^{\Delta})(1-\alpha\gamma u^{\Delta})}{\gamma(1-u^{\Delta})})$$

So the employment level l^{Δ} at equilibrium is given by

$$(14) \qquad l^{\Delta} = (1-u^{\Delta})\frac{N}{m}.$$

For example, let

$$\frac{N}{m} = 100, \alpha = .7, b = 200, \gamma = .9, b_u = .133$$

Then the involuntary unemployment rate at equilibrium $u^{\Delta} = 6\%$, the real wage at equilibrium $w^{\Delta} = 0.41$, and the effort of work at equilibrium $e^{\Delta} = 202.01$. Note that the unemployment benefit is only .133 but the equilibrium real wage is .41. An unemployed worker may indeed be involuntarily unemployed: He likes to work at wage $w = 0.41$ or lower but no firms have the vacancies for him.

Once again, note that the wage curve, with unbounded effort function, is upward sloping since

$$\frac{dw^{\Delta}}{du^{\Delta}} > 0.$$

Now assume that there is an upper bound $\bar{e}$ for $e(w,u)$ that is reached at equilibrium. As long as $F'(0)$ is large enough, there exists a unique symmetric Nash equilibrium such that $0 < u^{\Delta} < 1$. Let $\delta = \frac{1}{\gamma} ln\frac{\bar{e}}{b}$. Thus, the efficiency wage w^{Δ} at equilibrium is determined by

$$lnw^{\Delta} = lnb_u + \frac{\delta}{u^{\Delta}},$$

which is downward sloping. Moreover, it is a convex function in u^{Δ}. Thus, we obtain again the wage curve as in Fig.3 and 5. Unlike the wage curve in Fig.3, a change in the upper bound here changes the slope of the wage curve rather than shifts the curve. But an increase in the unemployment benefits b_u does shift the wage curve away from the origin.

7 Conclusion

Our simple efficiency wage model is quite standard except that unemployment rate has been endogenously determined at the symmetric Nash equilibrium. By doing so, we find that our simple efficiency wage model generates several novel aspects that are not shared by the existing efficiency wage models in the literature. We use it to show how it is possible to have "jobless recovering" in an economy. Moreover, we find that involuntary unemployment rate at equilibrium does not depend on the production function for a class of effort functions of work. Such a feature may be called as a "sticky" unemployment rate. With the same class of effort functions of work, we can provide insights on the issues related to the inter-industries wage differentials and the income inequality due to technical change experienced by the U.S. economy during the last several decades.

What we learned from the study of this paper is that involuntary unemployment should be a general phenomenon in an economy. Our study reveals why it is difficult to eliminate the involuntary unemployment. An economy with zero involuntary unemployment rate for a long period of time likely reduces workers' risk aversion for being unemployed. Such a reduction in the risk aversion means higher involuntary unemployment rate at equilibrium in the future. On the other hand, involuntary unemployment rate cannot sustain with a high level. A high level of involuntary unemployment rate makes workers more risk averse for being unemployed. Higher risk aversion implies lower involuntary unemployment at equilibrium in the future. It should be aware that this conclusion is based on the assumption that a high level of unemployment rate does not result in any change in the policy about labor markets such as the job protection and unemployment compensation. Any irrevocable change in the policy in favor of the unemployed due to the current high level in unemployment rate likely makes the current high unemployment rate a permanent phenomenon in the future, creating a kind of hysteresis of unemployment. The lesson we learned is this: Help but do not spoil unemployed!

Appendix

Table 1 Nonfarm Business Sector: Productivity, output, hourly compensation, seasonally adjusted. Percentage change from previous quarter at annual rate.

Year and quarter	Output per hour of all persons	Output	Hours of all persons	Nominal compensation per hour	Real compensation per hour
2001Q1	-1.4	-0.9	0.5	2.8	-0.9
Q2	-0.1	-2.7	-2.6	0.1	-3.0
Q3	2.1	-0.8	-2.9	1.0	0.1
Q4	7.2	2.9	-4.0	1.5	2.1
Annual	1.1	-0.1	-1.2	2.7	-0.1
2002Q1	8.6	6.2	-2.3	2.9	1.6
Q2	1.7	0.9	-0.7	4.0	0.4
Q3	5.5	5.2	-0.2	1.8	-0.3
Q4	0.7	1.7	0.9	3.9	1.9
Annual	4.8	2.7	-2.0	2.4	0.8
2003Q1	1.9	1.8	-0.1	3.4	-0.4

Source: U.S. BLS Current Population Survey Table 2 (June 3 2003).

Table 2 Unemployment rates by education[a], 25 yrs. & over

	93	94	95	96	97	98	99	00	01	02	03[b]
Less Than a High School Diploma	10.8	9.8	9.0	8.7	8.1	7.1	6.7	6.3	7.2	8.5	8.8
High School, No College	6.3	5.3	4.7	4.7	4.2	4.0	3.5	3.5	4.2	5.3	5.5
Some College or Associate Degree	5.2	4.4	4.0	3.8	3.3	3.0	2.8	2.7	3.3	4.5	4.8
Bachelor's Degree and Higher	3.0	2.6	2.5	2.3	2.0	2.2	1.8	1.7	2.3	2.9	3.1

a). Source. U.S. BLS.
b). From January to July.

Table 3 Unemployment and skill

Unemployment Rates by Occupations	April 1987[a]	2001[b]	2002[b]
Managerial and Professional Specialty	2.1	2.3	3.1
Technical, Sales, and Administrative Support	4.3	4.1	5.2
Service Occupations	7.6	5.8	6.7
Precision Production, craft, and repair	6.5	4.6	6.1
Operators, fabricators, and laborers	9.8	7.7	8.9

a). Source. Akerlof and Yellen (1990).
b). Source. U.S. BLS.

Table 4 Unemployment rates by industry and sex (Source: U.S.BLS Household data annual average Table 26 Unemployed persons by industry and sex).

			Men		Women	
Industry	2001	2002	2001	2002	2001	2002
Total, 16 years and over	4.7	5.8	4.8	5.9	4.7	5.6
Nonagricultural private wage and salary workers	5.0	6.2	5.0	6.2	4.9	6.1
Mining	4.7	6.2	4.7	6.1	4.2	6.8
Construction	7.3	9.2	7.5	9.5	5.1	7.3
Manufacturing	5.2	6.7	4.7	6.0	6.4	8.2
Durable goods	5.3	7.0	4.8	6.4	6.5	8.8
Lumber and wood products	6.4	6.9	6.6	7.0	5.2	6.8
Furniture and fixtures	5.0	7.2	4.5	6.1	6.0	9.8
Stone, clay, and glass products	5.4	5.6	5.3	4.8	5.7	8.8
Primary metal industries	5.3	7.8	4.3	7.7	9.5	8.2
Fabricated metal products	4.9	6.7	4.4	6.7	6.6	6.8
Machinery, except electrical	5.0	7.8	4.9	7.2	5.3	9.5
Electrical machinery, equipment, and supplies	6.0	8.4	4.8	7.2	7.8	10.4
Transportation equipment	4.5	5.1	4.1	4.7	5.8	6.4
Automobiles	5.1	5.4	4.7	4.8	6.5	7.4
Other transportation equipment	3.6	4.6	3.3	4.6	4.6	4.6
Professional and photographic equipment	3.9	6.1	3.3	5.4	4.9	7.5
Other durable goods industries	8.3	9.2	8.6	7.0	7.7	12.6
Nondurable goods	5.2	6.1	4.4	5.2	6.3	7.6
Food and kindred products	5.2	6.2	4.7	5.0	6.2	8.5
Textile mill products	8.3	9.3	7.4	7.9	9.3	11.0
Apparel and other textile products	9.7	10.6	7.6	7.9	10.9	12.3
Paper and allied products	4.0	3.5	3.2	3.1	6.5	4.8
Printing and publishing	4.1	5.5	4.1	5.2	4.1	6.0
Chemicals and allied products	3.9	4.9	3.6	4.7	4.4	5.2
Rubber and miscellaneous plastics products	4.8	6.1	4.1	5.7	6.2	6.9
Other nondurable goods industries	4.6	6.1	4.2	6.3	5.5	5.5
Transportation and public utilities	4.1	5.5	3.9	5.0	4.5	6.6
Transportation	4.5	5.4	4.4	5.1	5.0	6.3
Communications and other public utilities	3.4	5.7	3.1	4.9	3.9	7.0
Wholesale and retail trade	5.5	6.8	5.1	6.2	6.0	7.5
Wholesale trade	3.9	5.0	3.5	4.2	4.8	6.8
Retail trade	5.9	7.2	5.6	6.8	6.2	7.6
Finance, insurance, and real estate	2.7	3.3	2.7	2.9	2.8	3.5
Service industries	4.6	5.5	4.9	6.1	4.4	5.1
Professional services	2.9	3.5	2.7	3.3	2.9	3.6
Other service industries	7.0	8.6	6.6	8.5	7.5	8.7
Agricultural wage and salary workers	9.5	9.1	9.4	9.0	9.7	9.4
Government, self-employed and unpaid family workers	2.1	2.5	2.0	2.6	2.1	2.4

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