The War and Peace between the Snipe and the Clam

G. George Hwang (黃光雄)[*,**]

Department of Economics, Shih Hsin University

george@cc.shu.edu.tw

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Abstract

Will the possibility of a stalemate and the consequent predation by a common enemy (fisherman) bring about peace between the two rivals (snipe and clam) at war? According to a simple game-theoretic model of contest, the answer is no. If the equilibrium without a fisherman is a war, then a possible attack by a fisherman will not bring about peace in equilibrium. The fisherman effect on the war effort can be positive if war effort reduces the probability of stalemate. Furthermore, the fisherman can destroy a previously equilibrium state of peace. If the initial equilibrium is not peace, at least one contestant will fight harder in response to an increase in the probability of the fisherman's arrival or the strength of the fisherman.

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[*] Department of Economics, Shih Hsin University, No 1, Ln. 17, Sec. 1, Mu-Cha Rd., Taipei, 116 Taiwan, Tel: +886-2-22368225 x 3409, Email: george@cc.shu.edu.tw

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G. George Hwang

The fight of a snipe and a clam comes into a stalemate, and the fisherman gets the benefit.

A Chinese proverb

I. Introduction

The origin of the Chinese proverb is a diplomatic mission during the period of the Warring States (490-221BC).[1] According to the Legends of the Warring States (Chan-kuo T'se 戰國策), Diplomat Dai Hsu (蘇代) created the following parable to persuade the ruler of Chao (趙) not to attack the Kingdom of Yen (燕) (Crump (1979) p. 543).

> Today as I came here I crossed over the Yi River and a large mussel had just opened its shell to sun itself. Along came a heron to peck its flesh and the mussel closed up on the bird's beak. "If it does not rain today or tomorrow there will be a dead mussel here", said the heron. "If he does not leave today or tomorrow there will be a dead heron here", replied the mussel. Neither was willing to relax his grip, so along came a fisherman and bagged them both.

The ruler of Chao desisted upon hearing Hsu's warning that Chin (秦), another kingdom, will play the fisherman as "Yen and Chao will be able to hold each other off for a long while and exhaust their citizenry." This paper focuses on a weakness in Hsu's argument,

[1] A Warring States Map is available at http://www.umass.edu/wsp/conferences/wswg/16/chinamap1.html where Chao, Yen, and Ch'in are spelled as Zhao, Yan, and Qin, respectively.

namely, his assumption that Yen and Chao will necessarily hold each other off for a long while. If Chao can seize Yen before the attack by Chin, Hsu's argument does not hold. Suppose fighting harder can end the war sooner. Shouldn't the ruler of Chao send more troops to take over Yen rather than desist?

This paper offers a game-theoretic model of contest to examine when the fisherman effect should be positive.[2] The fisherman effect is defined as the effect of an increase in the likelihood of the presence of another predator (following the stalemate between the two rivals) on the equilibrium war efforts (of the snipe and the clam). The present analysis departs from Hsu's by endogenizing the probability of stalemate and the war effort. Another difference is that Hsu advices the ruler of Chao what he *should* do while the present game-theoretic analysis derives equilibrium prediction about what the snipe and the clam *will* do.

The rest of the paper is organized as follows. Section II illustrates the underlying decision tree when the probability of stalemate is a constant. It is to be shown that there is no fisherman effect in this case. Section III studies the equilibrium war efforts under a third-party stalemate probability function (SPF). Section IV concludes with a discussion on the implications of this revised story of the snipe and the clam.

II. Constant probability of stalemate: no fisherman effect

Assume that two players, player 1 and player 2, are contemplating spending resources to

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[2] The model in this paper is an extension of Hwang's (2002) loot-seeking war.

seize the stakes controlled by each other. In such a loot-seeking war, the winner's gain (the loot) is the loser's loss (the stake). The basic analytical framework is shown as the decision tree in Figure 1. In traditional models of contests, the game begins with node 2: there must be a winner in each of the two possible final outcomes (node 4 and 5). In contrast, node 1 in Figure 1 means that there is a probability determined by SPF (Stalemate Probability Function) that node 3 will be reached.[3]

[Figure 1]

To consider the possible arrival of an external predator (fisherman) after the stalemate, this paper adds node 6 and 7 to the decision tree. Node 7 means that the fisherman does come to the scene and captures both the snipe and the clam. As a result, both the snipe and the clam become losers. If the fisherman does not show up, there are no winners and no losers (node 6).

Let v and $1-v$ denote the stakes of the two contestants relative to the sum of the stakes prior to the war. Without loss of generality, assume that $v<1/2$. In other words, player 1's stake is smaller than that of player 2. If player 1 wins and player 2 loses (node 5), the payoff to player 1 is $1-v-x_1$ and the payoff to player 2 is $-(1-v)-x_2$ where x_1 and x_2 denote the war efforts (spending) by player 1 and player 2, respectively. Similarly, player 1's payoff is $-v-x_1$ and player 2's payoff is $v-x_2$ if player 1 loses and player 2 wins

[3] Hwang (2002) introduces node 1 and node 3 without considering the fisherman in this paper.

(node 4). At node 6, the payoffs to the two players are $-x_1$ and $-x_2$ because each player loses nothing to the rival or to the fisherman. By contrast, both players are losers to the fisherman and the consequent payoffs are $-v-x_1$ and $-(1-v)-x_2$ at node 7.

To solve for the equilibrium of the game, we assume that the SPF is a constant, *a*, in this section. In other words, the chance of reaching node 3 from node 1 is *a*. If node 2 is reached (the probability is *1-a*), the probability of reaching node 5 and node 4 are specified by the contest success functions (CSF) of player 1 and player 2. For analytical tractability, the models adopt Tullock's (1980) linear probit CSF. Under this assumption, each contestant's winning probability equals the ratio of her expenditure to the sum of all contestants' expenditures (given that the stalemate does not take place). In other words, the winning probability is $CSF_1=x_1/(x_1+x_2)$ for player 1 and $CSF_2=x_2/(x_1+x_2)$ for player 2.

To describe the role of the fisherman, let *f* be the probability that the fisherman will arrive at the scene and captures both the snipe and the clam. This means that the probability of reaching node 7 is *f* and node 6 is *1-f* from node 3. An alternative interpretation of *f* is the strength of the fisherman: the stronger the fisherman, the more likely he can catch both the snipe and the clam.

Based on the above assumptions, the expected payoffs to player 1 and player 2 are

$$y_1 = (1-a)*\frac{x_1}{x_1+x_2}*(1-v)-(1-a)*\frac{x_2}{x_1+x_2}*v-x_1-afv$$

$$y_2 = (1-a)*\frac{x_2}{x_1+x_2}*v-(1-a)*\frac{x_1}{x_1+x_2}*(1-v)-x_2-af(1-v) \qquad (1)$$

The objective of a contestant is to maximize the expected payoff by choosing the optimal amount of war spending given the war spending of the rival. Solving the first order conditions of maximization yields the best responses of the two players:

$$x_1 = -x_2 + \sqrt{(1-a)x_2}$$

$$x_2 = -x_1 + \sqrt{(1-a)x_1} \qquad (2)$$

Nash Equilibrium is the solution to the system of equations of the two best responses: *((1-a)/4, (1-a)/4)*.

Since the probability of the arrival of the fisherman does not enter the Nash Equilibrium war spending of either player, there is no fisherman effect:

Proposition 1. If the war efforts have no effects on the probability of stalemate, the fisherman has no effect on the equilibrium war efforts.

III. Third-party SPF

In the case of Third-Party SPF, the constant SPF in Section II is replaced by $w/(x_1+x_2+w)$ (if $x_1>0$ or $x_2>0$), where *w* represents a factor that increases the probability of a stalemate for a given level of war efforts by the two rivals. Examples of such third-party factors include the wind, the whirlpool, and the wilderness of the battlefield which make the outcome of the war less dependent on the war efforts by the contestants. The expected payoffs of player 1 and player 2 can be obtained by substituting *a* in (1) with

$w/(x_1+x_2+w)$:

$$y_1 = (1-\frac{w}{x_1+x_2+w})*\frac{x_1}{x_1+x_2}*(1-v)-(1-\frac{w}{x_1+x_2+w})*\frac{x_2}{x_1+x_2}*v-x_1-\frac{w}{x_1+x_2+w}fv$$

$$y_2 = (1-\frac{w}{x_1+x_2+w})*\frac{x_2}{x_1+x_2}*v-(1-\frac{w}{x_1+x_2+w})*\frac{x_1}{x_1+x_2}*(1-v)-x_2-\frac{w}{x_1+x_2+w}f(1-v)$$

(3)

Simplification yields:

$$y_1 = \frac{x_1}{x_1+x_2+w}*(1-v)-\frac{x_2+wf}{x_1+x_2+w}*v-x_1$$

$$y_2 = \frac{x_2}{x_1+x_2+w}*v-\frac{x_1+wf}{x_1+x_2+w}*(1-v)-x_2 \quad (4)$$

The best responses can be obtained accordingly:

$$x_1 = -x_2 - w + \sqrt{x_2+(1-v)w+vwf} \quad (5)$$

if $1-v+fv>w$ and $x_2 < \frac{1}{2} - w + \frac{\sqrt{1-4vw(1-f)}}{2}$;

$x_1=0$, (6)

otherwise.

$$x_2 = -x_1 - w + \sqrt{x_1+vw+(1-v)wf} \quad (7)$$

if $v+f(1-v)>w$ and $x_1 < \frac{1}{2} - w + \frac{\sqrt{1-4(1-v)w(1-f)}}{2}$;

$x_2=0$, (8)

otherwise.

A zero-effort best response means that the contestant should cease fire given the war spending by the opponent. Proposition 2 summarizes the three types of resulting equilibrium based on who chooses to cease fire.

Proposition 2.

Suppose the probability of stalemate is determined by a third-party SPF. The resulting equilibrium war efforts are as follows:

Case 1. Two-sided war: Neither player ceases fire.

If (i) f is sufficiently large (which implies w is sufficiently small): $v+f(1-v)>w$, (ii)

$$(1-4wf-8w(1-f)(1-v)+\sqrt{1+8wf})/8) < \frac{1}{2}-w+\frac{\sqrt{1-4vw(1-f)}}{2}$$, and (iii)

$$(1-4wf-8w(1-f)v+\sqrt{1+8wf})/8 < \frac{1}{2}-w+\frac{\sqrt{1-4(1-v)w(1-f)}}{2}$$

then

(x_1^{NE}, x_2^{NE})

$$=((1-4wf-8w(1-f)v+\sqrt{1+8wf})/8,(1-4wf-8w(1-f)(1-v)+\sqrt{1+8wf})/8) \qquad (9)$$

Case 2. One-sided war: Player 2 (the one with more to lose) ceases fire.

If both f and w are neither too large nor too small: $v+f(1-v)<w<1-v+fv$ or $(w-(1-v))/v<f<(w-v)/(1-v)$, then

$(x_1^{NE}, x_2^{NE}) = (-w + \sqrt{(1-v)w + vwf}\ , 0)$ (10)

Case 3. Peace: Both players cease fire.

If f is sufficiently small and w is sufficiently large, $1-v+fv<w$, then

$(x_1^{NE}, x_2^{NE}) = (0, 0)$ (11)

Proof:

Case 1.

(i) $v+f(1-v)>w \rightarrow$ the Best Response of Player 2 is given by (7).

(ii) $v<1/2 \rightarrow 1-v+fv> v+f(1-v) \rightarrow 1-v+fv>w \rightarrow$ the Best Response of Player 1 is given by (5) if $x_2 < \frac{1}{2} - w + \frac{\sqrt{1-4vw(1-f)}}{2}$ and 0 otherwise .

(iii) The Nash Equilibrium war spending is the solution to (5) and (7).

Case 2.

(i) $v+f(1-v)<w \rightarrow$ the Best Response of player 2 is given by (8).

(ii) $w<1-v+fv \rightarrow$ the Best Response of player 1 is given by (5).

(iii) The Nash Equilibrium war spending is the solution to (5) and (8).

Case 3.

(i) $1\text{-}v+fv<w \rightarrow$ the Best Response of player 1 is given by (6).

(ii) $v<1/2 \rightarrow 1\text{-}v+fv> v+f(1\text{-}v) \rightarrow v+f(1\text{-}v)<w \rightarrow$ the Best Response of player 2 is given by (8).

(iii) The Nash Equilibrium war spending is the solution to (6) and (8).

Figure 2 illustrates an example of case 1. Given the two parameter values: $v=0.25$ and $w=0.1$, the equilibrium war efforts of the two players are always positive as f increases from 0 to 1.

[Figure 2]

According to Case 3 of Proposition 2, peace equilibrium implies a lower bound on the third party factor, $w_L=1\text{-}v+fv<w$, or equivalently, an upper bound on the likelihood that the fisherman will arrive $f_U=(w\text{-}(1\text{-}v))/v$. In other words, peace can be destroyed if the probability of an attack by the fisherman increases.

Corollary 3: The lower bound on *w* for peace equilibrium is increasing in *f*. The upper bound on *f* for peace equilibrium is increasing in *w*.

An increase in the probability of fisherman's arrival or the strength of the fisherman, *f*, raises the minimum requirement on *w* for peace to be the equilibrium.

Proof:

The effect of *f* on the lower bound on *w* is $dw_L /df=d(1-v+fv)/df=v>0$. The effect of *w* on the upper bound on *f* is $df_U /df=d((w-(1-v))/v)/df=1/v>0$.

Corollary 4: The fisherman effect is positive in one-sided war.

An increase in *f* leads to an increase in the equilibrium war effort by the contestant who does not cease fire in a one-sided war.

Proof:

In case 2 (one-sided war), player 2 ceases fire and player 1 spends $-w+\sqrt{(1-v)w+vwf}$ in equilibrium. The first derivative of player 1's war expenditure with respect to *f* in is positive.

Corollary 5: The fisherman effects in two-sided wars.

Suppose the equilibrium is a two-sided war. The fisherman effect on the war spending of player 2 (the one with more to lose) is always positive. The fisherman effect on the war spending of player 1 is positive if her stake is sufficiently large and negative otherwise.

Proof:

In case 1 (two-sided war), the equilibrium war efforts by player 1 and player 2 are both positive. The first derivatives of the war efforts with respect to *f* are

$$dx_1^{NE}/df = (-1 + 2v + 1/\sqrt{1+8wf})w/2 \qquad (12)$$

$$dx_2^{NE}/df = (1 - 2v + 1/\sqrt{1+8wf})w/2 \qquad (13).$$

From (12), it follows that the fisherman effect on player 1 is positive if and only if player 1's stake is sufficiently large:

$dx_1^{NE}/df>0$

$$\Leftrightarrow (1 - 1/\sqrt{1+8wf})/2 < v \qquad (14).$$

Similarly, (13) implies that the fisherman effect on player 2 is positive if and only if player 2's stake is sufficiently large (player 1's stake is sufficiently small):

$dx_2^{NE}/df>0$

$$\Leftrightarrow v < (1 + 1/\sqrt{1+8wf})/2 \qquad (15).$$

(15) is always true given the assumption that player 2 is the one with the greater stake ($v<1/2$ and $1/2<1-v$).

Corollary 5 says that the fisherman has asymmetric effects on the two players when the difference in the stakes is sufficiently large: the one with a higher stake will increase war effort and her opponent will reduce war effort. In contrast, when the stakes at war are sufficiently close, both contestants will increase war efforts in response to an increase in f.

IV. Concluding remark: on the use of parables in moral persuasion

This paper offers a simple game-theoretic model to show that the threat of a common predator (fisherman) can encourage the contestants at war to fight harder in equilibrium. This conclusion is contrary to the intention of the inventor of the story of the snipe and the clam. This conclusion, however, does not constitute an alternative advice for the contestants at war: Game theoretic models assume that the players are rational and the equilibrium will obtain regardless of the suggestions by diplomats or game theorists. This does not imply the uselessness of this analysis. The implication of this paper can be useful for the "third-parties" who are interested in the contests fought by others. To reduce the war efforts or to build peace in equilibrium, one may have to keep the potential predator (fisherman) off the scene.

References

Hwang, G. George (2002) "Rent-Seeking and Loot-Seeking when Stalemate is a Possible Outcome." Public Choice Society Annual Meeting, San Diego, March 22-24, 2002.

Liu, Hsiang (1979) Chan-Kuo Ts'e. (Trans. J. I. Crump) San Francisco, CA: Chinese Materials Center, Inc.

Tullock, Gordon (1980) "Efficient Rent Seeking," in James. M. Buchanan, Robert D. Tollison and Gordon Tullock (eds), *Toward a Theory of the Rent Seeking Society*, Texas A&M Press, 1980, pp. 269-282.

Figure 1. The contest between the snipe and the clam.

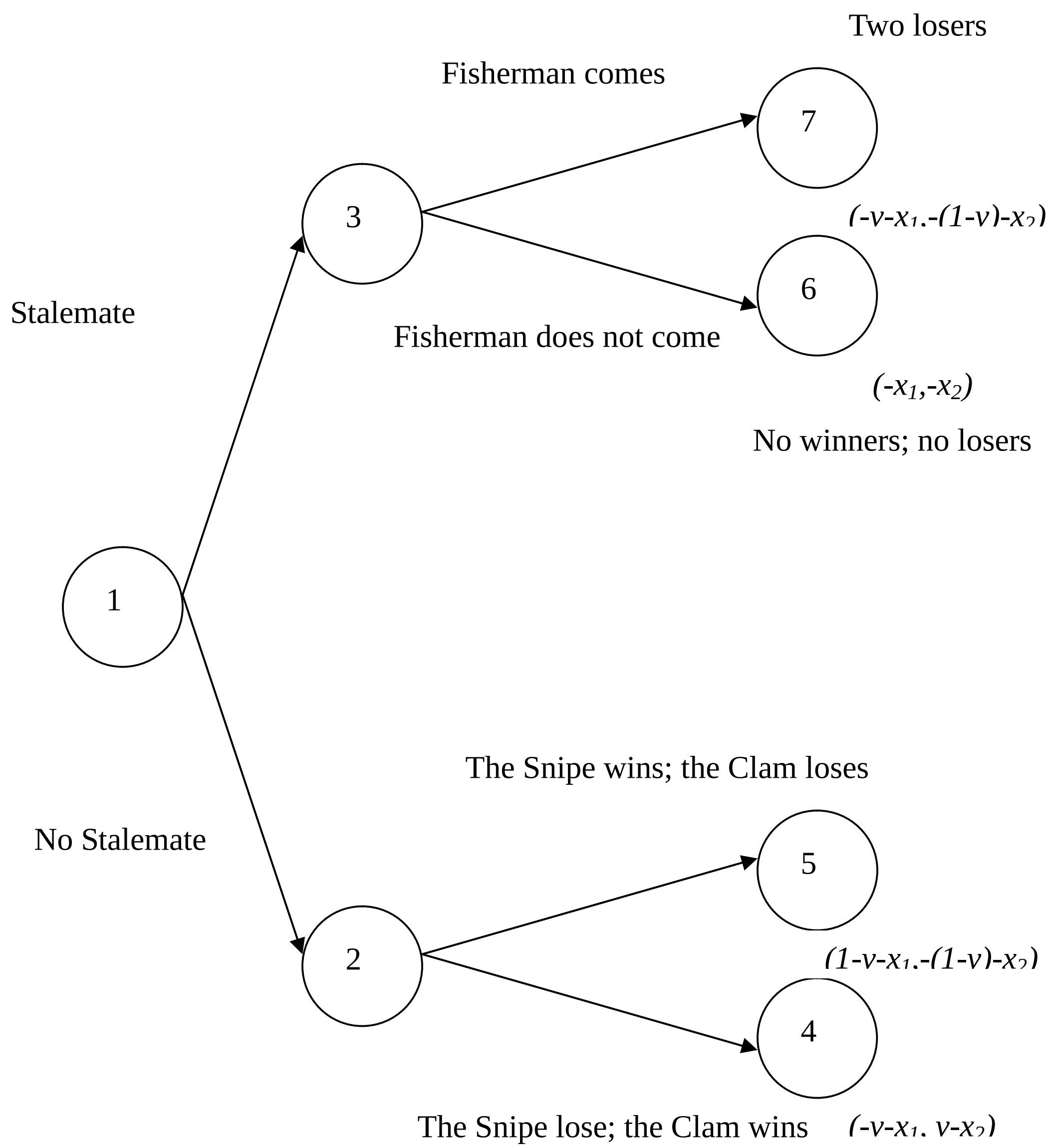

Figure 2. The equilibrium war efforts as functions of the probability of fisherman's arrival.