Partial Outsourcing in Cournot competition

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1 Case Study

Case 1. The outsourcing of microwave oven by US company General Electric(GE) to Samsung

In 1980's GE found it difficult to compete with its low-price, high-quality Japanese competitors, so it outsourced its production of the cheaper models to Samsung, a small Korean company at that time, while producing more advanced models at home. However, in the later years GE found that it was becoming harder to produce microwaves in the USA at a competitive price. In May 1985, GE decided to outsource all microwave production to Samsung. Shortly afterwards, GE quit the market altogether and Samsung became the world's largest manufacturer of microwave ovens. (*The contraction organization, a strategic guide to outsourcing*, by Simon Domberger, 1998, Oxford University Press)

*Case 2.*Outsourcing in the mobile phone industry.

Nokia, Motorola and Ericsson all engage in substantial outsourcing, but at different levels. Ericsson outsources basically all of its product of mobile handsets, Nokia 15-20%, and Motorola 30-40%. (*Partial subcontracting, monitoring cost, and market structure*, by OZ Shy and Rune stenbacka, 2003, working paper) All Ericsson branded phones are produced by an outside supplier Flextronics, who also produces phones for Siemens, Motorola, and Nokia. Ericsson believes that outsourcing allows manufacturers to exploit economies of scale and output flexibility, however, Nokia emphasizes that in-house production allows manufacturers to maximize control and minimize risk. (Cellular News, Oct03, 2002)

*Case 3.*Outsourcing in the car manufacturing industry.

"Japanese firms such as Toyota and Nissan purchase approximately 80 percent of the components required for their vehicles assembly from other firms. By contrast, American auto manufactures such as Ford and General Motors, at round 50 and 30 per cent respectively, buy in substantially smaller proportions of manufactured parts."(*The contraction organization, a strategic guide to outsourcing*, by Simon Domberger, 1998, Oxford University Press)

2 Previous Literature

Yossef Spiegel(1993). *Horizontal subcontracting.* Rand Journal of Economics. Firms with asymmetric convex costs can use horizontal subcontracting to allocate production more efficiently between them and subsequently generate a mutually beneficial surplus.

Morton I. Kamien; Lode Li; Dov Samet(1989). *Bertrand Competition with subcontracting.* Rand journal of economics. By constructing a two-stage game, this article investigates how the possibility of subsequently subcontracting to each other influences rival's initial competition to supply a contract or a market. The incentive for subcontracting comes from the strictly convex production cost.

3 Motivation

Every firm that breaks down its production process or its value chain into series of intermediate steps leading to a final output faces an important choice: how much intermediate output to produce and how much to buy in from other firms? It seems that there is a general trend to purchase more from outside and produce less in house of the requisite intermediate products. However, the ratio of intermediate input purchases to final product vary significantly from firm to firm, within particular industries. Management theorists suggest that sound outsourcing decisions involve identifying and securing core competencies. Core activities stay in-house and non-core activities can be contracted out:"Strategically outsource other activities—including many traditionally considered integral to any company—for which the firm has neither a critical strategic need nor special capabilities."(Quinn and Hilmer 1994:43, *Strategic Outsourcing*, Sloan management review, Summer:43-55)

Outsourcing is blooming. "Even the staid industrial giants of Germany and Japan are starting to sell off factories. They then award long-term contracts to outside suppliers–often the same companies that bought their plants." (Businessweek online, Aug28, 2000) However, one problem faced by the outsourcing firms is to prevent a contractor from becoming a direct competitor. As the case of mobile phone industry, "to avoid such conflicts", Flextronics promised not to make its own products."But as the role of contractors grows, the brand name on a product may not indicate the real power behind an industry." (Businessweek online, Aug28, 2000)

Besides, a lot of literature use strictly convex cost to explain the incentive for firms to outsource. However, outsourcing driven by pursuing economy of scale, which implies a concave cost, has not been deeply explored by theoretical research.

Several questions are forwarded:

1. When deciding whether or not to outsource, firms face a tradeoff between economies of scale and production control. What is the underlying strategic reason for firms to outsource?

2. Why several firms in an industry outsource from the same outside producer?

3. Why most of these firms only partly outsource, and why do they choose different levels of outsourcing?

Questions 1 and 2 can be explained by the incentive for firms to exploit economies of scale based on the concave cost assumption. However, the strictly convex cost assumption is difficult to explain question 2.

To deter entrance of the subcontractor may be an answer for question 3. Suppose that n firms produce both goods, one belonging to their core competency and the other one not. These n firms have incentive to fully outsource the good not in their core competency to an outside provider based on a strictly concave production cost function. Since the production of the good in core competency depends on the production of the good not in

core competency, the outside provider may have incentive to also produce the kind of good which in the n firms' core competence, because it has cost advantage in producing the good not in their core competency. In this case the n firms may restrict the outsourced quantities of the good not in core competency to enhance the average cost of the provider. They partly sacrifice economies of scale to prevent the provider's entrance into the market of the product of their core competency.

4 Model

A model is constructed to explore the conditions for the existence of partial outsourcing equilibria between downstream incumbents and an unique upstream provider. The focus is to investigate how the incumbents will strategically choose the quantity outsourced to deter the entry of the outside provider.

We begin from a model consisting of only two firms, F_0 and F_1. F_0 is a monopoly producing an upstream good, denoted as good A. F_1 is a monopoly producing the downstream good, denoted as good B. Assume the only input for producing good B is good A. F_1 can either produce good A inside or outsource to F_0. However, F_0 has cost advantage in producing good A, which enables it to offer a price to F_1 which is lower than F_1's marginal cost. Although investment of F_1 to produce good B is a sunk cost, F_0 needs to pay an entry fee K if it wants to enter the downstream market to produce good B. We have some basic assumptions.

Assumption 1: One unit of good A can produce one unit of good B. The constant marginal cost in producing good B from good A is normalized to be zero.

Assumption 2: F_1 *has a linear technology in producing good A. The cost function is given as* $C_1(q) = bq$, *with* $b > 0$.

Assumption 3: F_0 *has economies of scale in producing good A. The cost function is given as* $C_0(q) = -cq^2 + bq$, *with* $1 > c > 0, b > c$. *The production ability of* F_0 *is* $q \leq \frac{b}{2c}$.

Assumption 4: If F_0 *invests* K *and enters to produce good* B, *it can achieve the same technology in producing good* B *as* F_1.

Assumption 5: The inverse demand function for good B is $p = max\{0, a - Q\}$, *with* $a > b > ac$ *and* Q *is the total quantity produced by* F_0 *and* F_1.

Assumption 6: All the market structure and cost structures are common knowledge.

In assumption 3, $q \leq \frac{b}{2c}$ implies that F_0 always has a non-negative marginal cost within its production ability. $b > ac$ in assumption 5 is derived from F_0's production ability. Assume that if F_0 is a monopoly in producing good B, its monopoly quantity does not exceed its production ability. Since F_0's monopoly quantity is $q_0^M = \frac{a-b}{2(1-c)}$, $q_0^M < \frac{b}{2c}$ gives $b > ac$.

The timing of the game is:

Stage one. F_0 announces price d for good A to F_1;

Stage two. F_1 signs contract with F_0 on g_1, with g_1 the quantity it want to outsource to F_0.

Stage three. Simultaneously F_0 and F_1 decide $\{q_0, h_1\}$. Here $q_0 > 0$ is the quantity of good B F_0 produces if it enters, otherwise $q_0 = 0$. h_1 is the quantity of good A F_1 produces by itself. If F_0 enters, F_0 and F_1 engage in Cournot competition on quantities $\{q_0, h_1\}$.

F_1's advantage comes from its status as an incumbent and the downward sloping demand curve. In stage two it acts as a Stackelberg leader by setting g_1. However, the more it outsourced, the lower F_0's average cost is, therefore the higher the incentive for F_0 to enter. Connecting with the fact that F_1 can produce good A inside with marginal cost b, F_1 will not outsource if $d > b$. For any $d < b$, F_1 has incentive to outsource to decrease its cost, but a big enough g_1 may induce F_0 to enter. Therefore, instead of fully outsourcing all its demand on good A, F_1 may partly outsource g_1 to F_0 and partly produce inside, to prevent the entry of F_1.

With d and g_1 given, in stage three, F_0 will enter if its profit by entering is bigger than its profit as a pure provider, otherwise it will stay outside.

5 analysis

The solution concept adopted here is subgame perfect Nash equilibrium(SPNE).

Begin from stage 3. In this stage, profits for F_0 and F_1 are

$$\pi_1 = (a - g_1 - h_1 - q_0)(g_1 + h_1) - dg_1 - bh_1$$

and

$$\pi_0 = (a - g_1 - h_1 - q_0)q_0 + dg_1 + c(g_1 + q_0)^2 - b(q_0 + g_1) - KI(q_0 > 0).$$

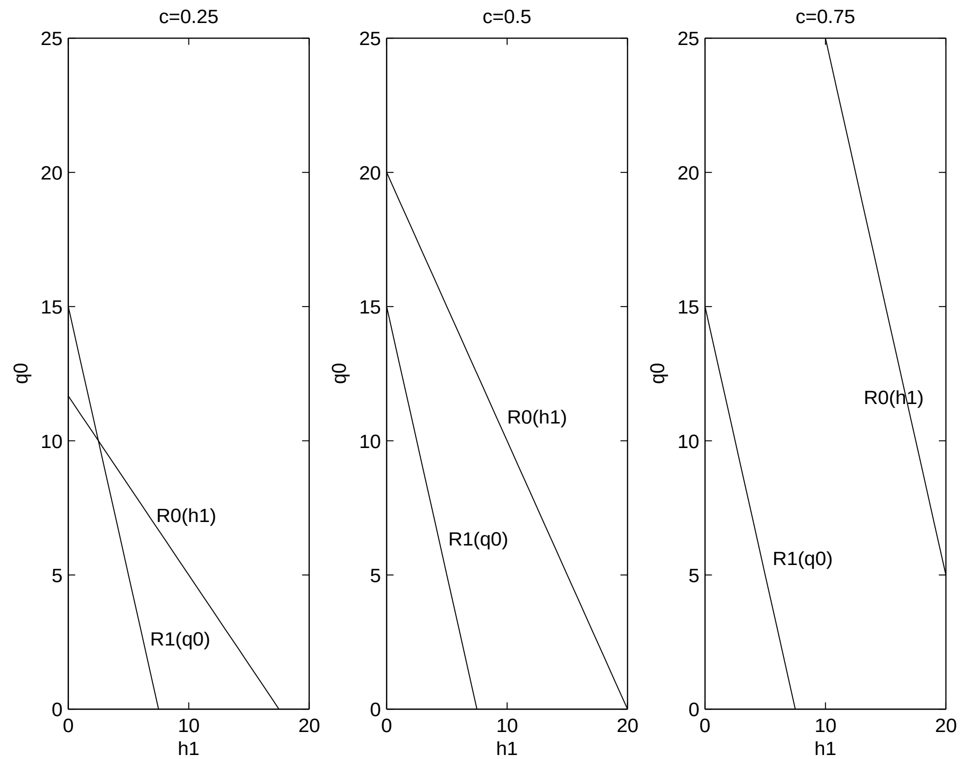
By first order condition, the reaction functions of h_1 and q_0 are

$$R_1(q_0) = \frac{a - g_1 - q_0 - b}{2}$$

and

$$R_0(h_1) = \frac{a - h_1 - g_1 + 2cg_1 - b}{2(1 - c)}.$$

Figure 5 shows the reaction functions according to different values of c. When $c \geq \frac{1}{2}$, F_1 will drop out from the market when it competes directly



with F_0. Therefore, If $c \geq \frac{1}{2}$ and F_0 enters in stage three, only F_0 produces, F_1 will set $h_1 = 0$. In this case, the optimal quantity of q_0 produced by F_0 is:

$$q_0(g_1) = \frac{a + (2c-1)g_1 - b}{2(1-c)}.$$

With $c > \frac{1}{2}$, $q_0(g_1)$ is increasing in g_1. Thus, the more quantity F_1 outsources, the more quantity F_0 produces if it enters.

If $c < \frac{1}{2}$, whether or not F_1 produces $h_1 > 0$ when F_0 enters depends on the value of g_1. By solving the reaction functions, the optimal productions for F_0 and F_1 in stage three are given as

$$h_1(g_1) = \frac{(a-b)(1-2c) - g_1}{3-4c}$$

and

$$q_0(g_1) = \frac{(4c-1)g_1 + (a-b)}{3-4c}.$$

If g_1 is small enough, F_1 will produce a positive value in stage three when it directly competes with F_0, otherwise if F_0 enters, F_1 will produce nothing and only F_0 is active in producing good B.

Assumption 7. $c > \frac{1}{2}$.

Assumption 7 simplifies our analysis for stage three when F_0 enters. Furthermore, when $c > \frac{1}{2}$, $q_0(g_1) = \frac{a+(2c-1)g_1-b}{2(1-c)}$ is increasing in g_1, which gives a straightforward illustration on the positive relationship between g_1 and F_0's incentive to enter.

6 F_0 and F_1's Strategies in Stage Three

In stage three F_0 chooses to enter or not by comparing its profits in these two cases. For any d and for any g_1, if F_0 stays outside, its profit is given by

$$\pi_0^{NE} = dg_1 + cg_1^2 - bg_1;$$

if it enters, its profit is given by

$$\pi_0^{E} = (a - g_1 - q_0)q_0 + dg_1 + c(g_1 + q_0)^2 - b(g_1 + q_0) - K.$$

The optimal q_0 which can be chosen by F_0 is restricted by its production ability, $q_0 + g_1 \leq \frac{b}{2c}$, and the market size, $a - Q \geq 0$. For F_0 to set its optimal quantity $q_0 = q_0(g_1) = \frac{a+(2c-1)g_1-b}{2(1-c)}$, with

$$q_0(g_1) + g_1 \leq \frac{b}{2c},$$

and

$$q_0(g_1) + g_1 \leq a$$

hold. From $a > b > ac$ in assumption 5, the constraint of production ability is more restrictive. To satisfy this constraint, we need $g_1 < \frac{b}{c} - a$. Therefore we have F_0's production when it enters as below.

$$q_0^* = \begin{cases} \dfrac{a + (2c-1)g_1 - b}{2(1-c)} & \text{if} \quad g_1 \leq \dfrac{b}{c} - a \\ \dfrac{b}{2c} - g_1 & \text{if} \quad \dfrac{b}{2c} \geq g_1 > \dfrac{b}{c} - a \end{cases}$$

Denote the surplus F_0 can win by entering as S, which is the difference between F_0's profits if it enters and if it stays outside:

$$\begin{aligned} S &= \pi_0^E - \pi_0^{NE} \\ &= (a - g_1 - q_0)q_0 + 2cg_1q_0 + cq_0^2 - bq_0 - K. \end{aligned}$$

When $S > 0$, F_0 for sure enters; when $S < 0$, F_0 for sure stays outside. When $S = 0$, F_0 is indifferent. Assume that F_0 will stay outside when $S = 0$. This holds when there are transaction costs correlated with investment and entering a market.

Lemma 1. *When $g_1 \leq \bar{g}_1$ or $g_1 \geq \bar{g}_1'$, F_0 will not enter. Here*

$$\bar{g}_1 = \begin{cases} \dfrac{2\sqrt{K(1-c)} - (a-b)}{2c-1} & \textit{if} \quad K \leq \dfrac{(1-c)(2ac-b)^2}{4c^2} \\ \dfrac{(1+2c)b - 2ac - \sqrt{(2ac-b)^2 - 16Kc^3}}{4c^2} & \textit{if} \quad K > \dfrac{(1-c)(2ac-b)^2}{4c^2} \end{cases}$$

and

$$\bar{g}_1' = \frac{(1+2c)b - 2ac + \sqrt{(2ac-b)^2 - 16Kc^3}}{4c^2}.$$

Proof. If F_0 enters, it produces $q_0 = q_0^*$. Inserting $q_0 = q_0^*$ into S, the total surplus of F_0 is

$$S^* = \begin{cases} \dfrac{[a-b+(2c-1)g_1]^2}{4(1-c)} - K & \text{if} \quad g_1 \leq \dfrac{b}{c} - a \\ \dfrac{(b-2cg1)(2ac-b-bc+2c^2g1)}{4c^2} - K & \text{if} \quad \dfrac{b}{2c} \geq g_1 > \dfrac{b}{c} - a \end{cases}$$

When $g_1 < \frac{b}{c} - a$, S^* is convex in g_1. In this range, F_0 will stay outside if

$$\begin{aligned} S^* \leq 0 &\Rightarrow \frac{[a-b+(2c-1)g_1]^2}{4(1-c)} - K \leq 0 \\ &\Rightarrow g_1 \leq \frac{2\sqrt{K(1-c)} - (a-b)}{2c-1}, \end{aligned}$$

with

$$\frac{2\sqrt{K(1-c)} - (a-b)}{2c-1} \leq \frac{b}{c} - a \Leftrightarrow K \leq \frac{(1-c)(2ac-b)^2}{4c^2}.$$

Therefore, if $K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$, F_0 will not enter if $g_1 \leq \frac{2\sqrt{K(1-c)}-(a-b)}{2c-1}$.

Suppose that instead $K > \frac{(1-c)(2ac-b)^2}{4c^2}$. Now F_0's surplus by entering is

$$S^* = \frac{(b-2cg_1)(2ac-b-bc+2c^2g_1)}{4c^2} - K,$$

which is concave in g_1. To deter F_0's entry,

$$\begin{aligned} S^* \leq 0 \Rightarrow & \frac{(b-2cg1)(2ac-b-bc+2c^2g1)}{4c^2} - K \leq 0 \\ & \Rightarrow g_1 \leq \frac{(1+2c)b-2ac-\sqrt{(2ac-b)^2-16Kc^3}}{4c^2}, \\ & \quad \text{or } \; g_1 \geq \frac{(1+2c)b-2ac+\sqrt{(2ac-b)^2-16Kc^3}}{4c^2}. \end{aligned}$$

To summarize, the critical values of g_1, denoted as $\bar{g}_1$ and $\bar{g}_1'$, is given as

$$\bar{g}_1 = \begin{cases} \dfrac{2\sqrt{K(1-c)}-(a-b)}{2c-1} & \text{if} \quad K \leq \dfrac{(1-c)(2ac-b)^2}{4c^2} \\ \dfrac{(1+2c)b-2ac-\sqrt{(2ac-b)^2-16Kc^3}}{4c^2} & \text{if} \quad K > \dfrac{(1-c)(2ac-b)^2}{4c^2} \end{cases}$$

and

$$\bar{g}_1' = \frac{(1+2c)b-2ac+\sqrt{(2ac-b)^2-16Kc^3}}{4c^2}.$$

If $g_1 \leq \bar{g}_1$ or $g_1 \geq \bar{g}_1'$, F_0 will not enter; Otherwise F_0 will enter. □

The maximized S^* is achieved as $S^* = \frac{(2ac-b)^2}{16c^3} - K$ when $g_1 = g_1^* = \frac{(1+2c)b-2ac}{4c^2}$. At $g_1 = g_1^*$, F_0 has the strongest incentive to enter. For $g_1 < g_1^*$, when $g_1 = 0$, the minimum S^* is achieved as $\frac{(a-b)^2}{4(1-c)} - K$, F_0's monopoly profit if it enters without any outsourcing. When $g_1 > \frac{b}{2c} - \frac{2ac-b}{4c^2}$, S^* is decreasing in g_1. This gives a possibility for F_1 to deter F_0's entry by over outsourcing, i.e. outsourcing a very big g_1 to exhaust F_0's production ability.

The range of K in which F_0 can either partial outsource or over outsource to deter F_1's entry, is $\frac{(a-b)^2}{4(1-c)} < K < \frac{(2ac-b)^2}{16c^3}$. Assume that F_0 will choose partial outsource if it has these two choices, because in stage two if F_0 has the power to decide whether or not to accept the g_1 offered by F_1, it may not accept a big amount since this makes entry unprofitable; even if it accepts, there exists holding-up risk which may make F_1 to be uncomfortable to deter entry by overoutsourcing.

Assumption 8. If F_1 can deter F_0's entry by either restricting or enlarging the quantity outsourced, it will choose restricting the quantity outsourced.

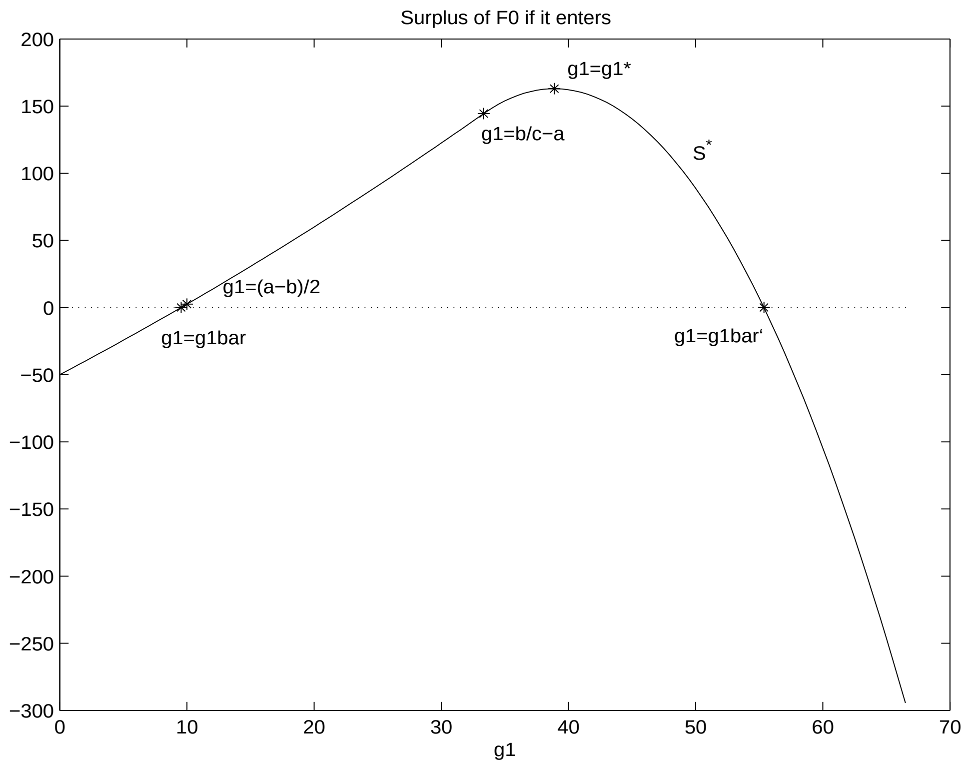


Figure 1: The graph sets $a = 100, b = 80, c = 0.6, K = 300$. The production ability of F_0 is $\frac{b}{2c} = 66.7$.

Figure 1 above illustrates the total surplus of F_1 when it enters. The curve of S^* is convex at first; after the point $g_1 = \frac{b}{c} - a$, it becomes concave. In the range $0 \leq g_1 \leq \frac{b}{2c} - \frac{2ac-b}{4c^2}$, S^* is increasing; then after $g_1 = \frac{b}{2c} - \frac{2ac-b}{4c^2}$, S^* is decreasing with g_1.

The lowest S^* is $\frac{(a-b)^2}{4(1-c)} - K$. If $K < \frac{(a-b)^2}{4(1-c)}$, $S^* > 0$ for any g_1, F_1 for sure enters at any quantity F_1 outsources. The highest S^* is $\frac{(2ac-b)^2}{16c^3} - K$. If $K \geq \frac{(2ac-b)^2}{16c^3}$, $S^* < 0$ for any g_1, F_0 for sure stays outside. In this case F_1 is better to fully outsource to fully utilize the cost advantage. Our analysis will exclude these ranges of K and focus on the values of K for which entry deterrence is necessary and possible.

If $\frac{(a-b)^2}{4(1-c)} \leq K < \frac{(2ac-b)^2}{16c^3}$, F_1 has the potential to deter F_0's entry by partial outsourcing $g_1 \leq \bar{g}_1$. From Lemma 1, $\bar{g}_1$ has different functional forms when $K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$ or not. Note that $\frac{(2ac-b)^2}{16c^3} > \frac{(1-c)(2ac-b)^2}{4c^2}$. Our following analysis focuses on $\frac{(a-b)^2}{4(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$, because including $\frac{(1-c)(2ac-b)^2}{4c^2} < K < \frac{(2ac-b)^2}{16c^3}$ will quite complicates our analysis without giving much new insight.

Assumption 9. $\frac{(a-b)^2}{4(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$.

In the last stage, F_0 has an unique pure strategy: enters if $\bar{g}_1 < g_1 < \bar{g}_1'$, otherwise stays outside.

Now look for F_1's strategy in stage three. F_1 has two choices: either $h_1 > 0$ or $h_1 = 0$. If F_1 expects that F_0 will enter in the last stage, $h_1 = 0$. If F_1 expects that F_0 will stay outside, h_1 may either be zero or positive.

F_1 knows F_0's strategy in stage three. Therefore, if $\bar{g}_1 < g_1 < \bar{g}_1'$, F_1 knows that F_0 will enter and F_1's best response is $h_1 = 0$. If $g_1 \leq \bar{g}_1$ or $g_1 \geq \bar{g}_1'$, F_1 knows that F_0 will not enter. In this case with d and g_1 given, F_1's problem is to choose h_1 to solve

$$\begin{aligned} \max \quad & \pi_1 = (a - g_1 - h_1)(g_1 + h_1) - dg_1 - bh_1 \\ \text{s.t.} \quad & h_1 \geq 0. \end{aligned}$$

Solving this gives

$$h_1 = \begin{cases} \dfrac{a-b}{2} - g_1 & \text{if} \quad g_1 \leq \dfrac{a-b}{2} \\ 0 & \text{if} \quad g_1 > \dfrac{a-b}{2} \end{cases}$$

If F_0 stays outside in stage three, F_1 will produce a positive quantity inside if and only if $g_1 < \frac{a-b}{2}$. Therefore, either F_0' entry or $g_1 > \frac{a-b}{2}$ induces F_1 to produce nothing in stage three, otherwise F_1 will produce inside in stage three, in this case it is partial outsourcing.

To summarize, in stage three there is an unique pure strategy Nash equilibrium. In equilibrium F_0's strategy is

{ Entering if $\bar{g}_1 < g_1 < \bar{g}_1'$; otherwise staying outside.}

F_1's strategy is:

{ No producing inside if $g_1 > \min\{\bar{g}_1, \frac{a-b}{2}\}$; producing $h_1 = \frac{a-b}{2} - g_1$ inside if $g_1 \leq \min\{\bar{g}_1, \frac{a-b}{2}\}$}.

7 F_1's Strategy in Stage Two

In step 2 F_1 is strategically setting g_1 taking into account the potential for F_0 to enter. There are two kinds of possibilities. The fist one is that F_1 outsources $g_1 > \bar{g}_1$, and in stage three $q_0 > 0$, $h_1 = 0$. The second one is that F_1 outsources $g_1 \leq \bar{g}_1$, in stage three $q_0 = 0$, and whether $h_1 > 0$ or $h_1 = 0$ depends on the value of g_1. There are three cases.

Case i. In case i, F_1 outsources $g_1 > \bar{g}_1$. Its problem is

$$\begin{aligned} \max \quad & \pi_1 = (a - g_1 - q_0 - d)g_1 \\ \text{s.t.} \quad & g_1 > \bar{g}_1 \qquad (1) \\ & q_0 = q_0^* = \frac{a + (2c-1)g_1 - b}{2(1-c)} \end{aligned}$$

Substituting $q_0 = q_0^*$ into the profit function, the optimal g_1 is solved by first order condition:

$$g_1^i(d) = \frac{a + b - 2ac}{2} - (1-c)d.$$

If condition (1) holds when $g_1 = g_1^i(d)$, it is necessary that

$$g_1^i(d) > \bar{g}_1 \Rightarrow d < \frac{a + b - 2ac}{2(1-c)} - \frac{2\sqrt{K(1-c)} - (a-b)}{(2c-1)(1-c)} = d_1.$$

Note that $d_1 < b$ is true. If $d < d_1$, $g_1 = g_1^i(d)$. If $d \geq d_1$, F_1 will outsource g_1 only a little bigger than $\bar{g}_1$, because π_1 is strictly concave in g_1 and with $g_1 > \bar{g}_1 > g_1^i(d)$, π_1 is decreasing in g_1. Write the quantity F_1 outsources when $d \geq d_1$ as $g_1 = \bar{g}_1 + \varepsilon_n$, with $\lim_{n\to\infty} \varepsilon_n = 0$.

If F_1 outsources $g_1 < \bar{g}_1$, F_0 will not enter. Depending on whether or not F_1 produces inside, we have case ii and case iii.

Case ii. In case ii, F_1 is outsourcing $g_1 \leq \bar{g}_1$ without production inside. Its problem is

$$\begin{aligned} \max \quad & \pi_1 = (a - g_1 - d)g_1 \\ \text{s.t.} \quad & g_1 \leq \bar{g}_1 \qquad (2) \\ & g_1 \geq \frac{a-b}{2} \qquad (3) \end{aligned}$$

Constraints (2) and (3) implies that $\bar{g}_1 \geq \frac{a-b}{2}$, which gives $K \geq \frac{(2c+1)^2(a-b)^2}{16(1-c)}$. Thus, case ii is possible only when $\frac{(2c+1)^2(a-b)^2}{16(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$. This implies that $b \geq \frac{3ac}{2+c}$, more restrictive than our assumption that $b > ac$. Assume this is true for case ii.

The optimal g_1 which maximizes π_1, denotes as $g_1^{ii}(d)$, is given by the first order condition:

$$g_1^{ii}(d) = \frac{a-d}{2}.$$

Note that $g_1^{ii}(d) > \frac{a-b}{2}$ with $d < b$. Case ii can be divided into two subcases:

Case ii.1. $g_1^{ii}(d) \leq \bar{g}_1$. Here $g_1^{ii}(d) \leq \bar{g}_1 \Rightarrow d \geq d_2$, with $d_2 = \frac{(1+2c)a-2b-4\sqrt{K(1-c)}}{2c-1}$. In this case $g_1 = g_1^{ii}(d) = \frac{a-d}{2}$.

Case ii.2. $\bar{g}_1 < g_1^{ii}(d)$, i.e. $d < d_2$. In this case F_1 sets $g_1 = \bar{g}_1$.

Therefore if it is optimal for F_1 to fully outsource with F_0 staying outside, F_1 either sets $g_1 = g_1^{ii}(d)$ or restricts $g_1 = \bar{g}_1$, according to the value of d.

Case iii. F_1 is outsourcing $g_1 \leq \bar{g}_1$ with production inside. Its problem is

$$\begin{aligned} \max \quad & \pi_1 = (a - g_1 - h_1)(g_1 + h_1) - dg_1 - bh_1 \\ \text{s.t.} \quad & g_1 \leq \bar{g}_1 && (4) \\ & g_1 < \frac{a-b}{2} && (5) \end{aligned}$$

Inserting $h_1 = \frac{a-b}{2} - g_1$ into F_1's problem, it can be rewritten as

$$\begin{aligned} \max \quad & \pi_1 = (a - \frac{a-b}{2})\frac{a-b}{2} - dg_1 - b(\frac{a-b}{2} - g_1) \\ \text{s.t.} \quad & g_1 \leq \bar{g}_1 && (4) \\ & g_1 < \frac{a-b}{2} && (5) \end{aligned}$$

Since $\frac{\partial \pi_1}{\partial g_1} = b - d \geq 0$, F_1 will outsource as much as possible, hence $g_1 = \min\{\bar{g}_1, \frac{a-b}{2} - \varepsilon_n\}$, with $\lim_{n\to\infty} \varepsilon_n = 0$. There are two subcases:

Case iii.1. If $\bar{g}_1 < \frac{a-b}{2} - \varepsilon_n$, i.e. $K < \frac{(2c+1)^2(a-b)^2}{16(1-c)}$, $g_1 = \bar{g}_1$.

Case iii.2. If $\bar{g}_1 \geq \frac{a-b}{2} - \varepsilon_n$, i.e. $K \geq \frac{(2c+1)^2(a-b)^2}{16(1-c)}$, $g_1 = \frac{a-b}{2} - \varepsilon_n$.

Next we compare F_1's profits in different cases, according to different values of K.

Category 1. $\frac{(a-b)^2}{4(1-c)} \leq K < \min\{\frac{(2c+1)^2(a-b)^2}{16(1-c)}, \frac{(1-c)(2ac-b)^2}{4c^2}\}$.

When K falls in this category, only case i and case iii.1 are possible. In case i, F_1's optimal profit is

$$\pi_1^i = (a - g_1 - q_0^* - d)g_1,$$

with $g_1 = g_1^i(d)$ if $d < d_1$; otherwise $g_1 = \bar{g}_1 + \varepsilon_n$, with $\lim_{n\to\infty} \varepsilon_n = 0$. In case iii.1, with $g_1 = \bar{g}_1$, F_1's optimal profit is

$$\pi_1^{iii.1} = (a - \frac{a-b}{2})\frac{a-b}{2} - d\bar{g}_1 - b(\frac{a-b}{2} - \bar{g}_1).$$

If $d \geq d_1$, compare $g_1 = \bar{g}_1 + \varepsilon_n$ in case i and $g_1 = \bar{g}_1$ in case iii.1. In case i, F_1 chooses g_1 as small as possible as long as $g_1 > \bar{g}_1$. Taking the limitation of ε_n, $g_1 = \bar{g}_1$ gives the sup of π_1 in case i. We can directly compare profits in these two cases by inserting $g_1 = \bar{g}_1$ into the profit functions:

$$\begin{aligned}&\pi_1^{iii.1}(\bar{g}_1) - \sup\ \pi_1^i \\ &= \frac{(a-b)^2(1-c) + 2(2c-1)(a-b)\bar{g}_1 + 2\bar{g}_1^2}{4(1-c)} > 0\end{aligned}$$

Therefore, case iii.1 dominates case i. We have lemma 1 below.

Lemma 2. *Suppose* $\frac{(a-b)^2}{4(1-c)} \leq K < min\{\frac{(2c+1)^2(a-b)^2}{16(1-c)}, \frac{(1-c)(2ac-b)^2}{4c^2}\}$. *If* $d \geq d_1$ *in stage 1,* F_1 *partial outsources* $g_1 = \bar{g}_1$ *in stage 2 to deter* F_0*'s entrance, then produces inside* $h_1 = \frac{a-b}{2} - \bar{g}_1$ *in stage 3.*

If $d < d_1$, in case i F_1 sets $g_1 = g_1^i(d)$, its optimal quantity when F_0 enters. In case iii.1, F_1 sets $g_1 = \bar{g}_1$ to deter F_0's entry, then produces $h_1 = \frac{a-b}{2} - \bar{g}_1$ in stage three. The intuition is, if d is low enough, F_1 may better to outsource more and let F_0 enter. If we calculate the difference of the profits in these two cases,

$$\begin{aligned}&\pi_1^{iii.1}(\bar{g}_1) - \pi_1^i(g_1 = g_1^i(d)) \\ &= (a - \frac{a-b}{2})\frac{a-b}{2} - d\bar{g}_1 - b(\frac{a-b}{2} - \bar{g}_1) - (a - g_1^i(d) - q_0(g_1^i(d)) - d)g_1^i(d).\end{aligned}$$

Since $\frac{\partial^2[\pi_1^{iii.1}(\bar{g}_1) - \pi_1^i(g_1 = g_1^i(d))]}{\partial d^2} = c - 1 < 0$, the optimal d to maximize this difference as $d^* = \frac{a+b-2ac-2\bar{g}_1}{2(1-c)}$, which is exactly d_1 if we insert the value of $\bar{g}_1$ into d^*. For any $d < d_1$, the difference is increasing. By letting the difference to be zero, there exists a $\hat{d}$, such that when $d = \hat{d}$, F_1 is indifferent with these two cases; when $d < \hat{d}$, F_1 is better off to fully outsource $g_1^i(d)$ with F_0 entering in stage 3; when $d > \hat{d}$, F_1 is better to partial outsource $\bar{g}_1$ to deter F_0's entrance, then produce $h_1 = \frac{a-b}{2} - \bar{g}_1$ inside. This is consistent with the intuition.

Lemma 3. *Suppose* $\frac{(a-b)^2}{4(1-c)} \leq K < min\{\frac{(2c+1)^2(a-b)^2}{16(1-c)}, \frac{(1-c)(2ac-b)^2}{4c^2}\}$. *If* d *is low enough, i.e.* $d < \hat{d}$, *in stage 2* F_1 *fully outsources* $g_1^i(d)$, *then* F_0 *enters* *in stage 3; if $\hat{d} \leq d < d_1$, F_1 partial outsources $\bar{g}_1$ to deter F_0's entry, and produces $\frac{a-b}{2} - \bar{g}_1$ inside.*

Assume $b \geq \frac{3ac}{2+c}$ is true, then we have category two in which case ii is also possible.

Category 2. $\frac{(2c+1)^2(a-b)^2}{16(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$.

In this category, we need to compare F_1's profit in case i, case ii, and case iii.2. Begin from comparing case ii and case iii.2.

In both cases F_1 is a monopoly with F_0 staying outside. In case ii, if $d \geq d_2$, $g_1 = g_1^{ii}(d) = \frac{a-d}{2}$; if $d < d_2$, $g_1 = \bar{g}_1$. Profit for F_1 in this case is

$$\pi_1 = (a - g_1 - d)g_1.$$

In case iii.2, $g_1 = \frac{a-b}{2} - \varepsilon_n$ for any d. Profit for F_1 in this case is

$$\pi_1 = (a - \frac{a-b}{2})\frac{a-b}{2} - dg_1 - b(\frac{a-b}{2} - g_1).$$

When $d \geq d_2$, in case ii F_1 is producing $\frac{a-d}{2}$ with marginal cost d. In case iii.2, F_1 restricts its total production to $\frac{a-b}{2}$, while producing ε_n inside with marginal cost b, which is greater than d. Therefore, case ii dominates case iii.2 for F_1.

When $d < d_2$, in case ii F_1 outsources $g_1 = \bar{g}_1$ to deter F_0's entry, then produce nothing inside. Since $\frac{a-b}{2} \leq \bar{g}_1$, case ii also dominates case iii.2 for F_1, because it produces more with a lower average cost in case ii.

Denote the outcome from comparing case ii and case iii.2 as case ii2:

If $d \geq d_2$, $g_1 = \frac{a-d}{2}$, $\pi_1 = (a - \frac{a-d}{2} - d)\frac{a-d}{2}$; if $d < d_2$, $g_1 = \bar{g}_1$, $\pi_1 = (a - \bar{g}_1 - d)\bar{g}_1$.

Now compare case i and case ii2. In case i, $\pi_1 = (a - g_1 - q_0(g_1) - d)g_1$. If $d < d_1$, $g_1 = g_1^i(d)$; if $d \geq d_1$, $g_1 = \bar{g}_1 + \varepsilon_n$. Note that $d_1 - d_2 = \frac{a-b-4\sqrt{K(1-c)}}{2(1-c)} < 0$, and $d_2 - b = \frac{(1+2c)(a-b)-4\sqrt{K(1-c)}}{2c-1} \leq 0$, hence $d_1 < d_2 \leq b$.

We need to consider three possibilities:

The first one is $d \geq d_2$. In case i, F_1 outsources $\bar{g}_1 + \varepsilon_n$ then in stage 3 F_0 enters. In case ii2, F_1 outsources $\frac{a-d}{2}$, and is a monopoly. Here $\frac{a-d}{2} < \bar{g}_1$. In both cases F_1 faces a marginal cost d, however, case ii2 gives F_1 its monopoly profit. Thus, case ii2 dominates case i for F_1.

Lemma 4. *Suppose $\frac{(2c+1)^2(a-b)^2}{16(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$. If $d \geq d_2$ in stage one, then F_1 fully outsources $g_1 = \frac{a-d}{2}$ in stage two. F_1 is a monopoly and produces nothing inside.*

The second possibility is $d_1 \leq d < d_2$. In case i F_1 outsources $g_1 = \bar{g}_1 + \varepsilon_n$ with F_0 entering; in case ii2, F_1 outsources $\bar{g}_1$ and is a monopoly. As our analysis before, in case i F_1 is better off if ε_n is becoming smaller, with its limitation to be zero. When $\varepsilon_n = 0$, F_1 becomes a monopoly just as in case ii2. Therefore, case ii2 dominates case i.1.

Lemma 5. *Suppose* $\frac{(2c+1)^2(a-b)^2}{16(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$. *If* $d_1 \leq d < d_2$ *in stage one,* F_1 *outsources* $g_1 = \bar{g}_1$ *in stage two.* F_1 *is a monopoly and produces nothing inside.*

The third possibility is $d < d_1$. In case i F_1 fully outsources $g_1 = g_1^i(d)$, its optimal quantity when F_0 enters. In case ii2 it restricts the quantity outsourced to be $\bar{g}_1$, which guarantees itself the monopoly status. Intuition is that when d is very low, F_1 may be better off to outsource more with F_0's entry. Calculating $\pi_1^i(g_1^i(d)) - \pi_1^{ii2}(\bar{g}_1)$, the difference between F_1's profits in these two cases, we can have that

$$\frac{\partial^2[\pi_1^i(g_1^i(d)) - \pi_1^{ii2}(\bar{g}_1)]}{\partial d^2} = 1 - c > 0,$$

which is strictly convex in d. Using first order condition, the optimal d which minimize the difference is exactly d_1. Solving $\pi_1^i(g_1^i(d)) - \pi_1^{ii2}(\bar{g}_1) = 0$, there exists a $\tilde{d}$ satisfying $\tilde{d} < d_1$, at which $\pi_1^i(g_1^i(d)) = \pi_1^{ii2}(\bar{g}_1)$. When $d < \tilde{d}$, $\pi_1^i(g_1^i(d)) > \pi_1^{ii2}(\bar{g}_1)$, F_1 is better off to fully outsource $g_1 = g_1^i(d)$, and let F_0 enters. When $d \geq \tilde{d}$, F_1 is better off to only outsource $\bar{g}_1$ to deter F_0's entry.

Lemma 6. *Suppose* $\frac{(2c+1)^2(a-b)^2}{16(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$. *If* $d < d_1$ *in stage 1, When* d *is low enough, i.e.* $d < \tilde{d}$, F_1 *fully outsources* $g_1^i(d)$, *then* F_0 *enters in stage three; otherwise* F_1 *only outsources* $\bar{g}_1$ *to be a monopoly. In both cases* F_1 *has no inside production.*

8 F_0's strategy in stage one

In stage three F_0 is setting d according to strategies of F_1 in stage two. As above, we analyze its strategy according to two categories.

Category 1. $\frac{(a-b)^2}{4(1-c)} \leq K < \min\{\frac{(2c+1)^2(a-b)^2}{16(1-c)}, \frac{(1-c)(2ac-b)^2}{4c^2}\}$.

If $d > d_1$, from Lemma 2, F_1 partial outsources $g_1 = \bar{g}_1$, and produce $\frac{a-b}{2} - \bar{g}_1$ inside. F_0's profit is

$$\pi_0 = (d - b)\bar{g}_1 + c\bar{g}_1^2.$$

If $\hat{d} < d \leq d_1$, from Lemma 3, F_1 outsources $\bar{g}_1$ without any production inside. F_0 will stay outside, and its profit function is the same as above.

If $d < \hat{d}$, from Lemma 3, F_1 will outsource $g_1 = g_1^i(d)$ with F_0 entering in stage 3. F_0's profit function is

$$\pi_0 = (a - g_1^i(d) - q_0)q_0 + dg_1^i(d) + c(g_1^i(d))^2 - bg_1^i(d) - K,$$

and in stage 3 F_0 is maximizing its profit by choosing $q_0 = q_0(g_1^i(d))$.

In stage one F_0 chooses its optimal d by comparing its possible profits. If d lies between $\hat{d}$ and d_1, F_0 faces the same profit function, with $g_1 = \bar{g}_1$ and total produciton cost fixed. Therefore, F_0 will charge d as high as possible, as long as it is lower than b. However, F_0 will compare its profit in this case with its profit when it enters by charging a very low d. However, the later case will not be optimal for F_0, if the value of K is not very small. Because in this case F_1 is outsourcing much since d is small, therefore when F_0 enters its quantity to produce for good B is restricted to be low. Besides, F_0 can not get much profit from outsourcing, or may even lose in outsourcing. Thus we have theory 1 below.

Theorem 1. *Suppose* $\frac{(a-b)^2}{4(1-c)} \leq K < min\{\frac{(2c+1)^2(a-b)^2}{16(1-c)}, \frac{(1-c)(2ac-b)^2}{4c^2}\}$. *when K is not very small, the unique SPNE is that F_0 charges d as high as possible, but lower than b; F_1 partial outsources $\bar{g}_1$ to F_0 and partial produces $\frac{a-b}{2} - \bar{g}_1$ inside.*

Category 2. $\frac{(2c+1)^2(a-b)^2}{16(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$.

In this category, F_0 compares its profit according to Lemma 4, 5 and 6. If $d \geq d_2$, $g_1 = \frac{a-d}{2}$, and F_0's profit is

$$\pi_0 = (d - b)\frac{a - d}{2} + c(\frac{a - d}{2})^2.$$

If $\tilde{d} \leq d < d_2$, $g_1 = \bar{g}_1$, and F_1's profit is

$$\pi_0 = (d - b)\bar{g}_1 + c\bar{g}_1^2.$$

If $d < \tilde{d}$, F_1 outsources $g_1 = g_1^i(d)$ with F_0 entering in stage 3. F_0's profit is

$$\pi_0 = (a - g_1^i(d) - q_0)q_0 + dg_1^i(d) + c(g_1^i(d))^2 - bg_1^i(d) - K,$$

and in stage 3 F_0 is maximizing its profit by choosing $q_0 = q_0(g_1^i(d))$. For any d there does not exist partial outsourcing.

Theorem 2. *If* $\frac{(2c+1)^2(a-b)^2}{16(1-c)} \leq K \leq \frac{(1-c)(2ac-b)^2}{4c^2}$, *there does not exist partial outsourcing equilibrium for F_0 entering or not. when K is not very small, the unique SPNE is that F_1 fully outsources with F_0 staying outside.*