General licensing schemes for a cost-reducing innovation

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Abstract

Optimal combinations of upfront fee and royalty are considered for a cost-reducing innovation in a Cournot oligopoly industry. The consumers are better off, firms are worse off and welfare is increased due to the innovation. The post-innovation price and payoff of any firm is higher with an incumbent innovator. An incumbent innovator sells the license to every firm except perhaps one. This is true for an outsider innovator only for less significant innovations, while for significant innovations, the license is sold to only two firms and a natural duopoly is created. The private value of the patent is increasing in the magnitude of the innovation for both types of innovators. Compared to an outsider, an incumbent producer has higher incentives to develop significant innovations if she assigns a high probability to the event that someone else would succeed to innovate in case she fails, while the converse holds if this probability is small. Finally, for significant innovations, the industry size that provides the highest incentive to innovate increases in the magnitude of the innovation.

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1 Introduction

Patent licensing by means of a combination of upfront fee and royalty is one of the most commonly observed licensing policies in practice [see, e.g., Taylor and Silberstone (1973), Rostoker (1984)]. The theoretical literature on patent licensing has mainly considered three methods of licensing: by means of royalty, upfront fee and auction. This paper attempts to fill the gap in the literature by considering optimal combinations of royalty with either upfront fee or auction for the licensing of a cost-reducing innovation. Further, we carry out the analysis for two possible scenarios: the innovator is an outsider to the industry and arguably the more natural case where she is one of the incumbent producers. For both cases, we characterize the optimal licensing policies and discuss their impact on the price and the structure of the market, payoffs of the agents, and incentives and dissemination of innovation.

The formal analysis of patent licensing was initiated by Arrow (1962). Considering licensing of a cost-reducing innovation by means of uniform linear royalty only, he showed that the innovator's licensing rent in a perfectly competitive industry exceeds the incremental profit of a monopolist due to the innovation and concluded that a perfectly competitive industry provides the innovator with a higher incentive to innovate than does the monopoly. However, Arrow (1962) did not consider the aspect of strategic interaction among firms, which plays a crucial role in an oligopoly. The strategic analysis depends, among other factors, on whether the innovator is an outsider or an incumbent firm.

A cost-reducing innovation is said to be *drastic* [Arrow (1962)] if the monopoly price under the new technology does not exceed the competitive price under the old technology; otherwise, it is *non-drastic*. Clearly, if an incumbent innovator is a monopolist, or if she is endowed with a drastic cost-reducing innovation, she extracts the entire monopoly profit with the new technology.

The same fact is true for an outsider innovator when the industry size is at least two. When an outsider innovator faces a monopolist, then irrespective of whether the innovation is drastic or not, the innovator obtains the difference between the respective monopoly profits with the new and the old technology. Thus, the issue of patent licensing is non-trivial only in case the innovation is non-drastic. For this reason, in this paper, we restrict ourselves to non-drastic innovations.

The interaction of an outsider innovator and the firms in an oligopoly was first studied in a game-theoretic setting independently by Katz and Shapiro (1985, 1986) [hereafter KS (85) and KS (86) respectively] and Kamien and Tauman (1984, 1986) [hereafter KT (84) and KT (86) respectively]. As we have already mentioned, the literature has mainly considered three standard policies of licensing, namely, (1) a flat pre-determined upfront fee, (2) a uniform per-unit royalty payment, and, (3) auctioning off a limited number of licenses through a first-price sealed-bid auction, where the highest bidders pay their bids and get licenses. In what follows, we summarize the main findings of our work and discuss it in relation to the existing literature. We refer to Kamien (1992) for an excellent survey on patent licensing. See also Reinganum (1989) for a comprehensive survey on various aspects of innovation, including licensing.

In this paper, we consider a Cournot oligopoly where the firms are equally efficient in the beginning. The innovator (who is either an outsider or an incumbent firm) has a cost-reducing innovation, which she can license to some or all other firms. Depending on the licensing scheme and the number of licensees, asymmetry arises among the firms. Specifically, we consider a three-stage game, where in the first stage, the innovator announces the royalty rate and the number of licenses to be sold, say m and the firms are invited to bid in a first-price sealed bid auction for the license.[1] Thus the licensing policy is a

[1]The innovator may include a minimum bid for the license, which is required only if she

combination of *auction and uniform linear royalty*, which we call the AR policy. In the second stage, each firm bids for the license and the m highest bidders win the license (ties are resolved at random). The winners pay the innovator their respective bids upfront, in addition to their future royalty payments. In the final stage, all firms compete in quantities. The AR policy can be viewed as a combination of upfront fee and linear royalty, where the firms determine the upfront fee through their bids in the auction.[2]

We show that for both cases of outsider and incumbent innovators, there is a unique subgame-perfect equilibrium outcome. The payoff of the innovator is increasing in the magnitude of the innovation. Moreover, the consumers are better off, firms are worse off and the total welfare is increased as results of the innovation. These conclusions are in line with the existing literature. We further show that the incumbent innovator always obtains strictly more than the total reduction in production cost of the pre-innovation competitive output. This is also true for an outsider innovator, except for the cases where she faces a monopoly or a duopoly. In KT (86) and Kamien, Oren and Tauman (1992) [hereafter KOT (92)], this level is in fact the upper bound of the innovator's payoff. We also show that the equilibrium rate of royalty is zero for both outsider and incumbent innovators for insignificant innovations[3] while for other innovations, the royalty in case of an incumbent innovator is at least as high

intends to sell the license to all firms. In such a case, in the absence of a minimum bid, each firm will bid zero, knowing that it will get the license irrespective of its bid.

[2] An alternative policy would be the combination of *upfront fee and uniform linear royalty* (FR), where the innovator herself fixes the upfront fee along with the rate of royalty. It can be shown that the AR policy is superior to the FR policy if the innovator sells limited number of licensees, while the two are equivalent if the license is sold to all firms. When the optimal AR policy involves selling licenses to all firms, then this policy is a standard combination of upfront fee and royalty and in effect, no auction takes place.

[3] Except for the cases of a duopoly or a triopoly with an incumbent innovator. Then, the licensing policy entails only a royalty and no upfront fee, with the rate of royalty equaling the magnitude of the innovation.

as that of an outsider. Moreover, the post-innovation Cournot price and the payoff of any firm (other than the innovator) are also higher in case of an incumbent innovator. The rest of our results depend on whether the innovator is an incumbent firm or not.

When an outsider innovator faces a monopolist, she extracts the difference between the respective monopoly profits with the new and the old technology through an upfront fee. In case of a duopoly, the license is sold to both firms. When the industry size is at least three, sufficiently significant innovations are sold to only two firms, a phenomenon that is somewhat in line with the empirical findings of Firestone (1971) and Caves, Crookell and Killing (1983), who pointed out that patents from independent innovators are often licensed exclusively. When the innovation is sold to only two firms, all non-licensee firms drop out of the market and a natural duopoly is created. As for less significant innovations, they are sold to all firms, except perhaps one.

In case of an incumbent innovator, when the number of other firms is at most two, each firm becomes a licensee and the licensing policy involves only royalty and no upfront fee, where the rate of royalty equals the magnitude of the innovation. When there are at least three other firms, the innovation is sold to all firms, except perhaps one. This result differs from that of KS (85), where it was found that while minor innovations will be licensed when firms are equally efficient prior to the innovation, exclusion will occur in case of major innovations. In contrast, we show that when the innovator is one of the incumbent firms in a duopoly (or a triopoly), it is never optimal for the innovator to exclude any firm. It should be noted that KS (85) have considered only upfront fee policy, while the optimal policy in case of a duopoly (or a triopoly) consists only of a royalty and its optimal level coincides with the magnitude of the innovation. As for general industry sizes, we show that it is never optimal for an incumbent innovator to exclude more than one rival.

Considering licensing by means of either just an upfront fee or just a linear royalty, KT (86) have shown that for a Cournot oligopoly with linear demand, licensing by upfront fee is better than royalty both for the innovator and from social point of view. KOT (92) have extended these results to general demand and have shown that among the three standard policies, licensing by means of royalty is inferior to the other two while upfront fee is inferior to auction not only for the innovator, but also for consumers. Although the theoretical literature shows the superiority of both auction and upfront fee to royalty, as licensing schemes, royalties and combination of upfront fee and royalty policies are more prevalent than other standard forms of licensing. In the oft-quoted survey of Rostoker (1984) of corporate licensing, upfront fee plus royalty policy was observed in 46% of cases, whereas licensing by means of exclusive royalty was observed in 39% of the firms surveyed. An attempt to bridge the discrepancy between empirical observations and theoretical predictions was made by Wang (1998), who considered a model of Cournot duopoly where the innovator is an incumbent firm. In this framework, licensing by means of royalty yields better payoff to the innovator than upfront fee licensing. Extending this work to a Cournot oligopoly, Kamien and Tauman (2002) have shown that royalty licensing is superior to both auction and upfront fee policies for an incumbent innovator when the innovation is sufficiently significant. By considering both outsider and incumbent innovators, we merge this new line of enquiry with the standard literature. Our result in case of a duopoly with an incumbent innovator strengthens the result of Wang (1998): we show that the royalty policy is optimal among all AR policies. Several other approaches have been taken to investigate the potential benefits of royalty licensing over licensing by upfront fee, e.g., licensing has been considered under asymmetric information [Gallini and Wright (1990), Beggs (1992), Bousquet, Cremer, Ivaldi and Wolkowicz (1998)] and under other forms of competition like the Stackelberg duopoly [Filippini (2002), Kabiraj (2002)]. Our analysis complements these approaches. We show that for some markets, it is always optimal to charge only royalty and no upfront fee. For other markets, it is optimal to charge a combination of upfront fee and royalty for significant innovations, while for relatively insignificant innovations, it is optimal to charge only upfront fee. Thus, we provide an explanation of the prevalence of only royalty, only upfront fee and combination of fee and royalty, as observed in practice. In a related work, Erutku and Richelle (2000) have shown that in an oligopoly with at least two firms, an outsider innovator can design an upfront fee plus royalty policy that enables her to extract the entire monopoly profit with the new technology. This result is obtained with a fairly complicated royalty structure. On the one hand, a licensee has to pay the innovator a positive per-unit royalty that depends on his individual output as well as the total industry output and on the other hand, the innovator pays him back an amount that is non-linear in his output. Such policies would be difficult to implement in practice.[4]

The impact of licensing on the industry structure is one important aspect in which our result significantly differs from the existing literature. In one form or another, it has been pointed out in earlier papers [KS (85), KT (86), KOT (92)] that if the innovation is k-drastic,[5] then an outsider innovator who uses only upfront fee policy or only auction policy will sell just k licenses. On the other hand, if the innovator uses only royalty, then the license is sold to all firms [KT (86)]. We show here that for significant 2-drastic innovations, the license is sold to only two firms and a natural duopoly is created. For other innovations,

[4]For other interesting, but less practical selling mechanisms that enable the innovator to obtain the total industry profit, see KOT (92) and Sen (2002b). See also Kamien, Tauman and Zamir (1990).

[5]In an industry of n firms, an innovation is *k-drastic* for $k < n$ if k is the minimum number such that the k-firm oligopoly price under the new technology does not exceed the competitive price under the old technology. Thus, if an innovation is k-drastic and k firms have the new technology, all other firms drop out of the market and a k-firm natural oligopoly is created.

including (less significant) 2-drastic innovations, the innovator sells the license to all firms, except perhaps one.

In the final section, we discuss two issues regarding the incentives to innovate. First, we compare the incentives of outsider and incumbent innovators to develop significant innovations. For any industry of size at least two, the payoff of an outsider innovator exceeds the incremental payoff of an incumbent innovator. This is compatible with KS (86). To obtain this result, the incremental payoff of an incumbent innovator is taken to be the difference between her post-innovation payoff (the sum of her licensing payoff and post-innovation Cournot profit) and the pre-innovation Cournot profit. This implicitly assumes that if she had failed to innovate, there would have been no innovation at all. However, if she believes that some other entity (either an outsider or another incumbent firm) will succeed in case she fails, then her opportunity cost is the post-innovation Cournot profit of a non-licensee.[6] Since every firm is worse off due to the innovation, the incremental payoff due to the innovation becomes larger. We show that in this case, an incumbent firm will have higher incentives to innovate compared to an outsider, at least for significant innovations.

The relation between industry size and incentives to innovate is one of the classic issues in economics [Schumpeter (1943) (Chapters VII and VIII), Arrow (1962)]. In this paper, we analyze the relation between the industry size and the incentive to innovate for any magnitude of innovation and show that the conclusions in this regard are qualitatively the same for both outsider and incumbent innovators. For royalty licensing, similar to Arrow (1962), it has been shown that the perfectly competitive industry provides the highest incentive for innovation [see KT (86) and KOT (92)]. For upfront fee licensing, however, there is no sharp conclusion. In KT (86), it was pointed out that

[6] Note that the innovator extracts the entire surplus through the auction, so that the post-innovation profit of any licensee firm is the same as that of a non-licensee.

for small innovations, the industry size that provides the highest incentive is decreasing in the magnitude of the innovation. In this paper, we show that for both outsider and incumbent innovators, the monopoly or the duopoly does not provide the highest incentive; the industry size that provides the highest incentive is decreasing in the magnitude of the innovation when the innovation is small [as in KT (86)], while it is increasing for significant innovations. Moreover, if the magnitude of the innovation is either very small or almost drastic, then a competitive industry provides the highest incentive to innovate. This non-monotonicity is the result of restricting the rate of royalty and upfront fee to non-negative values. Without these restrictions, the industry size that provides the highest incentive to innovate is always increasing in the magnitude of the innovation.[7] The duopoly provides the highest incentive when the innovation is sufficiently insignificant and as before, it increases indefinitely as the innovation approaches a drastic innovation.

The rest of the paper is organized as follows. In Section 2, we present the model and the licensing schemes. In Section 3, we derive the optimal licensing schemes. In Section 4, we study the incentives to innovate. All proofs have been relegated to the appendix.

2 The Model

Let us first describe the model with an outsider innovator in detail. We consider a Cournot oligopoly with n firms producing the same product, where $N = \{1, \ldots, n\}$ is the the set of firms. For $i \in N$, let q_i be the quantity produced by firm i and let $Q = \sum_{i \in N} q_i$. The inverse demand function of the industry is linear and is given by $Q = a - p$, for $p \leq a$ and $Q = 0$, otherwise. With the old technology, all n firms produce with the identical constant marginal cost c,

[7] Recall that we are considering only non-drastic innovations. For a drastic innovation, the incentive is the same for all industry sizes.

where $0 < c < a$. An outsider innovator has been granted a patent for a new technology that reduces the marginal cost from c to $c - \varepsilon$, where $0 < \varepsilon < c$. The innovator decides to license the new technology to some or all firms of the industry. In case of an incumbent innovator, there are $n + 1$ firms, where $N_I = \{I\} \cup N$ is the set of firms and firm I is the innovator.

2.1 The Licensing Schemes

The licensing schemes available to the innovator are combinations of upfront fee and uniform linear royalty.[8] On the basis of how the upfront fee is determined, there are two types of policies: (i) the *upfront fee plus royalty* (FR) policy and (ii) the *auction plus royalty* (AR) policy. A typical FR policy is given by $\langle m, r, f \rangle$, where m, $1 \leq m \leq n$, is the number of firms to whom the policy is offered, $r \geq 0$ is the per-unit uniform royalty, and $f \geq 0$ is the upfront fee that each licensee has to pay. For this policy, if a licensee firm produces q, it pays the innovator $f + rq$. A typical AR policy is given by $\langle m, r \rangle$, where m, $1 \leq m \leq n$, is the number of firms to whom the policy is offered and $r \geq 0$ is the per-unit (uniform) royalty that each licensee has to pay. When the policy $\langle m, r \rangle$ is announced, firms in N bid for the license in a first-price sealed-bid auction and m highest bidders win the license. If a firm has won the license with bid b and produces q, it pays the innovator $b + rq$.

2.2 The Willingness to Pay for the License

Now we determine the willingness to pay for a potential licensee. Notice that the Cournot output and profit of any firm depend on the number of licensees m

[8]Since we are interested in policies that are empirically observable, we exclude negative royalty and negative upfront fee. The restriction on upfront fee is binding only in case the innovator is an incumbent firm in a duopoly. However, the restriction on royalty is binding for both outsider and incumbent innovators in a general oligopoly for sufficiently insignificant innovations.

and the rate of royalty r, but it is independent of the upfront fee or the winning bid. Denote by $\Phi_L(m,r)$ and $\Phi_N(m,r)$ the Cournot profit of a licensee and a non-licensee respectively when the number of licensees is m and the rate of royalty is r.[9] First consider the FR policy. Suppose there are m licensees and the rate of royalty is r. Then, the willingness to pay for the license is

$$f(m,r) = \Phi_L(m,r) - \Phi_N(m-1,r). \tag{1}$$

Next, consider the AR policy and suppose there are m licensees and the rate of royalty is r, where $1 \leq m \leq n-1$ and $r \geq 0$. Then, the willingness to pay for the license is

$$b(m,r) = \Phi_L(m,r) - \Phi_N(m,r). \tag{2}$$

In contrast to (1), we subtract $\Phi_N(m,r)$ instead of $\Phi_N(m-1,r)$. This is because for the AR policy $\langle m,r\rangle$, a firm knows that irrespective of whether it becomes a licensee or not, there will be m licensees. When the AR policy $\langle m,r\rangle$ is announced, in equilibrium, at least $m+1$ firms will bid $b(m,r)$,[10] and m of them will be chosen at random. By (1) and (2),

$$b(m,r) - f(m,r) = \Phi_N(m-1,r) - \Phi_N(m,r). \tag{3}$$

Note that the effective marginal cost of a licensee firm is $c - \varepsilon + r$ and that of a non-licensee firm is c. Thus as long as $r \leq \varepsilon$, a licensee firm is at least as efficient as a non-licensee firm so that the Cournot profit of a non-licensee is decreasing in the number of licensees, implying that $\Phi_N(m-1,r) \geq \Phi_N(m,r)$.[11] Let $\Pi_{FR}(m,r)$ and $\Pi_{AR}(m,r)$ denote the payoff of the innovator for the FR and AR policy respectively when there are m licensees and the rate of royalty is r. The following lemma, in a less general form, was shown in KS (86).

[9] Of course, these expressions will be different depending on whether the innovator is an incumbent firm or not. We use the same notation to avoid notational complication.

[10] If the bids are ranked in descending order and the $(m+1)$-th bid is smaller than the m-th bid, then every one of m bidders could benefit from a small reduction of his bid.

[11] Clearly, a firm may accept a rate of royalty $r > \varepsilon$ only if it is compensated by the innovator. We rule this out since we exclude negative upfront fee.

Lemma 1. *For* $1 \leq m \leq n-1$, $\Pi_{FR}(m,r) \leq \Pi_{AR}(m,r)$ *and equality holds iff* $r = \varepsilon$.

In view of Lemma 1, if $1 \leq m \leq n-1$ and $r < \varepsilon$, the innovator is strictly better off using the AR policy. When $m = n$, then every firm bids zero since each firm is guaranteed to have a license irrespective of its bid. Thus, the AR policy can be used for $m = n$ only with a pre-specified minimum bid, $\underline{b}$. Let $\langle n, r, \underline{b} \rangle$ be the modified AR policy. When the license is sold to n firms, the number of licensees will be $n-1$ if a licensee firm declines to buy the license. So, the minimum bid is

$$\underline{b} = \Phi_L(n,r) - \Phi_N(n-1,r). \tag{4}$$

Clearly, in equilibrium, every firm bids exactly $\underline{b} = f(n,r)$, and the FR and AR policies coincide for $m = n$. If $r = \varepsilon$, then again every firm bids zero in both of these policies. It is thus enough to consider the AR policy $\langle m, r \rangle$ for $1 \leq m \leq n-1$ and the modified AR policy $\langle n, r, f(n,r) \rangle$ for $m = n$, where $f(n,r)$ is given by (1). For the rest of the paper, we shall only consider the AR policy, possibly modified when required.

2.3 The Games G^O and G^I

Next, we describe the three-stage licensing games for outsider and incumbent innovators. The licensing game with an outsider innovator, denoted by G^O, has three stages. In the first stage, the innovator either announces $\langle m, r \rangle$ for $1 \leq m \leq n-1$, or $\langle n, r, f(n,r) \rangle$. In the second stage, firms in N bid simultaneously for the license, where the m highest bidders win the license and pay their respective bids. Ties in the cutoff bid are resolved at random. The set of licensees become commonly known at the end of the second stage. In the third stage, all firms compete in quantities. The licensing game G^I with an incumbent innovator is defined similarly. Note that for the game G^I, in the third stage, the innovator produces with marginal cost $c-\varepsilon$ and competes with all other firms.

3 Optimal Licensing Schemes

3.1 An outsider innovator

First we analyze the game G^O, (with an outsider innovator) and state the results that are specific to an outsider innovator. The results that are common to both G^O and G^I are discussed in Proposition 3 later in this section.

Proposition 1. *Consider the game G^O. Suppose there is a negligible but positive cost of contracting for every licensee. Then, there exists a unique subgame-perfect equilibrium outcome. It has the following properties.*

[i] *In case of a monopoly or a duopoly, the innovator sells the license to all firm(s).*

[ii] *For industries of size $n \geq 3$, there is a number $1 < q(n) \leq 2$ such that when $\varepsilon \geq (a-c)/q(n)$, the license is sold to only two firms, a natural duopoly is created and the innovator obtains $(a-c)\varepsilon$. When $\varepsilon < (a-c)/q(n)$, the innovator sells the license to at least $n-1$ firms and all firms continue to operate. Moreover, $q(n)$ is decreasing in n and it approaches 1 as n increases indefinitely.*

[iii] *The optimal rate of royalty is strictly smaller than ε and hence the effective marginal cost of every licensee falls below c.*

The intuition behind Proposition 1 can be given as follows. For a significant innovation, say a k-drastic one, the innovator can create a k-firm natural oligopoly and by gradually increasing the rate of royalty, she can create a natural oligopoly of size larger than k. There are two conflicting effects. The innovator can extract the entire k-firm natural oligopoly profit, provided that k is smaller than the industry size, and earn a relatively small royalty payments, or she can increase the royalty rate at the cost of a larger size of natural oligopoly. The magnitude of the innovation plays a role in settling this trade-off between royalty and upfront fee. If ε is sufficiently large, namely $\varepsilon \geq (a-c)/q(n)$, then the innovator sells the license to only two firms and creates a natural duopoly. When $\varepsilon < (a-c)/q(n)$, the effect of royalty dominates. The payoff of the innovator from creating a k-firm natural oligopoly is increasing in k and the innovator eventually ends up selling the innovation to all firms, except perhaps one. That the effect of royalty is dominant is more evident from the fact that even when the innovator excludes a single firm, the royalty rate is set high enough so that the sole non-licensee continues to operate.

Remark. Suppose $\varepsilon \geq (a-c)/q(n)$ for $n \geq 3$. Auctioning off only two licenses is the unique optimal policy only in the presence of a (negligible) positive cost of contracting τ for every licensee. In the absence of such contracting cost, any AR policy $\langle m, \beta(m) \rangle$, where $2 \leq m \leq n-1$ and $\beta(m) = \varepsilon - (a-c)/m$, is optimal. It results in the Cournot price of the pre-innovation marginal cost c and the innovator obtains $(a-c)\varepsilon$. Taking the contracting cost into account, the payoff of the innovator is $(a-c)\varepsilon - m\tau$ and clearly the unique optimal policy is to sell the license to only two firms.

3.2 An incumbent innovator

Consider the game G^I involving an incumbent innovator and n other firms. The next proposition states properties that are specific to an incumbent innovator.

Proposition 2. *The game G^I has a unique subgame-perfect equilibrium outcome. It has the following properties.*

[i] *When the number of firms other than the innovator is at most two* ($n \leq 2$),

the innovator sells the license to all firms through a policy based only on royalty, and the rate of royalty equals the magnitude of innovation ε.

[ii] *When there are at least three firms other than the innovator* ($n \geq 3$), *then the license is sold to at least* $n-1$ *firms, the rate of royalty is smaller than* ε *and all firms continue to operate.*

The intuition of Proposition 1 can be carried over to Proposition 2, with one difference: in this case, the innovator is herself a producer. If the rate of royalty is low, on one hand, the effective marginal cost of a licensee firm is small and the innovator can extract more from licensee firms through auction. On the other hand, an efficient licensee hurts the Cournot profit of the innovator. Unlike an outsider innovator, it is never optimal for an incumbent innovator to create a natural oligopoly whose size is less than the industry size. An incumbent innovator always prefers to charge a sufficiently higher rate of royalty to weaken her competitors and as a consequence, there is no change in the market structure.

3.3 Comparison between G^O and G^I

In the next proposition, we compare and contrast the properties of G^O and G^I.

Proposition 3. *Consider the unique subgame-perfect equilibrium outcomes of* G^O *and* G^I *respectively.*

I. *The following hold in both of these outcomes.*

(a) *If the industry size is at least two, the Cournot price and the payoff of any firm* (*other than the innovator*) *fall below their respective pre-innovation levels.*

(b) *The social welfare is higher as a result of the innovation.*

(c) *The private value of the patent is increasing in* ε.

(d) *For insignificant innovations, all firms become licensees.*

II. *The two outcomes differ in the following way.*

(a) *If the industry size is at least two, the post-innovation Cournot price and the payoff of any firm are less in* G^O *compared to* G^I.

(b) *If the industry size is at least two, the equilibrium royalty rate* r_I^* *of* G^I *is at least as high as the equilibrium royalty rate* r_O^* *of* G^O. *If the industry size is two or three, then* $r_O^* < r_I^* = \varepsilon$. *For larger sizes of industry, and insignificant innovations,* $r_O^* = r_I^* = 0$ *while* $r_O^* < r_I^*$ *for all other innovations.*

(c) *An incumbent innovator always obtains at least* $(a-c)\varepsilon$ *while an outsider innovator obtains at least* $(a-c)\varepsilon$ *if the industry size is at least three.*

(d) *In* G^I, *the industry structure remains unchanged (namely, all firms continue to operate) and all firms, except perhaps one, become licensees. In* G^O, *this is true iff* $\varepsilon \leq (a-c)/q(n)$. *If* $\varepsilon > (a-c)/q(n)$, *the license is sold to only two firms and a natural duopoly is created.*

Certain conclusions of Proposition 3 can be easily verified. Consider an industry where there are n firms other than an incumbent innovator. If the innovator sells the license to all other firms using the royalty rate $r = \varepsilon$, then her payoff is

$$\Pi^I(n, \varepsilon) = (a-c-\varepsilon)^2/(n+2)^2 + (a-c)\varepsilon > (a-c)\varepsilon.$$

In case of an outsider innovator in an industry of n firms, it can be shown that for relatively significant innovations,[12] there is a licensing policy that enables the innovator to earn exactly $(a-c)\varepsilon$. Let $n \geq 3$ and suppose $\varepsilon \geq (a-c)/(n-1)$.

[12]The proof for less significant innovations appear in the appendix.

Let $r_O = \varepsilon - (a-c)/(n-1) \geq 0$. It can be easily shown that when the innovator sells the license to $n-1$ firms and the rate of royalty is r_O, then the Cournot price is c and hence an $(n-1)$-firm natural oligopoly is created. Every licensee then bids his entire Cournot profit. Let q_L be the Cournot output of a licensee. Then, $q_L = (a-c+\varepsilon-r_O)/n = (a-c)/(n-1)$ and the payoff of the innovator is $\Pi^O = (n-1)[p(Q)-(c-\varepsilon)]q_L$. Since $p(Q) = c$, we have $Q = a-c$ and $\Pi^O = (a-c)\varepsilon$. For $n = 1$, the innovator obtains less than $(a-c)\varepsilon$ since the monopolist can always guarantee his pre-innovation profit, $\Pi_M(c)$. Thus, the payoff of the innovator is the difference between post-innovation and pre-innovation monopoly profits, given by

$$\Pi_M(c-\varepsilon) - \Pi_M(c) = (a-c+\varepsilon)^2/4 - (a-c)^2/4 < (a-c)\varepsilon.$$

Note that for $n = 2$, $r_O = \varepsilon - (a-c) < 0$ and consequently the royalty rate r_O cannot be charged. The innovator obtains less than $(a-c)\varepsilon$. However, if we allow for negative royalties then an outsider innovator can obtain at least $(a-c)\varepsilon$ for all $n \geq 2$.

The private value of the patent is the post-innovation payoff of the innovator. In case of an outsider innovator it is the rent that she extracts by selling the license, while an incumbent innovator obtains her post-innovation Cournot profit in addition to the licensing rent. The private value of the patent is an increasing function in the magnitude of the innovation in both G^O and G^I. This result is rather intuitive and consistent with the existing literature. The intuition is especially simple in case of an outsider innovator. Suppose the magnitude of the innovation is $\varepsilon = \varepsilon_1$ and the rate of royalty is $r_1 \geq 0$. Then, the effective marginal cost of a licensee is $c-\varepsilon_1+r_1$. Now consider a magnitude of innovation $\varepsilon_2 > \varepsilon_1$ and let $r_2 = r_1+\varepsilon_2-\varepsilon_1 \geq 0$. Then, the effective marginal cost of a licensee is $c-\varepsilon_2+r_2 = c-\varepsilon_1+r_1$, which is the same as before. So, the Cournot outputs and willingness to pay for the license do not change and the innovator obtains the same payoff as before. This shows that the payoff of the innovator at ε_2 least as high as that at ε_1.

The post-innovation Cournot price is smaller than its pre-innovation level. For G^O, sufficiently significant innovations are sold to only two firms, thus creating a natural duopoly where the Cournot price is c (which is less than the pre-innovation price). For other innovations, for both G^O and G^I, all firms continue to operate (the incumbent innovator operates with marginal cost $c - \varepsilon$). The result then follows from the fact that all firms are at least as efficient as before and some are even more efficient. Next, note that the net payoff of every firm (other than the innovator) is the post-innovation Cournot profit of a non-licensee. This becomes smaller as a result of the innovation since a non-licensee firm competes with firms that are at least as efficient as before.

The innovation results in a higher total welfare. Let q_I, q_L, q_N be the post-innovation Cournot output of the innovator, a licensee and a non-licensee respectively. For an outsider innovator, $q_I = 0$. Now consider the policy $\langle m, r \rangle$. Then, the post-innovation industry output is $Q_2 = q_I + mq_L + (n-m)q_N$ and the post-innovation Cournot price is $p(Q_2) \geq c$. When computing the total welfare, we can ignore the upfront fee or the winning bid, since these are lump-sum transfers from one agent to the another. Similarly, the total royalty payment is transferred from the licensees to the innovator and its total effect can also be ignored. Consequently, the post-innovation sum of payoffs of the innovator and all firms is given by

$$PS_2 = [p(Q_2) - (c - \varepsilon)]q_I + m[p(Q_2) - (c - \varepsilon)]q_L + (n-m)[p(Q_2) - c]q_N$$

$$= [p(Q_2) - c]Q_2 + \varepsilon(q_I + mq_L) > [p(Q_2) - c]Q_2.$$

The consumers' surplus is

$$CS_2 = \left[\int_0^{Q_2} p(q)dq\right] - p(Q_2)Q_2,$$

so that the welfare W_2 satisfies

$$W_2 > \left[\int_0^{Q_2} p(q)dq\right] - p(Q_2)Q_2 + [p(Q_2) - c]Q_2 = \int_0^{Q_2} [p(q) - c]dq.$$

Let the pre-innovation industry output be Q_1. Then $p(Q_1) \geq c$ and the pre-innovation sum of payoffs of the innovator and all firms is given by $PS_1 = [p(Q_1) - c]Q_1$, while the consumers' surplus is given by

$$CS_1 = \left[\int_0^{Q_1} p(q)dq\right] - p(Q_1)Q_1.$$

The total welfare is

$$W_1 = \left[\int_0^{Q_1} p(q)dq\right] - p(Q_1)Q_1 + [p(Q_1) - c]Q_1 = \int_0^{Q_1} [p(q) - c]dq.$$

Since $Q_2 > Q_1$, it follows that $W_2 > W_1$.

4 Incentives to Innovate

In this section, we study the incentives to innovate. First, we compare the incentives of outsider and incumbent innovators and investigate who has a higher incentive to innovate significant innovations. Next, for both outsider and incumbent innovators, we determine the industry size that provides them with the highest incentives.

4.1 Comparison of Incentives

In this subsection, we attempt to answer the question of who has a higher incentive to innovate: an outsider innovator, or an incumbent firm? The incremental payoff of the innovator due to the innovation is the difference between her post-innovation payoff and her opportunity cost. The opportunity cost of an outsider innovator, denoted by α, is exogenously given. The opportunity cost of an incumbent innovator depends on the specific pre-innovation environment. For instance, if no entity other than the potential incumbent innovator is engaged in innovative activity, then the opportunity cost is the pre-innovation Cournot profit. A more realistic scenario would be the one where other entities also compete to develop a cost-reducing innovation. In that case, the opportunity cost depends on the belief of the potential incumbent innovator about the likelihood of success of other entities conditional on the event that she fails to innovate. Specifically, suppose that prior to the innovation, there are T entities, both inside and outside the industry, who are engaged in a patent race to develop a cost-reducing innovation. It is assumed that the race results in one of the two outcomes: either there is an innovation of magnitude $\varepsilon > 0$, or there is no innovation. Licensing of the innovation takes place at the end of the race provided there is a winner. If no one succeeds, then firms are engaged in Cournot competition with the old marginal cost. To further simplify our analysis, we assume that all potential innovators invest the same amount in R&D, so that the cost of investment plays no role in our comparison. Let W_t denote the event that entity t wins the patent race and $\overline{W}$ denote the event that no one wins. The events $W_1, \ldots, W_T, \overline{W}$ are mutually exclusive and exhaustive. Consider a potential incumbent innovator, I. Note that W_I is the event that I wins the patent race. Let $\lambda_t^I = P_I(W_t)$ and $\lambda_{\overline{W}}^I = P_I(\overline{W})$, where $P_I(W)$ is the probability that I assigns to W. Conditional on the event $\overline{W_I}$ that I does not win the race, the probability that there is a winner is $\tilde{\lambda}_I = P_I(\cup_{t \neq I} W_t | \overline{W_I})$. Applying Bayes' rule, we have

$$\tilde{\lambda}_I = \frac{P_I((\cup_{t \neq I} W_t) \cap \overline{W}_I)}{P_I(\overline{W_I})}.$$

Clearly, $\cup_{t \neq I} W_t \subseteq \overline{W_I}$ and $\{W_t\}_{t=1}^T$ are mutually exclusive. Thus,

$$\tilde{\lambda}_I = \frac{P(\cup_{t \neq I} W_t)}{P(\overline{W_I})} = \frac{\sum\limits_{t \neq I} \lambda_t^I}{1 - \lambda_I^I} = \frac{1 - \lambda_I^I - \lambda_{\overline{W}}^I}{1 - \lambda_I^I}.$$

Note that when I is the only entity to engage in innovation, then $1 - \lambda_I^I = \lambda_{\overline{W}}^I$, and $\tilde{\lambda}_I = 0$. In that case, the opportunity cost of I is the pre-innovation

Cournot profit. However, if I believes that there are other entities who are likely to succeed, then $\tilde{\lambda}_I$ is positive and I will earn the post-innovation profit of a non-licensee (which is also the net payoff of a licensee) in case someone else succeeds to innovate. This opportunity cost also depends on I's belief on who is more likely to win: an outsider or incumbent firm.

We begin with the scenario considered in KS (86), where the opportunity cost of an outsider innovator is zero and that of an incumbent innovator I is the pre-innovation Cournot profit ($\alpha = 0$ and $\tilde{\lambda}_I = 0$). We show that for this case, the payoff of an outsider innovator exceeds the incremental payoff of an incumbent innovator for any magnitude of innovation and for any industry of size at least two. This result is compatible with KS (86). However, if $\tilde{\lambda}_I$ is sufficiently close or equal to 1, then the opposite result holds for significant innovations, that is, the incremental payoff of an incumbent innovator exceeds the payoff of an outsider innovator.

Proposition 4. *Suppose that the industry size is at least two and α is sufficiently small. For any $0 < \varepsilon \leq c$, I has lower incentives to innovate than an outsider innovator if $\tilde{\lambda}_I$ is sufficiently small. If $\tilde{\lambda}_I$ is sufficiently large, for significant innovations, I has higher incentives.*

The conclusions of Proposition 4 are fairly intuitive. When an incumbent firm I believes that if it does not win the patent race, then someone else will win with high probability, then the opportunity cost of I is sufficiently low (and possibly zero), so that the incremental payoff is high. However, if she is the only potential innovator, then the incremental payoff due to the innovation and consequently the incentive to innovate is lower.

The second part of Proposition 4 does not necessarily hold for insignificant innovations. In particular, when the industry size is at least four, for sufficiently insignificant innovations, the royalty rate is zero and the license is sold to all firms for both incumbent and outsider innovators [see Proposition 2.3]. Hence in both cases, all firms have the same marginal cost $c - \varepsilon$ and obtain the same profit. Let $\Pi_n(n, c')$ denote the Cournot profit of a firm in an n-firm oligopoly when the marginal cost of each firm is c' and let $\tilde{\Pi}_n(n-1, c-\varepsilon)$ denote the Cournot profit of a firm that operates with marginal cost c in an n-firm oligopoly where the marginal cost of any other firm is $c-\varepsilon$. Then, every firm pays $\Pi_n(n, c-\varepsilon) - \tilde{\Pi}_n(n-1, c-\varepsilon)$ as the winning bid to the innovator. Hence, the post-innovation payoff of an outsider innovator is

$$\Pi^O = n[\Pi_n(n, c-\varepsilon) - \tilde{\Pi}_n(n-1, c-\varepsilon)], \tag{5}$$

while that of an incumbent innovator is

$$\Pi^I = \Pi_n(n, c-\varepsilon) + (n-1)[\Pi_n(n, c-\varepsilon) - \tilde{\Pi}_n(n-1, c-\varepsilon)]. \tag{6}$$

In both cases, the net payoff of any firm other than the innovator is given by $\tilde{\Pi}_n(n-1, c-\varepsilon) > 0$. Given that I fails to innovate, there are two possible events: (i) there is some entity other than I that succeeds to innovate, in which case I obtains $\tilde{\Pi}_n(n-1, c-\varepsilon)$ and (ii) no other entity succeeds to innovate, in which case I obtains $\Pi_n(n, c)$. Hence, the (expected) opportunity cost of I is given by

$$\Phi^I = \tilde{\lambda}_I \tilde{\Pi}_n(n-1, c-\varepsilon) + (1-\tilde{\lambda}_I)\Pi_n(n, c), \tag{7}$$

while the incremental payoff of I due to the innovation is given by

$$\Delta^I = \Pi^I - \Phi^I. \tag{8}$$

Noting that $\Pi^I - \Pi^O = \tilde{\Pi}_n(n-1, c-\varepsilon)$, from (5)-(8), it follows that

$$\Delta^I - \Pi^O = (1-\tilde{\lambda}_I)[\tilde{\Pi}_n(n-1, c-\varepsilon) - \Pi_n(n, c)].$$

Since $\Pi_n(n, c) > \tilde{\Pi}_n(n-1, c-\varepsilon)$, we conclude that $\Delta^I \le \Pi^O$ with equality if and only if $\tilde{\lambda}_I = 1$. Hence, when $\tilde{\lambda}_I < 1$, we have $\Delta^I < \Pi^O - \alpha$ for sufficiently small α. In other words, as long as there is a positive probability that all entities will fail to innovate, the incremental payoff of an incumbent innovator due to the innovation is less than the payoff of an outsider innovator. Thus, when the industry size is at least four, an incumbent firm has lower incentives to innovate insignificant innovations compared to an outsider.

4.2 Industry Size and Incentives

For an outsider innovator, the industry that provides the highest incentive to innovate a non-drastic cost-reducing innovation is neither a monopoly nor a duopoly. This follows directly from Proposition 3. We determine the industry size that provides the highest incentive for an incumbent innovator only for the case studied in KS (86), namely the case where the opportunity cost of an incumbent innovator is the pre-innovation Cournot profit (or, equivalently, $\tilde{\lambda}_I = 0$) and that of an outsider innovator is zero ($\alpha = 0$). When the incumbent innovator is a monopolist, the incremental payoff is given by

$$\Delta_0(\varepsilon) = (a - c + \varepsilon)^2/4 - (a - c)^2/4.$$

It can be shown that when there is only one firm other than the innovator, the incremental payoff is given by

$$\Delta_1(\varepsilon) = \Big[(a - c - \varepsilon)^2/9 + (a - c)\varepsilon\Big] - (a - c)^2/9.$$

If there are two firms other than the innovator, then

$$\Delta_2(\varepsilon) = \Big[(a - c - \varepsilon)^2/16 + (a - c)\varepsilon\Big] - (a - c)^2/16.$$

Noting that $a - c > \varepsilon$, it is easily seen that

$$\Delta_2(\varepsilon) > \max\{\Delta_0(\varepsilon), \Delta_1(\varepsilon)\}. \tag{9}$$

Thus, neither a monopoly, nor a duopoly provides an incumbent innovator with the highest incentive to innovate. In general, the conclusions are qualitatively the same for both types of innovators: the highest incentive to innovate is induced by an industry size that is increasing in the magnitude of the innovation when the innovation is significant, and it is decreasing for less significant innovations.

Proposition 5. *Let* $n^O(\varepsilon)$ *and* $n^I(\varepsilon)$ *denote the industry size that provides the highest incentive to innovate when the magnitude of the innovation is* ε *for outsider and incumbent innovators respectively. Then the following hold for* $J \in \{O, I\}$.

[i] $n^J(\varepsilon) > 2$ *for all* $0 < \varepsilon < a - c$ *and there is a constant* $k_J > 2$ *such that* $n^J(\varepsilon)$ *is decreasing in* ε *for* $0 < \varepsilon < (a-c)/k_J$ *and it is increasing for* $(a-c)/k_J \leq \varepsilon < a - c$. *Further,* $\lim_{\varepsilon \downarrow 0} n^J(\varepsilon) = \lim_{\varepsilon \uparrow a-c} n^J(\varepsilon) = \infty$.

[ii] *When the upfront fee or the rate of royalty are allowed to take negative values, then* $n^J(\varepsilon)$ *is increasing in* ε *for all* $0 < \varepsilon < a - c$. *Further, there is a constant* $d_J > 1$ *such that* $n^J(\varepsilon) = 2$ *for* $0 < \varepsilon \leq d_J$ *and* $\lim_{\varepsilon \uparrow a-c} n^J(\varepsilon) = \infty$.

The result that for significant innovations, the optimal industry size increases in the magnitude of the innovation has a simple intuition in case of an outsider innovator. By Proposition 1, if $\varepsilon \geq (a-c)/q(n)$ for $n \geq 3$, then the optimal policy is to sell the license to only two firms and the innovator obtains exactly $(a-c)\varepsilon$. However, she obtains more than $(a-c)\varepsilon$ when the innovation is relatively insignificant. Thus, for any ε, the innovator would rather prefer an industry size in which ε becomes relatively insignificant with respect to that industry size, specifically $\varepsilon < (a-c)/q(n)$. Since $q(n)$ is decreasing in n, as the magnitude of the innovation increases, the innovator needs higher values of n to make the innovation insignificant.

Appendix

To shorten the length of the proofs, some details are omitted. They are available from the first author upon request.

Notations. We denote by q and Φ, with suitable subscripts, the Cournot output and the Cournot profit respectively: $q_L(m,r)$ and $q_N(m,r)$ denote the Cournot output of a licensee and a non-licensee firm respectively when there are m licensees and the rate of royalty is r. Similarly, $\Phi_L(m,r)$ and $\Phi_N(m,r)$ denote the respective Cournot profits. When the innovator is an incumbent firm, then $q_I(m,r)$ and $\Phi_I(m,r)$ denote respectively the Cournot output and profit of the innovator. Recall that the effective marginal cost of a licensee is $c-\varepsilon+r$ while that of a non-licensee is c. When $r>\varepsilon$, clearly the Cournot profit of a licensee is less than that of a non-licensee, so it is a dominated strategy for a firm to accept any licensing policy where $r>\varepsilon$. Thus, it is sufficient to consider to $r\in[0,\varepsilon]$. For the AR policy $\langle m,r\rangle$, for $1\le m\le n-1$, the equilibrium winning bid is given by

$$b(m,r)=\Phi_L(m,r)-\Phi_N(m,r).$$

The policy $\langle n,r\rangle$ comes with a minimum bid $\underline{b}$. In equilibrium,

$$\underline{b}=f(n,r)=\Phi_L(n,r)-\Phi_N(n-1,r),$$

and all firms bid $\underline{b}$. For the proof of Proposition 1, we determine the respective Cournot outputs and profits in the next few lemmas. The proofs are straightforward, and omitted.

Lemma A1. *Suppose* $0<\varepsilon<min\{a-c,c\}$ *and let* $\beta(m)=\varepsilon-(a-c)/m$ *for* $1\le m\le n$. *Then,* $\beta(m-1)<\beta(m)<\varepsilon$ *for* $2\le m\le n$.

Lemma A2. *Let* $1\le m\le n-1$.

[1] *If* $r\le\beta(m)$, *then*

$$q_L(m,r)=\frac{a-c+\varepsilon-r}{m+1},\qquad q_N(m,r)=0.$$

[2] *If* $r\in[\beta(m),\varepsilon]$, *then*

$$q_L(m,r)=\frac{a-c+(n-m+1)(\varepsilon-r)}{n+1},\qquad q_N(m,r)=\frac{a-c-m(\varepsilon-r)}{n+1}.$$

Moreover, $\Phi_J(m,r) = [q_J(m,r)]^2$ *for* $J \in \{L, N\}$.

Lemma A3. *Let* $m = n$*. Then,* $q_L(n,r) = (a - c + \varepsilon - r)/(n+1)$ *for every* $r \in [0, \varepsilon]$*. Again,* $\Phi_L(n,r) = [q_L(n,r)]^2$.

Proof of Proposition 1. For $n \geq 2$, we denote by $r_n^*(m)$ the optimal royalty rate when there are n firms and the number of licensees is m. Let m^* be the equilibrium number of licensees and let $r_n^* = r_n^*(m^*)$.

The equilibrium payoff of an outsider innovator from the policy $\langle m, r\rangle$ is

$$\Pi_n^O(m,r) = \begin{cases} mrq_L(m,r) + m[\Phi_L(m,r) - \Phi_N(m,r)], & 1 \leq m \leq n-1, \\ nrq_L(n,r) + n[\Phi_L(n,r) - \Phi_N(n-1,r)], & m = n. \end{cases} \tag{10}$$

Part [i] of Proposition 1 has been proved in the main text for $n = 1$. Let $n = 2$. If $m = 2$, from Lemma A3 and (10), we have

$$\Pi_2^O(2,r) = 2r\left[\frac{a-c+\varepsilon-r}{3}\right] + 2\left[\left(\frac{a-c+\varepsilon-r}{3}\right)^2 - \left(\frac{a-c-\varepsilon+r}{3}\right)^2\right].$$

The unrestricted maximum is attained at $r = \bar{r}^O(2) \equiv [3\varepsilon - (a-c)]/6 < \varepsilon$. Since $\bar{r}^O(2) \geq 0$ iff $\varepsilon \geq (a-c)/3$, the maximum is attained at $r = \bar{r}^O(2)$ for $\varepsilon \geq (a-c)/3$ and

$$\Pi_2^O(2, \bar{r}^O(2)) = [(a-c)^2 + 42(a-c)\varepsilon + 9\varepsilon^2]/54 < (a-c)\varepsilon. \tag{11}$$

Next, consider the case $\varepsilon \leq (a-c)/3$. For this case, the maximum is attained at $r = 0$, and the payoff is given by

$$\Pi_2^O(2,0) = 2\left[(a-c+\varepsilon)^2/9 - (a-c-\varepsilon)^2/9\right] = 8(a-c)\varepsilon/9 < (a-c)\varepsilon. \tag{12}$$

Conclusion 1. *For* $n = 2$,

$$r_2^*(2) = \begin{cases} \dfrac{3\varepsilon - (a-c)}{6}, & \varepsilon \geq \dfrac{a-c}{3} \\ 0, & \textit{otherwise} \end{cases} \tag{13}$$

Suppose next that $m = 1$. From (10) and Lemma A2, we have

$$\Pi_2^O(1, r) = r\left[\frac{a - c + 2\varepsilon - 2r}{3}\right] + \left[\frac{a - c + 2\varepsilon - 2r}{3}\right]^2 - \left[\frac{a - c - \varepsilon + r}{3}\right]^2.$$

It can be easily shown that $\Pi_2^O(1, r)$ is decreasing in r, implying that $r_2^*(1) = 0$ and

$$\Pi_2^O(1, 0) = (a - c + 2\varepsilon)^2/9 - (a - c - \varepsilon)^2/9. \tag{14}$$

From (11) and (14), $\Pi_2^O(2, \bar{r}^O(2)) > \Pi_2^O(1, 0)$ for all $\varepsilon < a - c$. From (12) and (14), $\Pi_2^O(2, 0) - \Pi_2^O(1, 0) = [2(a - c) - 3\varepsilon]\varepsilon/9$. By (13), $\langle 2, 0\rangle$ could be optimal only if $\varepsilon < (a - c)/3$, in which case, $\Pi_2^O(2, 0) > \Pi_2^O(1, 0)$. Thus, $m^* = 2$ and $r_2^* = r_2^*(2)$.

Conclusion 2. *In case of a duopoly* ($n = 2$), *an outsider innovator always sells the license to both firms, where the rate of royalty is given by* (13).

This completes the proof of part [i] of Proposition 1. To prove part [ii] of the proposition, consider $n \geq 3$. First, suppose that $m = n$. It can be shown that for $r \leq \beta(n-1)$, the payoff of the innovator is maximized at $r = \beta(n-1)$, so that it is sufficient to consider $r \in [\max\{0, \beta(n - 1)\}, \varepsilon]$. In that case, the unrestricted maximum of $\Pi_n^O(n, r)$ is attained at $r = \bar{r}^O(n)$ where

$$\bar{r}^O(n) \equiv \frac{(n - 1)[(2n - 1)\varepsilon - (a - c)]}{2(n^2 - n + 1)} < \varepsilon. \tag{15}$$

It can be verified that for every n, $\bar{r}^O(n) > \beta(n - 1)$. Further, $\bar{r}^O(n) \geq 0$ iff $\varepsilon \geq (a-c)/(2n-1)$. Therefore if $\varepsilon \geq (a-c)/(2n-1)$, the maximum is attained at $r = \bar{r}^O(n)$ and

$$\Pi_n^O(n, \bar{r}^O(n)) = \frac{n[(n - 1)^2(a - c)^2 + 2(2n^3 + n^2 + 1)(a - c)\varepsilon + (n + 1)^2\varepsilon^2]}{4(n + 1)^2(n^2 - n + 1)}. \tag{16}$$

When $\varepsilon \leq (a - c)/(2n - 1)$, the maximum is attained at $r = 0$. Hence

$$\Pi_n^O(n, 0) = n(a - c + \varepsilon)^2/(n + 1)^2 - n[a - c - (n - 1)\varepsilon]^2/(n + 1)^2. \tag{17}$$

Moreover,

$$r_n^*(n) = \begin{cases} \dfrac{(n-1)[(2n-1)\varepsilon-(a-c)]}{2(n^2-n+1)} < \varepsilon, & \varepsilon \geq \dfrac{a-c}{2n-1} \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

Next, consider $1 \leq m \leq n-1$. If $r \leq \beta(m)$, the maximum is attained at $r = \beta(m)$. So, it is sufficient to consider $r \in [\max\{0, \beta(m)\}, \varepsilon]$. It can be shown that if $\varepsilon \geq (a-c)/2$, then for every $1 \leq m \leq n-1$, $\Pi^O(m,r)$ is decreasing in r and hence $r_n^*(m) = \max\{0, \beta(m)\}$. Noting that $\beta(1) < 0$ and $\beta(m) \geq 0$ for all $2 \leq m \leq n-1$ when $\varepsilon \geq (a-c)/2$, we have $r_n^*(1) = 0$ and $r_n^*(m) = \beta(m)$. In addition, $\Pi_n^O(m, r_n^*(m)) = (a-c)\varepsilon$ and $\Pi_n^O(1,0) < \Pi_n^O(1,\beta(1)) = (a-c)\varepsilon$. Consequently, if $\varepsilon \geq (a-c)/2$, then for $1 \leq m \leq n-1$, any $m \geq 2$ is optimal. Taking into account the negligible but positive cost of contracting per licensee, we conclude that the innovator will choose the policy $\langle 2, \beta(2)\rangle$. When $\varepsilon < (a-c)/2$, it can be verified that the payoff of the innovator is maximized at $\langle m, r\rangle = \langle n-1, \tilde{r}^O(n-1)\rangle$ where

$$\tilde{r}^O(n-1) = \frac{2(n-2)\varepsilon-(a-c)}{2(n-1)}. \tag{19}$$

Note that $\tilde{r}^O(n-1) \geq 0$ iff $\varepsilon \geq (a-c)/2(n-2)$.

Conclusion 3. *If $(a-c)/2(n-2) \leq \varepsilon \leq (a-c)/2$, then $m^* \geq n-1$, $r_n^*(n-1) = \tilde{r}^O(n-1)$, given by (19) and*

$$\Pi_n^O(n-1, \tilde{r}^O(n-1)) = \frac{(a-c)^2 + 4n(a-c)\varepsilon + 4\varepsilon^2}{4(n+1)} > (a-c)\varepsilon. \tag{20}$$

Consider next the case where $\varepsilon \leq (a-c)/2(n-2)$. It can be shown that for every $1 \leq m \leq n-1$, the maximum is attained at $r = 0$. Noting that $\Pi_n^O(m,0)$ is increasing in m, we have the following.

Conclusion 4. *If $\varepsilon \leq (a-c)/2(n-2)$, then $m^* \geq n-1$ and $r_n^*(n-1) = 0$.*

Moreover,

$$\Pi_n^O(n-1,0) = (n-1)\left[\left(\frac{a-c+2\varepsilon}{n+1}\right)^2 - \left(\frac{a-c-(n-1)\varepsilon}{n+1}\right)^2\right] > (a-c)\varepsilon. \quad (21)$$

From (16), (17), (20), and (21), we conclude the following.

Conclusion 5. *For* $n \geq 3$, *the innovator obtains at least* $(a-c)\varepsilon$ *if* $\varepsilon \geq (a-c)/2$ *and more than* $(a-c)\varepsilon$ *if* $\varepsilon < (a-c)/2$.

Also, it follows from (13), (18) and (19) that $r_n^* < \varepsilon$ as claimed in part [iii]. Denote

$$s(n) = \frac{n+1}{n}\left[\frac{1+\sqrt{n^2-n+1}}{n-1}\right]^2 \text{ and let}$$

$$q(n) = \min\{2, s(n)\}. \quad (22)$$

Note that $s(n)$ is strictly decreasing in n and $s(n) < 2$ iff $n \geq 7$. Consequently, $q(n) = 2$ for $3 \leq n \leq 6$ and $q(n) < 2$ for $n \geq 7$. Also, $q(n) \downarrow 1$ as $n \to \infty$.

It can be shown that for $n = 3$, there is a constant $k > 2$ such that when $\varepsilon \geq (a-c)/k$, the license is sold to only two firms and the innovator obtains $(a-c)\varepsilon$. Otherwise, the license is sold to all three firms and the innovator earns more than $(a-c)\varepsilon$. Further, the rate of royalty is positive if either $(a-c)/5 < \varepsilon < (a-c)/k$ or $\varepsilon > (a-c)/2$, and it is zero otherwise. Moreover, a natural duopoly is created with $m = 2$ when $\varepsilon \geq (a-c)/2$. Otherwise, all three firms continue to operate.

Finally, let $n \geq 4$. It can be shown that for $\varepsilon \geq (a-c)/q(n)$, the license is sold to only two firms, a natural duopoly is created and the innovator earns $(a-c)\varepsilon$. Otherwise, the license is sold to at least $n-1$ firms and all firms continue to operate. In particular, if $(a-c)/2 \leq \varepsilon \leq (a-c)/q(n)$, then the optimal policy is $\langle n, r_n^*(n)\rangle$, where r_n^* is given by the upper part of (18). This fact will be used in the proof of Proposition 5. This completes the proof of Proposition 1. ■

Proof of Proposition 2. Recall that the industry here consists of $n+1$ firms including the innovator.

Lemma A4. *Suppose* $0 < \varepsilon < min\{a-c, c\}$ *and let* $\theta(m) = [c-a+(m+1)\varepsilon]/m$ *for* $1 \leq m \leq n$. *Then, for* $n \geq 2$ *and* $2 \leq m \leq n$, $\theta(m-1) < \theta(m) < \varepsilon$.

Lemma A5. *Suppose* $n \geq 2$ *and* $1 \leq m \leq n-1$.
[1] *If* $0 \leq r \leq \theta(m)$, *then*

$$q_I(m,r) = \frac{a-c+\varepsilon+mr}{m+2}, \qquad q_L(m,r) = \frac{a-c+\varepsilon-2r}{m+2}, \qquad q_N(m,r) = 0.$$

[2] *If* $\max\{0, \theta(m)\} \leq r \leq \varepsilon$, *then*

$$q_I(m,r) = \frac{a-c+(n-m+1)\varepsilon+mr}{n+2},$$

$$q_L(m,r) = \frac{a-c+(n-m+1)\varepsilon-(n-m+2)r}{n+2},$$

$$q_N(m,r) = \frac{a-c-(m+1)\varepsilon+mr}{n+2}.$$

In all of these cases, $\Phi_J(m,r) = [q_J(m,r)]^2$ *for* $J \in \{I, L, N\}$, *where* I *stands for the incumbent innovator.*

Lemma A6. *Suppose* $n \geq 1$ *and* $m = n$. *Then for* $r \in [0, \varepsilon]$,

$$q_I(n,r) = \frac{a-c+\varepsilon+nr}{n+2}, \qquad q_L(n,r) = \frac{a-c+\varepsilon-2r}{n+2}.$$

Again, $\Phi_J(n,r) = [q_J(n,r)]^2$ *for* $J \in \{I, L\}$.

To prove part [i] of the proposition, first consider $n = 1$. This is the case where there is only one firm, firm 1, other than I. It can be shown that the optimal policy of I is to charge $r = \varepsilon$ and no upfront fee. The payoff of I is

$$\Pi_1^I(1,\varepsilon) = (a-c-\varepsilon)^2/9 + (a-c)\varepsilon > (a-c)\varepsilon. \tag{23}$$

Next, consider $n \geq 2$. Then, the equilibrium payoff of I for the policy $\langle m, r\rangle$ is

$$\Pi_n^I(m,r) = \begin{cases} \Phi_I(m,r) + mrq_L(m,r) + m[\Phi_L(m,r) - \Phi_N(m,r)], & 1 \leq m \leq n-1, \\ \Phi_I(n,r) + nrq_L(n,r) + n[\Phi_L(n,r) - \Phi_N(n-1,r)] & m = n. \end{cases} \tag{24}$$

The proof of the following lemma is straightforward and hence omitted.

Lemma A7. *Let* $n \geq 2$.

[1] *The policy* $\langle n, \varepsilon \rangle$ *weakly dominates any policy* $\langle m, r \rangle$ *such that* $1 \leq m \leq n-1$ *and* $0 \leq r \leq \theta(m)$, *namely* $\Pi_n^I(m, r) \leq \Pi_n^I(n, \varepsilon)$.

[2] *The policy* $\langle n, \varepsilon \rangle$ *weakly dominates any policy* $\langle n, r \rangle$ *if* $0 \leq r \leq \theta(n-1)$.

[3] *The policy* $\langle n, \varepsilon \rangle$ *weakly dominates every policy* $\langle 0, r \rangle$. *Consequently,* $m^* \geq 1$.

Now we can complete the proof of part [i]. Let $n = 2$ and consider first the case where $m = 2$. By Lemma A7, it is sufficient to consider $r \in [\max\{0, \theta(1)\}, \varepsilon]$. In what follows, we show that the maximum payoff in this interval is $\Pi_2^I(2, \varepsilon)$. From Lemma A6 and (24), we have

$$\Pi_2^I(2, r) = \left[\frac{a - c + \varepsilon + 2r}{4}\right]^2 + 2r\left[\frac{a - c + \varepsilon - 2r}{4}\right]$$
$$+2\left[\left(\frac{a - c + \varepsilon - 2r}{4}\right)^2 - \left(\frac{a - c - 2\varepsilon + r}{4}\right)^2\right],$$

which is increasing in r for $r \in [\max\{0, \theta(1)\}, \varepsilon]$, so that for $m = 2$, $\langle 2, \varepsilon \rangle$ is the best policy, and we have

$$\Pi_2^I(2, \varepsilon) = (a - c - \varepsilon)^2/16 + (a - c)\varepsilon > (a - c)\varepsilon. \tag{25}$$

Next, consider $m = 1$. By Lemma A7, it is sufficient to consider $r \in [\max\{0, \theta(1)\}, \varepsilon]$. From Lemma A5 and (24), we have

$$\Pi_2^I(1, r) = \left[\frac{a - c + 2\varepsilon + r}{4}\right]^2 + r\left[\frac{a - c + 2\varepsilon - 3r}{4}\right]$$
$$+\left[\frac{a - c + 2\varepsilon - 3r}{4}\right]^2 - \left[\frac{a - c - 2\varepsilon + r}{4}\right]^2.$$

The unrestricted maximum is attained at $\tilde{r}^I(1) \equiv [c - a + 2\varepsilon]/3 < \varepsilon$. Note that $\theta(1) = c - a + 2\varepsilon = 3\tilde{r}^I(1)$. If $\varepsilon \leq (a - c)/2$, then $\tilde{r}^I(1) \leq \theta(1) \leq 0$, so the maximum is attained at $r = 0$ and

$$\Pi_2^I(1, 0) = \left[\frac{a - c + 2\varepsilon}{4}\right]^2 + \left[\frac{a - c + 2\varepsilon}{4}\right]^2 - \left[\frac{a - c - 2\varepsilon}{4}\right]^2.$$

Since $\Pi_2^I(2,\varepsilon) - \Pi_2^I(1,0) = \varepsilon[2(a-c) - 3\varepsilon]/16 > 0$, if $\varepsilon \leq (a-c)/2$, then the optimal policy in this region is $\langle 2, \varepsilon \rangle$. If $\varepsilon > (a-c)/2$, then $0 < \tilde{r}^I(1) < \theta(1)$. Since $r \in [\theta(1), \varepsilon]$, the maximum is attained at $r = \theta(1)$. In this case, the non-licensee firm drops out of the market and $\Pi_2^I(1, \theta(1)) = (a-c)\varepsilon$. By (25), we have the following.

Conclusion 6. *Let $n = 2$. Then, $m^* = 2$, $r_2^* = \varepsilon$ and $\Pi_2^I(2,\varepsilon) > (a-c)\varepsilon$.*

The proof of part [i] of the proposition is thus complete.

For part [ii], consider $n \geq 3$. Using Lemma A7, it can be shown that when $m = n$, the unrestricted maximum of $\Pi_n^I(n,r)$ is attained at $r = \bar{r}^I(n)$, where

$$\bar{r}^I(n) = \frac{n(2n-1)\varepsilon - (n-2)(a-c)}{2(n^2-n+1)}. \tag{26}$$

Also, $\theta(n-1) < \bar{r}^I(n) < \varepsilon$. If $\varepsilon > (n-2)(a-c)/n(2n-1)$, then $\bar{r}^I(n) > 0$, so that the maximum is attained at $r = \bar{r}^I(n)$, and

$$\Pi_n^I(n, \bar{r}^I(n)) = \frac{(n^3+4)(a-c+\varepsilon)^2 + 4n^2(n+1)^2(a-c)\varepsilon}{4(n+2)^2(n^2-n+1)}. \tag{27}$$

If $\varepsilon \leq (n-2)(a-c)/n(2n-1)$, the maximum is attained at $r = 0$ and

$$\Pi_n^I(n,0) = \frac{(a-c+\varepsilon)^2}{(n+2)^2} + \frac{n(a-c+\varepsilon)^2}{(n+2)^2} - \frac{n(a-c-n\varepsilon)^2}{(n+2)^2}. \tag{28}$$

Thus, we have

$$r_n^*(n) = \begin{cases} \dfrac{n(2n-1)\varepsilon - (n-2)(a-c)}{2(n^2-n+1)}, & \varepsilon \geq \dfrac{(n-2)(a-c)}{n(2n-1)} \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

Next, let $1 \leq m \leq n-1$. If $\varepsilon \geq (a-c)/2$, then $\theta(m) \geq 0$ for all $1 \leq m \leq n-1$, and the unrestricted maximum of $\Pi_n^I(m,r)$ is attained at $r = \theta(m)$, where $\Pi_n^I(m,\theta(m)) = (a-c)\varepsilon < \Pi_n^I(n,\varepsilon)$.

Conclusion 7. *For $n \geq 3$ and $\varepsilon \geq (a-c)/2$, $m^* = n$.*

Suppose next that $\varepsilon < (a-c)/2$. If $1 \leq m \leq n-1$, the unrestricted optimal policy is $\langle m, r\rangle = \langle n-1, \tilde{r}^I(n-1)\rangle$, where

$$\tilde{r}^I(n-1) = \frac{2(n^2-2)\varepsilon - n(a-c)}{2(n^2-1)}. \tag{30}$$

If $n(a-c)/2(n^2-2) \leq \varepsilon < (a-c)/2$, then $\tilde{r}^I(n-1) \geq 0$ and the optimal royalty rate is $\tilde{r}^I(n-1)$. Then,

$$\Pi_n^I(n-1, \tilde{r}^I(n-1)) = \frac{(a-c)^2 + 4n(a-c)\varepsilon + 4\varepsilon^2}{4(n+1)}. \tag{31}$$

If $\varepsilon \leq n(a-c)/2(n^2-2)$, then it can be shown that for $1 \leq m \leq n-1$, the optimal policy is $\langle m, r\rangle = \langle n-1, 0\rangle$.

Conclusion 8. *For $n \geq 3$, if $\varepsilon \geq (a-c)/2$, then $m^* \geq n-1$.*

Remark. Suppose $n \geq 3$ and $\varepsilon < (a-c)/2$. Then $r_n^* = 0$ and $m^* = n$ for small values of ε. If ε is sufficiently close to $(a-c)/2$, then $m^* = n$ and r^* is positive and given by (29).

Part [ii] of Proposition 2 follows from Conclusions 7 and 8. Finally, from Lemmas A5 and A6, noting that $r^*(m^*) > \theta(m^*)$, it follows that all firms continue to operate. This completes the proof of Proposition 2. ■

Proof of Proposition 3. For part I, the proofs of (a) and (b) have been provided in the main text, while (c) and (d) follow from the proofs of Propositions 1 and 2. For part II, the proofs of (a) and (b) are standard, but long and tedious and hence omitted. Finally, (c) and (d) of II also follows from the proofs of Propositions 1 and 2. ■

Proof of Proposition 4. Let $N = n+1$ be the industry size in both G^O and G^I. That is, there are $n+1$ firms when the innovator is an outsider and n firms other than the innovator when she is an incumbent firm. Let O and I denote the outsider and the incumbent innovator respectively. Using the continuity argument, it is sufficient to prove that I has lower incentives to innovate compared to O if $\tilde{\lambda}_I = 0$, while for significant innovations, the converse is true if $\tilde{\lambda}_I = 1$. Similarly, we assume $\alpha = 0$. Suppose $\tilde{\lambda}_I = 0$, that is, I believes that there will be no innovation if she fails to innovate. By (11), when $n = 1$ and $\varepsilon \geq (a-c)/3$, the payoff of O is

$$\Pi_2^O(2, \bar{r}^O(2)) = [(a-c)^2 + 42(a-c)\varepsilon + 9\varepsilon^2]/54, \tag{32}$$

where $\bar{r}^O(2) = [3\varepsilon - (a-c)]/6$. Let $\Delta_n(m, r)$ be the incremental payoff of an incumbent innovator in an industry of size of $n+1$ from the policy $\langle m, r\rangle$.[13] For $n = 1$, from (23), the incremental payoff of I is

$$\Delta_1(1, \varepsilon) = (a-c-\varepsilon)^2/9 + (a-c)\varepsilon - (a-c)^2/9. \tag{33}$$

Since $\varepsilon < a - c$, it can be easily checked that $\Pi_2^O(2, \bar{r}^O(2)) > \Delta_1(1, \varepsilon)$. Next, let $n \geq 2$ and $\varepsilon \geq (a-c)/q(n+1)$. It can be shown that for this case, $m^* = n$ and $r_n^* = \bar{r}^I(n)$ for I. Then by (27), we have

$$\Delta_n(n, r_n^*) = \frac{(n^3+4)(a-c+\varepsilon)^2 + 4n^2(n+1)^2(a-c)\varepsilon}{4(n+2)^2(n^2-n+1)} - \frac{(a-c)^2}{(n+2)^2}.$$

From Proposition 1, it follows that O obtains $(a-c)\varepsilon$ for this case and it can be easily verified that $\Delta_n(n, r_n^*) < (a-c)\varepsilon$ in this region. This proves the first part of the proposition.

Next, suppose $\tilde{\lambda}_I = 1$. By Proposition 3, the post-innovation payoff of any firm is higher in G^I than in G^O. Thus, the lowest incremental payoff of I is attained when the innovator is another incumbent firm in case I fails to innovate. Let $n = 1$. If $\varepsilon \geq (a-c)/2$, then by (33)

$$\Delta_1(1, \varepsilon) = (a-c-\varepsilon)^2/9 + (a-c)\varepsilon - (a-c-\varepsilon)^2/9 = (a-c)\varepsilon.$$

On the other hand, Proposition 1 asserts that O obtains less than $(a-c)\varepsilon$ for $n = 1$, so the result is proved for $n = 1$. Next, let $n \geq 2$. If $\varepsilon \geq (a-c)/q(n+1)$,

[13]No confusion should arise from the fact that subscript n is used for $\Delta_n(.,.)$ while subscript $n+1$ is used for $\Pi_{n+1}(.,.)$. The subscript in both cases refer to the number of firms other than the innovator. We find this notation convenient for the proof of Proposition 5.

from (27), the post-innovation payoff of I is

$$\Pi_n^I(n, r_n^*) = \frac{(n^3+4)(a-c+\varepsilon)^2 + 4n^2(n+1)^2(a-c)\varepsilon}{4(n+2)^2(n^2-n+1)},$$

and the net payoff of any firm other than the innovator is

$$\tilde{\Pi} = \frac{n^2(n+1)^2(a-c-\varepsilon)^2}{4(n+2)^2(n^2-n+1)^2}.$$

Consequently, $\Delta_n(n, r_n^*) = \Pi_n^I(n, r_n^*) - \tilde{\Pi}$. By Proposition 1, if $\varepsilon \geq (a-c)/q(n+1)$, then O obtains exactly $(a-c)\varepsilon$. On the other hand, it can be easily verified that $\Delta_n(n, r_n^*) > (a-c)\varepsilon$. This completes the proof of Proposition 4. ■

Proof of Proposition 5. We prove Proposition 5 only in case the innovation is significant. We omit the proof of part [ii] of the proposition since we only deal with non-negative royalty. The industry size that provides the highest incentive to innovate when the magnitude of the innovation is ε is denoted by $n^O(\varepsilon)$ for an outsider innovator and $n^I(\varepsilon)$ for an incumbent innovator.

Outsider innovator. Consider $\varepsilon \geq (a-c)/2$. Denoting $x \equiv (a-c)/\varepsilon$, we then have $x \in (1, 2]$ (since we are only considering non-drastic innovation, i.e., $\varepsilon < a-c$). Let $\Lambda_n^O(\varepsilon)$ be the payoff of an outsider innovator for industry size n and magnitude of innovation ε. Then, $n^O(\varepsilon) = \arg\max_{n\geq 1} \Lambda_n^O(\varepsilon)$. From Proposition 3, we have $\max\{\Lambda_1^O(\varepsilon), \Lambda_2^O(\varepsilon)\} < (a-c)\varepsilon \leq \Lambda_n^O(\varepsilon)$ for $n \geq 3$. So, it is enough to consider $n \geq 3$. From (22), it follows that $\Lambda_n^O(\varepsilon) = (a-c)\varepsilon$ when $x \in (1, q(n)]$, and $\Lambda_n^O(\varepsilon) = \Pi_n^O(n, r_n^*) > (a-c)\varepsilon$ when $x \in (q(n), 2]$, where r_n^* is given by the upper part of (18). [See the last paragraph of the proof of Proposition 1]. Since $q(n) = 2$ for $3 \leq n \leq 6$ and $q(n) < 2$ for $n \geq 7$, we have $\Lambda_n^O(\varepsilon) = (a-c)\varepsilon$ for $3 \leq n \leq 6$, and $x \in (1, 2]$. For $n \geq 7$, and $x \in (1, q(n)]$, $\Lambda_n^O(\varepsilon) = (a-c)\varepsilon$, while for $x \in (q(n), 2]$, $\Lambda_n^O(\varepsilon) = \Pi_n^O(n, r_n^*) > (a-c)\varepsilon$. This, together with the facts that $q(n+1) < q(n)$ for $n \geq 7$ and $\lim_{n\to\infty} q(n) = 1$ imply that for every $x \in (1, 2]$, there exists an integer $N(x) \geq 6$ such that $x \in (q(N(x)+1), q(N(x))]$, so that $\Lambda_n^O(\varepsilon) = (a-c)\varepsilon$ for $n \leq N(x)$ and $\Lambda_n^O(\varepsilon) =$

$\Pi_n^O(n, r_n^*) > (a-c)\varepsilon$ for $n \geq N(x)+1$. Hence $n^O(\varepsilon) \geq N(x)+1$. It can be shown that for $n \geq 7$, there exists $\gamma(n) > q(n)$ such that $\Pi_{n+1}^O(n+1, r_{n+1}^*) \geq \Pi_n^O(n, r_n^*)$ when $x \in (1, \gamma(n)]$ with equality iff $x = \gamma(n)$ and the reverse inequality holds if $x > \gamma(n)$. Moreover, $\gamma(n+1) < \gamma(n)$ and $\lim_{n\to\infty} \gamma(n) = 1$. Thus, for every $x \in (q(N(x)+1), q(N(x))]$, there exists an integer $\bar{N}(x) \geq N(x)$ such that $x \in (\gamma(\bar{N}(x)+1), \gamma(\bar{N}(x)]$. Hence, $\gamma(n) \geq x$ for $N(x) \leq n \leq \bar{N}(x)$ and $\gamma(n) < x$ for $n \geq \bar{N}(x)+1$. This implies that if $t = \bar{N}(x)$, then

$$\Pi_{t+1}^O(t+1, r_{t+1}^*) \geq \Pi_t^O(t, r_t^*) > \ldots > \Pi_{N(x)}^O(N(x), r_{N(x)}^*),$$

$$\Pi_{t+1}^O(t+1, r_{t+1}^*) > \Pi_{t+2}^O(t+2, r_{t+2}^*) > \ldots >$$

so that the maximum of $\Pi_n^O(n, r_n^*)$ over $n \geq N(x)+1$ is attained at $n = t+1 = \bar{N}(x)+1$. Hence, $n^O(\varepsilon) = \bar{N}(x)+1$.

Observation 1. *For $x \in (1, 2]$, $\bar{N}(x)$ is decreasing in x.*

Proof. Consider $1 < x_1 < x_2 \leq 2$. Then there are integers $\bar{N}(x_1)$ and $\bar{N}(x_2)$ such that $x_1 \in (\gamma(\bar{N}(x_1)), \gamma(\bar{N}(x_1)+1)]$ and $x_2 \in (\gamma(\bar{N}(x_2)), \gamma(\bar{N}(x_2)+1)]$. Suppose to the contrary that $\bar{N}(x_1) < \bar{N}(x_2)$. Then, $\bar{N}(x_1)+1 \leq \bar{N}(x_2)$, so that

$$\gamma(\bar{N}(x_2)+1) < \gamma(\bar{N}(x_2) \leq \gamma(\bar{N}(x_1)+1) < \gamma(\bar{N}(x_1)). \tag{34}$$

Since $x_1 \in (\gamma(\bar{N}(x_1)), \gamma(\bar{N}(x_1)+1)]$ and $x_2 \in (\gamma(\bar{N}(x_2)), \gamma(\bar{N}(x_2)+1)]$, from (34), it follows that $x_2 \leq x_1$, a contradiction. ■

Since $n^O(\varepsilon) = \bar{N}(x)+1$, where $x = (a-c)/\varepsilon$, and $\bar{N}(x)$ is decreasing in x, $n^O(\varepsilon)$ is increasing in ε. Since $\gamma(n) \to 1$ as $n \to \infty$, and $x \in (\gamma(\bar{N}(x)+1), \gamma(\bar{N}(x)]$, then $\bar{N}(x) \to \infty$ as $x \downarrow 1$.

Conclusion 9. *$n^O(\varepsilon)$ is increasing in ε when $\varepsilon \geq (a-c)/2$ and $n^O(\varepsilon) \to \infty$ as $\varepsilon \uparrow a-c$.*

Incumbent innovator. For industry size $n+1$, let $\Lambda_n^I(\varepsilon)$ denote the post-innovation payoff of an incumbent innovator. Let $\tilde{\Lambda}_n$ be the pre-innovation

Cournot profit, i.e., $\tilde{\Lambda}_n = (a-c)^2/(n+2)^2$. In an industry of size $n+1$, the incremental payoff due to the innovation is $\Delta_n(\varepsilon) = \Lambda_n^I(\varepsilon) - \tilde{\Lambda}_n$. Hence, $n^I(\varepsilon) = \arg\max_{n\geq 0\}} \Delta_n(\varepsilon)$. Due to (9), it is enough to consider $n \geq 2$. Denote

$$h(n) = \frac{n^4 + 5n^3 + 2n^2 - 4n + 4 + (n+2)\sqrt{(n+1)(n^2-n+1)(n^3+4)}}{2n^3 + n^2 - 4n}.$$

It can be verified that $h(n)$ obtains its minimum at $n = 3$. We consider here only significant innovations, specifically, $\varepsilon \geq (a-c)/h(3)$, or equivalently $x \in (1, h(3)]$. From the proof of Proposition 2, it can be shown that for $n \geq 2$, $\Lambda_n^I(\varepsilon) = \Pi_n^I(n, r_n^*)$ where $r_n^* = \bar{r}^I(n)$, given by (26). Let $\Delta_n(n, r_n^*) = \Pi_n^I(n, r_n^*) - \tilde{\Lambda}_n$. It can be shown that for all $x > 1$, $\Delta_n(n, r_n^*)$ is increasing in n for $2 \leq n \leq 8$, so that it is enough to consider $n \geq 8$. For $n \geq 8$, there is a function $\phi(n) > 1$ such $\Delta_{n+1}(n+1, r_n^*) \geq \Delta_n(n, r_n^*)$ iff $x \in (1, \phi(n)]$. Moreover, it can be shown that $\phi(n+1) < \phi(n)$, $\phi(12) < h(3) < \phi(11)$, and $\lim_{n\to\infty} \phi(n) = 1$. All these facts imply that for every $x \in (1, h(3)]$, there is an integer $\tilde{N}(x) \geq 12$ such that $x \in [\phi(\tilde{N}(x)), \phi(\tilde{N}(x)-1))$, so that for this region $n^I(\varepsilon) = \tilde{N}(x) + 2$. Similar to Observation 1, we can show that $\tilde{N}(x)$ is decreasing in x and $\tilde{N}(x) \to \infty$ as $x \downarrow 1$.

Conclusion 10. *$n^I(\varepsilon)$ is increasing in ε when $\varepsilon \geq (a-c)/h(3)$ and $n^I(\varepsilon) \to \infty$ as $\varepsilon \uparrow a-c$.*

This completes the proof of Proposition 5 for significant innovations. ■

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