

# Party Formation and Coalitional Bargaining in a Model of Proportional Representation

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## Abstract

We present a model of multi-party democracy under proportional representation (PR) that is comprised of 4 stages: party formation, voting, coalition formation and policy-making. We highlight how the relative importance of ideology to benefits of holding office influence the formation of coalitions. The model explains the diversity of outcomes observed empirically under PR. We show that in equilibrium, the types of coalitions that are formed may be minimal winning, minority as well as surplus and they need not be ideologically ‘connected’. It is also possible to have a caretaker government. The type of government that is formed in equilibrium depends upon the relative importance of rents of office vis-a-vis ideology as well as the relative seat shares of the parties. We compare and contrast the coalition formation and policy making stage in our model with existing models and then look at how the coalition formation game gives rise to incentives for groups to strategically form parties. In general, we get multiple equilibria but for specific parameter values we are able to compare the results of party formation under PR with that under plurality rule and show that Duverger’s law may not hold. Thus, we integrate the ‘citizen candidate’ models of party formation with models of coalitional bargaining.

## 1 Introduction.

One of the salient features observed in democracies which follow the system of voting by Proportional Representation (PR) is the preponderance of coali-

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tion governments.<sup>1</sup> Empirically, the type of such coalition governments seen under PR is fairly diverse both in terms of size as well as the ideological closeness of the coalition partners. In particular, minority, minimal winning as well as surplus governments have been seen in various parliamentary democracies under PR. A study of 15 post world war 2 European democracies by Gallagher, Laver and Mair (1995) find that about 35% of coalitions are ‘minimal winning’, 36% are minority (i.e. the coalition members constitute less than half the seats in the legislature) coalitions and the rest of the coalitions (29%) are surplus (i.e. they have more than the required number of parties needed to constitute a majority).<sup>2</sup> From the point of view of ‘ideological closeness’ such democracies also show diversity in that coalitions are not necessarily ideologically ‘connected’ and may often leave out the ‘median’ party.<sup>3</sup> However, until recently, coalition government was modeled in the literature as a ‘50% or more coalition’<sup>4</sup>. No minority government was considered. Nor did surplus governments occur in equilibrium.

We construct a model which can explain the broad differences in coalition formation and the diversity in policy outcomes in democracies under the broad rubric of PR documented in the empirical literature. A key feature of our model is the endogenous determination of the number of candidates (or parties), as well as the post election ruling coalition. Our paper contributes to the literature on formal models of parliamentary democracies under PR in two ways. First, it provides an explanation for the various types of coalition governments in terms of the constituent parties’ ideologies and the benefits associated with holding political office. Second, it sheds light on the incentives under PR for various groups to form political parties. This provides a basis for reexamining the Duvergerian hypothesis (see Duverger(1964)) that the number of parties under PR is greater than those under the plurality rule.<sup>5</sup>

A recent paper by Diermeier and Merlo (2000) also allows for the di-

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<sup>1</sup>Diermeier and Merlo (1999) look at 313 European governments from 11 multiparty democracies out of which only 20 have one party controlling more than half the seats.

<sup>2</sup>See also Laver and Schofield (1990) chapters 4 and 5 for details. The distinction they make between the members of government and members who support the government is that the members of the government are directly involved in policy making and enjoy additional benefits of office by having control over various departments of the government.

<sup>3</sup>About 25% of coalitions do that. See Laver and Schofield (earlier cit.).

<sup>4</sup>See Austen Smith and Banks (1988), Baron (1989, 1991,1998) and Lupia and Strom (1995).

<sup>5</sup>It is worth noting that this was formalized much later. See Fey (1997), Feddersen (1992) and Palfrey (1989) for formalizations of the ‘plurality leads to 2 party rule’ part of the law.

versity in outcome in terms of size at the coalition formation stage. However, both their paper and another recent paper by Baron and Diermeier (2001) use ‘efficient’ bargaining in which coalition partners negotiate the efficient policy and redistribute the surplus using unlimited side payments. For this Baron-Diermeier-Merlo (henceforth BDM) assume that there is perfect transferability of ‘perks of office’ as well as perfect commitment at the coalition formation stage to making such transfers. These assumptions do generate the observed phenomena of diversity in size but they are unable to analyze the ideological diversity of coalitions. Our model can explain both the diversity in size as well as ideological distance by showing how these phenomena are perfectly rationalizable given the trade-offs that parties face between policy compromises and power sharing. Thus, we are able to set forth testable implications which further empirical work can verify or refute.

However, our analysis is not confined to the policy making stage alone. Even as we characterize the type of government emerging from a legislature, it is important to study whether such a legislature would emerge from rational agents forming parties, given the payoffs from the ex-post coalition formation game. In particular, the rules governing coalition formation and policy making give rise to certain incentives at the party formation stage for ideological groups to launch parties-i.e. stand for elections. We study how groups will act strategically in the first stage in deciding whether or not to form parties (and thereby incur a cost) taking their ex post payoffs into account. We find that an equilibrium in which the median group is the only one to form a party always exists. Of course, this equilibrium is not unique. As the equilibrium decision of each group depends on their beliefs about the behavior of other groups we get multiple equilibria. However, for certain parameter values we compare this entry decision with that under plurality rule. Hence, we shed some light on Duverger’s Law (see Duverger (earlier cit. )) which states that while the plurality system leads to a two party regime, PR shows no such tendency. In doing so, we also formalize some of the issues that political activists talk about when comparing PR with plurality rule and suggesting the superiority of one system over the other.<sup>6</sup>

A simple illustration will help clarify these issues. There are 3 potential parties named 1, 2 and 3. Suppose that in every district parties 1 and 2 each enjoy the support of about 40% of voters in each district while about 20 % support party 3. Assuming sincere voting, under plurality, either party 1 or

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<sup>6</sup>See the online PR library at <http://www.mtholyoke.edu/acad/polit/damy/prlib.htm>, Amy (2000) and Barber (2001)

party 2 wins in every district. and thus party 3 has no incentive to contest the elections. However, under PR each party will get seats proportional to vote shares and thus 3 will have about 20% of the total seats in the legislature. Hence, under plurality, 3 will have no incentive to stand because it knows it will not get represented. Under PR, 3 will have an effective role as it may well be asked by either 1 or 2 to be part of a coalition government which means that 3's supporters get effective representation. However, 3 may use its power to try to get 'disproportionate' gains or switch sides leading to political instability. This is of course a stylized example but it provides an intuitive understanding of some of the issues involved. Thus, this example shows that to get a clear understanding of these issues we need a model which can shed light both on political entry as well as post election coalition formation and policy making. Our paper attempts to do this by building a formal model of PR starting from the party formation stage to government policies undertaken.

There are two crucial features of our model with regard to our treatment of political power. First, we assume that political power is vested in various ministerial offices and the person in charge of a particular ministry is entitled to that power. Without explicitly modeling the issues regarding non-verifiability of such power, we assume that such power cannot be (ex-post) transferred or (ex-ante) credibly promised to the people outside the ministries. Second, bargaining within the parties forming a coalition leads to their agreeing to share the ministries in the proportion of their seat shares. This is an empirical regularity that is consistently observed across time and space in democracies with PR (see Browne and Fendreis (1980) and Laver and Schofield, 1990).<sup>7</sup>

Given the outcome associated with each coalition several viable coalitions may be possible. Thus, the prerogative to initiate the process of coalition formation becomes crucial. In practice, a party called 'the formateur' is conferred such a right. Across the world, there are different rules governing the selection of a formateur. We will consider two protocols that are commonly observed and previously modeled. The first protocol is due to Diermeier and Merlo (1999)<sup>8</sup>. According to this protocol, each party in the hung legislature is probabilistically recognized as a formateur with the recognition probability being equal to the fraction of seats won by that party. Diermeier and Merlo argue in favor of empirical support for this protocol. Another protocol is due to Austen-Smith and Banks (1988). According to this protocol, the

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<sup>7</sup>We describe the bargaining procedure in the next section.

<sup>8</sup>This is actually a modified version of Baron and Ferejohn (1989)

parties are sequentially recognized as formateurs, starting with the largest party, followed by the second largest party and so on. In each of the protocols, if the attempt at government formation fails, then the status-quo policy is implemented under a caretaker government. We characterize the coalition formation game as described above and then solve backwards for the election and party formation stage. Here is a brief summary of the main features of our model and the important results.

First, at the coalition formation stage we show how, in equilibrium, our model generates the types of coalitions that we see in the real world. Minority, minimal winning as well as surplus coalitions occur depending on the identity of the formateur, the relative importance of ideology to the ‘rents of office’ which we shall loosely refer to as power, the seat shares of parties and the status quo policy i.e. the policy implemented if a coalition cannot be successfully formed. Interestingly, coalition structure is responsive to the magnitude of power in the following sense. For a certain range of power, the type of coalitions proposed by a formateur both in terms of size and with respect to ideological closeness can keep changing. For example, at moderate ranges of power the coalitions formed may not be connected while at higher ranges the coalitions formed in equilibrium may again be connected. However, we find that there exists an upper bound on power beyond which the nature of coalitions formed do not change. Interestingly, beyond that range surplus coalitions do not form in equilibrium. Another important issue which emerges is that once equilibrium considerations are taken into account what appears as surplus coalitions in the data are not surplus in the sense that the coalition may not really be carrying extra parties. The reason is as follows-only one of party A and B may be necessary for the coalition to win the vote of confidence in the house. However, A and B will decide to join a coalition only if both are invited together as their joint presence is necessary to skew the policy sufficiently towards them for it to be more attractive to either than the status quo.

The bargaining protocol also has an important bearing on the outcome. If instead of the one shot random recognition protocol we follow selection in order, with low power, the unique subgame perfect equilibrium is for the median party to form a minority government. This is independent of the order of recognition and the location of the status quo. The result goes through with endogenous, costly party formation- only 1 party would form and as cost becomes small, it is formed around the voter median. However, as the gains from office increase both ‘connected’ and ideologically distant coalitions can form which is in keeping with the results of the random recognition protocol.

At the entry (or the party formation) stage the presence of multiple equilibria does not allow us to do a complete characterization of the political process. However, it is interesting to note that the ‘citizen candidate’<sup>9</sup> framework of candidate entry which we use is robust to the institutional details of coalition formation. In particular, the citizen candidate model looks at equilibrium candidate entry under plurality and depending on costs generate one, two and three candidate equilibria. The coalition formation and policy outcome of our model generates similar types of equilibria including equilibria in which the median candidate is the only candidate as well as equilibria involving spoiler candidates. However, we find that three or more parties can also form unlike plurality rule which lends some support to the theory of greater political competition under PR. We illustrate using some examples why this may not be necessarily true. Hence, our conclusion is that as far as the normative debate on PR vs. plurality is concerned we need to understand institutional details better to make meaningful comparisons.

The rest of the paper is divided as follows. Section 2 presents the related literature in more details. Section 3 presents the formal coalition formation and policy making stages and derives the results on the type of coalitions formed including examples in multidimensional policy space. In section 4 we add the group formation stage and prove existence and multiplicity of equilibria. Section 5 does a robustness analysis involving the bargaining procedure and assumptions about voting behavior. Section 6 discusses the empirical implications of our model and concludes.

## 2 Related literature.

Our work is related to several strands in the literature, particularly with several papers on coalitional bargaining, party positioning and endogenous party formation. It also draws on the insights provided by classic works on electoral systems, the activist literature on PR and several case studies which throw light on actual coalitional structure.

The study of what type of coalitions will form in equilibrium dates back to Riker (1962). However, the concern in Riker’s work is with the division of a fixed amount of benefits which coalitional members will have access to. Hence, not surprisingly, he predicts a minimal winning coalition. In fact, his prediction for most cases is even sharper—he predicts a minimum winning coalition i.e. the minimal winning coalition made of the smallest number of members. Till recently, notwithstanding the empirical evidence on minority

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<sup>9</sup>See Osborne and Slivinski (1996) and Besley and Coate (1996).

and surplus coalitions most formal work in this area predicted minimal winning coalitions. However, when ideology is considered there are no longer compelling reasons to predict a minimal winning coalition. Instead, following Axelrod (1970), the natural thing to predict would be a ‘minimum winning connected’ coalition-i.e. a coalition that does not leave out a partner who is in between two coalition partners on the ideological dimension. In practice, even that is not seen and ‘disconnected coalitions are not uncommon.’<sup>10</sup> Recent work by Diermeier and Merlo (earlier cit.) try to fill the gap by building a model which is capable of explaining the variation in size of the coalition. However, we have already pointed out why the assumptions they make are somewhat stark and in particular, their assumption about the ability of parties to make unlimited transfers implies that they are unable to analyze coalition formation as functions of party size as well as the relative importance of power in office to ideology. Our paper attempts to fill this gap. Interestingly, the institutional details governing coalition formation imply that even as power gets very high (and hence Riker’s intuition should apply) minimal coalitions are not the norm—depending on the identity of the formateur minority governments or caretaker governments can form. At the opposite end of the spectrum, these institutional procedures imply that even with low gains from office minimal winning connected coalitions may not occur.

There are also a fair number of models of party formation/strategic entry. Starting from the ‘citizen candidate’ models of endogenous candidate entry to more recent papers by Riviere (2000), Osborne and Tourky (2002) and Morelli (2001) (where a distinction is made between a candidate and a party) there are papers which look at party formation. However, they either assume that the winner is selected by plurality rule or they model the post election policy outcome as a majority rule game, thus missing one of the most important aspects of PR which is coalition formation. However, an important insight that we get from Morelli’s paper is the importance of the skewness of the population distribution across districts which crucially determines if PR promotes more entry than plurality rule. We comment on this further in section 4. There are also papers dealing with strategic entry under more general outcome functions most notably Dutta, Jackson and LeBreton (2000) but the generality of the paper makes it impossible for them to generate any sharp predictions except that under complete information at least one player (party) will behave strategically which seems to follow from the Gibbard Satterthwaite Theorem.

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<sup>10</sup>See Laver and Schofield (earlier cit.) Chapter 5 for details.

An attempt to model the fact that PR promotes diversity has been made by Ortuno Ortin (1997). However, this model is somewhat ad hoc in that it is assumed that the policy is a weighted average of the 2 parties policy with the weight being proportional to seat shares. The main aim seems to be to provide an explanation of why policies pursued might deviate from the median voters position. Given the assumption, the result is hardly surprising but as long as policies require a majority in the legislature it is hard to justify the weighted rule in a 2 party environment. Hamlin and Hjortland (2000) integrate this approach with the citizen candidate literature but the assumption of vote weighted average does not capture the institutional details of coalition formation and government policy making which is an important aspect of democracies under PR.

A somewhat complete analysis of the electoral process has been made by Austen Smith and Banks (earlier cit.), Baron (1993) and Baron and Diermeier (earlier cit.). All 3 papers look at models of parties committing themselves to announcing a policy, given their preferences and voter distributions. However, as pointed out by Alesina (1988) parties committing to positions other than their ideal point, in a multi period one shot game is clearly time inconsistent. They also note that with ‘strategic voting’ (Baron (1993) uses sincere voting) minority parliaments form in equilibrium and voters do not always vote for the party closest to their ideal point. The latter result is hardly surprising with strategic voting. However, the first result is of interest as it provides a justification for dealing with the coalition formation stage under a minority parliament. These papers do not deal with party formation and their parties are endogenous in the sense that they choose positions. We thus differ from these papers in our assumptions about coalitional outcomes and in analyzing the entry stage. In keeping with the spirit of the debate on PR promoting more representation we assume a voting population which votes sincerely though we do discuss some of the implications of strategic voting.

In summary, our model does not just study the Baron-Diermeier and Diermeier-Merlo papers with non transferable utility and no commitment but integrates it with models of endogenous party formation. Thus, it looks at the entire ‘political equilibrium’ starting from party formation to legislative bargaining which these papers do not. Thus, our paper integrates the ‘institution free’ citizen-candidate approach to politics with the rich institutional of parliamentary democracy under PR.



### 3 The Model.

#### Environment:

This is a multistage game of perfect information. There are a finite number of ideological groups which can, at a cost, decide to form a party. After parties are formed all groups vote for the party closest to them on the ideological plane. The form of PR we model (which is followed in most European countries) is the so called ‘party list’ PR where the party puts up a list of candidates and voters cast a single vote for a party. We assume sincere voting i.e. we assume that voters cast their vote for the party closest to their ideal point and randomize with equal probability if they are equally close to more than one party. In this paper we abstract away from integer problems and ‘minimum floor rules’.<sup>11</sup> Hence, seat shares are realized exactly in proportion to vote shares for each party. Once seat shares are realized, if any party has more than half the seats that party is called to form the government. Otherwise, parties are selected randomly to form a government with the probability of selection being equal to its relative seat share

Formally, there are  $N$  groups of citizens. We denote by  $\aleph = \{1, 2, \dots, N\}$  the set of groups of citizens. Let  $1 \leq N < \infty$ . It is assumed that all the members of a group are identical with regard to preferences on the policy space<sup>12</sup> and there are  $n_i$  of them in each group. The policy space is assumed to be the unit interval. Let  $u_i : [0, 1] \rightarrow \mathfrak{R}$  denote the utility function representing the preferences of a member of group  $i \in \aleph$ . We assume that parties and groups have convex distance preferences defined over the policy space given by a utility function  $u(|x_i - y|)$  where  $x_i$  is agent  $i$ ’s ideal point,  $y$  the implemented policy and  $u(\cdot)$  satisfies,  $u(0) = 0$ ,  $u' < 0$ ,  $u'' < 0$ . We denote by  $\delta \geq 0$  the cost of contesting an election for a party. Political parties also care for the benefits associated with office which we shall loosely refer to as power denoted by  $P$ . This can be enjoyed only by parties which serve in the government. Note this benefit is available to parties only if they are a part of the government.

#### The game:

The following time line is useful for understanding the stages of the game:

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<sup>11</sup>In practice, there is a ‘threshold’ number of votes that a party has to win to get elected in the legislature. However, if minority representation and diversity in representation is a goal of PR we see no reason to impose such rules.

<sup>12</sup>Hence, we do not model diversity within a group i.e. we assume that the group has solved its collective choice problem.

t=1	t=2	t=3	t=4
party formation	voting	coalition formation	policy making

In the candidate entry stage groups simultaneously choose whether they will launch a party or not. Let  $e^j \in \{0, 1\}$  denote group  $j$ 's entry decision where  $e^j = 1$  (respectively 0) means that  $j$  chooses to launch (respectively not to launch) a party. Let  $e = (e^1, e^2, \dots, e^N)$  denote the profile of entry decisions. Let  $\sigma^j$  denote the mixed strategy of group  $j$  with  $\sigma$  denoting the profile of mixed strategies. Formally,  $C = \{i \in \mathbb{N} : e^i = 1\}$  and by  $C(e)$  we denote the set of parties that is generated by the entry profile  $e$ .

Given an entry profile  $e$ , we can calculate for each  $e^i \in C$ , the vote share of  $i$ . Denote the vote share of entrant  $i$  by  $s_i$ . If the set of parties is  $C$ , let  $V_i(C)$  denote the expected utility of party  $i$  in the government formation stage. The reason we can uniquely map entry decisions into seat shares stems from our assumption about sincere voting. This, together with an assumption on single peakedness of voter preferences also enables us to calculate the vote transfers if any one party enters or drops out-i.e. the votes gained or lost by a party as a result of entry or exit by another party. The parties which gain or lose as a result of entry or exit by another party will be referred to as neighbors. For any one dimensional policy space this concept can be made more formal as follows.

For each group  $i$ , denote the set of its non-neighbors,  $M(i)$  as follows:

$$M(i) = \{j : \text{for each } j, \text{ there exists a party } k \text{ s.t. } x_i < x_k < x_j\}$$

Consequently, denote by  $N(i)$  the set of  $i$ 's neighbors, where  $N(i) = \mathbb{N} - \{i \cup M(i)\}$ . For any set of entrants  $C$  and  $C'$  such that  $i \in C, C'$  and  $N(i) \subset C, C'$   $s_i(C) = s_i(C')$  where  $s_i(C)$  and  $s_i(C')$  denote vote shares of party  $i$  when the set of parties is  $C$  and  $C'$  respectively. This captures the fact that the voters have single peaked preferences.

Let  $\{C, \{s_i\}_{i \in C}\}$  denote a legislature composed of parties in the set  $C$  where party  $i$ 's seat share is denoted by  $s_i$ . Following proportional representation, it is assumed that the parties get seats in the legislature in proportion to their vote shares. Since we abstract away from the minimum vote share requirements and from the integer constraints, we denote by  $s_i$  both the vote share as well as the seat share for party  $i \in C$ . A government is formed out of the parties that constitute the legislature. We assume that the following 'government formation game' describes the process of government formation. The government selection process has 3 stages: *Formateur Selection*, *Proto Coalition selection* and *Vote of Confidence*. We describe each stage briefly.

1) *Formateur Selection* If there exists a party  $i$  with  $s_i > \frac{1}{2}$ , then party  $i$  is

asked to be the formateur with probability 1. Otherwise, party  $i \in C$  is asked to be the formateur with probability  $s_i$ . 2) *Proto Coalition selection* The formateur makes a one shot offer to a subset  $D \subseteq C$  to form a government. If the offer is unanimously accepted by the constituents of  $D$ , then  $D$  proceeds to seek a vote of confidence of the legislature. If the offer is not unanimously accepted then a caretaker government is formed. 3) *Vote of Confidence* The members of the legislature vote for or against  $D$  forming the government. If strictly more than 50% of the legislature votes for  $D$ , then  $D$  forms a government. Otherwise a caretaker government is formed.

Following government formation we have the policy making stage. At the policy making stage the parties constituting the government bargain over the policy and share of the spoils of office. If  $D (\neq \emptyset)$  is the set of parties in the government and if  $\pi_i$  is the relative seat share of  $i$  with respect to  $D$ , then the implemented policy is  $x_D = \sum_{i \in D} \pi_i x_i$  and party  $i$  receives a share  $\pi_i P$  of political power. If there is a caretaker government in office, then a pre-specified policy  $x_\emptyset$  is implemented.<sup>13</sup> As cases of special interest, when  $D = \emptyset$ , we are said to have a ‘caretaker’ government; when  $D = C$ , we are said to have a ‘national’ government; when  $|D| = 1$ , we have a single party government; and when  $|D| > 1$ , we have a coalition government in office.

Given that this is a multistage game of perfect information we shall be looking at subgame perfect equilibria. We shall refer to the equilibrium of the whole game as the Political equilibrium and the equilibrium of the continuation game starting with any given legislature as the legislative equilibrium. We postpone a formal definition till we solve the game.

## 4 Solving the Legislative Model.

We will solve the game backwards. Thus, we will solve for the coalition formation and policy making stage without endogenizing the legislature. In the next section we shall look at party formation and study the incentives generated by the parliamentary game for party formation. Thus, in this section we start with a given seat share for each party. There are two stages in the legislative game viz. government formation and policy making.

### 4.1 The Government Formation and Policy Making game.

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<sup>13</sup>The caretaker government does not enjoy any  $P$ . This can be interpreted in several ways. The simplest interpretation is that the caretaker government does not have access to the patronage associated with office which a government usually enjoys. Note, in our analysis the question of where the resource (i.e.  $P$ ) comes from makes no difference as we do not do welfare analysis nor do we have strategic voting behavior.

We assume that each party in the legislature acts as a cohesive decision making unit which tries to maximize the payoff of its representative member. The bargaining procedure by which this occurs is the following. Once the coalition wins the investiture vote, policy making and division of the spoils of office is decided by the members bargaining over the division of weights assigned to their party. The bargaining protocol is a  $N$  person Rubinstein alternating offer game (see Rubinstein (1982)) where each member of the coalition gets to make an offer in a prespecified order. Hence, a bigger party gets more turns at proposing.<sup>14</sup> The stationary solution, when the discount factor between periods tend to 1, leads to the outcome being the weighted average of the coalition members.<sup>15</sup> Let  $v_i(D)$  denote the average payoff of a member of party  $i$ . If  $i \notin D$ , then  $v_i(D) = u(|x_i - x_D|)$  and if  $i \in D$ , then  $v_i(D) = u(|x_i - x_D|) + \frac{1}{s_D}P$  where  $s_D$  is the ‘size’ of coalition  $D$ .<sup>16</sup> Let  $v_i(\emptyset)$  denote the payoff of party  $i$  when there is a caretaker government. At the vote of confidence stage, the members of party  $i$  will vote for the proposed government  $D$  if  $v_i(D) \geq v_i(\emptyset)$  (we assume that when indifferent, a party member votes for the proposed government). Let  $A(D)$  denote the set of parties that would vote for the proposed government  $D$  and let  $s_{A(D)}$  denote its size. If  $s_{A(D)} > \frac{1}{2}$ , then  $D$  forms the government. Let  $W$  denote the set of proto coalitions that will win the vote of confidence. Formally,  $W \equiv \{D \in 2^C \text{ s.t. } s_{A(D)} > \frac{1}{2}\}$ . Now we come to the proto coalition selection stage. At this stage the formateur  $k$  must choose the proto coalition. Let  $Y$  denote the set of proto coalitions that are unanimously preferred by its constituents over the status quo. This unanimity feature is important as every coalition member has a veto power in that it can decide not to be in the coalition. Hence, unanimity is required among the selected members for a coalition to be formed. Let  $D_k$  denote the proto coalition selected by  $k$ , i.e.  $D_k = \operatorname{argmax}_{D \in W \cap Y} v_k(D)$ . We assume that  $D_k$  is unique for each  $k$ . Thus, associated with each formateur  $k$  we have an equilibrium government  $D_k$ . Thus, a legislative equilibrium can be formally defined as follows:

<sup>14</sup>See Chen-Ying Huang (2000) who uses the same idea that a bigger coalition gets more turns at proposing.

<sup>15</sup>This follows from the fact that we have concerned ourselves with a division over weights which is equivalent to a pie division problem. See the discussion in Osborne and Rubinstein (1990). Also, we need to assume that the break up of a coalition has sufficient cost so that the outside option is sufficiently unattractive to be binding for any member. Hence, any disagreement leads to a reversion to the status quo and coalition members suffer an additional loss in utility from the break up.

<sup>16</sup>Note that party  $i$  receives  $\pi_i P$  part of the power. Hence, the per party member share of power is  $\frac{\pi_i P}{s_i}$ . Since  $\pi_i = \frac{s_i}{s_D}$ , we have the per capita share to be  $\frac{1}{s_D}P$ .

**Definition 1** *A legislative equilibrium is a collection of proto coalition functions  $D^*(\cdot, \cdot) = (D_1(\cdot, \cdot), D_2(\cdot, \cdot), \dots, D_N(\cdot, \cdot))$  such that  $\forall k \in C, D_k = \operatorname{argmax}_{D \in W \cap Y} v_k(D)$*

Note that existence is not a problem as the sets  $W$  and  $Y$  are well defined. Hence,  $D_k$  is well defined. We have assumed that  $D_k$  is unique for each  $k$  for convenience. This does not change any result as whenever the proto coalition that maximizes a formateur's payoff is not unique we can assume that any of the proto coalitions is chosen with equal probability.

## 4.2 Results.

Before stating our main results on the parliamentary stage it is useful to make precise the types of coalitions we had described in the introduction. If  $s_D < \frac{1}{2}$  we have a minority government, otherwise the government may be minimum winning or surplus. If we find that there is a party  $i$  such that  $s_{D-i} > \frac{1}{2}$  we would normally say that the government is a surplus government, otherwise it is a minimum winning government. However,  $D-i$  may not belong to  $W \cap Y$ . In that case, it seems more appropriate to call such governments minimal winning and reserve the term surplus if in addition to  $s_{D-i} > \frac{1}{2}$ ,  $(D-i) \in W \cap Y$ . We will provide an illustration of this while describing one of our results. Regarding ideological closeness, the following definition of connected coalitions will be used.

**Definition 2** *A coalition is said to be connected if there does not exist a party  $i$  outside the coalition which lies inside the line joining the extreme points of the coalition members ideal points.*

We now provide the main results of our legislative game regarding coalition size and ideology. As the purpose of the paper is to be able to explain the diversity of coalitions we have a set of results showing what type of conditions lead to what type of government rather than one main result predicting a particular type of government. The first result concerns the size of coalitions that can form in equilibrium.

**Proposition 1** *An equilibrium government  $D_k$  may be minimal winning, minority, surplus or even caretaker.*

**Proof.** We shall show by constructing examples that each of the governments mentioned above can form.

1. *Minimal Winning* Let  $C = \{1, 2, 3\}$  with  $x_1 = 0$ ,  $x_2 = 0.4$ ,  $x_3 = 1$  and  $x_\emptyset = 0.7$ . Let  $s_1 = s_2 = s_3 = \frac{1}{3}$ . Assume that  $P = 0$ . Given these

parameters,  $W \cap Y = \{\{1, 2\}, \{2\}, \{2, 3\}, \{1, 3\}, \emptyset\}$ . Suppose that party 1 is the formateur. If 1 proposes the proto coalition  $\{1, 2\}$ , then the implemented policy is 0.2 which is closer to  $x_1$  than the policy of any other government in  $W \cup Y$ . Hence  $D_1 = \{1, 2\}$ . This is a minimal winning coalition.

2. *Minority* Take the same set up as above except that party 2 is the formateur. Since  $\{2\} \in W \cap Y$ , 2 will propose that it should form a minority government.

3. *Caretaker* Same set up as before except that  $x_0 = 0.35$ . Let 1 be the formateur.  $W \cap Y = \{\{2\}, \emptyset\}$ . Since  $x_0$  is closer to 1 than  $x_2$ , 1 will propose a caretaker government. (Alternatively, 1 will propose a minority government by it and that will get defeated at the investiture stage)

4. *Surplus* This requires a different set up. With 3 parties surplus government cannot take place. We construct an example using 5 parties. Let  $C = \{1, 2, 3, 4, 5\}$

$x_1 = 0, s_1 = 0.04, x_2 = 0.1, s_2 = 0.05, x_3 = 0.5, s_3 = 0.4, x_4 = 0.7, s_4 = 0.11, x_5 = 1, s_5 = 0.4, x_0 = 0.7$ . Let 3 be the formateur. There is a range of  $P$  for which 3 chooses a surplus coalition of  $\{2, 3, 4\}$  even though 2 is redundant in the sense that the coalition of  $\{3, 4\}$  can win the confidence vote. Notice that we need to check two things here. First, whether 3 will prefer the  $\{2, 3, 4\}$  surplus coalition to all other coalitions. Second, whether 4 will accept the offer. Calculations show that the surplus coalition will have a policy which is approximately 0.5. It is easy to see that 2 will support the coalition. Therefore, it is sufficient to check this against the minimal winning coalition i.e. the 3,4 coalition where the policy is 0.54. Thus, we require the following conditions to simultaneously hold.

a) 3 prefers this to the other coalitions in  $W \cap Y$  because  $U_2(|0.5 - 0.5|) + 0.71P > U(|0.5 - 0.54|) + 0.78P$ , b) acceptance of 4 given by  $U_3(|0.7 - 0.5|) + 0.2P > U_3(|0.7 - 0.7|)$  i.e. 4 prefers to than the status quo ■

While it is sufficient to construct examples to prove that these types of governments can occur in equilibrium it is worth noting that these are not knife edge results in the sense that changing the numbers do not change the results. Also, certain additional insights follow easily from our coalition formation game. The following propositions formalize them.

**Proposition 2** *When the median party is the formateur it always forms a minority government which includes only itself.*

**Proof.** It is easy to see that this belongs to  $W$  as a majority of members prefer this to the status quo. It is clearly in  $Y$  as the median party prefers this to the status quo. Since the median party is the formateur it maximizes its utility as it implements its ideal point and enjoys the entire  $P$ . ■

**Proposition 3** *Assume that  $P = 0$  and that  $x_\emptyset \in [x_{\min}, x_{\max}]$ , then there cannot be a national government unless the implemented policy of the national government exactly coincides with  $x_\emptyset$ .*

**Proof.** Let  $\bar{x}$  denote the seat-share weighted average of all parties' policy positions. There are two possibilities to consider, 1)  $\bar{x} > x_\emptyset$ , and 2)  $\bar{x} < x_\emptyset$ . In the first case the party with ideal point  $x_{\min}$  will refuse to be a part of the coalition whereas in the second case the party with ideal point  $x_{\max}$  will refuse to join the government. Hence the national government option, i.e., set  $C$  is not in  $Y$  and therefore not in  $W \cap Y$ . ■

A couple of comments are in order. First, if  $x_\emptyset \notin [x_{\min}, x_{\max}]$ , then one can get a national government even if  $P = 0$ . Consider the following example. Let the ideal points of the parties be 0, 0.5 and  $0.5 + \varepsilon$ . Let each party have an equal seat share and let the party with ideal point 0 be the formateur. Assume that  $x_\emptyset = 0.7$ . It is clear that if party 0 invites either of the parties with ideal points 0.5 or  $0.5 + \varepsilon$  to form a coalition, then they refuse it as the outside option is closer to them. But if the party with ideal point 0 invites both of them together, then the implemented policy is  $0.33 + \varepsilon/3$  which both the invitees prefer over 0.7. Hence, the party with ideal point 0 would propose a National government. Second, note that for small amount of power the national government is still the equilibrium. Again, if the outside option were to be *too* unattractive (e.g. 0.8) then there would be no national government. Hence, we see that national government forms for moderately unattractive outside option and low power. Moreover, this is an example of a scenario where by the usual definition we have a surplus government *but if we take equilibrium considerations the government is not surplus as the formateur with ideal point 0 can form a government only if both the parties are invited*. Thus, we also provide a new explanation for why we may see coalitions which have additional members who may be left out without the government losing a majority. Also, if  $P \neq 0$  then we can have national governments even when the status quo  $x_\emptyset \in [x_{\min}, x_{\max}]$ . The intuition is similar in that if the formateur is an extreme party other parties agree to join the coalition only if they are invited together in the coalition. As an example let  $x_\emptyset = 0.1 = x_2$ . Let  $x_1 = 0$  and  $x_3 = 0.8$ . Let 3 have 40% of the seats and 1 and 2 have 30% each. It is easy to see that for moderate  $P$  the coalition  $\{1, 2, 3\}$  proposed by 1 belongs to  $W \cap Y$  while the coalition  $\{1, 2\}$  or  $\{1, 3\}$  does not.

### 4.3 Political Power and Coalition structure

Now we turn our attention to another important question. What effect does the change in the amount of political power  $P$  have on the outcome of the government formation game? An increase in  $P$  makes it more lucrative for all the political parties to be a part of the government. Hence the set of coalitions that are feasible at a lower level of power are feasible at a higher level of power as well. Formally we have,

**Lemma 1**  *$W \cap Y$  is weakly increasing in  $P$ , i.e. if  $D \in W \cap Y$  for some  $P$ , then  $D \in W \cap Y$  for all  $P' > P$ .*

**Proof.** It suffices to show that both  $W$  and  $Y$  are non-decreasing in  $P$ .

First, let us consider  $D \in W$ . For any party  $i$ , such that  $i \in A(D)$  but  $i \notin D$ , the payoff from voting for  $D$  is unaffected by changes in  $P$ . However, for any party  $i \in D \cap A(D)$ ,  $v_i(D)$  is strictly increasing in  $P$ . This establishes that the number of parties voting for  $D$  is weakly increasing in  $P$ .

Now consider  $Y$ . For any  $i \in D (\neq \emptyset) \in Y$ ,  $v_i(D)$  is strictly increasing in  $P$ . Moreover, for any coalition  $D$ ,  $v_i(D)$  is strictly increasing in  $P$ . Hence, the set of proto coalitions that are unanimously preferred by its constituents over the status quo cannot go down. ■

The above lemma leads to a useful insight: an increase in political power gives the formateur a greater set of potential proto coalitions. Hence, the payoff of the formateur is higher with higher  $P$ , *even after controlling for the direct effect of a greater  $P$ .*

Given this lemma the following proposition is immediate.

**Proposition 4** *There exists an upper bound to  $P$  beyond which the equilibrium coalition  $D_k$  does not change with changes in  $P$ . Moreover, these 'limiting coalitions' cannot be surplus.*

**Proof.** We know that  $v_i(D)$  is strictly increasing in  $P$ . Moreover,  $v_i(D)$  is not bounded. Hence, for sufficiently large  $P$  we have  $v_i(D) > v_i(\emptyset)$  for all  $i \in D$ . Hence, as  $P$  increases, any  $D$  belongs to  $Y$ . We have also proved that  $W$  is non decreasing in  $P$ . Given that  $v_i(D)$  is strictly increasing in  $P$  and not bounded this implies that beyond some  $P$  all majoritarian coalitions are in  $W$ . Hence, beyond a critical  $P$  the set  $W$  does not change. This means that  $W \cap Y$  does not change. Thus, for every  $k$ ,  $D_k = \arg \max_{D \in W \cap Y} v_k(D)$  does not change.

It is easy to see that surplus coalitions cannot occur. Consider any surplus coalition and let  $D_{-i}$  denote the coalition without a surplus member



*i.* The ideological loss in utility to the formateur  $k$  is  $u_k(|x_i - x_D|) - u(|x_i - x_{D-i}|)$ , which is finite. The gain in power is  $\frac{1}{s_{D-i}}P - \frac{1}{s_D}P$ . For sufficiently large  $P$  this gain is greater than the ideological loss. ■

However, at intermediate levels of power coalition structure is responsive to changes in  $P$ . The following example illustrates this.

**Example 1** Consider  $x_1 = 0, x_2 = 0.5, x_3 = 1, s_i = \frac{1}{3}$  for all  $i$  and  $\phi = 0.4$ . When  $P = 0$  only caretaker or minority governments form. Let parties have quadratic loss functions viz  $u_i = -(x_i - x)^2 + P$ . Beyond  $P = 0.02$  the 1,3 coalition forms whenever 1 or 3 is the proposer. Beyond  $P = 0.025$  the 1,2 coalition forms when 1 is the proposer and the 1,3 coalition forms when 3 is the proposer. Beyond  $P = 0.0525$  ‘connected coalitions’ form each time an extreme party is the formateur.

An important issue to comment on is the relation between connectedness of coalitions and political power. Following Axelrod (earlier cit.) one would predict that at low power parties would form ideologically connected coalitions. However, that is not necessarily the case. We know for instance that minority governments are possible. Even when parties form coalitions the size of parties may cause the coalition to be disconnected even at zero power. The following example illustrates this.

**Example 2** There are 3 parties with ideal points 0, 0.80 and 1. The seat shares are respectively 0.4, 0.4 and 0.2. Suppose  $P=0$ . The status quo is 0.1. Let the party with ideal point 0 be the formateur. It is easy to see that the extreme party with ideal point 1 is chosen and the implemented policy is 0.33 which is better for 1 than the connected coalition which would lead to 0.5 as the chosen policy.

It is of course true that after controlling for size disconnected coalitions occur only after a certain minimum value of  $P$ . Moreover, when  $P$  becomes very large Riker’s prediction does not hold as minority governments can still occur. Riker’s minimal winning prediction only serves as an upper bound to the size of coalitions.

## 5 Party formation.

We are now able to define the political equilibrium. We first define the entry stage equilibrium and then the political equilibrium.

**Definition 3** *Entry-stage Equilibrium:* A profile  $e$  of entry decisions constitutes an equilibrium if, for all  $e^i \in C$ ,  $V(C) - \delta > V(C')$ , where  $V(C)$  (respectively  $V(C')$ ) is the expected utility of party  $i$  from contesting (respectively not contesting) and the set of entrants is denoted by  $C$  and  $C' = C - i$ .

**Definition 4** *Political Equilibrium:* A political equilibrium is a collection  $\{D^*(\cdot, \cdot), \sigma^*(\cdot)\}$  where  $D^*(\cdot, \cdot) = (D_1(\cdot, \cdot), D_2(\cdot, \cdot), \dots, D_N(\cdot, \cdot))$  is a collection of proto coalition decision functions, and  $\sigma^*$  is an entry-profile such that,

1.  $\forall k \in C, D_k = \operatorname{argmax}_{D \in W \cap Y} v_k(D)$  and
2.  $\sigma^*$  is an equilibrium of the entry game given the proto-coalition decision functions.

## 5.1 Results.

Given these definitions we can now easily show existence.

**Proposition 5** *A Political equilibrium exists.*

**Proof.** The number of players ( $1 \leq N < \infty$ ) and the strategy set is finite. Hence, the conditions for existence of a Nash equilibrium holds. In particular, the government formation subgame associated with each formateur also has an equilibrium as  $D_k$  is well defined. ■

Now since the entry decision of each party is dependent on the decisions by other parties it is not very difficult to see that we get multiple equilibria. We shall demonstrate this by giving examples of such multiplicity. However, as our next proposition shows that the median group being the only group to form a party is always an equilibrium.

**Proposition 6** *There exists a political equilibrium in which the median party stands uncontested and implements its ideal policy in parliament.*

**Proof.** If the median party forms a party no group can get more than half the votes by standing on its own.

As the median party retains its absolute majority it still becomes the formateur and implements its ideal point. Thus any group  $i$  by putting up a candidate incurs a net cost since its change of utility from standing is  $U(x_m) - U(x_m) - \delta = -\delta < 0$ . Hence, no unilateral deviation is profitable. Any group by forming a party only undergoes a cost.<sup>17</sup> ■

<sup>17</sup>This is not robust in the sense that it depends on the simultaneity of the game. Note this non robustness is true for the citizen candidate model as well.

However, as the next example makes clear there exists parameter values for the status quo for which we have groups which cannot form a government stand so as to increase the chances of the status quo being implemented. These are akin to the ‘spoiler candidates’ in the citizen candidate model. This also allows us to demonstrate multiplicity. We shall have more to say about what the interpretation of the status quo is in the last section.

**Example 3** Consider  $x_2 < x_\phi < x_3$ . There exists configurations in which party 3’s offer will never be accepted nor will it ever be offered by any other party to be part of the coalition for which 3 will still contest. To see this let  $x_1 = 0, x_2 = 0.2, x_3 = 1, x_\phi = 0.4, s = \frac{1}{3}$  for all  $i$ . Let  $P=0.1$ . It is easy to see that both 1 and 2 prefer the status quo to joining a coalition with 3. However, by withdrawing 3 will give 2 more than half the votes and 2 will implement its ideal point. That gives 3 a utility of  $-0.64$ . By forming a party it gets an expected utility of  $-0.6$ . Hence, if  $\delta < 0.04$  3 will contest.

It is useful to compare this with plurality voting. In that case  $x_3$  will not contest and the unique equilibrium will be the median group  $x_2$ . This example conforms to the popular notion that PR promotes more political entry while plurality voting leads to a tendency for the median group to get represented. Moreover, by increasing chances of the status quo PR may lead to a failure of successful coalition formation. It is also worth noting that unless groups are symmetrically placed we need voter uncertainty to generate 2 party rule seen in countries using plurality voting.

An interesting point made by Morelli (earlier cit.) is that in a multi-district model if the population distribution across districts is sufficiently dissimilar Duverger’s predictions are reversed. We demonstrate that even in a one district scenario we can get more parties under plurality voting than under PR.

**Example 4** Consider 3 groups with ideal points  $0, \frac{1}{2}$  and  $1$ . Let the status quo be located at  $\frac{1}{4}$ . Assume that the group sizes are symmetric. Let  $P = 0$ . Under PR it is easy to see that there is a unique equilibrium in which the median party with ideal point  $\frac{1}{2}$  will stand. This is because the group with ideal point  $1$  can never be successful in forming a government nor does it prefer the status quo to the median party’s ideal point Hence, it will not form a party. Clearly, all group members with ideal point  $1$  will then vote for  $\frac{1}{2}$ . Hence, the group with ideal point  $0$  will not find it worthwhile to form a party. However, under plurality the 2 extreme groups standing (and winning with equal probability) is an equilibrium.

The intuition for this is that the further extreme party (with ideal point 1) prefers 2's policy than a coalition of 1 and 2 and hence drops out to give 2 a majority. Given that, 1 also drops out as it has no chance of winning. Thus, while several formalizations of Duverger's law relied on voters behaving strategically, we have shown in both examples that even with sincere voting the intuition is not very different if there is strategic behavior on the part of parties.

## 5.2 A note on Multidimensional Analysis.

In practice, parties care for more than one dimension. This becomes important if there is no unique dimension over which all parties care most. For instance, the Green Party may care more about the environment while a Conservative party may want to promote religious values and a centrist may care more about the economy. Hence, multidimensional analysis is important if we are to have a complete understanding of the political process. We shall now show how the rules governing coalitional policy making that we adopt can be reasonably redefined for multidimensional policy spaces. An interesting by-product of this analysis is that the 'efficient' outcome implies 'log rolling' among parties-hence, except for very special utility functions parties do not get to implement their ideal point in any dimension for all multiparty coalitions. We appropriately define certain terms in multidimensional space.

### **Neighborhood property (generalization of single peakedness):**

As there is more than one dimension we need to put relative weights to the different dimensions and specify the particular form the preferences take. In the extreme case where preferences are lexicographic the one dimensional analysis is sufficient for identifying voting behavior in a group. Otherwise we need a multidimensional preference function. For convenience, we will use a quadratic loss function, with different weights being attached to different policies and utility is a weighted sum of the various policies. Thus the utility of group  $i$  if the implemented policy vector has elements  $(x_1, x_2, \dots, x_m)$  is given by  $u_i = \sum w_j^i (x_j^i - x_j)^2$ . Therefore, given a set of parties, a group votes for the party whose vector of ideal points minimizes its loss. Thus group  $i$  chooses  $\arg \min_{k \in C} (\sum w_j^i (x_j^i - x_j^k)^2)$ . However, the weights being different for different groups bring up fresh problems as this implies that different players will have different perceptions of distance. Hence, party  $i$  may believe that party  $j$  is closer to it than party  $k$  but party  $j$  may believe that party  $k$  is closer. This problem is of course an important one and Brams et al (2001) analyze this to show how even with single peaked preferences

‘disconnected’ coalitions form in a one dimensional policy space as different players have different perceptions about distance. As their policy space is uni dimensional they require more than 5 players to get ‘disconnected coalitions’-in multidimensional space using their methodology 3 is enough. However, we do not pursue this line of research here but assume a common set of weights which for simplicity we set to be equal for all dimensions. Hence group  $i$  has a loss function given by  $u_i = \sum (x_i - x)^2$  where  $x$  is the implemented policy vector.

Given this setup, we should be able to do a multidimensional analysis. Note that as in the unidimensional case the trade-off with power still remains and many of our propositions can be generalized. However, except under special circumstances there is no appropriate definition of a ‘median party’. The set of possible coalitions that can form in equilibrium given the random selection rule and the status quo vector can however be analyzed . It is interesting to note that a consequence of parties caring for more than one policy dimension is that the status quo may lie in a position which makes it impossible for any party to form a government. An example will clarify this point.

**Example 5** *Consider 3 parties 1, 2, 3 with ideal points located on a 2 dimensional policy space with bliss points  $(0,0)$  ,  $(0,1)$  and  $(1,0)$  Let the status quo be given by  $(0, \frac{1}{2})$  . Let the party seats shares be 0.1, 0.45 and 0.45 respectively. Let  $P=0$ . Notice that no 2 party coalition is possible. To see this notice that the 1,2 coalition leads to a position  $(0, \frac{9}{11})$  which gives 2 lower utility than the status quo. Similarly the 1,3 coalition gives  $(\frac{9}{11}, 0)$  which is worse for 2. The 1,3 coalition gives  $(\frac{1}{2}, \frac{1}{2})$  which is worse for 2. Now the grand coalition gives  $(\frac{9}{20}, \frac{9}{20})$  which is worse than the status quo for all 3. Notice that at low cost of entry all 3 parties entering to make sure that the status quo is maintained is an equilibrium.*

At the same time the status quo may be such that more than one party can form a minority government.

## 6 Robustness: How critical are the assumptions?

In this section we study the robustness of the equilibrium to the assumptions we made about parliamentary rules (formateur selection procedure/ bargaining outcomes), voting behavior and inability of parties to commit to positions other than their ideal points. We will not deal with the last two issues except make brief remarks about each. However, we shall talk in

some detail about two alternate ways to model legislative behavior which are common in the literature.

### 6.1 Majority Rule game.

Instead of the formateur selection and coalition formation procedure suppose Parliament operated by voting on each issue by majority rule. In a single dimensional policy space this would lead to the Condorcet winner. If a single issue is what matters to the groups we get fairly sharp results for the whole political process. Consider the arbitrary  $N$  groups and assume a unique Median exists. In that case we get the following results immediately.

**Proposition 7** *If after elections the median of the candidates (representing different party positions) is implemented, at most two groups put up candidates in equilibrium.*

**Proof.** First, note that more than 2 candidates standing cannot be an equilibrium. To see this note that if 3 (or more) candidates contest in the second stage there will be at least one candidate who will be non pivotal i.e. whose dropping out will not affect the implemented policy. hence, it is not optimal for that candidate to contest.

We now show that there can be 0, 1 or 2 candidate equilibria.

Now if cost are very high it is easy to see that no candidate will stand.

To get one candidate equilibria first note that there exists a  $c$  for which if the candidate with ideal point 0 stands, for all  $\epsilon$ ,  $1-\epsilon$  will not find it worthwhile to contest and win This implies that  $u(-1 + \epsilon) > P - c$ . This implies a continuum of one candidate equilibria where any candidate can stand in equilibrium. However, as  $c$  decreases the range decreases i.e. the marginal candidate who can stand uncontested moves towards the median. Further, as  $c$  decreases we can get two candidate equilibria symmetrically around the median.<sup>18</sup> ■

Notice, that this contrasts with Duverger's hypothesis that PR leads to a multiparty (more than two) system. Moreover, this range around which symmetric 2 candidate equilibria can occur also keeps shrinking.

**Proposition 8** *As costs go to zero (and the median is unique) the unique equilibrium is for the median citizen to form a party.*

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<sup>18</sup>We have assumed that the group sizes are the same for convenience and they are at the same distance from their neighbors, hence 2 candidate equilibria are possible. Otherwise, as we have shown before we need to introduce voter uncertainty to get 2 candidate equilibria.

**Proof.** We need to consider only 1 candidate or 2 candidate equilibria.

Consider a 1 candidate equilibrium with a group  $x_i \neq x_m$  where  $x_m$  is the ideal point of the median candidate. WLOG, let  $x_i < x_m$ . Clearly, any  $j$  such that  $x_i < x_j \leq x_m$  can form a party and get more than half the votes. The net gain to group  $j$  is  $U(x_j) - U(x_i) + P - \delta > 0$  when  $\delta \rightarrow 0$ .

Now consider 2 candidate equilibria. We already know that they must be symmetric around the median. let us denote the utility to the median group in these symmetric equilibria by  $U(x_s)$ . Since, the post election policy gets selected by majority rule, if the median group deviated and formed a party it will get its ideal point in stage 2. Hence, by deviating the median group gets  $U(x_m) - U(x_s) + P - \delta > 0$  when  $\delta \rightarrow 0$ .

We already know that the median group being the only group to form a party is an equilibria. We have shown no other equilibria exists. Hence, as cost go to zero this is the unique equilibrium. ■

A comparison with plurality voting is quite interesting. For different cost levels we get 1 or 2 candidate equilibria as in the citizen candidate model with plurality voting. As cost decrease our prediction is extremely sharp under PR unlike plurality voting and it predicts a unique outcome. However, this seems hardly representative of how Parliament works. In particular, even if this were taken to be a way to make decisions we run into problems if the policy space is multidimensional. Different results obtain depending on how voting on different issues take place.

## 6.2 Selection in order.

Another rule which is sometimes seen in formateur selection (and mandated by law in Greece) is selection in order analyzed by Austen Smith and Banks (earlier cit.). We now look at ex post coalitions under the ‘Selection in order’ rule (Austen Smith and Banks Protocol). Briefly, this involves a fixed order of asking parties to be the formateur - starting with the largest (in terms of vote shares) and then if the largest fails to form a government the second largest and so on. If all parties fail a national government is formed and the policy implemented is a status quo policy which is implemented by a caretaker government which enjoys no ego rents.

We now present some results which contrast with random selection.

**Proposition 9** *If power and entry costs are low the unique equilibrium of the political game is for the median group to be the unique party to form.*

**Proof.** We first show that if power is ‘low’ in the parliamentary game only the median party will be able to command a majority support. This is

because any coalition will have an implemented policy  $x_j \neq x_m$  where  $x_m$  is the median party's ideal point. Hence, a majority of members prefer  $x_m$  to  $x_j$ . Hence, the optimal coalition when the median party proposes is for it to propose a coalition consisting only of itself which will be accepted. Therefore, the unique equilibrium of the parliamentary game is for  $x_m$  to get implemented.

Clearly, if this is the outcome in the legislature no other group will launch a party in the party formation stage. ■

**Proposition 10** *If the status quo policy  $\phi$  is implemented by a caretaker government which enjoys no power that government will not form in equilibrium.*

**Proof.** We are required to show that at least 1 party can form a successful coalition when it is the formateur. Notice that as  $\phi$  lies between  $x_1$  and  $x_n$  a coalition of the median party is preferred by a majority to the status quo. So there exists a feasible coalition which dominates the status quo. ■

We notice that this result contrasts with that under proportional selection. In fact, while caretaker governments are not unheard of it is usually the case that even though governments may not form at the first attempt it is almost always the case that some coalition comes to power. The one shot version of the random recognition protocol by cutting off the game in one stage does not allow for any other party to get a chance to propose leading to this 'extreme' situation. A further insight that we get is the following.

**Corollary 1** *An extreme party cannot form a minority government.*

**Proof.** To see this notice that the middle party's minority government is preferred by a majority of members. Thus there exists at least one coalition which Hence, parties commanding a majority of seats will not accept the proposal of a minority government by an extreme party. ■

A couple of remarks at this point are in order.

**Remark 1** *Minimal winning ,minority and surplus governments are possible in equilibrium. Moreover, the coalitions may be connected or disconnected.*

The trade-offs involved are similar to the proportional selection model.

**Remark 2** *The first party may not be able to form a coalition. Hence, delays may occur in equilibrium. However, it is worth noting that for every equilibrium involving delay there is an equilibrium without delay which leads to the same government.*



Notice that the formateur may not be able to form a government including itself and hence the offer it makes to other parties to join a coalition with it will be turned down. However, an equivalent outcome can be achieved by the formateur proposing a coalition excluding itself which lies in  $W \cap Y$ .

Some points of difference are worth noting.

First, in the random recognition protocol we may get caretaker governments as well as minority governments. Selection in order never leads to a caretaker government in equilibrium. Moreover, only a median party can form a minority government and that too only when power is very low. At the empirical level, while selection in order is not borne out, it is worth investigating if the predictions of the one period random recognition model used in recent papers by Baron and Diermeier (2000) and Diermeier and Merlo (2001) which we have adopted here as well captures important features of the data. Clearly, finite periods of these protocols change the results but it is still not clear what institutional details correspond to this random recognition protocol. Thus, when the largest party is not selected we need to see if this is because a party other than the largest has indicated that it has the support of other parties which would enable it to form a government. Another important thing to look at is how well the ‘random recognition’ model fits the data after accounting for an incumbency bias i.e. where the last party in power is first asked to form the government. The ‘selection in order’ protocol is however something that can be observed and legislated on (as in Greece). However, there is certainly a lot to be said for this ‘random selection’ procedure in terms of capturing the inherent uncertainty that is associated with the political environment in government formation in most countries. Moreover, this is random selection model of BDM we have analyzed under modified assumptions lead to fairly interesting results.

Clearly, there are issues which are important in the political process which we have left out. While there is evidence that voters may behave strategically there is no compelling reason to choose one over the other. As long as representation per se is important there are less reasons for voters to behave strategically under PR than under plurality. There is enough evidence in the recent runoff in the French Presidential elections to believe that voters behaved sincerely.<sup>19</sup> Further work is certainly needed in this area. We have embedded a citizen candidate model in the institutional framework of coalition government formation. Thus, we do not allow parties to credibly commit to positions other than their own. Given that parties may

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<sup>19</sup>Candidates to the left of Jospin received enough votes which, if combined, could have ensured Chirac’s defeat.

have access to a credible commitment device (often repeated play ensures that, see Alesina (earlier cit.)). it would be interesting to see if this would lead to more divergence or more convergence of party policies. This remains a fascinating area of future research.

## **7 A discussion of our results and their empirical relevance.**

We have presented a model of parliamentary democracy under PR which looks at political coalition formation as a function of party size and the relative importance of power to ideology. Moreover, by endogenizing the political entry stage we have shown how our legislature is consistent with a party formation game under the assumption of sincere voting. Our coalition and policy making stages in particular give rise to certain predictions which contrast with those existing in the literature. In particular, two limiting cases arise, one when parties care only for ideology and another where the rents of office become very large. However, these limiting cases do not give rise to the coalitions predicted by Axelrod (earlier cit.) or Riker (earlier cit.). Part of this comes from an explicit consideration of party size. Thus, when parties care only for ideology they may leave out an ideologically close partner because a large party can tilt the policy too close towards its ideal point because of its increased bargaining strength. Again as power becomes very large Riker's 'size principle' does not apply because the institutional details of what happens if no party can form a proto-coalition demand that some policy continues to be in effect under caretaker governments. Hence, if policy matters at all (even if lexicographically) and the inability to form coalitions leads to high but asymmetric costs to different partners, minority coalitions survive. However, Riker's size principle applies as an upper bound on the size of coalitions that can be formed at high levels of power which is why surplus coalitions cannot occur beyond a certain threshold of power.

We can usefully compare our paper to the papers by Baron-Diermeier-Merlo (BDM) which use efficient bargaining and perfect commitment within the coalition. Given any coalition, our model predicts a policy orientation that is skewed towards larger parties while BDM predicts that party size will be irrelevant. This stems from the bargaining procedure in BDM giving equal weights to all parties. With different weights in proportion to size, our results will match. The more fundamental difference comes from the assumption of no commitment which leads to substantive differences in predictions for coalitions for a given a composition of the legislature. Em-

pirically, we do not believe that perfect commitment or the converse are observed. However, our results are robust to some degree of commitment. We believe that apart from the usefulness of analyzing the polar opposite of BDM no commitment at the coalition formation stage is often a good approximation-this implies that the proposer cannot make a take it or leave it offer such that anything in the status quo's majority win set would be accepted. This seems consistent with the observed phenomenon of power sharing that is seen in coalitions around the world. As we had pointed out earlier Laver and Schofield (earlier cit.) state that cabinet seats being allotted roughly in proportion to seat shares is one of the most observed empirical regularities of coalition governments. They also provide a discussion for why at the coalition formation stage the manifestos written are not binding and that actual policies are a result of intricate bargaining inside the coalition.<sup>20</sup> Hence, both as an approximation to what happens in the real world and in terms of predictive power we argue that no commitment is not a bad assumption.

Another issue that we address but which BDM cannot because of their assumption of efficient bargaining is the ideological connectedness of coalitions as functions of power. We are not aware of too many studies which look at this systematically. A paper by Indridason (2001) examines the nature of disconnected coalitions in the context of a few northern European countries and tentatively concludes that this is related to whether the office seeking model is more appropriate or whether the coalition is governed by ideological considerations. Our model makes a set of predictions which are more precise and can provide a useful basis for further case studies along these lines.

An extremely important issue is the formateur selection procedure. Diermeier and Merlo (1999) provide the first systematic empirical study but they study only the first stage i.e. they look at whether it is the case that the first party is always asked to form the government against the alternate that this is roughly in proportion to seat shares and find empirical support for the latter. However, some things are worth further investigation-when the party elected to be the formateur is not the largest it is worth looking at whether they have informally waived the right to form the government. Moreover, whether it is the case that the party selected to be the formateur has an informal pre electoral understanding with a set of parties which together can win a majority in parliament. Also, given an incumbency bias in selection we need to check how well the two alternatives perform after controlling for

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<sup>20</sup>See also Laver and Shepsle (1995).

that. Another issue worth pursuing is to see how well proportional selection fits the data after controlling for the number of parties. These issues together with a more specific procedure incorporating institutional details of bargaining among parties inside a coalition will lead the way to a more complete understanding of formateur selection considerations as well as why delays in bargaining over government formation occur.

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