Customer Information Sharing Among Rival Firms

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Abstract

The recent rapid growth of the Internet as a medium of communication and commerce, combined with the development of sophisticated software tools, are to a large extent responsible for producing a new kind of information: databases with detailed records about consumers' preferences. These databases have become part of a firm's assets, and as such they can be sold to competitors. This possibility has raised numerous concerns from consumer privacy advocates and regulators, who have entered into a heated debate with business groups and industry associations about whether the practice of customer information sharing should be banned, regulated, or left unchecked. This paper investigates the incentives of rival firms to share their customer-specific information and evaluates the welfare implications if such exchanges are banned, in the context of a perfect price discrimination model.

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1 Introduction

Earlier literature on information sharing among rival firms has mainly focused on two types of information exchanges: i) firms share - directly or indirectly - their private signals about demand conditions [e.g. Gal-Or (1985), Vives (1990) and Villas-Boas (1994)], or ii) firms exchange cost data [e.g. Shapiro (1986) and Armantier and Richard (2001)]. The recent rapid growth of the Internet as a medium of communication and commerce, combined with the development of sophisticated software tools, are to a large extent responsible for producing a new kind of information: databases with detailed records about consumers' preferences. Such data are gleaned from a customer's transactions with a firm and public records and are used to assemble a detailed picture of consumers.[1] Firms can utilize this information to improve the focus of their marketing campaigns, to design products that better fit the needs of their customers and to tailor their price offers according to each consumers' brand preferences. These databases have become part of a firm's assets, and as such they can be sold to competitors. This possibility has raised numerous concerns from consumer privacy advocates and regulators, who have entered into a heated debate with business groups and industry associations about whether the practice of customer information sharing should be banned, regulated, or left unchecked.[2] Nevertheless, there is very little theoretical work done on this issue. This paper is a step in this direction. We look at rival firms' incentives to share their customer-specific information and we evaluate the welfare implications if such exchanges are banned, in the context of a perfect price discrimination model.

We formulate a dynamic (two-period) location model of horizontal and vertical differentiation with two rival firms. In the first period firms know only the distribution of brand preferences and each charges a uniform price. At the beginning of the second period each firm collects detailed (perfect) information about its own customers (i.e., the ones who purchased its product in period 1). Then, each firm decides whether to sell its customer database to the rival firm. The customer information enables a firm, in the next stage, to price discriminate among consumers with different degrees of brand loyalty.

[1] "*Few consumers could write down even 1% of the amount of data that companies have about them*," Customer Data Means Money, www.informationweek.com, August 20, 2001.

[2] For example see, "*Senator takes aim at e-commerce data-sharing effort*," www.computerworld.com, December 7, 2000. Based on the Online Privacy Protection Act consumers should give their consent to firms before they share customer information with a third party, e.g. "*A very public battle over privacy*," Business Week, May 23, 2002. However, firms make every effort to safeguard valuable consumer information and their option to sell it to third parties. According to the latter article above, "...most companies burry the opt-out notices within masses of legal jargon at the bottom of monthly mailings."

We show that a necessary condition for some type of information sharing to be part of a subgame perfect equilibrium is firm asymmetry. More specifically, when firms have equal customer bases (i.e., pure horizontal differentiation), then in equilibrium, neither firm finds it profitable to sell its database to the rival firm. With *enough* firm asymmetry, in the unique subgame perfect equilibrium, the firm with the lower customer base (i.e., the low quality firm) sells its information to the firm with the higher customer base (i.e., the high quality firm). The high quality firm never sells, in equilibrium, its information to the low quality firm, regardless of the difference in the customer bases between the two firms. If sharing of customer information is banned, then social welfare may decrease or increase depending upon the degree of firm asymmetry and how heavily the future is discounted. In addition, firms always become worse off, while consumers always become better off, when sharing is banned.

Chen, Narasimhan and Zhang (2001) is a paper most closely related to our work. The authors investigate the incentives of rival firms to sell customer information of imperfect targetability (accuracy). The main differences between our model and theirs are: i) in their model there are three types of consumers (loyal to a firm and switchers), while in ours there is a continuum of consumers, ii) our information identifies the preferences of each consumer with perfect accuracy, while in their model information may also be imperfect and iii) their model is static, whereas ours is dynamic. They show that information sharing will take place provided that the size of the two firms' loyal customers is not too different and moreover the seller of information is the firm with the low level of targetability. This is in contrast with our conclusion, where information sharing occurs if and only if the two firms' customer bases are sufficiently different. Fudenberg and Tirole (2000) use a similar to ours two-period location model with symmetric firms, where in the second period firms can segment the consumers into two groups (own customers and rival firm's customers) depending upon a consumer's purchasing decision in period 1. Firms do not collect any further information about their own customers and consequently the issue of information sharing does not arise. The authors focus on the use of short-term and long-term contracts as part of a firm's equilibrium strategy.

The rest of the paper is organized as follows. In section 2 we present the model. The two-period game is analyzed in section 3, where we search for a subgame perfect equilibrium which entails some type of customer information sharing. In section 4, we solve the game assuming that information sharing is banned and we assess the welfare implications of such a policy. We conclude in section

5. The appendix contains the proofs of propositions 2, 3 and 4.

2 The description of the model

There are two firms A and B who produce competing nondurable goods A and B respectively with constant per-unit marginal cost of c and are located at the two end points of the unit interval $[0,1]$. There are two periods, $t = 1,2$, and a common discount factor $\delta \in [0,1]$. The market is comprised of a continuum of consumers uniformly distributed on the unit interval. Each consumer buys one unit of either good A or B or neither in each period. A consumer located at point x derives utility, in period t, $V_{\mathsf{A}} - \zeta x$ if he buys from firm A and utility $V_{\mathsf{B}} - \zeta(1-x)$ if he buys from firm B, and a zero utility if he buys from neither firm, where $\zeta > 0$ is the per-unit of distance transportation cost.[3] We assume that $V_{\mathsf{A}} \geq V_{\mathsf{B}}$, allowing the firms to be asymmetric. Hence, consumers x's relative preferences over the two goods is given by, $\ell = (V_{\mathsf{A}} - V_{\mathsf{B}}) + \zeta(1-2x)$. There is a one-to-one correspondence between x and ℓ and therefore from now on the representative consumer is identified by ℓ. We call ℓ consumer ℓ's degree of loyalty, which is uniformly distributed on the interval $I = [-\ell_{\mathsf{B}}, \ell_{\mathsf{A}}]$, with $\ell_{\mathsf{A}} = (V_{\mathsf{A}} - V_{\mathsf{B}}) + \zeta$, $\ell_{\mathsf{B}} = -(V_{\mathsf{A}} - V_{\mathsf{B}}) + \zeta$ and density 1. Clearly, $\ell_{\mathsf{A}} \geq \ell_{\mathsf{B}}$ and $\ell_{\mathsf{A}} > 0$. Consumers with positive loyalty prefer brand A, while consumers with negative loyalty prefer brand B, all else equal. We further assume that $\ell_{\mathsf{B}} \geq 0$. If $\ell_{\mathsf{A}} = \ell_{\mathsf{B}}$, the model is analogous to the standard model of horizontal differentiation. If $\ell_{\mathsf{B}} = 0$, it becomes the standard model of vertical differentiation. If $\ell_{\mathsf{A}} > \ell_{\mathsf{B}} > 0$, the setup has elements of both horizontal and vertical differentiation. Let $p_{\mathsf{A}}^{\mathsf{t}}(\ell)$ and $p_{\mathsf{B}}^{\mathsf{t}}(\ell)$ denote the price offers to consumer ℓ, from firm A and B respectively, in period t. We denote by $\pi_{\mathsf{A}}^{\mathsf{t}}$ and $\pi_{\mathsf{B}}^{\mathsf{t}}$ firm A's and B's profits respectively in period t. By Π_{A} and Π_{B} we denote the sum of discounted profits over the two periods. The firms act to maximize Π_{A} and Π_{B}. Consumer ℓ maximizes her discounted sum of period utilities, using the same discount factor δ as the firms.[4]

At the beginning of period 1 firms know only the distribution of consumer preferences. At the end of period 1 each firm collects detailed (perfect) information only about its own consumers' brand preferences (i.e., the ones who purchased its product). A firms can sell its information

[3] We assume that there is no aggregate demand response. In other words, the value of the goods is sufficiently high and therefore the market is always covered.

[4] A similar model, but in a static version, has been employed by Shaffer and Zhang (2002).

directly to the rival firm, or indirectly by first selling it to a market-research company, knowing that the latter will sell it to the rival.[5] This distinction does not make a difference, in our model, and we will assume that selling is direct. Also, we assume that firms follow a *simple* strategy in regards to the information sharing: a firm sells its entire customer database as it is. In other words, we do not search for an optimal selling mechanism. For example, we have excluded strategies on part of the information seller such as: selling only part of the information, or even damaging the information by "throwing in" some noise. This issue is certainly very interesting, but it goes beyond the scope of the present paper. Furthermore, to simplify the analysis, we have assumed that a firm has no information about the brand preferences of the rival firm's own customers prior to information sharing (besides, of course, knowing that these customers are not its own). In practice, firms may possess such data, albeit this information is most likely to be more noisy than the rival's corresponding information. Therefore, our implicit assumption in this paper is that this noise is sufficiently high, so that a firm cannot segment the consumers of its rival (before information sharing takes place).

There are two types of customer information sharing that will be considered in this paper,

- Two-way information sharing, where firms exchange their customer databases (the net price may be strictly positive).
- One-way information sharing, where only one firm sells its customer information to its rival.

Firms can price discriminate by (say) sending coupons with different face values to different consumers. There are three distinct types of pricing strategies that a firm can adopt, in our context: i) *uniform* pricing, where each consumer on the $[-\ell_{\mathsf{B}}, \ell_{\mathsf{A}}]$ interval receives the same price, ii) *blanket* couponing, where a group of consumers (strictly smaller than the whole $[-\ell_{\mathsf{B}}, \ell_{\mathsf{A}}]$ interval) receives the same price and iii) *targeted* couponing, where each individual consumer receives a different price. If firms do not share their customer information, then each firm distributes blanket coupons to the customers of the rival firm and targeted coupons to its own customers. If there is a one-way information sharing, then the firm with all the information sends targeted coupons to all consumers, while the other firm sends targeted coupons only to its own customers and blanket coupons to the

[5]Recently, Wal-Mart decided to stop selling customer data to market-research companies, since this practice benefits Wal-Mart's competitors more, Customer Data Means Money, Aug. 20, 2001, www.informationweek.com.

customers of its rival. Finally, if there is a two-way information sharing, then each firm sends targeted coupons to all consumers.

The game we will analyze unfolds as follows:

Period 1

- In stage 1, firms, simultaneously and independently, choose their uniform prices.
- In stage 2, consumers decide from which firm to buy.

Period 2

- In stage 1, each firm decides whether to sell its customer information to its rival firm.
- In stage 2, firms, simultaneously and independently, choose their blanket coupons.
- In stage 3, firms, simultaneously and independently, choose their targeted coupons.
- In stage 4, consumers decide from which firm to buy.

We assume that targeted promotions, in period 2, are chosen *after* firms have decided about the value of their blanket coupons. This set up parallels the multistage games that have been examined in the literature [e.g. Banks and Moorthy (1999), Rao (1991), Shaffer and Zhang (1995 & 2002) and Thisse and Vives] where firms choose their promotional strategies (targeted coupons) *after* they have chosen their regular (uniform) prices. This assumption serves two purposes. First it is consistent with the common view that a firm's regular price can be adjusted slower than the choice of targeted coupons and second if both decisions are made simultaneously no pure strategy equilibrium exists. Although, blanket coupons in period 2 are not exactly the same as a uniform price, blanket coupons have an element of stickiness, relative to targeted coupons, similar to that of a regular price. Moreover, a pure strategy equilibrium does not exist when firms choose blanket and targeted coupons simultaneously (for the same reason that it does not exist when firms choose regular prices and targeted coupons simultaneously).[6] Hence, we model these two strategic choices in period 2 sequentially. Furthermore, there is no need to include a regular price in period 2.[7]

Remark 1. We completely ignore other possible utilizations of consumer information by the firms and we focus entirely on its use as a facilitator of price discrimination. Furthermore, we overlook possible non-economic effects (e.g. pure privacy issues) that information sharing may have on consumers. Also, we assume away the likelihood that a firm's customer database can also be shared with a non-rival firm, which is very likely to benefit both parties. Moreover, there is no *demand creation* in our model, another positive aspect of having detailed information. Our purpose in this paper is to identify equilibrium strategies regarding sharing of customer information in the most competitive environment, where only the *business stealing* effect is present.

Remark 2. We assume that the information enables the firms to learn the location of each consumer with perfect accuracy (perfect information). In reality, firms can identify each consumer's brand loyalty with some noise, which depends on the quantity and quality of the available information. In Liu and Serfes (2003), we solve a symmetric (pure horizontal differentiation) price discrimination model with imperfect information about consumer brand preferences. The imprecision with which each consumer's loyalty is identified depends on the quality of the available information. As the quality increases the noise is reduced. The limit of this process is the perfect information paradigm. Although this modeling approach seems more realistic, it renders the model intractable when it is coupled with firm asymmetry (a very crucial assumption in the present model). Hence, one can view the results in the present paper as the solution to an interesting limiting case.

In the next section, we search for a subgame perfect equilibrium (SPE). In particular, we are interested in a SPE in pure strategies where some type of information sharing takes place.

3 Analysis

In this section, we proceed as follows. After the first period ends, there are four subgames following the firms' decisions about whether to share information or not: i) no information exchange (NE), ii)

[6]Proof is available upon request.

[7]See the discussion in Liu and Serfes as to why a regular price does not play a sheltering role in the absence of targeting costs (an assumption that we maintain in this paper).

one-way sharing, where only firm A sells its information to B ($A \rightarrow B$), iii) one-way sharing, where only firm B sells to A ($B \rightarrow A$) and iv) two-way sharing ($A \longleftrightarrow B$). We analyze each subgame by finding the equilibrium prices and profits. We assume that an information selling transaction will occur if and only if there are gains from trade (GFT), i.e., joint firm profits *strictly* increase over the profits prior to that transaction.[8] Furthermore, firm A's and B's bargaining powers over the surplus from the information sharing are $(1-\sigma)>0$ and $\sigma>0$ respectively. Then, we move up to period 1, where firms choose their uniform prices to maximize the discounted sum of profits over the two periods. Consumers also act strategically in period 1. In particular, they maximize the discounted sum of their utility over the two periods by correctly anticipating the firms' information sharing decisions and the period 2 equilibrium prices.[9]

3.1 Period 2: Information sharing and pricing decisions

Let ℓ^* denote the marginal consumer in period 1. We assume that if a consumer is indifferent between the two brands, then she buys the product that she prefers if the prices were the same. If $\ell^* \geq 0$, then firm A collects information about its own consumers, who are the ones located in $[\ell^*, \ell_{\mathsf{A}}]$, while firm B collects information about the consumers located in $[-\ell_{\mathsf{B}}, \ell^*)$. If $\ell^* < 0$, then the marginal consumer belongs in firm B's database. We begin with the case where $\ell^* \geq 0$. Let $I_1 = [\ell^*, \ell_{\mathsf{A}}]$ and $I_2 = [-\ell_{\mathsf{B}}, \ell^*)$.

- **Subgame 1**: No exchange of information (NE).

Each firm first sends blanket coupons to the customers of the rival firm and then distributes targeted price promotions to its own customers.[10] Lets' first examine the interval I_1. Since the cutoff point is in firm A's territory, firm B will send the same price offer $p^2_{\mathsf{B}} = c$ to all consumers.

[8]In this paper, we have assumed that consumers do not care about how information about them is used and whether it is shared or not. The other extreme is to assume that consumers must give their consent before firms share (or sell) their information. Then, consumers (or a party representing them) essentially enter, along with firms, into the bargaining process over the distribution of surplus from sharing information. This scenario is also likely given recent regulatory efforts to impose an "opt-in" standard, by which firms must obtain permission before a consumer's information is shared with third parties (see, Report for Congress, "*Internet Privacy: Overview and Pending Legislation*," Feb. 6, 2003, www.epic.org). We reserve this interesting topic for future research.

[9]This modeling assumption is also made in Fudenberg and Tirole.

[10]A firm is allowed to send blanket coupons to its own customers as well, but this possibility is ignored due to the flexibility of charging individualized prices in the next stage. In addition, blanket coupons - like a regular price - serve no sheltering role in the absence of targeting costs. This holds true for all the subgames we analyze.

Firm A's best response is to offer $p_A^2(\ell) = \ell + c$. Each consumer is indifferent between the two firms and therefore buys from firm A. The profits are,

$$\pi_A^2 = \int_{\ell^*}^{\ell_A} \left(p_A^2(\ell) - c\right) d\ell = \frac{\ell_A^2 - (\ell^*)^2}{2} \text{ and } \pi_B^2 = 0, \text{ (in segment } I_1). \tag{1}$$

Therefore, the joint profits in the interval I_1 are,

$$\pi^{NE} = \frac{\ell_A^2 - (\ell^*)^2}{2}, \text{ (in segment } I_1). \tag{2}$$

Now we examine the firms' strategies and profits in the interval I_2. Firm A sends blanket coupons, p_A^2, to the consumers in this interval, while firm B responds by sending targeted price offers. Given p_A^2, firm B's best response is to set $p_B^2(\ell) = p_A^2 - \ell \geq c$. Clearly the marginal consumer $\hat{\ell}$ is located in $(0, \ell^*)$ and is given by $\hat{\ell} = p_A^2 - c$. Since firm B knows the location of each consumer perfectly it charges a price equal to marginal cost to the marginal consumer. Firm A chooses p_A^2 to maximize,

$$\pi_A^2 = \left(p_A^2 - c\right) \int_{\hat{\ell} = p_A - c}^{\ell^*} d\ell = \left[\ell^* \left(p_A^2 - c\right) - \left(p_A^2 - c\right)^2\right].$$

The first order necessary and sufficient condition is,

$$\frac{d\pi_A}{dp_A} = \ell^* - 2\left(p_A - c\right) = 0 \Longrightarrow p_A^2 = \frac{\ell^*}{2} + c. \tag{3}$$

Hence, $\hat{\ell} = p_A^* - c = \ell^*/2$. Therefore, firm A's profit in the interval I_2 is,

$$\pi_A^2 = \left(p_A^2 - c\right) \int_{\frac{\ell^*}{2}}^{\ell^*} d\ell = \frac{(\ell^*)^2}{4}, \text{ (in segment } I_2). \tag{4}$$

Firm B's profit in I_2 is,

$$\pi_B^2 = \int_{-\ell_B}^{\frac{\ell^*}{2}} \left(p_B^2(\ell) - c\right) d\ell = \int_{-\ell_B}^{\frac{\ell^*}{2}} \left(p_A^2 - \ell - c\right) d\ell = \frac{(\ell^* + 2\ell_B)^2}{8}, \text{ (in segment } I_2). \tag{5}$$

The joint profits in the interval I_2 are,

$$\pi^{NE} = (4) + (5) = \frac{3(\ell^*)^2 + 4\ell_B^2 + 4\ell^*\ell_B}{8}, \text{ (in segment } I_2). \tag{6}$$

The joint profits in $[-\ell_B, \ell_A]$ are,

$$\pi^{NE} = (2) + (6) = \frac{4\ell_A^2 - (\ell^*)^2 + 4\ell_B^2 + 4\ell^*\ell_B}{8}. \tag{7}$$

- Subgame 2: Firm A sells its information to firm B ($A \to B$).

In the interval I_1 firm B sets $p_B^2 = c$ and firm A charges $p_A^2(\ell) = c + \ell$. The profits of each firm in this interval are,

$$\pi_A^2 = \int_{\ell^*}^{\ell_A} \ell d\ell = \frac{\ell_A^2 - (\ell^*)^2}{2} \text{ and } \pi_B^2 = 0, \text{ (in segment } I_1\text{)}.$$

The joint profits in this interval are,

$$\pi^{A \to B} = \frac{\ell_A^2 - (\ell^*)^2}{2}, \text{ (in segment } I_1\text{)}. \tag{8}$$

In the interval I_2, joint profits are the same as in (6) since firm A has no information about firm B's customers. Hence the joint profits in $[-\ell_B, \ell_A]$ are,

$$\pi^{A \to B} = (6) + (8) = \frac{4\ell_A^2 - (\ell^*)^2 + 4\ell_B^2 + 4\ell^*\ell_B}{8}. \tag{9}$$

Note that $\pi^{NE} = \pi^{A \to B}$ [i.e., (7)=(9)] and therefore no gains from trading information exist when firm A sells its information to firm B.

- Subgame 3: Firm B sells its information to firm A ($B \to A$).

Firm B captures all the consumers in $[-\ell_B, 0]$ where firm A charges a price equal to $p_A^2 = c$ and firm B offers individualized prices $p_B^2(\ell) = c - \ell$. Firm A captures all the consumers in $[0, \ell_A]$ where firm B charges a uniform price equal to $p_B^2 = c$ and firm A offers individualized prices $p_A^2(\ell) = \ell + c$. The profits are,

$$\pi_A^2 = \int_0^{\ell_A} \ell d\ell = \frac{\ell_A^2}{2} \text{ and } \pi_B^2 = -\int_{-\ell_B}^0 \ell d\ell = \frac{\ell_B^2}{2}. \tag{10}$$

Hence, the joint profits are,

$$\pi^{B \to A} = \pi_A^2 + \pi_B^2 = \frac{\ell_A^2 + \ell_B^2}{2}. \tag{11}$$

Firm B will sell its information to firm A if and only if $\pi^{B \to A} > \pi^{NE}$. In other words, if and only if,

$$\begin{aligned} \frac{\ell_A^2 + \ell_B^2}{2} &> \frac{\left(4\ell_A^2 - (\ell^*)^2 + 4\ell_B^2 + 4\ell^*\ell_B\right)}{8} \Longleftrightarrow \\ GFT &= \frac{\ell^*(\ell^* - 4\ell_B)}{8} > 0 \Longleftrightarrow \ell^* > 4\ell_B. \end{aligned} \tag{12}$$

- Subgame 4: Firms exchange their information ($A \longleftrightarrow B$).

The joint profits are the same as in subgame 3. Joint profits remain unchanged when firm A sells its information to firm B (see subgame 2). Hence, no such transaction will take place.

Next, assume that $\ell^* < 0$. It can be easily seen that firm A has no incentive to acquire firm B's information, for the same reason that firm B has no incentive to acquire firm A's information when $\ell^* \geq 0$ (see subgame 2). The gains from trade when firm B acquires firm A's information are,[11]

$$GFT = \frac{\ell^* \left(\ell^* + 4\ell_{\mathsf{B}}\right)}{8} > 0 \Longleftrightarrow \ell^* < -4\ell_{\mathsf{B}}.$$

Since ℓ^* cannot be less than $-\ell_{\mathsf{B}}$, gains from trading information are negative when $\ell^* < 0$.

The next proposition summarizes the results regarding information exchanges.

Proposition 1 (Information sharing in period 2). *When firms are of the same quality (i.e., $V_{\mathsf{A}} = V_{\mathsf{B}}$), then no exchange of consumer information takes place at the beginning of period 2. When firm A is the higher quality firm (i.e., $V_{\mathsf{A}} > V_{\mathsf{B}}$), then: i) firm A never sells its customer information to firm B, and ii) firm B sells its information to firm A if and only if $\ell^* > 4\ell_{\mathsf{B}}$.*

Proof. The proof is based on the results from the analysis of the four subgames. Note that only in subgame 3 gains from trading information may be positive. Therefore, we focus on that subgame. When $V_{\mathsf{A}} > V_{\mathsf{B}}$ and more precisely $\ell_{\mathsf{A}} > 4\ell_{\mathsf{B}}$, information sharing (where B is selling to A) is possible, provided that the first period marginal consumer ℓ^* is located at a point greater than $4\ell_{\mathsf{B}}$ [see (12)]. If, on the other hand, $V_{\mathsf{A}} = V_{\mathsf{B}}$, then $\ell_{\mathsf{A}} = \ell_{\mathsf{B}}$ and consequently the highest possible ℓ^* is ℓ_{A} which is less than $4\ell_{\mathsf{B}}$. Hence, there are no gains from trading information. ■

The intuition behind the above result goes as follows. When $\ell^* > 0$ some of firm A's loyal customers purchase in period 1 from firm B. Consequently, firm A does not have these consumers in its database. This forces firm A to treat these consumers the same as the consumers who are loyal to firm B. Now suppose that firm A obtains information from firm B. This gives firm A the flexibility to charge customized prices to all consumers which creates two opposing effects that govern market interaction: first, competition intensifies since firm A follows a more aggressive

[11] The derivations are similar to the ones when `$^* \geq 0$ and are omitted.

pricing strategy in firm B's territory (negative effect) and second, profits for firm A from its more loyal customers increase due to the surplus extraction effect (positive effect). Next, we look at these two opposing effects more closely in each one of the four distinct market segments (see also figure 1). We compare the difference in joint profits between sharing and no sharing of information.

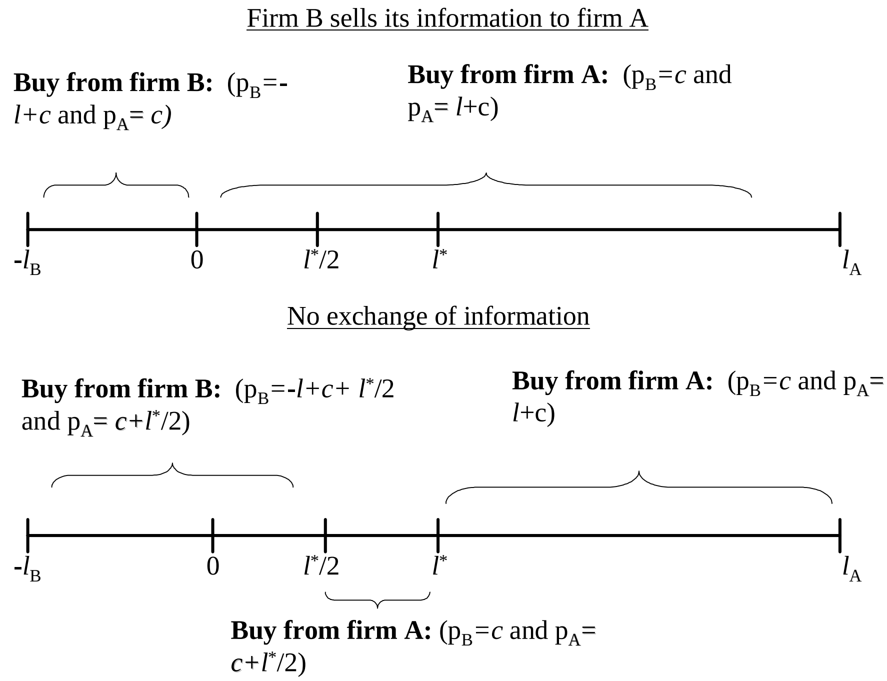

Figure 1: Second period prices and market shares

1. Interval $[-\ell_B, 0]$. Joint profits decrease (*only the negative effect is present*). Firm A, without firm B's information, finds it in its best interest to *focus* on its own loyal customers. This in turn helps firm B to raise its price to its own loyal customers and therefore firm B's profits increase compared to the outcome when B sells its information to A. Firm A makes no sales in this interval, with or without information.

2. Interval $[0, \ell^*/2]$. No change in joint profits (*both effects are present, but cancel each other out*). When B sells its information to A, these consumers buy from firm A, otherwise they buy from B. When both firms possess information about these consumers, the competition is very intense for those located close to zero, but for the ones closer to $\ell^*/2$ firm A extracts

more surplus, compared to the outcome where only firm B has information. It turns out that there is a reduction in joint profits in the interval $[0, \ell^*/4]$ (due to intensified competition) and an increase in joint profits in the interval $[\ell^*/4, \ell^*/2]$ (due to surplus extraction), when both firms possess information. Moreover, these two opposing effects cancel each other out and the net effect on joint profits in the interval $[0, \ell^*/2]$ is zero.

3. Interval $[\ell^*/2, \ell^*]$. Joint profits increase (*only the positive effect is present*). Firm B looses nothing since in any case it charges a price equal to marginal cost and its profits are zero, but firm A gains since it can tailor its prices to each individual consumer.

4. Interval $[\ell^*, \ell_A]$. No change in joint profits (*neither effect is present*). These consumers buy from firm A and firm B prices at marginal cost, under any type of information structure that we have allowed for.

When the interval $[\ell^*/2, \ell^*]$ is sufficiently greater than $[-\ell_B, 0]$, the positive effect dominates the negative. Practically, this means that firm A's customer base is sufficiently greater than firm B's and moreover when firm A is forced to charge a uniform price, in period 1, it does not find it profitable to serve all of its loyal customers. Rather, it charges a relatively high price to extract more rents from its relatively more loyal customers. As a result, some of the customers who, all else equal, prefer firm A's product to firm B's, end up buying from B in the first period.[12] But once firms have the flexibility of charging discriminatory prices, firm A finds it profitable to reclaim these consumers. If the size of this franchise is relatively big, then the gain in profits that firm A experiences outweighs firm B's losses. This finding echoes the result in Shaffer and Zhang (2002), who show that the firm with the larger loyal following may become better off when firms move from uniform to discriminatory pricing, i.e., the game need not be a prisoners' dilemma. The idea in that paper is that the *market share effect* - which benefits the firm with the larger customer base - may dominate the intensified competition effect. This market share effect clearly plays a critical role in our framework as well. Moreover, we take it a step further by comparing the gains of the larger firm with the losses of its smaller rival.

It remains to be shown that the first period uniform pricing strategy that we described in the above paragraph is indeed part of a SPE. This is what we do next.

[12]This pricing strategy, on part of the firm with the larger loyal following, has been shown to be an equilibrium strategy by Shaffer and Zhang (2002), in a static model.

3.2 Period 1: Uniform pricing

In this section, we mainly search for a SPE where sharing of information takes place. As we proved in the previous subsection, the only possibility is for firm B to sell its information to firm A. Thus, in period 2 the game play is in subgame 3. Consumers have rational expectations. They know how their purchasing decisions in period 1 will affect the information each firm has about them and consequently the price offers they will receive from each firm in period 2. The marginal consumer ℓ^* (with $\ell^* > 0$, since otherwise sharing of information is not profitable) in period 1 must be indifferent between buying from firm A today at price p^1_{A} (and entering firm A's database) and then buying again in period 2 from firm A at price $\ell^* + c$, or buying from firm B in period 1 (and entering firm B's database) at price p^1_{B} and then buying from A in period 2 at price $\ell^* + c$. Thus the indifferent consumer must satisfy,

$$\left[p^1_{\mathsf{A}} + \delta\left(\ell^* + c\right)\right] - \left[p^1_{\mathsf{B}} + \delta\left(\ell^* + c\right)\right] = \ell^* \Longrightarrow \ell^* = p^1_{\mathsf{A}} - p^1_{\mathsf{B}}.$$

Each consumer located to the right of ℓ^* purchases firm A's product in both periods. Consumers located in $[0, \ell^*)$ purchase from firm B in period 1 and then switch to firm A in period 2 (since firm B sells information about these customers to firm A and since they are loyal to firm A, firm B cannot get them). Finally, consumers in $[-\ell_{\mathsf{B}}, 0)$ buy from firm B in both periods.

The information price (IP) firm A pays to firm B for acquiring firm B's database is,

$$IP = \sigma GFT = \sigma \frac{\ell^*\left(\ell^* - 4\ell_{\mathsf{B}}\right)}{8},$$

where σ is firm B's bargaining power and the other term represents the gains from trade [see Eq.(12)]. The profits of firm A and B in period 2 are given by Eq.(10). Therefore, firm A's and B's discounted sum of profits are,

$$\Pi_{\mathsf{A}} = \left(p^1_{\mathsf{A}} - c\right)\left(\ell_{\mathsf{A}} - \ell^*\right) + \delta\left[\frac{\ell^2_{\mathsf{A}}}{2} - IP\right] \text{ and} \tag{13}$$

$$\Pi_{\mathsf{B}} = \left(p^1_{\mathsf{B}} - c\right)\left(\ell^* + \ell_{\mathsf{B}}\right) + \delta\left[\frac{\ell^2_{\mathsf{B}}}{2} + IP\right]. \tag{14}$$

Firms in period 1 choose their uniform prices to maximize (13) and (14). The first period price is chosen by a firm to strike an optimal balance in the trade-off between: i) losing (gaining) marginal consumers in period 1 and having a smaller (larger) customer database in period 2 and

ii) gaining (losing) inframarginal rents in period 1. A SPE where firm B sells its information to firm A is summarized in the proposition below.

Proposition 2 (SPE with information sharing). *When $\ell_{\mathsf{A}} > 13\ell_{\mathsf{B}}$ the unique SPE can be described as follows:*

- *Period 1: The firms' uniform prices are,*

$$\begin{aligned} p_{\mathsf{A}}^1 &= \frac{\ell_{\mathsf{A}}(8-\delta\sigma)}{12} + \frac{\ell_{\mathsf{B}}(4+7\delta\sigma)}{12} + c \text{ and} \\ p_{\mathsf{B}}^1 &= \frac{\ell_{\mathsf{A}}(4-\delta\sigma)}{12} + \frac{\ell_{\mathsf{B}}(8+7\delta\sigma)}{12} + c. \end{aligned}$$

The marginal consumer is located at,

$$\ell^* = \frac{(\ell_{\mathsf{A}} - \ell_{\mathsf{B}})}{3}. \tag{15}$$

- *Period 2: Firm B sells its customer database to firm A at price,*

$$IP = \frac{\sigma(\ell_{\mathsf{A}} - \ell_{\mathsf{B}})(\ell_{\mathsf{A}} - 13\ell_{\mathsf{B}})}{72}.$$

The prices that each firm charges to each consumer are the same as in subgame 3.

- *Both periods: The sum of discounted equilibrium profits are,*

$$\Pi_{\mathsf{A}} = \frac{4}{9}\ell_{\mathsf{A}}^2 + \frac{1}{9}\ell_{\mathsf{B}}^2 + \frac{1}{2}\delta\ell_{\mathsf{A}}^2 + \frac{5}{9}\delta\sigma\ell_{\mathsf{A}}\ell_{\mathsf{B}} + \frac{4}{9}\ell_{\mathsf{A}}\ell_{\mathsf{B}} - \frac{5}{72}\delta\sigma\ell_{\mathsf{A}}^2 + \frac{1}{72}\delta\sigma\ell_{\mathsf{B}}^2 \text{ and} \tag{16}$$

$$\Pi_{\mathsf{B}} = \frac{4}{9}\ell_{\mathsf{B}}^2 + \frac{1}{9}\ell_{\mathsf{A}}^2 + \frac{1}{2}\delta\ell_{\mathsf{B}}^2 - \frac{1}{18}\delta\sigma\ell_{\mathsf{A}}\ell_{\mathsf{B}} + \frac{4}{9}\ell_{\mathsf{A}}\ell_{\mathsf{B}} - \frac{1}{72}\delta\sigma\ell_{\mathsf{A}}^2 + \frac{41}{72}\delta\sigma\ell_{\mathsf{B}}^2. \tag{17}$$

Proof. See appendix. ¥

Firm A finds it profitable to set a relatively high price in the first period, so that some of its loyal consumers buy from firm B [i.e., the ones in the interval $[0, \ell^*)$]. Firm B collects perfect information about the consumers who purchased its product in period 1 and are in the segment $[-\ell_{\mathsf{B}}, \ell^*)$. Firm A collects perfect information for those in $[\ell^*, \ell_{\mathsf{A}}]$. Then firm B sells its information to firm A. In the second period, each consumer buys from the firm she likes most.

We know, from propositions 1 and 2, that when $\ell_{\mathsf{A}} \leq 13\ell_{\mathsf{B}}$ then $\ell^* \leq 4\ell_{\mathsf{B}}$ and information sharing is *not* part of a SPE. We do not pursue the solution of the game under the assumption that $\ell^* \leq 13\ell_{\mathsf{B}}$, as this goes beyond the purpose of this paper.

4 Information sharing is banned

In this section, we assess the welfare implications when a regulator does not allow firms to share their information. Since when $\ell_A \leq 13\ell_B$, firms do not have incentives to share their information anyway, we assume that $\ell_A > 13\ell_B$. Further, we assume that firms at the beginning of period 2, do not share information and we compare this equilibrium outcome to the one where information sharing is unregulated (see proposition 2). The profit functions (and the logic behind their derivation) are the same as the ones given by Eqs. (A13) and (A14), when $\ell^* \geq 0$, or the ones given by Eqs. (A15) and (A16), when $\ell^* \leq 0$. The next proposition summarizes the SPE.

Proposition 3 (SPE when information sharing is banned). *When $\ell_A > 13\ell_B$ and information sharing is banned, the unique SPE can be described as follows:*

- *Period 1: The firms' uniform prices are,*

$$\begin{aligned} p_A^1 &= \frac{5\ell_A\delta^2 - 8\ell_B\delta - 18\ell_A\delta + 24c - 10c\delta + 16\ell_A + 8\ell_B}{2(12-5\delta)} \text{ and} \\ p_B^1 &= \frac{3\ell_A\delta^2 - 20\ell_B\delta - 10\ell_A\delta + 24c - 10c\delta + 8\ell_A + 16\ell_B + 4\ell_B\delta^2}{2(12-5\delta)}. \end{aligned}$$

The marginal consumer is located at

$$\ell^{**} = \frac{2\left[\ell_A(2-\delta) - 2\ell_B(1-\delta)\right]}{12-5\delta}. \tag{18}$$

- *Period 2: The prices that each firm charges to each consumer are the same as in subgame 1, where no exchange of information takes place.*

- *Both periods: The sum of discounted equilibrium profits are,*

$$\begin{aligned} \Pi_A &= (4\ell_A^2\delta^3 - 6\ell_A\ell_B\delta^3 - 4\ell_B\delta^3 + 46\ell_A\ell_B\delta^2 - 9\ell_A^2\delta^2 + 24\ell_B^2\delta^2 \\ &\quad -28\ell_A^2\delta - 36\ell_B^2\delta - 104\ell_A\ell_B\delta + 64\ell_A\ell_B + 64\ell_A^2 + 16\ell_B^2)/(12-5\delta)^2 \text{ and} \quad (19) \\ \Pi_B &= -(5\ell_A^2\delta^3 + 5\ell_A\ell_B\delta^3 - 4\ell_B\delta^3 - 58\ell_A\ell_B\delta^2 - 28\ell_A^2\delta^2 + 8\ell_B^2\delta^2 \\ &\quad +52\ell_A^2\delta + 76\ell_B^2\delta + 160\ell_A\ell_B\delta - 128\ell_A\ell_B - 32\ell_A^2 - 128\ell_B^2)/\left[2(12-5\delta)^2\right]. \quad (20) \end{aligned}$$

Proof. See appendix. ¥

The market share of firm A in period 1 is larger when information sharing is banned than when it is not [i.e., $\ell^* \geq \ell^{**}$, provided that $\ell_\mathsf{A} \geq 7\ell_\mathsf{B}$, a condition that is satisfied given our assumptions]. When information sharing is banned, firm A lowers its price in an attempt to gain a larger share and consequently to increase the number of consumers in its database, given that in period 2 the possibility of buying these names from its rival firm is non-existent. In response, firm B lowers its price as well, but not as aggressively as firm A.

Social welfare. With unit demands and a covered market, only the disutility from not buying the most preferred brand matters. The possibility of sharing customer databases with the rival firm distributes the dead-weight loss differently across the two periods than when this possibility is absent. In the former case, firms price less aggressively in the first period, which allows the lower quality firm to capture some of its rival's customers, resulting in an inefficient outcome. In the second period, though, this inefficiency disappears, since each consumer buys her most preferred brand. In the latter case, the higher quality firm fights more for market share, surrendering fewer consumers to the rival, which reduces the first period inefficiency. On the other hand, the second period inefficiency does not vanish (see figures 2 and 3). These opposing effects create an interesting trade-off for a regulatory authority who wishes to regulate customer information sharing. This trade-off we have identified is likely to be present in a context more general than ours. Next, we compute the social welfare over the two periods.

In period 1, the social welfare when information sharing is banned is greater than that when sharing is allowed by,

$$\int_{\ell^{**}}^{\ell^*} \ell d\ell = \frac{(\ell^*)^2 - (\ell^{**})^2}{2}.$$

In this case, the extra inefficiency when information sharing is allowed arises because the group of consumers in the interval $[\ell^{**}, \ell^*]$ do not buy their most preferred brand, whereas when information sharing is banned they do (see figure 2).

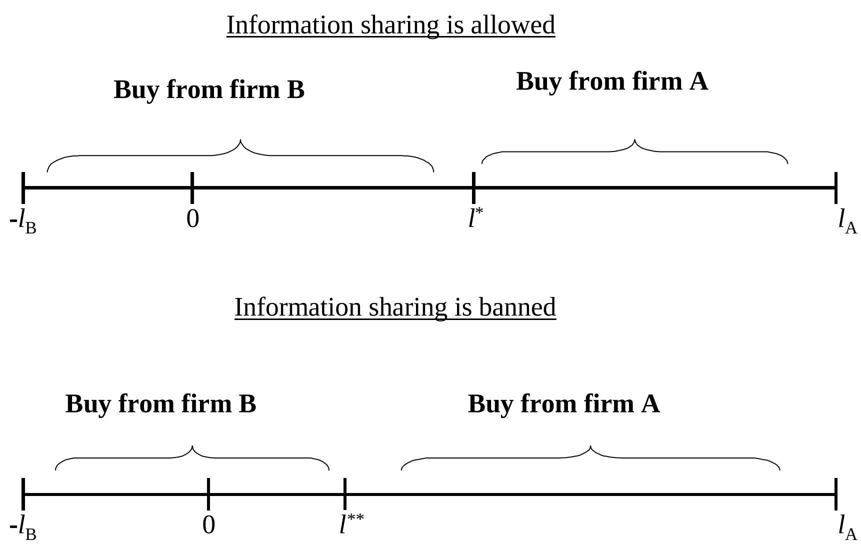

Figure 2: First period market shares

In period 2, the social welfare when information sharing is allowed is greater than that when sharing is banned by,

$$\int_0^{\frac{\ell^{**}}{2}} \ell d\ell = \frac{(\ell^{**})^2}{8}.$$

When information sharing is allowed, the second period outcome is efficient. Hence, the inefficiency when sharing is banned comes from the group of consumers in the interval $[0, \ell^{**}/2]$ who buy from firm B, while their favored firm is A (see figure 3).

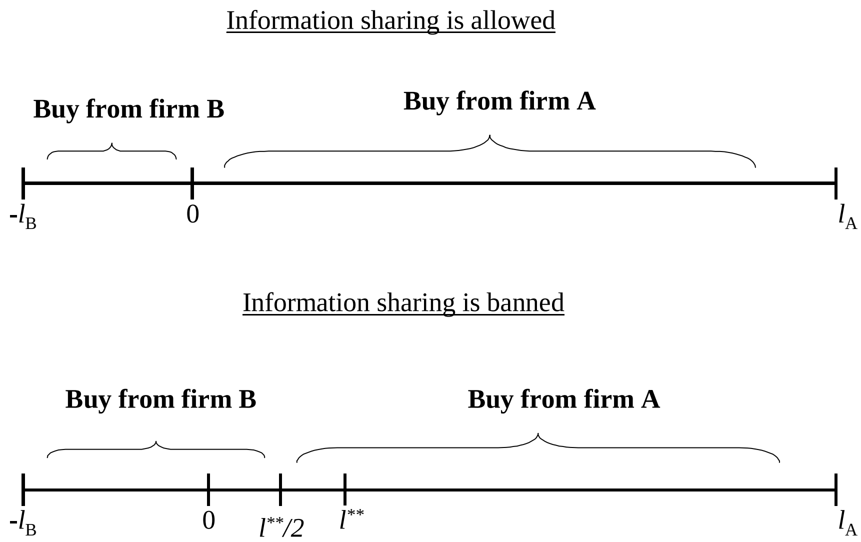

Figure 3: Second period market shares

Therefore, the discounted social welfare change when information sharing in banned is,

$$\frac{\left[(\ell^*)^2 - (\ell^{**})^2\right]}{2} - \delta\frac{(\ell^{**})^2}{8}. \tag{21}$$

If (21) is positive, then the outcome when information sharing is banned is more efficient than when it is not. The next proposition presents the social welfare comparison.

Proposition 4 (Social welfare comparison). *If $13\ell_{\mathsf{B}} < \ell_{\mathsf{A}} < \frac{7}{2}\left(\frac{7}{2}+\frac{3}{2}\sqrt{5}\right)\ell_{\mathsf{B}}$, then the social welfare when information sharing is banned decreases. If $\ell_{\mathsf{A}} > \frac{7}{2}\left(\frac{7}{2}+\frac{3}{2}\sqrt{5}\right)\ell_{\mathsf{B}}$, then the social welfare when information sharing is banned decreases for any $\delta < \hat{\delta} < 1$, while for any $\delta > \hat{\delta} > 0$ it increases. Moreover,*

$$\hat{\delta} = \frac{25\ell_{\mathsf{A}}^2 - 14\ell_{\mathsf{A}}\ell_{\mathsf{B}} - 47\ell_{\mathsf{B}}^2 - \sqrt{\left(433\ell_{\mathsf{B}}^2 - 590\ell_{\mathsf{A}}\ell_{\mathsf{B}} + 193\ell_{\mathsf{A}}^2\right)\left(\ell_{\mathsf{A}} - 7\ell_{\mathsf{B}}\right)^2}}{18\left(\ell_{\mathsf{A}}^2 - 4\ell_{\mathsf{A}}\ell_{\mathsf{B}} + 4\ell_{\mathsf{B}}^2\right)}.$$

Proof. See appendix. ¥

When the two firms do not have very different in size customer bases, banning information sharing lowers social welfare. When the customer base of one firm is significantly larger than that of the rival firm, banning information lowers social welfare only for relatively low discount factors. On the other hand, for relatively high discount factors, welfare increases.

Profits. Both firms become worse off when information sharing is banned [i.e., (16) $>$ (19) and (17) $>$ 20].[13]

Consumer welfare. The change in consumer welfare is simply the difference between the change in social welfare and the change in profits. We have shown that consumer welfare always decreases when information sharing is allowed.[14]

5 Concluding remarks

We develop a parsimonious two-period model with two rival firms who produce horizontally and vertically differentiated products. Our main purpose in this paper is to identify the necessary and sufficient conditions under which firms will share, in some way, their customer-specific information. The information is about the consumers' location (brand preferences) and enables the firms who possess it to engage in perfect price discrimination. In the first period, firms know only the distribution of preferences and consequently charge uniform prices. At the beginning of the second period they collect perfect information about their own customers (i.e., the ones who purchased their product in period 1) and decide whether to sell this information to the rival firm. We show that a necessary and sufficient condition for information sharing to be part of the unique subgame perfect equilibrium is sufficient firm asymmetry. In this case, the low quality firm finds it in its best interest to sell its customer database to the high quality firm. On the other hand, the high quality firm never sells its information to its low quality rival. If information sharing is banned, the social welfare decreases when the degree of firm asymmetry is below a certain threshold. When this threshold is exceeded social welfare decreases only when the discount factor is below a threshold, while it increases when the discount factor is above that threshold. Finally, when sharing is banned, profits decrease, while consumers surplus increases.

[13]The proof is very straightforward and it is omitted. It is available upon request.

[14]The proof is very straightforward and it is omitted. It is available upon request.

Appendix: Proofs of propositions

Proof of proposition 2. The proof consists of three parts: 1) first, we solve the system of reaction functions, 2) then, we check all possible deviations and finally 3) we prove uniqueness of SPE.

1) Firm A's and B's first order conditions (foc) are,

$$\frac{\partial \Pi_A}{\partial p_A^1} = 0 \Longrightarrow \ell_A - 2p_A^1 + p_B^1 - \frac{\delta\sigma p_A^1}{4} + \frac{\delta\sigma p_B^1}{4} + \frac{\delta\sigma\ell_B}{2} + c = 0 \text{ and} \tag{A1}$$

$$\frac{\partial \Pi_B}{\partial p_B^1} = 0 \Longrightarrow \ell_B - 2p_B^1 + p_A^1 - \frac{\delta\sigma p_A^1}{4} + \frac{\delta\sigma p_B^1}{4} + \frac{\delta\sigma\ell_B}{2} + c = 0. \tag{A2}$$

Note that the second order condition is satisfied. By solving (A1) and (A2) with respect to p_A^1 and p_B^1, we obtain the first period equilibrium (uniform) prices,

$$p_A^1 = \frac{\ell_A\,(8-\delta\sigma)}{12} + \frac{\ell_B\,(4+7\delta\sigma)}{12} + c \text{ and} \tag{A3}$$

$$p_B^1 = \frac{\ell_A\,(4-\delta\sigma)}{12} + \frac{\ell_B\,(8+7\delta\sigma)}{12} + c. \tag{A4}$$

By plugging (A3) and (A4) back into the objective functions, we obtain the sum of discounted equilibrium profits,

$$\Pi_A = \frac{4}{9}\ell_A^2 + \frac{1}{9}\ell_B^2 + \frac{1}{2}\delta\ell_A^2 + \frac{5}{9}\delta\sigma\ell_A\ell_B + \frac{4}{9}\ell_A\ell_B - \frac{5}{72}\delta\sigma\ell_A^2 + \frac{1}{72}\delta\sigma\ell_B^2 \text{ and} \tag{A5}$$

$$\Pi_B = \frac{4}{9}\ell_B^2 + \frac{1}{9}\ell_A^2 + \frac{1}{2}\delta\ell_B^2 - \frac{1}{18}\delta\sigma\ell_A\ell_B + \frac{4}{9}\ell_A\ell_B - \frac{1}{72}\delta\sigma\ell_A^2 + \frac{41}{72}\delta\sigma\ell_B^2. \tag{A6}$$

Based on the first period equilibrium prices, the marginal consumer is located at

$$\ell^* = p_A^1 - p_B^1 = \frac{(\ell_A - \ell_B)}{3}. \tag{A7}$$

By plugging (A7) into $IP = \sigma\frac{\ell^*(\ell^*-4\ell_B)}{8}$ we obtain the equilibrium information price, which is,

$$IP = \frac{\sigma\,(\ell_A-\ell_B)\,(\ell_A-13\ell_B)}{72}.$$

Recall from proposition 1, that for information sharing to be part of a SPE (where firm B sells its information to A) it must be the case that $\ell^* > 4\ell_B$. Using (A7), it follows that $\ell^* > 4\ell_B$ if and only if $\ell_A > 13\ell_B$.

2) To conclude that the above pair of prices constitutes a SPE, we must demonstrate that unilateral deviations are unprofitable. There are two types of deviations in our model: i) a firm changes its first period price but firm B still finds it profitable in period 2 to sell its information to firm A (i.e., the assumed structure remains unchanged) and ii) a price change leads to a no sharing of information in period 2 (i.e., the assumed structure changes). The first type of deviation has already been proved that it is not profitable, since the price pair is a solution to the system of best response functions.

Next we show that the second type of deviation is not profitable either. We first look at firm A's incentives to deviate and then at firm B's.

- Firm A's deviation in period 1.

For the structure to change it must be the case that firm A lowers its price to the point that $\ell^* < 4\ell_{\mathsf{B}}$ and therefore information sharing in period 2 is not profitable. There are two distinct interior cases: i) $4\ell_{\mathsf{B}} > \ell^* > 0$ and ii) $\ell^* < 0$ and two distinct boundary cases: i) $\ell^* = 0$ and ii) $\ell^* = -\ell_{\mathsf{B}}$. An interior deviation can be ruled out for one of the following two reasons: either it leads to lower profits than the ones before deviation, or it produces a contradiction in terms of ℓ^*. If the contradiction is in terms of ℓ^* (e.g., $\ell^* > 4\ell_{\mathsf{B}}$), then we have to check the boundary cases. We start with the first interior case.

This deviation on part of firm A leads the game play to subgame 1 in period 2. The marginal consumer ℓ^* in period 1 must be indifferent between buying from firm A today at price p^1_{A} (in which case she is in firm A's database) and then buying again in period 2 from firm A at price $\ell^* + c$, or buying from firm B in period 1 at price p^1_{B} (in which case she is in firm B's database) and then buying from A in period 2 from firm A at price $\ell^*/2 + c$. Thus the indifferent consumer must satisfy,

$$\left[p^1_{\mathsf{A}} + \delta\left(\ell^* + c\right)\right] - \left[p^1_{\mathsf{B}} + \delta\left(\frac{\ell^*}{2} + c\right)\right] = \ell^* \Longrightarrow \ell^* = \frac{2\left(p^1_{\mathsf{A}} - p^1_{\mathsf{B}}\right)}{(2-\delta)}. \tag{A8}$$

Firm A's sum of discounted deviation profits are,

$$\Pi^{\mathsf{d}}_{\mathsf{A}} = \left(p^1_{\mathsf{A}} - c\right)\left(\ell_{\mathsf{A}} - \ell^*\right) + \delta\left[\frac{2\ell^2_{\mathsf{A}} - (\ell^*)^2}{4}\right], \tag{A9}$$

where the second period profits come from subgame 1 and in particular (1) and (4), ℓ^* is given by (A8) and p^1_{B} is fixed at the level given by (A4). The deviating firm chooses p^1_{A} to maximize (A9). We show that as long as $\ell_{\mathsf{A}} > 13\ell_{\mathsf{B}}$ such deviation is not profitable. In particular, the maximized profits as given by (A5) are greater than the maximum deviation profits $\max_{\mathsf{p}^1_{\mathsf{A}}} \Pi^{\mathsf{d}}_{\mathsf{A}}$. To keep the paper within acceptable limits, in the remaining of this proof, we do not present the calculations regarding the comparison of firm profits before and after a deviation. These calculations are straightforward and are available upon request.

We continue with the second interior deviation, i.e., $\ell^* < 0$. No information sharing takes place in period 2. This case leads to a subgame similar to subgame 1. Following the same steps as in that subgame, we can show that firm B, in period 2, charges a price of $c - \ell^*/2$ to the consumers in segment $(\ell^*, \ell_{\mathsf{A}}]$ and an individualized price $c - \ell$ to the consumers in $[-\ell_{\mathsf{B}}, \ell^*]$. Firm A's profits in period 2 are analogous to those given by Eq.(5), i.e.,

$$\pi^2_{\mathsf{A}} = \frac{(\ell^* - 2\ell_{\mathsf{A}})^2}{8}. \tag{A10}$$

The marginal consumer ℓ^* in period 1 must be indifferent between buying from firm A today at price p^1_{A} and then buying in period 2 from firm B at price $c - \ell^*/2$, or buying from firm B in period 1 at price p^1_{B} and then buying again from B in period 2 at price $c - \ell^*$. The indifferent consumer must satisfy,

$$\left[p^1_{\mathsf{A}} + \delta\left(c - \frac{\ell^*}{2}\right)\right] - \left[p^1_{\mathsf{B}} + \delta\left(c - \ell^*\right)\right] = \ell^* \Longrightarrow \ell^* = \frac{2\left(p^1_{\mathsf{A}} - p^1_{\mathsf{B}}\right)}{(2-\delta)}. \tag{A11}$$

Using (A10) and (A11), firm A's sum of discounted deviation profits are,

$$\Pi^{\mathsf{d}}_{\mathsf{A}} = \left(p^1_{\mathsf{A}} - c\right)(\ell_{\mathsf{A}} - \ell^*) + \delta\frac{\left((\ell^*)^2 + 4\ell^2_{\mathsf{A}} - 4\ell^*\ell_{\mathsf{B}}\right)}{8}. \tag{A12}$$

We differentiate (A12) with respect to p^1_{A} and we solve the (foc). The function is strictly concave. The resulting ℓ^*, however, is positive which leads to a contradiction.

Next, we look at the boundary deviations. When $\ell^* = 0$, from (A8), p^1_{A} must be equal to p^1_{B}, where the latter is fixed at the level given by (A4). We plug the p^1_{A} which solves $\ell^* = 0$, into (A9) and we show that it is lower than (A5). Then we look at $\ell^* = -\ell_{\mathsf{B}}$. We use (A11) and (A12) to find firm A's profits. Again we show that they are lower than the ones given by (A5).

- Firm B's deviation in period 1.

For the structure to change it must be the case that firm B increases its price to the point that $\ell^* < 4\ell_{\mathsf{B}}$ and therefore information sharing in period 2 is not profitable. Similarly to the deviation of firm A, there are two interior deviations and two boundary. We begin with $4\ell_{\mathsf{B}} > \ell^* > 0$. The marginal consumer is the same as in (A8). The first period price of firm A is fixed at its level given by (A3). The sum of discounted deviation profits are calculated in a similar manner as those of firm A in the same type of deviation, using also the equilibrium profits from subgame 1. We obtain a contradiction, i.e., the resulting ℓ^* is greater than $4\ell_{\mathsf{B}}$.

Then we turn to $\ell^* < 0$. The marginal consumer is the same as in (A11). The first period price of firm A is fixed at its level given by (A3). The sum of discounted deviation profits are calculated in a similar manner as those of firm A in the same type of deviation. We find that $\ell^* > 0$ as long as $\ell_{\mathsf{A}} > 13\ell_{\mathsf{B}}$, which leads to a contradiction.

The two boundary deviations are computed in the same manner as those for firm A. Profits are lower than the ones given by (A6).

3) So far, we have proved that when $\ell_{\mathsf{A}} > 13\ell_{\mathsf{B}}$, the strategies that are described in the statement of proposition 2 constitute a SPE. Now we will show that this SPE is unique. Given that information will be shared in period 2, the pricing strategies, as we have demonstrated above, are indeed unique. But is information sharing the only outcome when $\ell_{\mathsf{A}} > 13\ell_{\mathsf{B}}$? To be more precise, we will prove that no sharing of information is not a SPE under the assumptions of proposition 2.

Let $\ell_{\mathsf{A}} > 13\ell_{\mathsf{B}}$, but nevertheless no sharing of information takes place in period 2. For this to be the case, it must be that $\ell^* \leq 4\ell_{\mathsf{B}}$ (otherwise firm B will sell its information to A). There are two cases: i) $4\ell_{\mathsf{B}} > \ell^* \geq 0$ and ii) $\ell^* \leq 0$. In the first case the marginal consumer is the same as in (A8), while in the second she is the same as in (A11). When $\ell^* \geq 0$, the profit functions are (using the results from subgame 1),

$$\Pi_{\mathsf{A}} = \left(p_{\mathsf{A}}^1 - c\right)\left(\ell_{\mathsf{A}} - \ell^*\right) + \delta\left[\frac{2\ell_{\mathsf{A}}^2 - (\ell^*)^2}{4}\right] \text{ and} \tag{A13}$$

$$\Pi_{\mathsf{B}} = \left(p_{\mathsf{B}}^1 - c\right)\left(\ell^* + \ell_{\mathsf{B}}\right) + \delta\left[\frac{(\ell^* + 2\ell_{\mathsf{B}})^2}{8}\right]. \tag{A14}$$

The above profit functions are strictly concave in p_{A}^1 and p_{B}^1 respectively. We derive the two reaction functions and we find the unique intersection point. When $\ell_{\mathsf{A}} > 14\ell_{\mathsf{B}}$, the ℓ^* we find using the solution to the system of the reaction functions is greater than $4\ell_{\mathsf{B}}$, which is inconsistent with the no information sharing presumption. When $13\ell_{\mathsf{B}} < \ell_{\mathsf{A}} \leq 14\ell_{\mathsf{B}}$, firm A has an incentive to deviate by increasing its price, thereby lowering its first period market share to the point where information sharing is profitable in period 2, a contradiction to the no information sharing assumption.

When $\ell^* \leq 0$, the profit functions are,

$$\Pi_{\mathsf{A}} = \left(p^1_{\mathsf{A}} - c\right)\left(\ell_{\mathsf{A}} - \ell^*\right) + \delta\left[\frac{\left(\ell^* - 2\ell_{\mathsf{A}}\right)^2}{8}\right] \text{ and} \tag{A15}$$

$$\Pi_{\mathsf{B}} = \left(p^1_{\mathsf{B}} - c\right)\left(\ell^* + \ell_{\mathsf{B}}\right) + \delta\left[\frac{\left(3\left(\ell^*\right)^2 + 4\ell^2_{\mathsf{B}} + 4\ell^*\ell_{\mathsf{B}}\right)}{8}\right]. \tag{A16}$$

The above profit functions are strictly concave in p^1_{A} and p^1_{B} respectively. We derive the two reaction functions and we find the unique intersection point. The ℓ^* we find, using the solution to the system of the reaction functions is strictly positive, a contradiction. ¥

Proof of proposition 3. We begin our search for a SPE, assuming that $\ell^{**} \geq 0$. Differentiate (A13) and (A14) with respect to p^1_{A} and p^1_{B}. The second order conditions are satisfied. The unique solution to the system of the two reaction functions is,

$$p^1_{\mathsf{A}} = \frac{5\ell_{\mathsf{A}}\delta^2 - 8\ell_{\mathsf{B}}\delta - 18\ell_{\mathsf{A}}\delta + 24c - 10c\delta + 16\ell_{\mathsf{A}} + 8\ell_{\mathsf{B}}}{2\left(12 - 5\delta\right)} \text{ and} \tag{A17}$$

$$p^1_{\mathsf{B}} = \frac{3\ell_{\mathsf{A}}\delta^2 - 20\ell_{\mathsf{B}}\delta - 10\ell_{\mathsf{A}}\delta + 24c - 10c\delta + 8\ell_{\mathsf{A}} + 16\ell_{\mathsf{B}} + 4\ell_{\mathsf{B}}\delta^2}{2\left(12 - 5\delta\right)}. \tag{A18}$$

The first period marginal consumer, based on the above prices, is located at,

$$\ell^{**} = \frac{2\left[\ell_{\mathsf{A}}\left(2 - \delta\right) - 2\ell_{\mathsf{B}}\left(1 - \delta\right)\right]}{12 - 5\delta}.$$

To conclude that the above pair of prices constitutes a SPE, we must demonstrate that unilateral deviations are unprofitable. There are two types of deviations in our model: i) a firm changes its first period price, but still $\ell^{**} \geq 0$ and ii) a price change leads to $\ell^{**} < 0$. The first type of deviation has already been proved that it is not profitable, since the price pair is a solution to the system of best response functions.

Next we show that the second type of deviation is not profitable either. We first look at firm A's incentives to deviate and then at firm B's.[15]

- Firm A's deviation in period 1.

[15] As in the proof of proposition 2, the details pertaining to the comparison of profits before and after a deviation are straightforward and therefore are omitted. They are, however, available upon request.

Firm B's price is fixed at the level given by (A18). Firm A's profit function is the same as the one given by (A15), since now $\ell^{**} < 0$. We show that this type of deviation is not profitable.

- Firm B's deviation in period 2.

Firm A's price is fixed at the level given by (A17). Firm B's profit function is the same as the one given by (A16), again since $\ell^{**} < 0$. We show that this deviation is not profitable either.

Next, we search for an equilibrium when $\ell^{**} < 0$. The firms' profit functions are the same as the ones given by (A15) and (A16). We derive the two reaction functions and we solve them to obtain a pair of prices. We then show that firm A has always an incentive to deviate to a price such that the resulting ℓ^{**} is strictly positive. Contradiction. Hence, there does not exist such an equilibrium. ¥

Proof of proposition 4. From (21), and after we use (15) and (18), we obtain,

$$\begin{aligned} & \frac{\left[(\ell^*)^2 - (\ell^{**})^2\right]}{2} - \delta\frac{(\ell^{**})^2}{8} \\ = & \frac{\delta\left(25\delta\ell_{\mathsf{A}}^2 - 12\ell_{\mathsf{A}}^2 - 14\delta\ell_{\mathsf{A}}\ell_{\mathsf{B}} - 120\ell_{\mathsf{A}}\ell_{\mathsf{B}} - 47\delta\ell_{\mathsf{B}}^2 + 132\ell_{\mathsf{B}}^2 - 9\delta^2\ell_{\mathsf{A}}^2 + 36\delta^2\ell_{\mathsf{A}}\ell_{\mathsf{B}} - 36\delta^2\ell_{\mathsf{B}}^2\right)}{18\left(12 - 5\delta\right)^2}. \end{aligned} \tag{A19}$$

If (A19) is positive, the social welfare when information sharing is banned increases. We set $(A19) = 0$ and we solve with respect to δ. This yields the following two non-zero solutions,

$$\begin{aligned} \delta_1 &= \frac{25\ell_{\mathsf{A}}^2 - 14\ell_{\mathsf{A}}\ell_{\mathsf{B}} - 47\ell_{\mathsf{B}}^2 - \sqrt{\left(433\ell_{\mathsf{B}}^2 - 590\ell_{\mathsf{A}}\ell_{\mathsf{B}} + 193\ell_{\mathsf{A}}^2\right)\left(\ell_{\mathsf{A}} - 7\ell_{\mathsf{B}}\right)^2}}{18\left(\ell_{\mathsf{A}}^2 - 4\ell_{\mathsf{A}}\ell_{\mathsf{B}} + 4\ell_{\mathsf{B}}^2\right)} \\ \delta_2 &= \frac{25\ell_{\mathsf{A}}^2 - 14\ell_{\mathsf{A}}\ell_{\mathsf{B}} - 47\ell_{\mathsf{B}}^2 + \sqrt{\left(433\ell_{\mathsf{B}}^2 - 590\ell_{\mathsf{A}}\ell_{\mathsf{B}} + 193\ell_{\mathsf{A}}^2\right)\left(\ell_{\mathsf{A}} - 7\ell_{\mathsf{B}}\right)^2}}{18\left(\ell_{\mathsf{A}}^2 - 4\ell_{\mathsf{A}}\ell_{\mathsf{B}} + 4\ell_{\mathsf{B}}^2\right)}. \end{aligned}$$

First note that $\delta_1 < \delta_2$. $(A19)$ is negative for any $\delta < \delta_1$ and any $\delta > \delta_2$. It is positive for $\delta \in (\delta_1, \delta_2)$. It can be checked that if $\ell_{\mathsf{A}} > \frac{7}{2}\left(\frac{7}{2} + \frac{3}{2}\sqrt{5}\right)\ell_{\mathsf{B}}$, then $\delta_1 < 1$. Also, for $\ell_{\mathsf{A}} \in \left(13\ell_{\mathsf{B}}, \frac{7}{2}\left(\frac{7}{2} + \frac{3}{2}\sqrt{5}\right)\ell_{\mathsf{B}}\right)$, $\delta_1 > 1$. Moreover, when $\ell_{\mathsf{A}} > 13\ell_{\mathsf{B}}$, $\delta_2 > 1$.

Hence, if $\ell_{\mathsf{A}} < \frac{7}{2}\left(\frac{7}{2} + \frac{3}{2}\sqrt{5}\right)\ell_{\mathsf{B}}$, for any δ, $(A19)$ is positive which implies that welfare if information sharing is banned decreases. On the other hand, if $\ell_{\mathsf{A}} > \frac{7}{2}\left(\frac{7}{2} + \frac{3}{2}\sqrt{5}\right)\ell_{\mathsf{B}}$, then $\delta_1 < 1$ and consequently for any $\delta > \delta_1 = \hat{\delta}$, $(A19)$ is negative which implies that welfare if information sharing is banned increases, while when $\delta < \delta_1 = \hat{\delta}$ it increases. ¥

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