On two-point Nash equilibrium in concave bimatrix games

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The concavity of payoff functions is a very often used assumption, both in theoretical considerations and in practical applications. The classical result for games on R^k with continuous concave payoffs belongs to Nash. However, in many applications we have not continuous strategy spaces. More exactly, the decision goods are nondivisible, e.g. people, cars, shares. In such case we have finitely many actions and we obtain matrix games. Radzik studied optimal strategies in zero-sum matrix games with concavity-convexity properties. He shown that in such games there always exists a pair of optimal strategies with at most two membered carriers.

In this paper we generalize the Radzik's results to two-person non-zero-sum matrix games. The games described by such payoff matrices well approximate continuous games on the unit square with payoff functions $F_1(x, y)$ concave in x for each y, and $F_2(x, y)$ concave in y for each x. It is shown, that the optimal strategies in such games have very simple structure and a search procedure is given.

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