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RATIONAL EXPECTATIONS EQUILIBRIUM IN ECONOMY FOR MULTI-MODAL LOGIC

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ABSTRACT. We investigate a pure exchange economy under uncertainty with emphasis on the logical point of view; the traders are assumed to have non-partitional information structures corresponding to a multi-modal logic. We propose a pure exchange economy for the multi-modal logic **KT**, and give the generalized notion of rational expectations equilibrium for the economy. We establish the finite model property for the logic **KT** and the existence of the generalized rational expectation equilibrium. Further, we characterize welfare under the generalized rational expectations equilibrium in an economy $\mathcal{E}^{KT}$ for logic **KT**.

Keywords: Multi-modal logics, Pure exchange economy under uncertainty, Rational expectations equilibrium, Fundamental theorem in welfare economics.
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1. INTRODUCTION

This article relates economy and multi-agent modal logic. Let us consider a pure exchange economy under uncertainty. As far as the standard notion of economy either with complete information or with incomplete information, the role of traders' knowledge and beliefs remains obscured: The economy has not been investigated from the epistemic point of view.

The purposes of this article are as follows: First we propose the multi-modal logic **KT** by which the traders make their decision, second we give the extended notion of rational expectations equilibrium in the economy and we establish the existence of the equilibrium with emphasis on modal logical point of view. Finally we extend the fundamental theorem for welfare in the economy.

The stage is set by the following: Suppose that the trader have the multi-agent modal logic **KT**: It is an extension of the propositional logic with traders' modal operators requiring only the axiom (T) "each trader does not know a sentence whenever it is not true." The logic has non-partitional information structures, each of which gives an interpretation of the logic. Each trader has own utility function

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which is measurable, but he/she is not assumed to know the function completely. It is shown that

Existence of REE. *In a pure exchange economy under generalized information, assume that the traders have the multi-modal logic* **KT** *and they are risk averse. There exists a rational expectations equilibrium in the economy.*

Many authors have investigated the properties of a rational expectations equilibrium and its consequences in an economy under uncertainty (e.g., Kreps [8], Milgrom and Stokey [12], Morris [13], Einy et al [5] and others). The serious limitations of the analysis in these researches are its use of the 'partition' structure by which the traders receive information.

The structure is the Kripke semantics for the modal logic **S5**;[1] it is obtained if each trader t's possibility operator $P_t : \Omega \to 2^\Omega$ assigning to each state ω in a state space Ω the information set $P_t(\omega)$ that t possesses in ω is reflexive, transitive and symmetric. From the epistemic point of view, this entails t's knowledge operator $K_t : 2^\Omega \to 2^\Omega$ that satisfies 'Truth' axiom **T**: $K_t(E) \subseteq E$ (what is known is true), the 'positive introspection' axiom **4**: $K_t(E) \subseteq K_t(K_t(E))$ (we know what we do) and the 'negative introspection' axiom **5**: $\Omega \setminus K_t(E) \subseteq K_t(\Omega \setminus K_t(E))$ (we know what we do not know).[2]

One of these requirements, symmetry (or the equivalent axiom **5**), is indeed so strong that describes the hyper-rationality of traders, and thus it is particularly objectionable. The recent idea of 'bounded rationality' suggests dropping such assumption since real people are not complete reasoners. In this article we weaken both transitivity and symmetry imposing only reflexivity. As has already been pointed out in the literature, this relaxation can potentially yield important results in a world with imperfectly Bayesian agents (e.g. Geanakoplos [7]).

The idea has been performed in different settings. Among other things Geanakoplos [7] showed the no speculation theorem in the extended rational expectations equilibrium under the assumption that the information structure is reflexive, transitive and *nested* (Corollary 3.2 in Geanakoplos [7]). The condition 'nestedness' is interpreted as a requisite on the 'memory' of the trader.

However all those researches have been lacked the logics that represents the traders' knowledge. This article proposes the multi-modal logic of the traders and the economies under generalized information structure as the models for the logic. In the structure we shall relax the transitivity, and we establish the existence theorem in this generalized environment.

This article is organized as follows: In Section 2 we present the multi-modal logic **KT** and give its finite model property. Further we introduce the notion "economy for logic **KT**", a generalized notion of rational expectations equilibrium. Section 3 gives the existence theorem of rational expectations equilibrium. In Section 4 we extend the fundamental theorem for welfare in the economy for logic **KT**. Finally we conclude by giving some remarks about the assumptions of the theorem.

2. Pure exchange economy for multi-modal logic

2.1. **Logic of knowledge KT.** Let T be a set of n *traders* $\{1, 2, 3, \dots, t, \dots, n\}$. Let us consider *multi-modal logics* for traders T as follows: The *sentences* of the

[1] C.f.: Chellas [3], Fagin, Halpern et al [6].

[2] C.f.: Bacharach [2], Fagin, Halpern et al [6].

language form the least set containing each *atomic* sentence $\mathbf{P}_m (m = 0, 1, 2, \dots)$ closed under the following operations:

- nullary operators for *falsity* $\bot$ and for *truth* $\top$;
- unary and binary syntactic operations for *negation* $\neg$, *conditionality* $\to$ and *conjunction* $\wedge$, respectively;
- unary operation for *modality* $\Box_t$ with $t \in T$.

Other such operations are defined in terms of those in usual ways. The intended interpretation of $\Box_t \varphi$ is the sentence that 'trader t knows a sentence φ.'

By a *multi-modal logic* we mean a set of sentences, L, containing all truth-functional tautologies and closed under substitution and modus ponens. A multi-modal logic L' is an *extension* of L if $L \subseteq L'$. A sentence φ in a multi-modal logic L is a *theorem* of L (or *provable* in L), written by $\vdash_L \varphi$. Other proof-theoretical notions such as *L-deducibility, L-consistency, L-maximality* are defined in usual ways. (See, Chellas [3].)

A *system of traders' knowledge* is a multi-modal logic L closed under the rule of inference ($\mathrm{RE}_\Box$) and containing the schema (N), (M), (C), and (T): For every $t \in T$,

($\mathrm{RE}_\Box$) $$\frac{\varphi \longleftrightarrow \psi}{\Box_t \varphi \longleftrightarrow \Box_t \psi}$$

(N) $\Box_t \top$;

(M) $\Box_t(\varphi \wedge \psi) \longrightarrow (\Box_t \varphi \wedge \Box_t \psi)$;

(C) $(\Box_t \varphi \wedge \Box_t \psi) \longrightarrow \Box_t(\varphi \wedge \psi)$;

(T) $\Box_t \varphi \longrightarrow \varphi$.

Definition 1. The *multi-modal logic* **KT** is the minimal system of trades' knowledge.

2.2. **Information and Knowledge**[3]**.** Trader t's information structure is a couple $\langle \Omega, P_t \rangle$, in which Ω be a non-empty set called a *state-space* whose elements are called *states* and P_t is a mapping of Ω into 2^Ω. It is said to be *reflexive* if

Ref: $\omega \in P_t(\omega)$ for every $\omega \in \Omega$,

and it is said to be *transitive* if

Trn: $\xi \in P_t(\omega)$ implies $P_t(\xi) \subseteq P_t(\omega)$ for any $\xi, \omega \in \Omega$.

An *information structure* is a structure $\langle \Omega, (P_t)_{t \in T} \rangle$ where Ω is common for all trader, and it is called an *RT-information structure* if each P_t is reflexive and transitive.

Given our interpretation, a trader t for whom $P_t(\omega) \subseteq E$ knows, in the state ω, that some state in the event E has occurred. In this case we say that at the state ω the trader t knows E. i's *knowledge operator* K_t on 2^Ω is defined by $K_t(E) = \{\omega \in \Omega | P_t(\omega) \subseteq E\}$. The set $P_t(\omega)$ will be interpreted as the set of all the states of nature that t knows to be possible at ω, and $K_t E$ will be interpreted as the set of states of nature for which t knows E to be possible. We will therefore call P_t t's *possibility operator* on Ω and also will call $P_t(\omega)$ t's *possibility set* at ω. A possibility operator P_t is determined by the knowledge operator K_t such as $P_t(\omega) = \bigcap_{K_t E \ni \omega} E$. However it is also noted that the operator P_t cannot be

[2]See Fagin, Halpern et al [6].

uniquely determined by the knowledge operator K_t when P_t does not satisfy the both conditions **Ref** and **Trn**.

A *partitional* information structure is an *RT*-information structure $\langle\Omega, (P_t)_{t\in T}\rangle$ with the additional condition: For each $t \in T$ and every $\omega \in \Omega$,

Sym: $\xi \in P_t(\omega)$ implies $P_t(\xi) \ni \omega$.

2.3. **Truth.** A *model on* an information structure $\langle\Omega, (P_t)_{t\in T}\rangle$ is a triple $\mathcal{M} = \langle\Omega, (P_t)_{t\in T}, V\rangle$, in which a mapping V assigns either **true** or **false** to every $\omega \in \Omega$ and to every atomic sentence $\mathbf{P}_m$. The model $\mathcal{M}$ is called *finite* if Ω is a finite set.

Definition 2. By $\models^{\mathcal{M}}_{\omega} \varphi$, we mean that a sentence φ is *true* at a state ω in a model $\mathcal{M}$. *Truth at a state* ω in $\mathcal{M}$ is defined by the inductive way as follows:

(1) $\models^{\mathcal{M}}_{\omega} \mathbf{P}_m$ if and only if $V(\omega, \mathbf{P}_m) = \mathbf{true}$, for $m = 0, 1, 2, \dots$;

(2) $\models^{\mathcal{M}}_{\omega} \top$, and not $\models^{\mathcal{M}}_{\omega} \bot$;

(3) $\models^{\mathcal{M}}_{\omega} \neg\varphi$ if and only if not $\models^{\mathcal{M}}_{\omega} \varphi$;

(4) $\models^{\mathcal{M}}_{\omega} \varphi \longrightarrow \psi$ if and only if $\models^{\mathcal{M}}_{\omega} \varphi$ implies $\models^{\mathcal{M}}_{\omega} \psi$;

(5) $\models^{\mathcal{M}}_{\omega} \varphi \wedge \psi$ if and only if $\models^{\mathcal{M}}_{\omega} \varphi$ and $\models^{\mathcal{M}}_{\omega} \psi$;

(6) $\models^{\mathcal{M}}_{\omega} \Box_t\varphi$ if and only if $P_t(\omega) \subseteq ||\varphi||^{\mathcal{M}}$, for $t \in T$;

Where $||\varphi||^{\mathcal{M}}$ denotes the set of all the states in $\mathcal{M}$ at which φ is true; this is called the *truth set of* φ. We say that a sentence φ is *true in the model* $\mathcal{M}$ and write $\models^{\mathcal{M}} \varphi$ if $\models^{\mathcal{M}}_{\omega} \varphi$ for every state ω in $\mathcal{M}$. A sentence is said to be *valid in* an information structure if it is true in every model on the information structure.

2.4. **Finite model property.** Let Σ be a set of sentences. We say that $\mathcal{M}$ is a *model for* Σ if every member of Σ is true in $\mathcal{M}$. An information structure is said to be *for* Σ if every member of Σ is valid in it. Let $\boldsymbol{R}$ be the class of all models on any *reflexive* information structure. A multi-modal logic L is *sound with respect to* $\boldsymbol{R}$ if every member of $\boldsymbol{R}$ is a model on an information structure for L. It is *complete with respect to* $\boldsymbol{R}$ if every sentence valid in all members of $\boldsymbol{R}$ is a theorem of L. We say that L is *determined by* $\boldsymbol{R}$ if L is sound and complete with respect to $\boldsymbol{R}$.

A multi-modal logic L is said to have the *finite model property* if it is determined by the class of all finite models in $\boldsymbol{R}$.

Theorem 1. *The multi-modal logic* **KT** *has the finite model property.*

Proof. Will be given in Appendix A. □

From now on we consider the structure $\langle\Omega, (P_t)_{t\in T}, V\rangle$ as a finite model for **KT**.

2.5. **Economy for logic KT.** Let Ω be a non-empty *finite* set called a *state space*, and let 2^{Ω} denote the field of all subsets of Ω. Each member of 2^{Ω} is called an *event* and each element of Ω a *state*.

A *pure exchange economy under uncertainty* is a tuple $\langle T, \Omega, \boldsymbol{e}, (U_t)_{t\in T}, (\pi_t)_{t\in T}\rangle$ consisting of the following structure and interpretations: There are l commodities in each state of the state space Ω, and it is assumed that Ω is *finite* and that the consumption set of trader t is $\mathbb{R}^l_+$;

- $\boldsymbol{e} : T \times \Omega \to \mathbb{R}^l_+$ is t's *initial endowment*;
- $U_t : \mathbb{R}^l_+ \times \Omega \to \mathbb{R}$ is t's von-Neumann and Morgenstern utility function;
- π_t is a subjective prior on Ω for a trader $t \in T$.

For simplicity it is assumed that (Ω, π_t) is a finite probability space with π_t *full support*[4] for all $t \in T$.

Definition 3. An *pure exchange economy for logic* **KT** is a structure $\mathcal{E}^{KT} = \langle \mathcal{E}, (P_t)_{t\in T}, V\rangle$, in which $\mathcal{E}$ is a pure exchange economy such that $\langle \Omega, (P_t)_{t\in T}, V\rangle$ is a finite model for the logic **KT**. Furthermore it is called an economy *under RT-information structure* if each P_t is a reflexive and transitive information structure.

Remark 1. An economy under asymmetric information is an economy $\mathcal{E}^{KT}$ under partitional information structure (i.e., each P_t satisfies the three conditions **Ref**, **Trn** and **Sym**.)

Let $\mathcal{E}^{KT}$ be a pure exchange economy for logic **KT**. We denote by $\mathcal{F}_t$ the field generated by $\{P_t(\omega) \mid \omega \in \Omega\}$ and by $\mathcal{F}$ the join of all $\mathcal{F}_t (t \in T)$; i.e. $\mathcal{F} = \vee_{t\in T}\mathcal{F}_t$. We denote by $\{A(\omega) \mid \omega \in \Omega\ \}$ the set of all atoms $A(\omega)$ containing ω of the field $\mathcal{F} = \vee_{t\in T}\mathcal{F}_t$.

Remark 2. The set of atoms $\{A_t(\omega) \mid \omega \in \Omega\}$ of $\mathcal{F}_t$ does not necessarily coincide with the partition induced by P_t.

We shall often refer to the following conditions: For every $t \in T$,

- **A-1:** The function $e_t(\cdot)$ is $\mathcal{F}_t$-measurable with $\sum_{t\in T} e_t(\omega) \gneqq 0$ for all $\omega \in \Omega$.
- **A-2:** For each $x \in \mathbb{R}^l_+$, the function $U_t(x, \cdot)$ is $\mathcal{F}_t$-measurable.
- **A-3:** For each $\omega \in \Omega$, the function $U_t(\cdot, \omega)$ is strictly increasing on $\mathbb{R}^l_+$.
- **A-4:** For each $\omega \in \Omega$, the function $U_t(\cdot, \omega)$ is continuous, increasing, strictly quasi-concave and *non-saturated*[5]on $\mathbb{R}^l_+$.

Remark 3. It is plainly observed that **A-4** implies **A-3**. We note also that **A-3** does not mean that trader t knows his/her utility function $U_t(\cdot, \omega)$.[6]

2.6. **Allocations.** An *assignment* $\boldsymbol{x}$ is a mapping from $T \times \Omega$ into $\mathbb{R}^l_+$ such that for every $\omega \in \Omega$ and for each $t \in T$, the function $\boldsymbol{x}(t, \cdot)$ is at most $\mathcal{F}$-measurable. We denote by $\mathcal{A}ss(\mathcal{E}^{KT})$ the set of all assignments for the economy $\mathcal{E}^{KT}$.

By an *allocation* we mean an assignment $\boldsymbol{a}$ such that for every $\omega \in \Omega$,

$$\sum_{t\in T} \boldsymbol{a}(t, \omega) \leqq \sum_{t\in T} \boldsymbol{e}(t, \omega).$$

We denote by $\mathcal{A}lc(\mathcal{E}^{KT})$ the set of all allocations, and for each $t \in T$ we denote by $\mathcal{A}lc(\mathcal{E}^{KT})_t$ the set of all the functions $\boldsymbol{a}(t, \cdot)$ for $\boldsymbol{a} \in \mathcal{A}lc(\mathcal{E}^{KT})$.

2.7. **Expectation and Pareto optimality.** Let $\mathcal{E}^{KT}$ be the pure exchange economy for logic **KT**. We denote by $\mathbf{E}_t[U_t(\boldsymbol{x}(t, \cdot)]$ the *ex-ante* expectation defined by

$$\mathbf{E}_t[U_t(\boldsymbol{x}(t, \cdot)] := \sum_{\omega\in\Omega} U_t(\boldsymbol{x}(t, \omega), \omega)\pi_t(\omega)$$

for each $\boldsymbol{x} \in \mathcal{A}ss(\mathcal{E}^{KT})$. We denote by $\mathbf{E}_t[U_t(\boldsymbol{x}(t, \cdot))|P_t](\omega)$ the *interim* expectation defined by

$$\mathbf{E}_t[U_t(\boldsymbol{x}(t, \cdot)|P_t](\omega) := \sum_{\xi\in\Omega} U_t(\boldsymbol{x}(t, \xi), \xi)\pi_t(\xi|P_t(\omega)).$$

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[4] I.e., $\pi_t(\omega) \neq 0$ for every $\omega \in \Omega$.

[6] That is, $\omega \notin K_t([U_t(\cdot, \omega)])$ for some $\omega \in \Omega$, where $[U_t(\cdot, \omega)] := \{\ \xi \in \Omega \mid U_t(\cdot, \xi) = U_t(\cdot, \omega)\}$. This is because the information structure is not a partitional structure.

Definition 4. An allocation $\boldsymbol{x}$ in an economy $\mathcal{E}^{KT}$ is said to be *ex-ante Pareto-optimal* if there is no allocation $\boldsymbol{a}$ with the two properties as follows:

PO-1: For all $t \in T$, $\mathbf{E}_t[U_t(\boldsymbol{a}(t,\cdot)] \geqq \mathbf{E}_t[U_t(\boldsymbol{x}(t,\cdot)]$;

PO-2: There exists a trader $s \in T$ such that

$$\mathbf{E}_s[U_s(\boldsymbol{a}(s,\cdot)] \gneqq \mathbf{E}_s[U_s(\boldsymbol{x}(s,\cdot)].$$

2.8. **Rational expectations equilibrium.** Let $\mathcal{E}^{KT}$ be a pure exchange economy $\langle T, \Omega, \boldsymbol{e}, (U_t)_{t\in T}, (\pi_t)_{t\in T}, (P_t)_{t\in T}, V\rangle$ for logic **KT**. A *price system* is a non-zero $\mathcal{F}$-measurable function $p : \Omega \to \mathbb{R}^l_+$. We denote by $\sigma(p)$ the smallest σ-field that p is measurable, and by $\Delta(p)(\omega)$ the atom containing ω of the field $\sigma(p)$. The *budget set* of a trader t at a state ω for a price system p is defined by

$$B_t(\omega, p) := \{\ x \in \mathbb{R}^l_+ \mid p(\omega)\cdot x \leqq p(\omega)\cdot \boldsymbol{e}(t,\omega)\ \}.$$

Let $\Delta(p) \cap P_t : \Omega \to 2^\Omega$ be defined by $(\Delta(p) \cap P_t)(\omega) := \Delta(p)(\omega) \cap P_t(\omega)$; it is plainly observed that the mapping $\Delta(p) \cap P_t$ satisfies **Ref**. We denote by $\sigma(p) \vee \mathcal{F}_t$ the smallest σ-field containing both the fields $\sigma(p)$ and $\mathcal{F}_t$, and by $A_t(p)(\omega)$ the atom containing ω. It is noted that

$$A_t(p)(\omega) = (\Delta(p) \cap A_t)(\omega).$$

Remark 4. If P_t satisfies **Ref** and **Trn** then $\sigma(p) \vee \mathcal{F}_t$ coincides with the field generated by $\Delta(p) \cap P_t$.

We shall give the extended notion of rational expectations equilibrium for an economy $\mathcal{E}^{KT}$.

Definition 5. A *rational expectations equilibrium* for an economy $\mathcal{E}^{KT}$ under reflexive information structure is a pair $(p, \boldsymbol{x})$, in which p is a price system and $\boldsymbol{x}$ is an allocation satisfying the following conditions:

RE 1: For all $t \in T$, $\boldsymbol{x}(t,\cdot)$ is $\sigma(p) \vee \mathcal{F}_t$-measurable.

RE 2: For all $t \in T$ and for every $\omega \in \Omega$, $\boldsymbol{x}(t,\omega) \in B_t(\omega, p)$.

RE 3: For all $t \in T$, if $\boldsymbol{y}(t,\cdot) : \Omega \to \mathbb{R}^l_+$ is $\sigma(p) \vee \mathcal{F}_t$-measurable with $\boldsymbol{y}(t,\omega) \in B_t(\omega, p)$ for all $\omega \in \Omega$, then

$$\mathbf{E}_t[U_t(\boldsymbol{x}(t,\cdot))|\Delta(p) \cap P_t](\omega) \geqq \mathbf{E}_t[U_t(\boldsymbol{y}(t,\cdot))|\Delta(p) \cap P_t](\omega)$$

pointwise on Ω.

RE 4: For every $\omega \in \Omega$, $\sum_{t\in T} \boldsymbol{x}(t,\omega) = \sum_{t\in T} \boldsymbol{e}(t,\omega)$.

The allocation $\boldsymbol{x}$ in $\mathcal{E}^{KT}$ is called a *rational expectations equilibrium allocation.*

We denote by $RE(\mathcal{E}^{KT})$ the set of all the rational expectations equilibria of a pure exchange economy $\mathcal{E}^{KT}$ for logic **KT**, and denote by $\mathcal{R}(\mathcal{E}^{KT})$ the set of all the rational expectations equilibrium allocations for the economy.

3. Existence theorem

We shall give the existence theorem of the generalized rational expectations equilibrium for a pure exchange economy $\mathcal{E}^{KT}$ for logic **KT**. Let $\mathcal{E}^{KT}(\omega)$ be the economy with complete information for each $\omega \in \Omega$. We set by $W(\mathcal{E}^{KT}(\omega))$ the set of all the competitive equilibria for $\mathcal{E}^{KT}(\omega)$, and we denote by $\mathcal{W}(\mathcal{E}^{KT}(\omega))$ the set of all the competitive equilibrium allocations for $\mathcal{E}^{KT}(\omega)$.

Theorem 2. *Let $\mathcal{E}^{KT}$ be a pure exchange economy for logic* **KT** *satisfying the conditions* **A-1**, **A-2** *and* **A-4**. *Then there exists a rational expectations equilibrium for the economy; i.e.,* $RE(\mathcal{E}^{KT}) \neq \emptyset$.

Proof. In view of the conditions **A-1**, **A-2** and **A-4**, it follows from the existence theorem of a competitive equilibrium that for each $\omega \in \Omega$, there exists a competitive equilibrium $(p^*(\omega), \boldsymbol{x}^*(\cdot,\omega)) \in W(\mathcal{E}^{KT}(\omega))$.[7] We take a set of strictly positive numbers $\{k_\omega\}_{\omega\in\Omega}$ such that $k_\omega p^*(\omega) \neq k_\xi p^*(\xi)$ for any $\omega \neq \xi$. We define the pair $(p, \boldsymbol{x})$ as follows: For each $\omega \in \Omega$ and for all $\xi \in A(\omega)$, $p(\xi) := k_\omega p^*(\omega)$ and $\boldsymbol{x}(t,\xi) := \boldsymbol{x}^*(t,\omega)$. It is noted that $\boldsymbol{x}(\cdot,\xi) \in W(\mathcal{E}^{KT}(\omega))$ because $\mathcal{E}^{KT}(\xi) = \mathcal{E}^{KT}(\omega)$, and we note that $\Delta(p)(\omega) = A(\omega)$.

We shall verify that $(p, \boldsymbol{x})$ is a rational expectations equilibrium for $\mathcal{E}^{KT}$: In fact, it is easily seen that p is $\mathcal{F}$-measurable with $\Delta(p)(\omega) = A(\omega)$ and that $\boldsymbol{x}(t,\cdot)$ is $\sigma(p)\vee\mathcal{F}_t$-measurable, so **RE 1** is valid. Because $(\Delta(p)\cap P_t)(\omega) = A(\omega)$ for every $\omega \in \Omega$, it can be plainly observed that $\boldsymbol{x}(t,\cdot)$ satisfies **RE 2**, and it follows from **A-2** that for all $t \in T$,

$$\mathbf{E}_t[U_t(\boldsymbol{x}(t,\cdot))|\Delta(p)\cap P_t](\omega) = U_t(\boldsymbol{x}(t,\omega),\omega) \tag{3.1}$$

On noting that $\mathcal{E}^{KT}(\xi) = \mathcal{E}^{KT}(\omega)$ for any $\xi \in A(\omega)$, it is plainly observed that $(p(\omega), \boldsymbol{x}(t,\omega)) = (k_\omega p^*(\omega), \boldsymbol{x}^*(t,\omega))$ is also a competitive equilibrium for $\mathcal{E}^{KT}(\omega)$ for every $\omega \in \Omega$, and it can be observed by Eq (3.1) that **RE 3** is valid for $(p, \boldsymbol{x})$, in completing the proof. □

Remark 5. Matsuhisa and Ishikawa [10] shows Theorem 2 for an economy under *RT*-information structure.

4. Fundamental theorem for welfare economics

We shall characterize welfare under the generalized rational expectations equilibrium for an economy $\mathcal{E}^{KT}$ for logic **KT**.

Theorem 3. *Let $\mathcal{E}^{KT}$ be a pure exchange economy for logic* **KT** *satisfying the conditions* **A-1**, **A-2** *and* **A-4**. *An allocation is ex-ante Pareto optimal if and only if it is a rational expectations equilibrium allocation relative to some price system.*

Proof. Follows immediately from Propositions 1 and 3 below. □

Proposition 1. *Let $\mathcal{E}^{KT}$ be a pure exchange economy for logic* **KT** *satisfying the conditions* **A-1**,**A-2** *and* **A-3**. *If an allocation $\boldsymbol{x}$ is ex-ante Pareto optimal then it is a rational expectations equilibrium allocation relative to some price system.*

Proof. Is given by the same way in the proof of Proposition 4 in Matsuhisa and Ishikawa [10]. We shall give it in Appendix B for readers' convenience. □

Proposition 2. *Let $\mathcal{E}^{KT}$ be a pure exchange economy for logic* **KT** *satisfying the conditions* **A-1**, **A-2** *and* **A-3**. *If there exists a rational expectations equilibrium for $\mathcal{E}^{KT}$ then the set of all rational expectations equilibrium allocations $\mathcal{R}(\mathcal{E}^{KT})$ coincides with the set of all the assignments $\boldsymbol{x}$ such that $\boldsymbol{x}(\cdot,\omega)$ is a competitive equilibrium allocation for the economy with complete information $\mathcal{E}^{KT}(\omega)$ for all $\omega \in \Omega$; i.e.,*

$$\mathcal{R}(\mathcal{E}^{KT}) = \{\boldsymbol{x} \in \mathcal{A}lc(\mathcal{E}^{KT}) \mid \quad \textit{There isa price system } p \textit{ such that } (p(\omega), \boldsymbol{x}(\cdot,\omega)) \in W(\mathcal{E}^{KT}(\omega)) \textit{ for all } \omega \in \Omega\}.$$

[7]C.f.: Debreu [4].

Proof. Will be given in Appendix B. □

Proposition 3. *Let $\mathcal{E}^{KT}$ be a pure exchange economy for logic* **KT** *satisfying the conditions* **A-1**,**A-2** *and* **A-3**. *Then an allocation* $\boldsymbol{x}$ *is ex-ante Pareto optimal if it is a rational expectations equilibrium allocation relative to a price system.*

Proof. Will be given in Appendix B. □

5. Concluding remarks

We shall give a remark about the ancillary assumptions in results in this article. Could we prove the theorems under the generalized information structure removing out the reflexivity? The answer is no vein. If trader t's possibility operator does not satisfy **Ref** then his/her expectation with respect to a price cannot be defined at a state because it is possible that $\Delta(p)(\omega) \cap P_t(\omega) = \emptyset$ for some state ω.

Could we prove the theorems without four conditions **A-1**, **A-2**, **A-3** and **A-4**. The answer is no again. The suppression of any of these assumptions renders the existence theorem of rational expectations equilibrium (Theorem 2) vulnerable to the discussion and the example proposed in Remarks 4.6 of Matsuhisa and Ishikawa [10].

Matsuhisa [9] presents the notion of Ex-post core in the economy for logic **KT** equipped with non-atomic measure on the traders space T, and he establishes the core equivalence theorem based on Matsuhisa, Ishikawa and Hoshino [11]: The ex-post core in the economy for logic **KT** coincides with the set of all its rational expectations equilibrium allocations.

Appendix A

This appendix establishes the finite model property of the logic **KT** (Theorem 1). The proof will be given by the same way as described in Chellas [3].

A.1. **Proof sets.** Let L be a system of traders' knowledge. We recall the *Lindenbaum's lemma*:

Lemma 1. *Let L be a system of traders' knowledge. Every L-consistency set of sentences has an L-maximal extension.*

This is because L includes the ordinary propositional logic. We call the extension an *L-maximally consistent set*.

As a consequence, we can observe that *a sentence in L is deducible from a set of sentences Γ if and only if it belongs to every L-maximally consistent set of Γ*, and thus

Theorem 4. *Let L be a system of traders' knowledge. A sentence is a theorem of L if and only if it is a member of every L-maximally consistent set of sentences.*

We denote by $|\varphi|_L$ the class of L-maximally consistent sets of sentences containing the sentence φ; this is called the *proof set* of φ. We note that

Corollary 1. *Let L be a system of traders' knowledge.*

- (i): *A sentence φ is a theorem of L if and only if $|\varphi|_L = \Omega_L$;*
- (ii): *A sentence $\varphi \to \psi$ is a theorem of L if and only if $|\varphi|_L \subseteq |\psi|_L$.*

A.2. **Canonical Model.** Let L be a system of traders' knowledge. The *canonical model* for L is the model $\mathcal{M}_L = \langle T, \Omega_L, ({P^L}_t)_{t\in T}, V^L\rangle$ for L consisting of:

(1) Ω_L is the set of all the L-maximally consistent sets of sentences;
(2) $P_t^L : \Omega_L \to 2^{\Omega_L}$ is given by

$$P_t^L(\omega) = \{\xi \in \Omega_L \mid \text{For each } \varphi \in L, \Box_t\varphi \in \omega \text{ implies } \varphi \in \xi\}$$

(3) V_L is the mapping such that $V_L(\omega, \mathbf{P}_m) = \{\omega \in \Omega^L | \mathbf{P}_m \in \omega\}$ for $m = 0, 1, 2, \dots$.

We can easily observe that

Proposition 4. *The canonical model $\mathcal{M}_L$ is a model for a system L of traders' knowledge.*

A.3. **Filtration of Model.** Let $\mathcal{M}$ be a model for a system L of traders' knowledge. For each set of sentences Γ, we define the equivalence relation $\equiv$ on Ω by

$$\omega \equiv \xi \quad \text{if and only if for every sentence } \psi \text{ of } \Gamma, \models^{\mathcal{M}}_{\omega} \psi \Longleftrightarrow \models^{\mathcal{M}}_{\xi} \psi \ .$$

We denote by $[\omega]_\Gamma$ the equivalence class of ω and denote by $[X]_\Gamma$ the set of equivalence classes $[\omega]_\Gamma$ for all ω of X whenever X is a subset of Ω. By the Γ-*filtration* $\mathcal{M}^\Gamma$ (or *filtration of* $\mathcal{M}^\Gamma$ *through* Γ), we mean the model for L

$$\mathcal{M}^\Gamma = \langle T, \Omega^\Gamma, (P_t^\Gamma)_{t\in T}, V^\Gamma\rangle$$

consisting of: For each $t \in T$,

(1) $\Omega^\Gamma = [\Omega]_\Gamma$;
(2) $P_t^\Gamma : \Omega_\Gamma \to 2^{\Omega_L}$ is given by $P_t^\Gamma([\omega]) = [P_t^\Gamma(\omega)]_\Gamma$
(3) $V^\Gamma([\omega]_\Gamma, \mathbf{P}_m) = V_L(\omega, \mathbf{P}_m)$.

Remark 6. The Γ-*filtration* $\mathcal{M}^\Gamma$ is a well-defined model for the system L; i.e., it is actually a model for L in which the both mappings P_t^Γ and V^Γ are independent of the choices of states in each equivalence class.

A.4. By induction on the complexity of a sentence φ we can plainly verify that

Proposition 5. *Let $\mathcal{M}$ be a model for a system L and Γ a set of sentences closed under subsentences. Then the following two properties are true:*

(1) *For every sentence φ in Γ,*

$$\models^{\mathcal{M}} \varphi \quad \text{if and only if} \quad \models^{\mathcal{M}^\Gamma} \varphi \ .$$

(2) *The model $\mathcal{M}^\Gamma$ is finite if so is Γ.*

□

A.5. The important result about a canonical model is the following:

Basic Theorem. *Let $\mathcal{M}_L$ be the canonical model for a system L of traders' knowledge. Then for every sentence φ,*

$$\models^{\mathcal{M}_L} \varphi \quad \text{if and only if} \quad \vdash_L \varphi \ .$$

In other words,

$$||\varphi||^{\mathcal{M}_L} = |\varphi|_{\mathcal{M}_L}.$$

Proof. By induction on the complexity of φ. We treat only that φ is $\Box_t\psi$. As an inductive hypothesis we assume that $||\psi||^{\mathcal{M}_L} = |\psi|_{\mathcal{M}_L}$. Then for ever $\omega \in \Omega_L$,

$$\begin{aligned} \models_{\omega}^{\mathcal{M}_L} \Box_t\psi \quad & \text{if and only if} \quad ||\varphi||^{\mathcal{M}_L} \in P_t^L(\omega), \\ & \qquad \text{by the definition of validity ;} \\ & \text{if and only if} \quad |\varphi|^{\mathcal{M}_L} \in P_t^L(\omega),, \\ & \qquad \text{by the inductive hypothesis as above;} \\ & \text{if and only if} \quad \vdash_L \Box_t\psi \\ & \qquad \text{by the definition of canonical model.} \end{aligned}$$

□

A.6. **Proof of Theorem 1.** Let $\boldsymbol{R}_{FIN}$ denote the class of all finite models in $\boldsymbol{R}$. To prove the completeness with respect to $\boldsymbol{R}$. Soundness has been already observed in Proposition 4. The completeness will be shown by the way of contradiction as follows: Suppose that some sentence φ is not a theorem in L. In view of Basic Theorem, it follows that φ is not valid for a canonical model $\mathcal{M}_C$. Let Γ be the set of all subsentences of φ. By Proposition 5 we can observe that φ is not valid for the Γ-filtration $\mathcal{M}_L^\Gamma \in \boldsymbol{R}_{FIN}$, in contradiction. □

Appendix B

B.1. **Proof of Proposition 1.** For each $\omega \in \Omega$ we denote by $G(\omega)$ the set of all the vectors $\sum_{t\in T} \boldsymbol{x}(t,\omega) - \sum_{t\in T} \boldsymbol{y}(t,\omega)$ with an assignment $\boldsymbol{y} : T \times \Omega \to \mathbb{R}^l_+$ such that $U_t(\boldsymbol{y}(t,\omega),\omega) \geqq U_t(\boldsymbol{x}(t,\omega),\omega)$ for all $t \in T$; i.e.,

$$G(\omega) = \{ \sum_{t\in T} \boldsymbol{x}(t,\omega) - \sum_{t\in T} \boldsymbol{y}(t,\omega) \in \mathbb{R}^l \mid \boldsymbol{y} \in \mathcal{A}ss(\mathcal{E}^{KT}) \text{ and } U_t(\boldsymbol{y}(t,\omega),\omega) \geqq U_t(\boldsymbol{x}(t,\omega),\omega) \quad \text{for all } t \in T \}.$$

First, we note that that $G(\omega)$ is convex and closed in $\mathbb{R}^l_+$ by the conditions **A-1**, **A-2** and **A-4**. It can be shown that

Claim 1: For each $\omega \in \Omega$ there exists $p^*(\omega) \in \mathbb{R}^l_+$ such that $p^*(\omega)\cdot v \leqq 0$ for all $v \in G(\omega)$.

Proof of Claim 1: By the separation theorem,[8] we can plainly observe that the assertion immediately follows from that $v \leqq 0$ for all $v \in G(\omega)$: Suppose to the contrary that there exist $\omega_0 \in \Omega$ and $v_0 \in G(\omega_0)$ with $v_0 \gneqq 0$. Take an assignment $\boldsymbol{y}^0$ for $\mathcal{E}^{KT}$ such that for all t, $U_t(\boldsymbol{y}^0(t,\omega),\omega_0) \geqq U_t(\boldsymbol{x}(t,\omega_0),\omega_0)$ and $v_0 = \sum_{t\in T} \boldsymbol{x}(t,\omega_0) - \sum_{t\in T} \boldsymbol{y}^0(t,\omega_0)$. Consider the allocation $\boldsymbol{z}$ defined by

$$\boldsymbol{z}(t,\xi) := \begin{cases} \boldsymbol{y}^0(t,\omega_0) + \frac{v_0}{n} & \text{if } \xi \in A(\omega_0), \\ \boldsymbol{x}(t,\xi) & \text{if not.} \end{cases}$$

[8]See Lemma 8, Chapter 4 in Arrow and Hahn [1] pp.92.

It follows that for all $t \in T$,

$$\begin{aligned}\mathbf{E}_t[U_t(\boldsymbol{z})] &= \sum_{\xi \in A(\omega_0)} U_t(\boldsymbol{y}^0(t,\omega_0) + \frac{v_0}{n}, \xi)\pi_t(\xi) \\ &\quad + \sum_{\xi \in \Omega \setminus A(\omega_0)} U_t(\boldsymbol{x}(t,\xi),\xi)\pi_t(\xi) \\ &\gneqq \sum_{\xi \in A(\omega_0)} U_t(\boldsymbol{y}^0(t,\omega_0),\xi)\pi_t(\xi) \\ &\quad + \sum_{\xi \in \Omega \setminus A(\omega_0)} U_t(\boldsymbol{x}(t,\xi),\xi)\pi_t(\xi) \quad \text{because of } \mathbf{A\text{-}4} \\ &\geqq \mathbf{E}_t[U_t(\boldsymbol{x})].\end{aligned}$$

This is in contradiction to which $\boldsymbol{x}$ is ex-ante Pareto optimal as required.

Secondly, let p be the price system defined as follows: We take a set of strictly positive numbers $\{k_\omega\}_{\omega \in \Omega}$ such that $k_\omega p^*(\omega) \neq k_\xi p^*(\xi)$ for any $\omega \neq \xi$. We define the price system p such that for each $\omega \in \Omega$ and for all $\xi \in A(\omega)$, $p(\xi) := k_\omega p^*(\omega)$. It can be observed that $\Delta(p)(\omega) = A(\omega)$. To conclude the proof we shall show

Claim 2: The pair $(p, \boldsymbol{x})$ is a rational expectations equilibrium for $\mathcal{E}^{KT}$.

Proof of Claim 2: We first note that for every $t \in T$ and for every $\omega \in \Omega$,

$$(\Delta(p) \cap P_t)(\omega) = \Delta(p)(\omega) = A(\omega),$$

Therefore it follows from **A-2** that for every allocation $\boldsymbol{x}$,

$$\mathbf{E}_t[U_t(\boldsymbol{x}(t,\cdot))|(\Delta(p) \cap P_t)](\omega) = U_t(\boldsymbol{x}(t,\omega),\omega) \tag{B.1}$$

To prove Claim 2 it suffices to verify that $\boldsymbol{x}$ satisfies **RE 3**. Suppose to the contrary that there exists a non-empty set $S \in \Sigma$ with the two properties:

1. For all $s \in S$, there is a $\sigma(p) \vee \mathcal{F}_s$-measurable function $\boldsymbol{y}(s,\cdot) : \Omega \to \mathbb{R}^l_+$ such that $\boldsymbol{y}(s,\omega) \in B_s(\omega,p)$ for all $\omega \in \Omega$;
2. $\mathbf{E}_s[U_s(\boldsymbol{y}(s,\cdot))|(\Delta(p) \cap P_s)](\omega_0) \gneqq \mathbf{E}_s[U_s(\boldsymbol{x}(s,\cdot)|(\Delta(p) \cap P_s)](\omega_0)$ for some $\omega_0 \in \Omega$.

In view of Eq (B.1) it immediately follows from Property 2 that $U_s(\boldsymbol{y}(s,\omega_0),\omega_0) \gneqq U_s(\boldsymbol{x}(s,\omega_0),\omega_0)$, and thus $\boldsymbol{y}(s,\omega_0) \gneqq \boldsymbol{x}(s,\omega_0)$ by **A-5**. Therefore we obtain that for all $s \in S$, $p(\omega_0) \cdot \boldsymbol{y}(s,\omega_0) \gneqq p(\omega_0) \cdot \boldsymbol{x}(s,\omega_0)$, in contradiction. This completes the proof. □

B.2. **Proof of Proposition 2.** Let $\boldsymbol{x} \in \mathcal{R}(\mathcal{E}^{KT})$ and $(p,\boldsymbol{x})$ a rational expectations equilibrium for $\mathcal{E}^{KT}$. We shall show that $(p(\omega), \boldsymbol{x}(\cdot,\omega)) \in W(\mathcal{E}^{KT}(\omega))$ for any $\omega \in \Omega$: Suppose to the contrary that there exist a state $\omega_0 \in \Omega$ and non-empty set $S \subseteq T$ with the property: For each $s \in S$ there is an $a(s,\omega_0) \in B_s(\omega_0,p)$ such that $U_s(a(\omega_0),\omega_0) \gneqq U_s(\boldsymbol{x}(s,\omega_0),\omega_0)$. Define the function $\boldsymbol{y} : T \times \Omega \to \mathbb{R}^l_+$ by

$$\boldsymbol{y}(t,\xi) := \begin{cases} a(t,\omega_0) & \text{for } \xi \in A_t(p)(\omega_0) \text{ and } t \in S; \\ \boldsymbol{x}(t,\xi) & \text{otherwise.} \end{cases}$$

It is easily observed that $\boldsymbol{y}(t,\cdot)$ is $\sigma(p) \vee \mathcal{F}_t$-measurable for every $t \in T$. On noting that $\mathcal{E}^{KT}(\xi) = \mathcal{E}^{KT}(\omega)$ for any $\xi \in A_t(p)(\omega)$, it immediately follows that $B_t(\xi,p) = B_t(\omega,p)$ for every $\xi \in A_t(p)(\omega)$, so $\boldsymbol{y}(t,\omega) \in B_t(\omega,p)$ for all $t \in T$ and any $\omega \in \Omega$. Therefore it can be obtained that for all $s \in S$,

$$\mathbf{E}_s[U_s(\boldsymbol{x}(s,\cdot))|\Delta(p) \cap P_s](\omega_0) \lneqq \mathbf{E}_s[U_s(\boldsymbol{y}(s,\cdot))|\Delta(p) \cap P_s](\omega_0),$$

in contradiction for $(p, \boldsymbol{x}) \in \mathcal{R}(\mathcal{E}^{KT})$.

The converse will be shown as follows: Let $\boldsymbol{x}$ be an assignment with $(p(\omega), \boldsymbol{x}(\cdot, \omega)) \in W(\mathcal{E}^{KT}(\omega))$ for any $\omega \in \Omega$. We take a set of strictly positive numbers $\{k_\omega\}_{\omega \in \Omega}$ such that $k_\omega p(\omega) \neq k_\xi p(\xi)$ for any $\omega \neq \xi$. We define the price system $p^* : \Omega \to \mathbb{R}^l_+$ such that for each $\omega \in \Omega$ and for all $\xi \in A(\omega)$, $p^*(\xi) := k_\omega p(\omega)$. We shall show that $(p^*, \boldsymbol{x}) \in RE(\mathcal{E}^{KT})$: In fact, it is first noted that $\Delta(p^*)(\omega) = A(\omega)$ and that $(p^*(\xi), \boldsymbol{x}(\cdot, \xi)) \in W(\mathcal{E}^{KT}(\omega))$ for every $\xi \in A(p^*)(\omega)$ because $\mathcal{E}^{KT}(\xi) = \mathcal{E}^{KT}(\omega)$. Therefore $\boldsymbol{x}(t, \cdot)$ is $\sigma(p) \vee \mathcal{F}_t$-measurable for every $t \in T$, and $\boldsymbol{x}(t, \omega) \in B_t(\omega, p^*)$ for all $t \in T$. Let $\boldsymbol{y}(t, \cdot) : \Omega \to \mathbb{R}^l_+$ be a $\sigma(p^*) \vee \mathcal{F}_t$-measurable function with $\boldsymbol{y}(t, \omega) \in B_t(\omega, p^*)$ for all $\omega \in \Omega$. In viewing that $(\Delta(p^*) \cap P_t)(\omega) = A(\omega)$ it can be obtained from **A-3** that

$$\mathbf{E}_t[U_t(\boldsymbol{x}(t, \cdot))|\Delta(p^*) \cap P_t](\omega) = U_t(\boldsymbol{x}(t, \omega), \omega)$$

and

$$\mathbf{E}_t[U_t(\boldsymbol{y}(t, \cdot))|\Delta(p^*) \cap P_t](\omega) = U_t(\boldsymbol{y}(t, \omega), \omega).$$

Since $(p^*(\omega), \boldsymbol{x}(\cdot, \omega)) \in W(\mathcal{E}^{KT}(\omega))$ it can be observed that $U_t(\boldsymbol{x}(t, \omega), \omega) \geqq U_t(\boldsymbol{y}(t, \omega), \omega)$ for all $t \in T$ and for each $\omega \in \Omega$, from which it follows from **A-2** that

$$\mathbf{E}_t[U_t(\boldsymbol{x}(t, \cdot))|\Delta(p^*) \cap P_t](\omega) \geqq \mathbf{E}_t[U_t(\boldsymbol{y}(t, \cdot))|\Delta(p^*) \cap P_t](\omega).$$

Therefore $(p^*, \boldsymbol{x}) \in RE(\mathcal{E}^{KT})$ and $\boldsymbol{x} \in \mathcal{R}(\mathcal{E}^{KT})$, in completing the proof. □

B.3. **Proof of Proposition 3.** Let $(p, \boldsymbol{x})$ be a rational expectations equilibrium for $\mathcal{E}^{KT}$. It follows from Proposition 2 that $(p(\omega), \boldsymbol{x}(\cdot, \omega))$ is a competitive equilibrium for the economy with complete information $\mathcal{E}^{KT}(\omega)$ at each $\omega \in \Omega$. Therefore in viewing the fundamental theorem of welfare in the economy $\mathcal{E}^{KT}(\omega)$, we can plainly observe that for all $\omega \in \Omega$, $\boldsymbol{x}(\cdot, \omega)$ is Pareto optimal in $\mathcal{E}^{KT}(\omega)$, and thus $\boldsymbol{x}$ is ex-ante Pareto optimal. □

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