# Project Screening with Tiered Evaluation* 

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#### Abstract

We study a Bayesian game of two-sided incomplete information in which an agent, who owns a project of unknown quality, considers proposing it to an evaluator, who has the choice of whether or not to accept it. There exist two distinct tiers of evaluation that differ in the benefits they deliver to the agent upon acceptance of a project. The agent has to select the tier to which the project is submitted for review. Making a proposal incurs a cost on the agent in the form of a submission cost. We examine the effect of changes in the payoff parameters at the two tiers of evaluation on the efficiency of the equilibrium outcome. We show that changes in these parameters that are aimed at increasing the level of self-screening exerted by the agent do not necessarily have beneficial effects either on the quality of projects submitted for review or on the quality of projects that are implemented. From a methodological viewpoint, our paper proposes a novel method of performing comparative statics in games whose equilibria are defined by a system of equations with no closed-form solution.


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Keywords: Evaluation, Project Screening.

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## 1 Introduction

On July 5, 2011 the jury reached its verdict in the case State of Florida vs. Casey Marie Anthony and found the defendant not guilty in the death of her daughter. The verdict surprised the media and the general public as strong circumstantial evidence implicated the defendant in the crime. The outcome was deemed by some as a symptomatic failure of the judicial system, but many law experts viewed it, instead, to be a consequence of prosecutorial overreach in that particular trial. For instance, Scott Bonn, a professor of criminology at Drew University, argued in a New York Daily News article that "the prosecutor employed an extremely high-risk strategy by charging her with first-degree murder and, in addition, asking for capital punishment", and therefore that the "strenuous burden of proof weighed heavily on the state throughout the trial". ${ }^{1}$ He concludes that the prosecutors should have filled a lesser charge that was more likely to be accepted by the jury. Instead, because the "no double jeopardy" principle guarantees that a person cannot be prosecuted a second time for the same crime once a jury returns a verdict, the defendant walked away free.

It was often argued in the aftermath of the jury decision in the Casey Anthony trial that the apparent prosecutorial overreach may have been caused by the media hype surrounding the trial which increased the stakes on the prosecution to obtain a maximum penalty so as to appease the public opinion. ${ }^{2}$ At the same time, in other more routine trials, when a prosecutor makes his decision, he may also account for the cost of filing more severe charges (or of filing any charges at all), in terms of resources spent on collecting and organizing evidence and of time spent in court. ${ }^{3}$ These raise the questions of how payoffs impact the decisions of parties involved in such situations, and how these payoffs should be adjusted, when feasible, so as to improve the efficiency of the equilibrium outcome. Addressing these questions is the main objective of our paper.

We model these situations as a Bayesian game of two-sided incomplete information where an agent (the prosecutor), who owns a project (the case) of unknown quality (guilty or not guilty), considers submitting it to an evaluator (the jury), who upon receiving a project for review, has the choice of whether or not to accept it. The different charges that the prosecutor may file are

[^1]modelled as ranked tiers of evaluation to which the project can be submitted. In this paper we analyze the simplest case of two tiers, which we refer to in the following as the upper and the lower tier, respectively. The upper tier delivers higher benefits to the agent upon acceptance, and higher losses to the evaluator upon acceptance of a low-quality project. For instance, in a trial like Casey Anthony, filing more severe charges leads to better rewards to the prosecutor following a favorable ruling, but also increases the stakes on the jury, making it less likely to return a guilty verdict. In our model, we abstract away from incorporating the actual trial proceedings, and instead assume that prior to taking their actions, each player performs an assessment of the project that yields a private signal of quality. ${ }^{4}$ The agent then has to choose whether or not to submit the project, and in the relevant case, the tier to which to submit it. Upon receiving a project for review into a certain tier, the evaluator observes his signal and then decides whether or not to accept it.

The scope of applications captured by the model of project screening with tiered evaluation that we study in this paper extends beyond our particular motivating example. This framework can describe, in general, situations where an agent contemplates making a costly ${ }^{5}$ proposal of a project of unknown quality to an evaluator while facing a trade-off between the risk of being rejected and the higher benefits resulting from a better placement of the project. As an example, consider a manufacturer of a newly invented product whose value to consumers takes time to perfectly assess. In this case, when launching the product, the firm has to decide on the claims to make regarding its value to the users and on its price. Marketing the product as a high-value item generates larger revenues if the product is adopted, but also makes it more likely to fail as consumers may be reluctant to pay the higher price. ${ }^{6}$ Another example is the selling of an used car (or of any other pre-owned item whose quality cannot be perfectly evaluated, such as a house), where the owner can choose different degrees of disclosure of its mechanical issues. Disclosing no issues (submitting to a higher tier) reduces the likelihood of a sale, as the buyer may identify them, in which case he would distrust the seller and walk away, but if that does not happen, the selling price is higher. On the other hand, being more honest (submitting to the lower tier) reduces the sale price, but as long

[^2]as these mechanical issues are not too severe, results in a sale with higher probability. ${ }^{7}$ Because of the variety of these situations, we will present our analysis in the framework of a generic Bayesian game and then identify the relevant implications of its findings for applications of interest.

This paper contributes to the literature that investigates the role of payoffs on the efficiency of project screening initiated by Leslie (2005), who argued in the context of the academic scholarship review process that the optimal submission fees are strictly positive ${ }^{8}$ because they reduce the burden on editors and referees by discouraging long-shot submissions. ${ }^{9}$ Departing from earlier papers in this literature, in a framework with one tier of evaluation, Barbos (2013) considers the case of two-sided incomplete information where not only the agent's, but also the evaluator's assessment of the project is imperfect. Under this specification, while a higher submission cost does increase the quality of projects that the agent submits, it may not always be beneficial, as it also induces the evaluator to weaken his standards of acceptance, and under certain conditions, a higher submission fee decreases the expected quality of projects that are implemented.

Our analysis unveils four main insights which we present next.

1. We first investigate the assortative matching between the agent's signal of quality and the rank of the tier to which he submits the project. We show that if the benefit upon acceptance at the upper tier is high enough, then negative assortative matching may emerge in equilibrium, where projects with low signals of quality are submitted to the upper tier while projects of higher quality are submitted to the lower tier. In certain applications, negative assortative matching may be noticeable with a sufficiently large sample of outcomes, and thus likely to be averted in the long run through an exogenous adjustment in payoffs, but this finding suggests that it may still emerge in the short run following a sudden shift in payoff parameters. In other situations, negative matching could persist even in the long run. For instance, if the potential profits following a successful product launch as a high-end item are sufficiently high, manufacturers of products that have a relatively long shot at success may decide to pursue that avenue just in case the favorable outcome realizes, while manufacturers of products that are more likely to succeed may choose to

[^3]follow a safer approach and have their product be adopted with almost certainty. ${ }^{10}$ The same outcome may realize in the used car selling example if the payoff from deceiving is sufficiently high.
2. Focusing on equilibria with positive assortative matching, we next examine the effect of payoff parameters on the quality of projects submitted for review. Previous literature, analyzing the case of a single tier of evaluation, argued that an increase in the submission cost (or, equivalently, a decrease in the agent's benefit upon acceptance) induces more self-screening on the part of the agent, increasing the quality of projects that he submits. We show that this is no longer necessarily the case with a tiered system of evaluation. In particular, a higher submission cost at the upper tier actually decreases the quality of projects submitted between the two tiers. In our main motivating application, this suggests that when prosecutors have less incentive to file the more severe charges (either because of an increase in the cost of collecting the additional evidence or because of a decrease in the potential rewards), they will end up filing the lesser charges in cases that otherwise they would not have been pursued, increasing the burden on the judicial system.

On the other hand, a higher cost at the lower tier does discourage marginal submissions to that tier, and therefore increases the quality of projects received between the two tiers, suggesting a mechanism for reducing the number of projects submitted for evaluation. However, it has an indeterminate effect on the quality of projects submitted at either tier individually, and in particular it may actually even lower the average quality of projects received by both tiers.
3. The third insight of the paper relates payoff parameters with the expected quality of projects that are implemented, i.e. of projects that are submitted and accepted. We show that the effect of changes in payoffs at the upper tier can be elicited solely from the evaluator's information structure in the neighborhood of his equilibrium strategy. In particular, for the same class of information structures identified by Barbos (2013) in the case of a system of evaluation with a single tier, a higher submission cost at the upper tier increases the expected quality of projects that are implemented by each of the two tiers. On the other hand, the effect of changes in the payoff parameters at the lower tier is a function of the underlying information structure and the direction of the effects on the quality of projects that are submitted. This illustrates again the differential impact on equilibrium efficiency of the payoff parameters at the two tiers.

Similarly to the other findings in this paper, the potential decrease in the quality of projects that

[^4]are implemented, following an increase in the level of self-screening exerted by the agent, hinges on the assumption of imprecise evaluation, which renders the evaluator a strategic player who adjusts his acceptance policy in response to changes in the quality of projects submitted for review. From a policy perspective, our findings suggest that in those situations where there is reason to believe that the evaluator may infer information from the agent's decision, such as if a prosecutor's choice of charges in a trial is likely to influence the jury's beliefs, then when adjusting payoff parameters, a policy designer has to account for that fact that an increase in the perceived quality of projects submitted for review may come at the expense of the evaluator's own judgment of the project.
4. The last result of the paper compares the equilibria of games with one and two tiers of evaluation. Introducing a new upper tier in a system of evaluation where only one tier had existed increases both the quality of projects submitted for review and the quality of projects implemented between the two tiers. Introducing a new lower tier decreases the burden of evaluation at the upper tier suggesting a mechanism for reducing the overall cost of evaluation when the cost of reviewing a project in the upper tier is sufficiently higher than in the lower tier. These insights provide support for a tiered system of evaluation as a more efficient project-screening mechanism.

From a methodological viewpoint, our paper proposes a novel method of performing comparative statics in games whose equilibria are defined by a system of equations with no closed-form solution.

Several other papers from the literature examined optimal submission strategies when facing a tiered system of evaluation, in particular in a context where economic agents seek certification for their products from information intermediaries, such as ratings agencies. For instance, Farhi, Lerner, and Tirole (2008) study how the market structure and in particular the evaluator's rejection disclosure policy affect the choices of such agents, Lerner and Tirole (2006) investigate the role of biased technology standard setting authorities as certifiers, while Gill and Sgroi (2012) consider the case where an agent who submits a project for certification also has the ability to set the price for that item, thus endogenizing the payoffs from acceptance. On the other hand, in the academic publishing context, Heintzelman and Nocetti (2009) confirm the insight from Leslie (2005) in a search theoretical model where an author facing multiple journals has to decide on the optimal submission path. Our paper differs from the papers in this literature in that we consider a set-up with two-sided incomplete information where evaluation is imperfect and thus the agent has to account for the fact that the evaluator learns from his decision. ${ }^{11}$ At a formal level, the paper

[^5]closer to ours is Taylor and Yildirim (2011), who study a model of project proposals in which an agent chooses the amount of effort to exert in generating a project that is then submitted for review. A blind review system, in which payoff relevant information about the proposer is hidden from the reviewer, is compared with an informed regime in which the proposer's type is public information. While their results are driven by the moral hazard effects of the potentially available public information, we consider the effort level as sunk, and the agent's decision to be whether and where to submit a project, as a function of the available public and private information.

The rest of the paper is organized as follows. Section 2 defines the model, while in section 3 we characterize the equilibrium of the game. In section 4 we investigate the effect of submission fees on equilibrium strategies and on the efficiency of the outcome. Section 5 concludes.

## 2 The model

There are two players, an agent $(\mathcal{A})$ and an evaluator $(\mathcal{E})$. $\mathcal{A}$ owns a project and considers proposing it to $\mathcal{E}$. The project is of either high $(h)$ or low $(l)$ quality. The common prior probability of state $h$ is $\pi$. There are two tiers of evaluation, A and B , and when $\mathcal{A}$ submits a project, he has to select the tier to which to submit it. Upon receiving a project for review into a certain tier, $\mathcal{E}$ has the choice of whether or not to accept it. Submitting the project to tier $t \in\{\mathrm{~A}, \mathrm{~B}\}$ incurs a fee $c_{t}$ on $\mathcal{A}$. Irrespective of its ex-post observed quality, a project accepted in tier $t$, yields $\mathcal{A}$ a payoff $b_{t}$. $\mathcal{A}$ also has the option to not submit the project; the corresponding payoff is normalized to zero. $\mathcal{E}$ 's payoff from accepting a high-quality project in either tier is 1 , while the loss incurred by $\mathcal{E}$ from accepting a low-quality project in tier $t$ is $L_{t}$. $\mathcal{E}$ 's payoff from rejecting a project is normalized to zero. ${ }^{12}$ A project that is rejected once cannot be resubmitted for review to either tier. ${ }^{13}$ We make the following assumption on the payoff parameters of the model.

Assumption 1 (i) $b_{\mathrm{A}}>b_{\mathrm{B}}$; (ii) $\frac{b_{\mathrm{A}}}{c_{\mathrm{A}}}>\frac{b_{\mathrm{B}}}{c_{\mathrm{B}}}$; (iii) $L_{\mathrm{A}}>L_{\mathrm{B}}$.

[^6]By (i) and (ii), tier A delivers a better absolute and relative reward to $\mathcal{A}$ from an accepted project than tier B. We will refer to A and B as the upper and lower tier, respectively. Part (iii) implies that $\mathcal{E}$ is aversely affected more by the acceptance of a low-quality project in the upper tier.

Prior to making their decisions, $\mathcal{A}$ and $\mathcal{E}$ perform assessments of the project that result in subjective evaluations of its quality. $\mathcal{A}$ 's assessment yields a private signal $\mu \in[0,1] ; \mathcal{E}$ 's assessment yields a private signal $\sigma \in[0,1]$. For quality $q \in\{h, l\}$, let $G^{q}(\mu)$ and $F^{q}(\sigma)$ denote the cumulative distribution functions of the agent and the evaluators' signals, respectively. Also, let $g^{q}(\mu)>0$ and $f^{q}(\sigma)>0$ be the corresponding probability density functions.

Assumption 2 (i) $f^{q}$ and $g^{q}$ are bounded and twice continuously differentiable for $q \in\{h, l\}$; (ii) $\frac{d}{d \sigma}\left[\frac{f^{h}(\sigma)}{f^{l}(\sigma)}\right]>0, \frac{d}{d \mu}\left[\frac{g^{h}(\mu)}{g^{l}(\mu)}\right]>0$.

Part (ii) of the assumption is the usual monotone likelihood ratio, essentially implying that a higher signal is more informative of a high-quality project.

## 3 The equilibrium

Consider some arbitrary strategies of $\mathcal{A}$ and $\mathcal{E}$, respectively, $\mathcal{S}^{a g}:[0,1] \rightarrow\left\{s_{\mathrm{A}}, s_{\mathrm{B}}, n\right\}$ and $\mathrm{S}^{e v}$ : $\{\mathrm{A}, \mathrm{B}\} \times[0,1] \rightarrow\{a, r\}$, that map signals into actions, with the obvious interpretation of the action labels. Upon observing a project submitted to tier $t$, and after acquiring the signal $\sigma, \mathcal{E}$ accepts the project if and only if

$$
\begin{equation*}
\operatorname{Pr}\left(h \mid\left\{s_{t}\right\}, \sigma\right) \cdot 1+\left[1-\operatorname{Pr}\left(h \mid\left\{s_{t}\right\}, \sigma\right)\right]\left(-L_{t}\right) \geq 0 \Longleftrightarrow \operatorname{Pr}\left(h \mid\left\{s_{t}\right\}, \sigma\right) \geq \frac{L_{t}}{1+L_{t}} \tag{1}
\end{equation*}
$$

where the event $\left\{s_{t}\right\} \equiv\left\{\mu \in[0,1]: \mathcal{S}^{a g}(\mu)=s_{t}\right\}$. Denoting the event $\left\{a_{t}\right\} \equiv\{\sigma \in[0,1]:$ $\left.\mathcal{S}^{e v}(t, \sigma)=a\right\}$, it follows that $\mathcal{A}$ submits a project with quality signal $\mu$ to tier A if

$$
\begin{equation*}
b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right)-c_{\mathrm{A}} \geq \max \left\{0, b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right)-c_{\mathrm{B}}\right\} \tag{2}
\end{equation*}
$$

to tier B if

$$
\begin{equation*}
b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right)-c_{\mathrm{B}} \geq \max \left\{0, b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right)-c_{\mathrm{A}}\right\} \tag{3}
\end{equation*}
$$

and does not submit the project in the remaining case.

In appendix A 1 we show that in any Bayesian Nash Equilibrium, $\mathcal{E}$ adopts a cutoff strategy with respect to his informative signal, according to which he accepts a project if and only if his signal is higher than a threshold specific to each tier. Since $\mathcal{E}$ 's equilibrium strategy can be defined in terms of the corresponding thresholds, for the rest of the paper, we will use ( $\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}$ ) to denote a generic cutoff strategy, with thresholds of acceptance for the two tiers $\sigma_{\mathrm{A} s}$ and $\sigma_{\mathrm{B} s}$, respectively. We also show in appendix A 1 that the set of values of $\mu$ for which $\mathcal{A}$ submits projects to a particular tier is an interval (possibly empty), and that the set of values of $\mu$ for which $\mathcal{A}$ does not submit a project consists of either one (possibly empty) or two disjoint intervals.

The next lemma states that, under assumption 1 , if $\mathcal{E}$ adopts a cutoff strategy $\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)$, with $\sigma_{\mathrm{B} s} \geq \sigma_{\mathrm{A} s}$, then $\mathcal{A}$ 's best response is to never submit to tier B . In other words, if $\mathcal{E}$ is more stringent at the lower tier, then $\mathcal{A}$ never submits projects to that tier. Its proof from appendix A 2 shows that if $\sigma_{\mathrm{B} s} \geq \sigma_{\mathrm{A} s}$, then whenever $\mathcal{A}$ has a higher expected payoff from submitting to tier B than to A , then that payoff is in fact negative. In the following we thus examine the interesting equilibria where the evaluator is more stringent at the upper tier, i.e., where $\sigma_{\mathrm{B} s}<\sigma_{\mathrm{A} s}$.

Lemma 3.1 If $\sigma_{\mathrm{B} s} \geq \sigma_{\mathrm{A} s}$, then $\mathcal{A}$ either submits the project to tier A or does not submit it at all.

In appendix A3, we examine the agent's best response function and provide the necessary and sufficient condition for positive assortative matching between the agent's signal of quality and the rank of the tier to whom he submits a project. More precisely, we identify the condition on $\sigma_{\text {As }}$, $\sigma_{\mathrm{B} s}$, and the payoff parameters of the model under which for a given prior $\pi$, the set of signals for which $\mathcal{A}$ submits to the upper tier A is above the set of signals for which he submits to $\mathrm{B} .{ }^{14}$ The condition is not always satisfied, and thus negative assortative matching may emerge, where $\mathcal{A}$ submits projects with low signals to the upper tier, and projects with high signals to the lower tier. The following remark presents the conditions under which this occurs. The precise formal definitions of these conditions follow from the analysis presented in the appendix.

Remark 3.1 Negative assortative matching emerges when the following conditions are satisfied: (i) $\frac{b_{\mathrm{A}}}{c_{\mathrm{A}}}$ is high, (ii) $\sigma_{\mathrm{A} s}$ is high, (iii) $\sigma_{\mathrm{B} s}$ belongs to a subset of moderate values of $[0,1]$.

Moderate values of $\sigma_{\mathrm{B} s}$ allow a high level of identification of the quality of the project when

[^7]submitting it to B because, under the monotone likelihood property, the difference $F^{l}\left(\sigma_{\mathrm{B} s}\right)$ $F^{h}\left(\sigma_{\mathrm{B} s}\right)$ is in its highest range, implying that $\mathcal{E}$ will observe a signal $\sigma \geq \sigma_{\mathrm{B} s}$ with high probability, conditional on $h$, and with a low probability, conditional on $l$. Thus, when the conditions (ii) and (iii) of Remark 3.1 are satisfied, if $\mathcal{A}$ has a project with a high signal, he prefers submitting it to tier B, to have it identified as of high quality and thus accepted, rather than submitting it to A, where the probability of acceptance is small because of the stringent standards implied by the high value of $\sigma_{\mathrm{A} s}$. When $\mathcal{A}$ 's signal is lower (but not too low), he will submit to A because the high benefit/cost ratio, as suggested by condition ( $i$ ), will allow for a non-negative payoff in spite of the low probability of acceptance. For the lowest signals, $\mathcal{A}$ will refrain from submitting the project. On the other hand, given this strategy adopted by $\mathcal{A}, \mathcal{E}$ 's best response is precisely to employ very high standards of acceptance to the upper tier and moderate ones to the lower tier. Thus, negative assortative matching may occur in equilibrium.

This is an interesting and surprising insight, as it suggests that when the project evaluation relies insufficiently on the agent's self-screening mechanism at the upper tier (i.e., when $\frac{b_{\mathrm{A}}}{c_{\mathrm{A}}}$ is high), thus requiring the evaluator to rely heavily on his own assessment of the project at that tier by imposing very high acceptance standards, then negative assortative matching may emerge. In many applications this type of matching may be noticeable in the long run once a sufficiently large sample of outcomes is observed, and thus payoffs may be exogenously adjusted so as to restore a natural positive matching. However, Remark 3.1 suggests that following sudden changes in payoffs, such as when $b_{\mathrm{A}}$ increases sufficiently so as to induce the upper tier to attract low quality projects, forcing the evaluator to impose very strict standards of acceptance at the upper tier, then owners of high quality projects may prefer to submit their projects at the lower tier where acceptance is almost guaranteed. Such a sudden shift may occur, for instance, when a criminal trial draws the national media attention and, becoming emotionally overcharged, increases the pressure on prosecutors to seek a higher penalty. On the other hand, as argued in the Introduction, it is possible that in other situations negative assortative matching may persist as a long run equilibrium behavior if submitting to the upper tier continues to remain highly attractive.

We focus the rest of the analysis on the interesting case of interior equilibria with positive assortative matching where both tiers receive submissions and where the set of values of $\mu$ for which the agent does not submit the project is an interval. ${ }^{15}$ More precisely, we investigate properties of

[^8]equilibria in which $\mathcal{A}$ adopts a cutoff strategy characterized by two thresholds ( $\mu_{\mathrm{A} s}, \mu_{\mathrm{B} s}$ ), with $\mu_{\mathrm{As}} \in$ $(0,1), \mu_{\mathrm{B} s} \in\left(\mu_{\mathrm{A} s}, 1\right)$, such that $\mathcal{A}$ submits to tier A for $\mu \in\left[\mu_{\mathrm{A} s}, 1\right]$, to tier B for $\mu \in\left[\mu_{\mathrm{B} s}, \mu_{\mathrm{A} s}\right)$, and does not submit the project for $\mu \in\left[0, \mu_{\mathrm{B} s}\right)$. We will assume thus implicitly throughout the rest of the paper that the parameters of the model are such that the corresponding equilibria satisfy this regularity property, without explicitly mentioning this assumption each time.

The next two lemmas present the equations that define implicitly the two players' best response functions in these Bayesian Nash equilibria. Their proofs are in appendices A4 and A5. ${ }^{16}$

Lemma 3.2 Given $\mathcal{E}$ 's cutoff strategy, $\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)$, with $\sigma_{\mathrm{A} s}>\sigma_{\mathrm{B} s}$, $\mathcal{A}$ 's best response is characterized by two thresholds $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)$ and $\bar{\mu}_{\mathrm{B}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)$, with $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)>\bar{\mu}_{\mathrm{B}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)$, implicitly defined by the equations in $\bar{\mu}_{\mathrm{A}}$ and $\bar{\mu}_{\mathrm{B}}$, respectively

$$
\begin{gather*}
\frac{\pi}{1-\pi} \frac{g^{h}\left(\bar{\mu}_{\mathrm{A}}\right)}{g^{l}\left(\bar{\mu}_{\mathrm{A}}\right)}=\frac{b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}}  \tag{4}\\
\frac{\pi}{1-\pi} \frac{g^{h}\left(\bar{\mu}_{\mathrm{B}}\right)}{g^{l}\left(\bar{\mu}_{\mathrm{B}}\right)}=\frac{c_{\mathrm{B}}-b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]}{b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]-c_{\mathrm{B}}} \tag{5}
\end{gather*}
$$

such that $\mathcal{A}$ submits to tier A if $\mu \geq \bar{\mu}_{\mathrm{A}}(\cdot, \cdot)$, to tier B if $\mu \in\left[\bar{\mu}_{\mathrm{B}}(\cdot, \cdot), \bar{\mu}_{\mathrm{A}}(\cdot, \cdot)\right)$, and forgoes submitting the project if $\mu<\bar{\mu}_{\mathrm{B}}(\cdot, \cdot)$.

Lemma 3.3 Given $\mathcal{A}$ 's cutoff strategy $\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B} s}\right), \mathcal{E}$ accepts a project submitted to tier A if and only if $\sigma \geq \bar{\sigma}_{\mathrm{A}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B} s}\right)$, with $\bar{\sigma}_{\mathrm{A}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B} s}\right)$ given implicitly by the equation in $\bar{\sigma}_{\mathrm{A}}$

$$
\begin{equation*}
\frac{\pi}{1-\pi} \frac{f^{h}\left(\bar{\sigma}_{\mathrm{A}}\right)}{f^{l}\left(\bar{\sigma}_{\mathrm{A}}\right)} \frac{1-G^{h}\left(\mu_{\mathrm{A} s}\right)}{1-G^{l}\left(\mu_{\mathrm{A} s}\right)}=L_{\mathrm{A}} \tag{6}
\end{equation*}
$$

and accepts a project submitted to tier B if and only if $\sigma \geq \bar{\sigma}_{\mathrm{B}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B} s}\right)$, with $\bar{\sigma}_{\mathrm{B}}\left(\mu_{\mathrm{As}}, \mu_{\mathrm{B} s}\right)$ given implicitly by the equation in $\bar{\sigma}_{\mathrm{B}}$

$$
\begin{equation*}
\frac{\pi}{1-\pi} \frac{f^{h}\left(\bar{\sigma}_{\mathrm{B}}\right)}{f^{l}\left(\bar{\sigma}_{\mathrm{B}}\right)} \frac{G^{h}\left(\mu_{\mathrm{A} s}\right)-G^{h}\left(\mu_{\mathrm{B} s}\right)}{G^{l}\left(\mu_{\mathrm{A} s}\right)-G^{l}\left(\mu_{\mathrm{B} s}\right)}=L_{\mathrm{B}} \tag{7}
\end{equation*}
$$

[^9]The best-response functions, as elicited by equations (4), (5), (6) and (7), determine the equilibrium strategies of the two players denoted by $\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}\right)$ and $\left(\mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)$. The next lemma, whose proof is in appendix A6, presents the monotonicities of these best-response functions.

Lemma 3.4 (i) $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)$ is decreasing in $\sigma_{\mathrm{B} s}$ and increasing in $\sigma_{\mathrm{A} s}$; (ii) $\bar{\mu}_{\mathrm{B}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)$ is constant in $\sigma_{\mathrm{A} s}$ and increasing in $\sigma_{\mathrm{B} s}$; (iii) $\bar{\sigma}_{\mathrm{A}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B} s}\right)$ is decreasing in $\mu_{\mathrm{A} s}$ and constant in $\mu_{\mathrm{B} s}$; (iv) $\bar{\sigma}_{\mathrm{B}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B} s}\right)$ is decreasing in $\mu_{\mathrm{A} s}$ and $\mu_{\mathrm{B} s}$, when $\mu_{\mathrm{A} s}>\mu_{\mathrm{B} s}$.

For generic payoff parameters and information structures, the Bayesian Nash equilibrium of the game is not necessarily unique. As in other models, multiple equilibria may emerge because there exist different sets of self-fulfilling expectations for the same set of fundamentals of the model. The next proposition identifies a consistency requirement across different equilibria. Its corollary provides a sufficient condition for equilibrium uniqueness.

Proposition 1 If $\xi \equiv\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)$ and $\xi^{\prime} \equiv\left(\sigma_{\mathrm{A}}^{* \prime}, \sigma_{\mathrm{B}}^{* \prime}, \mu_{\mathrm{A}}^{* \prime}, \mu_{\mathrm{B}}^{* \prime}\right)$ are two Bayesian Nash equilibria with $\sigma_{\mathrm{B}}^{* \prime}>\sigma_{\mathrm{B}}^{*}$, then it must be that $\sigma_{\mathrm{A}}^{* \prime}>\sigma_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{* \prime}>\mu_{\mathrm{B}}^{*}$ and $\mu_{\mathrm{A}}^{* \prime}<\mu_{\mathrm{A}}^{*}$.

Corollary 3.1 Consider a Bayesian Nash equilibrium $\xi \equiv\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)$ and assume that for fixed values of $\sigma_{\mathrm{A}}^{*}$ and $\mu_{\mathrm{B}}^{*}$, the two best-response functions $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B} s}\right)$ and $\bar{\sigma}_{\mathrm{B}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B}}^{*}\right)$, as defined by (4) and (7), have the unique fixed point $\left(\mu_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}\right)$. Then, if

$$
\begin{equation*}
\frac{\partial \bar{\mu}_{\mathrm{A}}}{\partial \sigma_{\mathrm{B} s}}\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}\right) \cdot \frac{\partial \bar{\sigma}_{\mathrm{B}}}{\partial \mu_{\mathrm{A} s}}\left(\mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)<1 \tag{8}
\end{equation*}
$$

the equilibrium $\xi$ is unique.

Before presenting the proof of these two results, we introduce the three panels in Figure 1 on which we rely heavily in the rest of the analysis. In each panel, we depict the pairwise best-response functions defined by (4)-(7), when the two variables not considered in the respective panel are kept fixed. A solid curve represents a best-response function when the remaining variables are fixed at the values in $\xi$. A dashed curve depicts a best-response function when the remaining variables are fixed at the values in $\xi^{\prime}$. For instance, in panel (a), the solid curve $\bar{\mu}_{\mathrm{A}}^{\mathrm{o}}\left(\sigma_{\mathrm{A} s}\right)$ represents the best-response function $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B}}^{*}\right)$, i.e., the implicit function defined by $\frac{\pi}{1-\pi} \frac{g^{h}\left(\bar{\mu}_{\mathrm{A}}\right)}{g^{l}\left(\bar{\mu}_{\mathrm{A}}\right)}=\frac{b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B}}^{*}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B}}^{*}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}}$ as an equation in $\bar{\mu}_{\mathrm{A}}$. Similarly, the dashed curve
$\bar{\mu}_{\mathrm{A}}^{\mathbf{Z}}\left(\sigma_{\mathrm{A} s}\right)$ represents the best-response function $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B}}^{* \prime}\right)$, i.e., the implicit function defined by the same equation only that $\sigma_{\mathrm{B}}^{* \prime}$ replaces $\sigma_{\mathrm{B}}^{*}$. When there is no dashed curve, the function is the same in the two equilibria. For instance, in panel (a), $\bar{\sigma}_{\mathrm{A}}^{\mathbf{o}}\left(\mu_{\mathrm{A} s}\right)$ represents the best-response functions $\bar{\sigma}_{\mathrm{A}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B}}^{*}\right)$ and $\bar{\sigma}_{\mathrm{A}}\left(\mu_{\mathrm{As} s}, \mu_{\mathrm{B}}^{* \prime}\right)$, which by lemma $3.4(i i i)$ are the same (both functions are defined implicitly by the equation $\frac{\pi}{1-\pi} \frac{f^{h}\left(\bar{\sigma}_{\mathrm{A}}\right)}{f^{l}\left(\bar{\sigma}_{\mathrm{A}}\right)} \frac{1-G^{h}\left(\mu_{\mathrm{As}}\right)}{1-G^{l}\left(\mu_{\mathrm{As}}\right)}=L_{\mathrm{A}}$, which is independent of $\left.\mu_{\mathrm{B}}^{*}\right) .{ }^{17}$

All curves from the three panels are generic and are depicted only so as to exhibit the salient monotonicity property. In panel (c), since both curves are decreasing, they are presented as crossing each other twice, so as to allow for either of them crossing from below. To save on notation, we define a partial order $\succ$ on these curves by saying that for instance $\bar{\mu}_{\mathrm{A}}^{\mathbf{z}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathbf{o}}\left(\sigma_{\mathrm{B} s}\right)$ or $\bar{\sigma}_{\mathrm{B}}^{\mathbf{o}}\left(\mu_{\mathrm{A} s}\right) \succ$ $\bar{\sigma}_{\mathrm{B}}^{\mathrm{Z}}\left(\mu_{\mathrm{A} s}\right)$ if, as is the case in panel (c), the first curve is above the second one in a panel with $\sigma$ on the horizontal axis and $\mu$ on the vertical axis.


Figure 1

Proof of Proposition 1 and Corollary 3.1. To prove proposition 1, first note in panel (a) that $\sigma_{\mathrm{B}}^{* \prime}>\sigma_{\mathrm{B}}^{*}$ implies by lemma $3.4(i)$ that $\bar{\mu}_{\mathrm{A}}^{\mathrm{o}}\left(\sigma_{\mathrm{A} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathbf{Z}}\left(\sigma_{\mathrm{A} s}\right)$. Since $\bar{\sigma}_{\mathrm{A}}^{\mathrm{Z}}\left(\mu_{\mathrm{A} s}\right)$ is the same as $\bar{\sigma}_{\mathrm{A}}^{\mathrm{o}}\left(\mu_{\mathrm{As}}\right)$, it follows immediately by inspecting panel (a) that $\sigma_{\mathrm{A}}^{* \prime}>\sigma_{\mathrm{A}}^{*}$ and $\mu_{\mathrm{A}}^{* \prime}<\mu_{\mathrm{A}}^{*}$. Second, in panel (b), $\mu_{\mathrm{A}}^{* \prime}<\mu_{\mathrm{A}}^{*}$ implies by lemma $3.4(i v)$ that $\bar{\sigma}_{\mathrm{B}}^{\mathrm{Z}}\left(\mu_{\mathrm{B} s}\right) \succ \bar{\sigma}_{\mathrm{B}}^{\mathrm{O}}\left(\mu_{\mathrm{B} s}\right)$. Since $\bar{\mu}_{\mathrm{B}}^{\mathrm{z}}\left(\sigma_{\mathrm{B} s}\right)$ is the same as $\bar{\mu}_{\mathrm{B}}^{\mathrm{o}}\left(\sigma_{\mathrm{B} s}\right)$, it must be that $\mu_{\mathrm{B}}^{* \prime}>\mu_{\mathrm{B}}^{*}$ and $\sigma_{\mathrm{B}}^{* \prime}>\sigma_{\mathrm{B}}^{*}$. The second implication is consistent with the initial assumption from the text of the proposition. Finally, in panel (c), $\mu_{\mathrm{B}}^{* \prime}>\mu_{\mathrm{B}}^{*}$ implies by lemma $3.4(i v)$ that $\bar{\sigma}_{\mathrm{B}}^{\mathrm{o}}\left(\mu_{\mathrm{A} s}\right) \succ \bar{\sigma}_{\mathrm{B}}^{\mathrm{z}}\left(\mu_{\mathrm{A} s}\right)$, while $\sigma_{\mathrm{A}}^{* \prime}>\sigma_{\mathrm{A}}^{*}$ implies by lemma $3.4(i)$ that $\bar{\mu}_{\mathrm{A}}^{\mathbf{z}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathbf{o}}\left(\sigma_{\mathrm{B} s}\right)$.

To show corollary 3.1, consider an equilibrium $\xi=\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)$ such that for fixed values

[^10]of $\sigma_{\mathrm{A}}^{*}$ and $\mu_{\mathrm{B}}^{*}$, the curves $\bar{\sigma}_{\mathrm{B}}^{\mathrm{o}}\left(\mu_{\mathrm{A} s}\right)$ and $\bar{\mu}_{\mathrm{A}}^{\mathrm{o}}\left(\sigma_{\mathrm{B} s}\right)$ satisfy a single-crossing property, with $\bar{\sigma}_{\mathrm{B}}^{\mathrm{o}}\left(\mu_{\mathrm{A} s}\right)$ having a steeper downward slope at the intersection of the two curves. In panel (c), this is the case of the point of intersection that is in the upper left corner. Assume by contradiction that there exists another equilibrium of the game, $\xi^{\prime}=\left(\sigma_{\mathrm{A}}^{* \prime}, \sigma_{\mathrm{B}}^{* \prime}, \mu_{\mathrm{A}}^{* \prime}, \mu_{\mathrm{B}}^{* \prime}\right)$, and without loss of generality, that $\sigma_{\mathrm{B}}^{* \prime}>\sigma_{\mathrm{B}}^{*}$. In this case, by proposition $1, \sigma_{\mathrm{B}}^{* \prime}>\sigma_{\mathrm{B}}^{*}$ implies $\mu_{\mathrm{B}}^{* \prime}>\mu_{\mathrm{B}}^{*}$ and $\sigma_{\mathrm{A}}^{* \prime}>\sigma_{\mathrm{A}}^{*}$. In turn, these imply $\bar{\sigma}_{\mathrm{B}}^{\mathbf{o}}\left(\mu_{\mathrm{A} s}\right) \succ \bar{\sigma}_{\mathrm{B}}^{\mathbf{Z}}\left(\mu_{\mathrm{A} s}\right)$ and $\bar{\mu}_{\mathrm{A}}^{\mathbf{Z}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathbf{o}}\left(\sigma_{\mathrm{B} s}\right)$. By inspecting panel (c) of Figure 1 (more precisely, the intersection of the two curves in the upper left corner) it follows that it must be that $\sigma_{\mathrm{B}}^{* \prime}<\sigma_{\mathrm{B}}^{*}$ and $\mu_{\mathrm{A}}^{* \prime}>\mu_{\mathrm{A}}^{*}$. This is inconsistent with the initial assumption that $\sigma_{\mathrm{B}}^{* \prime}>\sigma_{\mathrm{B}}^{*}$. Thus the initial equilibrium is unique. Now, note that the slope of $\bar{\sigma}_{\mathrm{B}}^{\mathrm{o}}\left(\mu_{\mathrm{A} s}\right)$ at the equilibrium values equals $\left[\frac{\partial \bar{\sigma}_{\mathrm{B}}}{\partial \mu_{\mathrm{A} s}}\left(\mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)\right]^{-1}$. Thus $\bar{\sigma}_{\mathrm{B}}^{\mathbf{o}}\left(\mu_{\mathrm{A} s}\right)$ is steeper than $\bar{\mu}_{\mathrm{A}}^{\mathbf{o}}\left(\sigma_{\mathrm{B} s}\right)$ if and only if
$$
\frac{\partial \bar{\mu}_{\mathrm{A}}}{\partial \sigma_{\mathrm{B} s}}\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}\right)>\left[\frac{\partial \bar{\sigma}_{\mathrm{B}}}{\partial \mu_{\mathrm{A} s}}\left(\mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)\right]^{-1}
$$
which, since both sides are negative, can be rewritten as in (8).
Proposition 1 shows that if $\mathcal{E}$ is more stringent in his acceptance policy for tier B in equilibrium $\xi^{\prime}$, (i.e., $\sigma_{\mathrm{B}}^{* \prime}>\sigma_{\mathrm{B}}^{*}$ ), then first, $\mathcal{A}$ is more reluctant to submit marginal projects to tier B , $\left(\mu_{\mathrm{B}}^{* \prime}>\mu_{\mathrm{B}}^{*}\right)$, and second, $\mathcal{A}$ is more inclined to submit marginal projects to tier $\mathrm{A},\left(\mu_{\mathrm{A}}^{* \prime}<\mu_{\mathrm{A}}^{*}\right)$, since the alternative is less appealing. Given these, $\mathcal{E}$ is also more stringent in his acceptance policy at tier $\mathrm{A},\left(\sigma_{\mathrm{A}}^{* \prime}>\sigma_{\mathrm{A}}^{*}\right)$, to make up for the lower expected quality of projects submitted. While these feed-forward effects make the result intuitive, proposition 1 ensures that the feed-back effects, such as the effect of the increase in $\sigma_{\mathrm{A}}^{*}$ on $\mu_{\mathrm{A}}^{*}$, or of the decrease in $\mu_{\mathrm{A}}^{*}$ on $\sigma_{\mathrm{B}}^{*}$, do not offset them.

To understand corollary 3.1 , consider two equilibria, $\xi$ and $\xi^{\prime}$, with corresponding strategies as in the text of proposition 1. Note then that for a fixed value of $\sigma_{\mathrm{B}}^{*}$, a higher value of $\sigma_{\mathrm{A}}^{*}$, $\left(\sigma_{\mathrm{A}}^{* \prime}>\sigma_{\mathrm{A}}^{*}\right)$, would induce $\mu_{\mathrm{A}}^{*}$ to increase; in other words, if $\mathcal{E}$ is more stringent at tier A , then $\mathcal{A}$ is more reluctant to submit marginal projects to that tier. A higher $\mu_{\mathrm{A}}^{*}$, together with a higher $\mu_{\mathrm{B}}^{*}$, ( $\mu_{\mathrm{B}}^{* \prime}>\mu_{\mathrm{B}}^{*}$ ), would increase the quality of projects received by tier B , and thus induce a decrease in $\sigma_{\mathrm{B}}^{*}$. To instead have $\sigma_{\mathrm{B}}^{*}$ increasing and $\mu_{\mathrm{A}}^{*}$ decreasing (since $\sigma_{\mathrm{B}}^{* \prime}>\sigma_{\mathrm{B}}^{*}$ and $\mu_{\mathrm{A}}^{* \prime}<\mu_{\mathrm{A}}^{*}$ in $\xi$ and $\left.\xi^{\prime}\right), \sigma_{\mathrm{B}}^{*}$ and $\mu_{\mathrm{A}}^{*}$ should feed off each other. This means that $\sigma_{\mathrm{B}}^{*}$ should be higher because $\mu_{\mathrm{A}}^{*}$ is lower, and $\mu_{\mathrm{A}}^{*}$ should be lower because $\sigma_{\mathrm{B}}^{*}$ is higher. Thus, to have multiple equilibria, $\sigma_{\mathrm{B}}^{*}$ has to be very responsive to a decrease in $\mu_{\mathrm{A}}^{*}$, while $\mu_{\mathrm{A}}^{*}$ has to be very responsive to an increase in $\sigma_{\mathrm{B}}^{*}$; these would offset the effects of the increases in $\mu_{\mathrm{B}}^{*}$ and $\sigma_{\mathrm{A}}^{*}$. In panel (c), this is the case precisely when $\bar{\sigma}_{\mathrm{B}}^{\mathrm{O}}\left(\mu_{\mathrm{A} s}\right)$ and $\bar{\mu}_{\mathrm{A}}^{\mathrm{o}}\left(\sigma_{\mathrm{B} s}\right)$ intersect in the lower right corner where both have a steeper slope.

Equation (8) is the mathematical representation of the same condition.

## 4 Results

The main objective of this paper is to investigate the impact of the agent's preference parameters on the equilibrium outcome. Upon inspecting (4) and (5), one can notice that the sign of the effect of an increase in the benefit from having a project accepted into a given tier is the same with that of a decrease in the corresponding submission cost. ${ }^{18}$ Since most papers from the related literature examine the role of submission costs, we will focus our analysis on the same parameters, while keeping in mind that its results are immediately interpretable in terms of changes in benefits from acceptance in the relevant tier.

This section is organized as follows. In section 4.1, we present as a benchmark the main result from Barbos (2013) for the model with one tier of evaluation. In section 4.2, we examine the effect of increases in the submission costs at the two tiers of evaluation on the equilibrium strategies, and then in section 4.3, we employ these comparative statics results to investigate the effect of changes in payoffs on the efficiency of the equilibrium outcome. Finally, in section 4.4 we proceed in a different direction and examine the efficiency effect of the introducing of a second tier in a system of evaluation where initially only one tier had existed.

### 4.1 The model with one tier of evaluation

Consider a model as in section 2, only that with one tier of evaluation. In this case the agent has to decide only on whether or not to submit the project for review. The next lemma states that in the resulting game, an equilibrium exists, is unique, and must be in cutoff strategies.

Lemma 4.1 (Barbos (2013)) There exists a unique equilibrium of the game with one tier of evaluation. This equilibrium is completely characterized by two values $\left(\mu^{*}, \sigma^{*}\right) \in[0,1) \times(0,1)$ such that $\mathcal{A}$ submits a project if and only if $\mu \geq \mu^{*}$, and $\mathcal{E}$ accepts a project if and only if $\sigma \geq \sigma^{*}$.

[^11]The expected quality of projects that are implemented in this equilibrium, $h \operatorname{Pr}\left(h \mid \mu \geq \mu^{*}, \sigma \geq \sigma^{*}\right)+$ $l \operatorname{Pr}\left(l \mid \mu \geq \mu^{*}, \sigma \geq \sigma^{*}\right)$, is isomorphic to the probability $\operatorname{Pr}\left(h \mid \mu \geq \mu^{*}, \sigma \geq \sigma^{*}\right) .{ }^{19}$ The next proposition elicits the effect of an increase in the submission cost, $c$, on this measure.

Proposition 2 (Barbos (2013)) $\frac{d}{d c} \operatorname{Pr}\left(h \mid \mu \geq \mu^{*}, \sigma \geq \sigma^{*}\right)>0$ if and only if

$$
\begin{equation*}
\frac{d}{d \sigma} \ln \frac{f^{h}\left(\sigma^{*}\right)}{f^{l}\left(\sigma^{*}\right)}>\frac{d}{d \sigma} \ln \frac{1-F^{h}\left(\sigma^{*}\right)}{1-F^{l}\left(\sigma^{*}\right)} \tag{9}
\end{equation*}
$$

The term $\frac{f^{h}\left(\sigma^{*}\right)}{f^{l}\left(\sigma^{*}\right)}$ is the likelihood of the state $h$ as inferred from $\mathcal{E}^{\prime}$ 's equilibrium minimum acceptance standard $\sigma^{*}$. On the other hand, given $\mathcal{E}$ 's cutoff strategy, $\frac{1-F^{h}\left(\sigma^{*}\right)}{1-F^{l}\left(\sigma^{*}\right)}$ is the likelihood of state $h$ as inferred from the fact that $\mathcal{E}$ accepted a project. Thus, in a model with one tier of evaluation, a higher submission cost increases the quality of projects that are implemented if and only if the elasticity of the likelihood of a high quality project that is inferred from the $\mathcal{E}$ 's minimum standard is higher than the elasticity of the likelihood of a high quality project that is inferred from the fact that $\mathcal{E}$ accepted a project. Since the intuition of this result resembles those of the corresponding results from a model with multiple tiers, we defer presenting it to section 4.3.

### 4.2 The effects of submission fees on equilibrium strategies

In the model with multiple tiers of evaluation, with generic signal distributions, it is not tractable to obtain a closed-form solution for the equilibrium strategies amenable to direct comparative statics analysis. Instead, we employ a novel strategy of performing a comparative statics analysis in the following three steps. First, we identify all correlations between the signs of the changes in the equilibrium strategies that are imposed by (4)-(7) under the assumed change in the underlying parameter. Second, we identify the paths of the equilibrium strategies that are consistent with these correlations. Finally, for each equilibrium path, we verify that the shifts in the best-response functions that are imposed by the changes in the underlying parameter and in the equilibrium strategies are consistent with the assumed changes in the equilibrium strategies. At this step, we identify the equilibrium paths that may emerge only because of the multiplicity of equilibria.

[^12]The case of a change in $c_{\mathrm{A}}$ We start with the case of an increase in $c_{\mathrm{A}} \cdot{ }^{20}$ Assuming that $c_{\mathrm{A}}$ increases by $d c_{\mathrm{A}}>0$, by inspecting (4)-(7), one can infer the following necessary correlations among the possible changes in the equilibrium strategies.
(a) From (4), if $d \sigma_{\mathrm{A}}^{*}>0$ and $d \sigma_{\mathrm{B}}^{*}<0$, then $d \mu_{\mathrm{A}}^{*}>0$.
(b) From (5), if $d \sigma_{\mathrm{B}}^{*}>(<) 0$, then $d \mu_{\mathrm{B}}^{*}>(<) 0$.
(c) From (6), if $d \mu_{\mathrm{A}}^{*}>(<) 0$, then $d \sigma_{\mathrm{A}}^{*}<(>) 0$.
(d) From (7), if $d \mu_{\mathrm{A}}^{*}>(<) 0$ and $d \mu_{\mathrm{B}}^{*}>(<) 0$, then $d \sigma_{\mathrm{B}}^{*}<(>) 0$.

The result in (a) reads as follows. If an increase in $c_{\mathrm{A}}$ leads to an increase the equilibrium value of $\sigma_{\mathrm{A}}^{*}$ and to a decrease in $\sigma_{\mathrm{B}}^{*}$, then by (4), it must be that it also leads to an increase in $\mu_{\mathrm{A}}^{*}$. On the other hand, (b) means that if following the same increase in $c_{\mathrm{A}}$, the equilibrium value of $\sigma_{\mathrm{B}}^{*}$ increases then $\mu_{\mathrm{B}}^{*}$ must increase, while if $\sigma_{\mathrm{B}}^{*}$ decreases then $\mu_{\mathrm{B}}^{*}$ must decrease. Using these results, we have the following possible equilibrium paths following an increase in $c_{\mathrm{A}}$.

1. Assume $d \mu_{\mathrm{A}}^{*}>0$. $\mathrm{By}(\mathrm{c})$, it follows that $d \sigma_{\mathrm{A}}^{*}<0$. We have two cases to consider regarding the sign of $d \sigma_{\mathrm{B}}^{*}$. If $d \sigma_{\mathrm{B}}^{*}>0$, then by $(\mathrm{b}) d \mu_{\mathrm{B}}^{*}>0$. But by $(\mathrm{d})$, if $d \mu_{\mathrm{A}}^{*}>0$ and $d \mu_{\mathrm{B}}^{*}>0$, then it must be that $d \sigma_{\mathrm{B}}^{*}<0$. This contradicts the previous assumption. Therefore, it must be that $d \sigma_{\mathrm{B}}^{*}<0$, and thus by (b) that $d \mu_{\mathrm{B}}^{*}<0$.
2. Assume $d \mu_{\mathrm{A}}^{*}<0$. By (c), this implies that $d \sigma_{\mathrm{A}}^{*}>0$. By (a), this implies that $d \sigma_{\mathrm{B}}^{*}>0$. By (b), this implies that $d \mu_{\mathrm{B}}^{*}>0$.

The first of the two equilibrium paths is intuitive. Upon facing a higher $c_{\mathrm{A}}, \mathcal{A}$ is less inclined to submit marginal products to tier A , and thus $\mu_{\mathrm{A}}^{*}$ increases. The increase in $\mu_{\mathrm{A}}^{*}$ leads to an increase in the expected quality of projects received by tier A , which allows $\mathcal{E}$ to lower the corresponding standards, and thus $\sigma_{\mathrm{A}}^{*}$ decreases. The increase in $\mu_{\mathrm{A}}^{*}$ also leads to an increase in the expected quality of projects submitted to tier B , which allows $\mathcal{E}$ to also lower $\sigma_{\mathrm{B}}^{*}$. This makes $\mathcal{A}$ more willing to submit marginal projects to tier B , and thus $\mu_{\mathrm{B}}^{*}$ decreases. We depict these in Figure 2 below.

[^13]

The solid curves represent the pairwise best-response functions when the remaining variables are fixed at the values from the initial equilibrium $\xi \equiv\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)$, and the submission fee is $c_{\mathrm{A}}$. The partially dashed curves $\bar{\mu}_{\mathrm{A}}^{\mathrm{x}}\left(\sigma_{\mathrm{A} s}\right)$ and $\bar{\mu}_{\mathrm{A}}^{\mathrm{x}}\left(\sigma_{\mathrm{B} s}\right)$ represent the best-response functions when the fee is $c_{\mathrm{A}}^{\prime} \equiv c_{\mathrm{A}}+d c_{\mathrm{A}}$, but the values of the remaining variables are still fixed at $\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right) .^{21}$ For instance, from (4), it follows that for fixed values of $\sigma_{\mathrm{B}}^{*}$ and $\mu_{\mathrm{B}}^{*}$, to the same cutoff strategy $\sigma_{\mathrm{A} s}, \mathcal{A}$ responds with a higher $\bar{\mu}_{\mathrm{A}}$ when $c_{\mathrm{A}}$ increases to $c_{\mathrm{A}}^{\prime}$. Thus, $\bar{\mu}_{\mathrm{A}}^{\mathrm{x}}\left(\sigma_{\mathrm{A} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathrm{O}}\left(\sigma_{\mathrm{A} s}\right)$ in panel (a). Similarly, $\bar{\mu}_{\mathrm{A}}^{\mathbf{x}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathbf{o}}\left(\sigma_{\mathrm{B} s}\right)$ in panel (c). The dashed curves $\bar{\mu}_{\mathrm{A}}^{\mathbf{z}}\left(\sigma_{\mathrm{A} s}\right), \bar{\sigma}_{\mathrm{B}}^{\mathbf{z}}\left(\mu_{\mathrm{B} s}\right), \bar{\mu}_{\mathrm{A}}^{\mathbf{z}}\left(\sigma_{\mathrm{B} s}\right)$ and $\bar{\sigma}_{\mathrm{B}}^{\mathbf{Z}}\left(\mu_{\mathrm{A} s}\right)$ represent the best-response functions that correspond to $c_{\mathrm{A}}^{\prime}$, and to values of strategies from the new equilibrium $\xi^{\prime} \equiv\left(\sigma_{\mathrm{A}}^{* \prime}, \sigma_{\mathrm{B}}^{* \prime}, \mu_{\mathrm{A}}^{* \prime}, \mu_{\mathrm{B}}^{* \prime}\right)$. For instance, since $d \sigma_{\mathrm{B}}^{*}<0$, from lemma 3.4(i), it follows that at $c_{\mathrm{A}}^{\prime}, \mathcal{A}$ 's best-response $\bar{\mu}_{\mathrm{A}}$ is higher when $\mathcal{E}^{\prime}$ 's cutoff for tier B is fixed at $\sigma_{\mathrm{B}}^{* \prime}$ than at $\sigma_{\mathrm{B}}^{*}$. This implies that $\bar{\mu}_{\mathrm{A}}^{\mathbf{z}}\left(\sigma_{\mathrm{A} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathbf{x}}\left(\sigma_{\mathrm{A} s}\right)$ in panel (a). Similarly, in panel (b), from lemma $3.4(i v)$ it follows that $\bar{\sigma}_{\mathrm{B}}^{\mathrm{o}}\left(\mu_{\mathrm{B} s}\right) \succ \bar{\sigma}_{\mathrm{B}}^{\mathrm{z}}\left(\mu_{\mathrm{B} s}\right)$ because $d \mu_{\mathrm{A}}^{*}>0$. Finally, in panel (c), $d \mu_{\mathrm{B}}^{*}<0$ implies $\bar{\sigma}_{\mathrm{B}}^{\mathbf{Z}}\left(\mu_{\mathrm{A} s}\right) \succ \bar{\sigma}_{\mathrm{B}}^{\mathbf{0}}\left(\mu_{\mathrm{A} s}\right)$, while $d \sigma_{\mathrm{A}}^{*}<0$ implies $\bar{\mu}_{\mathrm{A}}^{\mathbf{x}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathbf{Z}}\left(\sigma_{\mathrm{B} s}\right)$. As seen in the figure, the equilibrium path is consistent with either type of initial equilibrium.

As we show next, the second equilibrium path can arise only when the initial equilibrium is not unique. Essentially, this scenario emerges as a consequence of a coordination of expectations on a different equilibrium in response to a change in the parameters of the model, rather than being

[^14]driven by an adjustment of players' strategies within the same equilibrium. Note that in panel (c) of Figure $3, d c_{\mathrm{A}}>0$ and $d \sigma_{\mathrm{B}}^{*}<0$ imply $\bar{\mu}_{\mathrm{A}}^{\mathrm{Z}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathrm{x}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathrm{o}}\left(\sigma_{\mathrm{B} s}\right)$, while $d \mu_{\mathrm{B}}^{*}>0$ implies $\bar{\sigma}_{\mathrm{B}}^{\mathbf{o}}\left(\mu_{\mathrm{A} s}\right) \succ \bar{\sigma}_{\mathrm{B}}^{\mathrm{Z}}\left(\mu_{\mathrm{A} s}\right)$. Thus, the only ways to have $d \sigma_{\mathrm{B}}^{*}>0$ and $d \mu_{\mathrm{A}}^{*}<0$ are either if the initial equilibrium is in the upper left corner and the two curves do not satisfy the single crossing condition, or if the initial equilibrium is in the lower right corner where $\bar{\mu}_{\mathrm{A}}^{\mathbf{o}}\left(\sigma_{\mathrm{B} s}\right)$ crosses $\bar{\sigma}_{\mathrm{B}}^{\mathbf{o}}\left(\mu_{\mathrm{A} s}\right)$ from above. These are precisely the conditions under which the equilibrium is not necessarily unique.


Figure 3

We collect these results in the following proposition.

Proposition 3 Consider an equilibrium $\xi \equiv\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)$ and assume dc$c_{\mathrm{A}}>0$. If $\xi$ is unique, then $d \mu_{\mathrm{A}}^{*}>0, d \mu_{\mathrm{B}}^{*}<0, d \sigma_{\mathrm{A}}^{*}<0$ and $d \sigma_{\mathrm{B}}^{*}<0$. If $\xi$ is not unique, then it may also happen that $d \mu_{\mathrm{A}}^{*}<0, d \mu_{\mathrm{B}}^{*}>0, d \sigma_{\mathrm{A}}^{*}>0$ and $d \sigma_{\mathrm{B}}^{*}>0$.

Focusing on the case when the equilibrium is unique, note that while a higher $c_{\mathrm{A}}$ does increase the quality of projects submitted to tier A (as $d \mu_{\mathrm{A}}^{*}>0$ ), it also decreases the quality of projects submitted between the two tiers ( as $d \mu_{\mathrm{B}}^{*}<0$ ). Therefore, unlike the case of a system of evaluation with one tier, in a system with multiple tiers, a higher submission cost is not unequivocally beneficial for the quality of projects submitted for review because higher submission costs at the upper tiers decrease the quality of projects submitted to the lower tiers. This insight hinges on the underlying assumption that evaluation is imprecise; if evaluation was precise, a higher $c_{\mathrm{A}}$ has no effect on the agent's decision at the margin on whether to submit a project to tier B or to forgo submitting it.

In addition to the interpretation of this finding in the context of a trial that was mentioned in the Introduction, proposition 3 has interesting implications in other situations captured by our
model. For instance, in the case of a new product launch, it implies that a higher cost of launching a product as a high-end item (or a decrease in the potential benefit from successfully marketing a product as such) leads the firm to launch as low-end items some products that would have otherwise been discarded. Intuitively, because buyers are more confident in products advertised as low-end products (since they are aware that the firm has less incentive to skim the better products to launch them as high-end), the firm is more willing to launch such products of marginal quality as low-end products as the buyers are more likely to adopt them. On the other hand, in the car selling example, it implies that when the benefit from hiding a mechanical issue is higher, then fewer cars are being put up for sale since buyers expect cars with disclosed issues to be of lower quality.

The case of a change in $c_{\mathrm{B}}$ Similarly to the previous analysis, assuming $d c_{\mathrm{B}}>0$ (or $d b_{\mathrm{B}}<0$ ), one can infer the following necessary correlations among equilibrium strategies.
(a) From (4), if $d \sigma_{\mathrm{B}}^{*}>0$ and $d \sigma_{\mathrm{A}}^{*}<0$, then $d \mu_{\mathrm{A}}^{*}<0$.
(b) From (5), if $d \sigma_{\mathrm{B}}^{*}>0$, then $d \mu_{\mathrm{B}}^{*}>0$.
(c) From (6), if $d \mu_{\mathrm{A}}^{*}>(<) 0$, then $d \sigma_{\mathrm{A}}^{*}<(>) 0$.
(d) From (7), if $d \mu_{\mathrm{A}}^{*}>(<) 0$ and $d \mu_{\mathrm{B}}^{*}>(<) 0$, then $d \sigma_{\mathrm{B}}^{*}<(>) 0$.

Therefore, the equilibrium paths that can emerge when $c_{\mathrm{B}}$ increases are the following.

1. Assume $d \mu_{\mathrm{B}}^{*}>0$ and $d \sigma_{\mathrm{B}}^{*}>0$. By (d), it follows that $d \mu_{\mathrm{A}}^{*}<0$, and then by (c) that $d \sigma_{\mathrm{A}}^{*}>0$.
2. Assume $d \mu_{\mathrm{B}}^{*}>0$ and $d \sigma_{\mathrm{B}}^{*}<0$. If $d \mu_{\mathrm{A}}^{*}<0$, then by (c) $d \sigma_{\mathrm{A}}^{*}>0$.
3. Assume $d \mu_{\mathrm{B}}^{*}>0$ and $d \sigma_{\mathrm{B}}^{*}<0$. If $d \mu_{\mathrm{A}}^{*}>0$, then by (c) $d \sigma_{\mathrm{A}}^{*}<0$.
4. Assume $d \mu_{\mathrm{B}}^{*}<0$. Then by (b), $d \sigma_{\mathrm{B}}^{*}<0$. By (d) it follows that $d \mu_{\mathrm{A}}^{*}>0$, which then by (c) implies that $d \sigma_{\mathrm{A}}^{*}<0$.

The third step of the analysis is along the lines of the case of an increase in $c_{\mathrm{A}}$ and is omitted. On the first three equilibrium paths, when $d c_{\mathrm{B}}>0, \mathcal{A}$ is more reluctant to submit low-signal marginal projects to tier B , and thus $d \mu_{\mathrm{B}}^{*}>0$. On the first two of these paths, $\mathcal{A}$ also abstains from submitting high-signal marginal projects to tier B , and thus $d \mu_{\mathrm{A}}^{*}<0$. If the net effect on
the quality of projects submitted to tier B is negative, $\mathcal{E}$ becomes more stringent in his acceptance policy at tier B , and so $d \sigma_{\mathrm{B}}^{*}>0$, as on the first equilibrium path. If the net effect is positive, $\mathcal{E}$ is less stringent, and so $d \sigma_{\mathrm{B}}^{*}<0$, as on the second equilibrium path. On both paths $\mathcal{E}$ becomes more stringent at tier A since the expected quality of projects received at that tier is lower. The third equilibrium path occurs when the quality of projects submitted to tier B increases significantly following the increase in $\mu_{\mathrm{B}}^{*}$. In this case, $\sigma_{\mathrm{B}}^{*}$ decreases sufficiently so as to induce an increase in $\mu_{\mathrm{A}}^{*}$, and a consequent decrease in $\sigma_{\mathrm{A}}^{*}$. The last equilibrium path emerges again only when the initial equilibrium is not necessarily unique. ${ }^{22}$ We collect these results in the next proposition.

Proposition 4 Consider an equilibrium $\xi \equiv\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)$ and assume dc$c_{\mathrm{B}}>0$. If $\xi$ is unique, then $d \mu_{\mathrm{B}}^{*}>0$, and one of the following three equilibrium paths occurs: (i) $d \sigma_{\mathrm{B}}^{*}>0, d \mu_{\mathrm{A}}^{*}<0$ and $d \sigma_{\mathrm{A}}^{*}>0$; (ii) $d \sigma_{\mathrm{B}}^{*}<0, d \mu_{\mathrm{A}}^{*}<0$ and $d \sigma_{\mathrm{A}}^{*}>0$; (iii) $d \sigma_{\mathrm{B}}^{*}<0, d \mu_{\mathrm{A}}^{*}>0$ and $d \sigma_{\mathrm{A}}^{*}<0$. If $\xi$ is not unique, then it may also happen that $d \mu_{\mathrm{B}}^{*}<0, d \mu_{\mathrm{A}}^{*}>0, d \sigma_{\mathrm{A}}^{*}<0$, and $d \sigma_{\mathrm{B}}^{*}<0$.

It deserves noting at this time that the equilibrium path selection is a local property, in that for given values of the payoff parameters, it is determined exclusively from the local properties of the signal structures in a neighborhood of the initial equilibrium. In particular, depending on the amount of additional information extracted with an infinitesimal change in the strategy of each player, the equilibrium may follow at each starting point any of these paths. Further regularities on the equilibrium paths can only be identified under additional assumptions on the payoff parameters of the model and the information structure beyond that imposed by assumptions 1 and 2.

Focusing again on the case of unique equilibria, the first insight of proposition 4 is that a higher cost of submitting to the lower tier increases the quality of projects submitted between the two tiers (as $\mu_{\mathrm{B}}^{*}$ increases). Therefore, in a situation with a tiered system of evaluation such as the one from our motivating example of a prosecutor's choice of charges in a criminal trial, a mechanism designer interested in reducing the number of projects submitted for review can proceed by increasing the cost of submission to the lower tier (or by reducing the corresponding benefits). This is in contrast to an increase in the cost at the upper tier, which, as elicited in proposition 3, has the opposite effect. However, note that while the quality of projects received between the two tiers increases, the average quality of projects received individually by both tiers may simultaneously decrease. This

[^15]is the case on path $(i)$ where $d \sigma_{\mathrm{B}}^{*}>0$ and $d \sigma_{\mathrm{A}}^{*}>0$ imply that the evaluator needs to be more stringent at both tiers since he receives projects of lower quality in each of them. ${ }^{23}$

Proposition 4 also suggests that in those situations where the sign of the effect on one equilibrium strategy is observed after a change in payoffs at the lower tier, then it may be possible to infer the signs of the effects on the other strategies without observing them. For instance, if after an increase in prosecutors' cost of filing the lesser charges $\left(d c_{\mathrm{B}}>0\right)$, it is observed that prosecutors are less inclined to file not only the lesser charges ( $d \mu_{\mathrm{B}}^{*}>0$, which is expected), but also the more severe charges $\left(d \mu_{\mathrm{A}}^{*}>0\right)$, then it can be inferred that juries require a lower burden of proof for both types of charges $\left(d \sigma_{\mathrm{B}}^{*}<0\right.$ and $\left.d \sigma_{\mathrm{A}}^{*}<0\right)$. This is because $d \mu_{\mathrm{A}}^{*}>0$ identifies path (iii) out of the three possible paths elicited in proposition 4. Similarly, if after an increase in $c_{\mathrm{B}}$, juries are observed to demand a higher burden of proof for the lesser charges ( $d \sigma_{\mathrm{B}}^{*}>0$ ), it must be that juries are also more demanding for the severe charges ( $d \sigma_{\mathrm{A}}^{*}>0$ ), and that prosecutors are more inclined to file the severe charges $\left(d \mu_{\mathrm{A}}^{*}<0\right)$. Again, the decrease in $\sigma_{\mathrm{A}}^{*}$ identifies path $(i)$. Finally, a decrease in the burden of proof required for the severe charges identifies path (iii).

### 4.3 The effects of submission fees on the equilibrium expected quality of the projects that are implemented

In this section, we examine the effect of a change in the two submission costs on the efficiency of the equilibrium outcome. ${ }^{24}$ The measures of efficiency that we employ here are the expected qualities of projects implemented by the evaluator in the two tiers, which are isomorphic with $\operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)$, for tier A , and $\operatorname{Pr}\left(h \mid \mu_{\mathrm{A}}^{*} \geq \mu \geq \mu_{\mathrm{B}}^{*}, \sigma \geq \sigma_{\mathrm{B}}^{*}\right)$, for tier B . The next proposition elicits the effect of an increase in $c_{\mathrm{A}}$ on these two values. We restrict attention again to the interesting case where the initial equilibrium is unique, and thus the comparative statics are driven by the fundamentals of the model rather than equilibrium selection.

Proposition 5 Assume that the equilibrium ( $\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}$ ) is unique. Then

[^16](i) $\frac{d}{d c_{\mathrm{A}}} \operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)>0$ if and only if $\frac{d}{d \sigma} \ln \frac{f^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}>\frac{d}{d \sigma} \ln \frac{1-F^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}$.
(ii) $\frac{d}{d c_{\mathrm{A}}} \operatorname{Pr}\left(h \mid \mu_{\mathrm{A}}^{*} \geq \mu \geq \mu_{\mathrm{B}}^{*}, \sigma \geq \sigma_{\mathrm{B}}^{*}\right)>0$ if and only if $\frac{d}{d \sigma} \ln \frac{f^{h}\left(\sigma_{\mathrm{B}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{B}}^{*}\right)}>\frac{d}{d \sigma} \ln \frac{1-F^{h}\left(\sigma_{\mathrm{B}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{B}}^{*}\right)}$.

Proof. By Bayes' Rule, we have

$$
\begin{align*}
\operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right) & =\frac{\operatorname{Pr}\left(\mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*} \mid h\right) \operatorname{Pr}(h)}{\operatorname{Pr}\left(\mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*} \mid h\right) \operatorname{Pr}(h)+\operatorname{Pr}\left(\mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*} \mid l\right) \operatorname{Pr}(l)} \\
& =\frac{\pi}{\pi+(1-\pi) \frac{\operatorname{Pr}\left(\mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*} \mid l\right)}{\operatorname{Pr}\left(\mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*} \mid h\right)}}=\frac{\pi}{\pi+\frac{1-\pi}{\frac{1-G^{h}\left(\mu_{\mathrm{A}}^{*}\right)}{1-G^{l}\left(\mu_{\mathrm{A}}^{*}\right)} \frac{\left.1-\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}}} \tag{10}
\end{align*}
$$

where for the third equality we used the conditional independence of the two players' signals. Therefore, $\operatorname{Pr}\left(h \mid \mu \geq \mu^{*}, \sigma \geq \sigma^{*}\right)$ increases following an increase in $c_{\mathrm{A}}$ if and only if the sum $\ln \frac{1-G^{h}\left(\mu_{\mathrm{A}}^{*}\right)}{1-G^{l}\left(\mu_{\mathrm{A}}^{*}\right)}+$ $\ln \frac{1-F^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}$ increases. From (6), written in equilibrium, we have $\ln \frac{1-G^{h}\left(\mu_{\mathrm{A}}^{*}\right)}{1-G^{l}\left(\mu_{\mathrm{A}}^{*}\right)}=\ln L_{\mathrm{A}}-\ln \frac{\pi}{1-\pi}-$ $\ln \frac{f^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}$, so $\operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)$ increases if and only if $\ln \frac{1-F^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}-\ln \frac{f^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}$ increases. But

$$
\begin{equation*}
\frac{d}{d c_{\mathrm{A}}}\left[\ln \frac{1-F^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}-\ln \frac{f^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}\right]=\frac{d}{d \sigma}\left[\ln \frac{1-F^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}-\ln \frac{f^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}\right]\left(\frac{d \sigma_{\mathrm{A}}^{*}}{d c_{\mathrm{A}}}\right) \tag{11}
\end{equation*}
$$

Since by proposition 3, we have $\frac{d \sigma_{\mathrm{A}}^{*}}{d c_{\mathrm{A}}}<0$ the proof of part (i) is complete. The proof of part (ii) follows the same steps. ${ }^{25}$

To understand these results, consider the effect of an increase in $c_{\mathrm{A}}$ on $\operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)$. By proposition 3, a higher $c_{\mathrm{A}}$ has a positive effect on the expected quality of projects that are implemented in tier A by increasing the quality of projects that are submitted ( $\mu_{\mathrm{A}}^{*}$ increases), and a negative effect by decreasing $\mathcal{E}$ 's standards of acceptance ( $\sigma_{\mathrm{A}}^{*}$ decreases). On net, the cost increase has a positive effect if $\operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)$ is more responsive to the induced increase in $\mu_{\mathrm{A}}^{*}$ than to the decrease in $\sigma_{\mathrm{A}}^{*}$. Now, as seen in $(10), \operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)$ is a monotone transformation of the product of the likelihoods of state $h$ inferred from the fact that $\mathcal{A}$ submitted the project, $\frac{1-G^{h}\left(\mu_{\mathrm{A}}^{*}\right)}{1-G^{l}\left(\mu_{\mathrm{A}}^{*}\right)}$, and from the fact that $\mathcal{E}$ accepted it, $\frac{1-F^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}$. Therefore, the responsiveness of $\operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)$ with respect to $\mu_{\mathrm{A}}^{*}$ can be elicited from the elasticity of the likelihood $\frac{1-G^{h}\left(\mu_{\mathrm{A}}^{*}\right)}{1-G^{l}\left(\mu_{\mathrm{A}}^{*}\right)}$ with respect to $\mu_{\mathrm{A}}^{*}$, while the responsiveness of $\operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)$ with respect to $\sigma_{\mathrm{A}}^{*}$ can be elicited from the elasticity of the likelihood $\frac{1-F^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}$ with respect to $\sigma_{\mathrm{A}}^{*}$. In turn, the former elasticity can be elicited in equilibrium from $\mathcal{E}$ 's decision problem, described by (6), as a function

[^17]of the elasticity of $\frac{f^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}$ with respect to $\sigma_{\mathrm{A}}^{*}$. It follows that the sign of $\frac{d}{d c_{\mathrm{A}}} \operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)$ can be elicited by comparing the two elasticities as in the text of the proposition. ${ }^{26}$

Propositions 2 and 5 reveal that the effects of a higher submission cost to the upper tier of a tiered system of evaluation on the expected qualities of projects implemented by both tiers are qualitatively similar to the effect of a higher submission cost in a model with one tier of evaluation. More precisely, since a higher $c_{\mathrm{A}}$ leads to unambiguous decreases in both $\sigma_{\mathrm{A}}^{*}$ and $\sigma_{\mathrm{B}}^{*}$, the effect of a higher $c_{\mathrm{A}}$ on the quality of projects implemented by the two tiers can be elicited solely by investigating the elasticities of the two likelihoods at the equilibrium values of $\sigma_{\mathrm{A}}^{*}$ and $\sigma_{\mathrm{B}}^{*}$.

The next proposition presents the effect of an increase in $c_{\mathrm{B}}$. Its proof shares the same steps as the proof of proposition 5 up to equation (11) and is thus omitted.

Proposition 6 Assume that the equilibrium $\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{B}}^{*}, \mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B}}^{*}\right)$ is unique. Then
(i) $\frac{d}{d c_{\mathrm{B}}} \operatorname{Pr}\left(h \mid \mu \geq \mu_{\mathrm{A}}^{*}, \sigma \geq \sigma_{\mathrm{A}}^{*}\right)>0$ if and only if $\left[\frac{d}{d \sigma} \ln \frac{1-F^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}-\frac{d}{d \sigma} \ln \frac{f^{h}\left(\sigma_{\mathrm{A}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{*}\right)}\right] \frac{d \sigma_{\mathrm{A}}^{*}}{d c_{\mathrm{B}}}>0$
(ii) $\frac{d}{d c_{\mathrm{B}}} \operatorname{Pr}\left(h \mid \mu_{\mathrm{A}}^{*} \geq \mu \geq \mu_{\mathrm{B}}^{*}, \sigma \geq \sigma_{\mathrm{B}}^{*}\right)>0$ if and only if $\left[\frac{d}{d \sigma} \ln \frac{1-F^{h}\left(\sigma_{\mathrm{B}}^{*}\right)}{1-F^{l}\left(\sigma_{\mathrm{B}}^{*}\right)}-\frac{d}{d \sigma} \ln \frac{f^{h}\left(\sigma_{\mathrm{B}}^{*}\right)}{f^{l}\left(\sigma_{\mathrm{B}}^{*}\right)}\right] \frac{d \sigma_{\mathrm{B}}^{*}}{d c_{\mathrm{B}}}>0$.

Note that by proposition 4 , the endogenous condition $\frac{d \sigma_{t}^{*}}{d c_{\mathrm{B}}}<0$ occurs when a higher $c_{\mathrm{B}}$ increases the quality of projects submitted to tier $t \in\{\mathrm{~A}, \mathrm{~B}\}$ (due to $\mathcal{A}$ shifting some high-signal marginal projects from B to $A$ ). Thus, the effect of an increase in $c_{\mathrm{B}}$ on the quality of projects implemented by the two tiers depends on the elasticities of the two likelihoods in the neighborhoods of $\sigma_{\mathrm{A}}^{*}$ and $\sigma_{\mathrm{B}}^{*}$, respectively, and on the sign of the change in the evaluator's strategy (as determined by the corresponding effect on the quality of projects submitted for review at each tier).

As illustrated by counter-example in Barbos (2013), the condition on the two elasticities, $\frac{d}{d \sigma} \ln \frac{f^{h}(\sigma)}{f^{l}(\sigma)}>\frac{d}{d \sigma} \ln \frac{1-F^{h}(\sigma)}{1-F^{l}(\sigma)}$, is not always satisfied. Therefore, the two results of this section reaffirm the main insight in Barbos (2013), and show that when evaluation is imperfect, an increase in the level of self-screening exerted by the agent may be detrimental to the quality of projects implemented in either tier because of the induced decrease in standards of acceptance. Therefore, a policy maker interested in improving the efficiency of the equilibrium outcome in a situation where the evaluator learns from the agent's decision, has to account for the fact that enhancing the contribution of the agent in the screening of the projects comes at the cost of a diminishing

[^18]contribution of the evaluator, which may lead to a decrease in the quality of projects that are implemented.

In the car transaction example, this means, for instance, that if new regulations lower the seller's potential benefit from withholding negative information about his car (i.e., $b_{\mathrm{A}}$ decreases), then the average quality of cars transacted under both scenarios may actually decrease. Intuitively, since the seller is less likely to be dishonest ( $\mu_{\mathrm{A}}^{*}$ increases), the buyer is more confident in cars advertised as in perfect condition, and is thus less stringent when evaluating them ( $\sigma_{\mathrm{A}}^{*}$ decreases). If the information loss resulting from the buyer's lower standards outweighs the information gain generated by the seller's higher standards, the quality of cars transacted as in perfect condition (tier A) decreases. Moreover, since the lower temptation to be dishonest makes the seller less likely to present some of the better cars as in perfect condition ( $\mu_{\mathrm{A}}^{*}$ increases), the buyer also has higher expectations of the quality of cars presented as with some mechanical issues (tier B). Thus, the buyer is more inclined to accept such cars ( $\sigma_{\mathrm{B}}^{*}$ decreases), and so the seller is less inclined to sell a car for scrap ( $\mu_{\mathrm{B}}^{*}$ increases). The lower standards adopted by both players at tier B do not necessarily imply a decrease in the quality of cars transacted as with issues since cars of better quality are also sold with disclosure of their issues ( $\mu_{\mathrm{A}}^{*}$ increases). However, this quality does decrease when the condition identified in proposition $5(\mathrm{~b})$ is not satisfied.

On the other hand, proposition 6 unveils the differential impact of a change in payoff parameters at the lower tier. In particular, since a higher value of $c_{\mathrm{B}}$ may increase the standards of acceptance at either tier, when these standards do increase, one will observe a decrease in the quality of projects implemented in that tier for the same information structure that would lead to an increase in this quality under an increase in the value of $c_{\mathrm{A}}$.

### 4.4 Introducing a second tier of evaluation

We close by presenting a proposition that compares the equilibrium of a game with one tier of evaluation with the equilibrium from the game with both tiers. More precisely, we analyze the impact of introducing an additional upper or lower tier in a system of evaluation in which only one tier had existed. ${ }^{27}$ The proof of the proposition is in appendix A7.

[^19]Proposition $7 \operatorname{Let}\left(\sigma_{\mathrm{A}}^{1 *}, \mu_{\mathrm{A}}^{1 *}\right)$ and $\left(\sigma_{\mathrm{B}}^{1 *}, \mu_{\mathrm{B}}^{1 *}\right)$ be the equilibria of the games with only tier of evaluation A or B , respectively. Also, let $\left(\sigma_{\mathrm{A}}^{2 *}, \sigma_{\mathrm{B}}^{2 *}, \mu_{\mathrm{A}}^{2 *}, \mu_{\mathrm{B}}^{2 *}\right)$ be the equilibrium of the game with both tiers. Then, $\mu_{\mathrm{A}}^{2 *}>\mu_{\mathrm{A}}^{1 *}, \sigma_{\mathrm{A}}^{2 *}<\sigma_{\mathrm{A}}^{1 *}, \mu_{\mathrm{B}}^{2 *}>\mu_{\mathrm{B}}^{1 *}$ and $\sigma_{\mathrm{B}}^{2 *}>\sigma_{\mathrm{B}}^{1 *}$.

Thus, the introduction of a lower tier B in a system of evaluation in which only tier A had existed induces $\mathcal{A}$ to be more selective in submitting to tier $\mathrm{A},\left(\mu_{\mathrm{A}}^{2 *}>\mu_{\mathrm{A}}^{1 *}\right)$, which allows $\mathcal{E}$ to be less stringent in his standards of acceptance at that tier, $\left(\sigma_{\mathrm{A}}^{2 *}<\sigma_{\mathrm{A}}^{1 *}\right)$. On the other hand, the introduction of an upper tier A in a system in which only tier B had existed lowers the expected quality of projects received by tier B , inducing $\mathcal{E}$ to become more stringent, $\left(\sigma_{\mathrm{B}}^{2 *}>\sigma_{\mathrm{B}}^{1 *}\right)$. In turn, this makes $\mathcal{A}$ more selective in submitting marginal projects to tier $\mathrm{B},\left(\mu_{\mathrm{B}}^{2 *}>\mu_{\mathrm{B}}^{1 *}\right)$.

These results have two policy implications. First, $\mu_{\mathrm{A}}^{2 *}>\mu_{\mathrm{A}}^{1 *}$ suggests an additional intuitive mechanism to induce more self-screening by the agent at tier A. Thus, by introducing a new lower benefit tier of evaluation, tier A receives for review projects of higher quality. The quality of projects submitted between the two tiers does decrease in this case ( $\mu_{\mathrm{B}}^{2 *}<\mu_{\mathrm{A}}^{1 *}$ ), so this lowers the overall burden on the evaluator if the cost of evaluation at the upper tier is sufficiently higher than at the lower tier. On the other hand, $\mu_{\mathrm{B}}^{2 *}>\mu_{\mathrm{B}}^{1 *}$ implies that by introducing a new upper tier, $\mathcal{A}$ will refrain from submitting low-quality projects to the lower tier B , increasing the quality of projects submitted between the two tiers. Moreover, since $\sigma_{\mathrm{B}}^{2 *}>\sigma_{\mathrm{B}}^{1 *}$ it also follows that introducing tier A increases the quality of projects implemented between the two tiers. These findings lend additional support for a tiered system of evaluation as an efficient mechanism of project screening.

## 5 Conclusion

In this paper we investigate the effect of changes in payoff parameters on the efficiency of the equilibrium outcome in a game where the owner of a project of unknown quality faces a tiered system of evaluation to which he can submit his project for review. We consider a setup where evaluation is imperfect, and thus the evaluator is a strategic player who adjusts his strategy in response to changes in the quality of projects that are submitted. When the agent's payoff parameters at the upper tier makes submitting to this tier highly appealing, we show that negative assortative matching may emerge in equilibrium where projects of lower quality are submitted to the upper tier, and those of higher quality to the lower tier. Unlike previous results from the literature, in a system of evaluation with multiple tiers, a higher submission cost may decrease the quality of projects that
are submitted. In particular, a higher submission cost at the upper tier decreases the quality of projects submitted between the two tiers, while a higher cost at the lower tier may simultaneously decrease the average quality of projects submitted at both tiers. We also investigate the effect of payoff parameters on the quality of projects that are implemented, and show that changes in these payoffs that induce the agent to exert a higher level of self-screening may not be beneficial because of the ensuing relaxation of the standards of acceptance. Finally, by comparing the equilibrium outcomes from a tiered system of evaluation with the outcome from a system with only one tier, we provide support for a tiered system of evaluation as a more efficient project screening mechanism.

## Appendix

## Appendix A1.

First, for $\mathcal{E}$ 's beliefs, by Bayes' Rule we have

$$
\operatorname{Pr}\left(h \mid\left\{s_{t}\right\}, \sigma\right)=\frac{j\left(\left\{s_{t}\right\}, \sigma \mid h\right) \operatorname{Pr}(h)}{j\left(\left\{s_{t}\right\}, \sigma \mid h\right) \operatorname{Pr}(h)+j\left(\left\{s_{t}\right\}, \sigma \mid l\right) \operatorname{Pr}(l)}
$$

where $j(\cdot \mid \cdot)$ denotes the conditional probability density function of the relevant continuous random variable. Since $\mathcal{A}$ 's action and the signal $\sigma$ are conditionally independent, it follows that $j\left(\left\{s_{t}\right\}, \sigma \mid q\right)=\operatorname{Pr}\left(\left\{s_{t}\right\} \mid q\right) f^{q}(\sigma)$, and thus that

$$
\begin{align*}
\operatorname{Pr}\left(h \mid\left\{s_{t}\right\}, \sigma\right) & =\frac{\operatorname{Pr}\left(\left\{s_{t}\right\} \mid h\right) f^{h}(\sigma) \pi}{\operatorname{Pr}\left(\left\{s_{t}\right\} \mid h\right) f^{h}(\sigma) \pi+\operatorname{Pr}\left(\left\{s_{t}\right\} \mid l\right) f^{l}(\sigma)(1-\pi)} \\
& =\frac{\operatorname{Pr}\left(\left\{s_{t}\right\} \left\lvert\, h \frac{f^{h}(\sigma)}{f^{\prime}(\sigma)} \frac{\pi}{1-\pi}\right.\right.}{\operatorname{Pr}\left(\left\{s_{t}\right\} \mid h\right) \frac{f^{h}(\sigma)}{f^{l}(\sigma)} \frac{\pi}{1-\pi}+\operatorname{Pr}\left(\left\{s_{t}\right\} \mid l\right)} \tag{12}
\end{align*}
$$

Since, the last term is increasing in $\frac{f^{h}(\sigma)}{f^{l}(\sigma)}$, the fact that $\frac{d}{d \sigma}\left[\frac{f^{h}(\sigma)}{f^{l}(\sigma)}\right]>0$ implies $\frac{d}{d \sigma} \operatorname{Pr}\left(h \mid\left\{s_{t}\right\}, \sigma\right)>0$. Thus, given (1), it follows that for any $\mathcal{S}^{a g}, \mathcal{E}$ responds with a cutoff strategy by accepting a project submitted to tier $t$ if and only if $\sigma \geq \bar{\sigma}_{t}\left(\mathcal{S}^{a g}\right)$, with $\bar{\sigma}_{t}\left(\mathcal{S}^{a g}\right) \in[0,1]$. Thus, in any equilibrium, the evaluator uses a cutoff strategy.

On the other hand, for $\mathcal{A}$ 's belief we have

$$
\begin{align*}
\operatorname{Pr}\left(\left\{a_{t}\right\} \mid \mu\right) & =\operatorname{Pr}\left(\left\{a_{t}\right\} \mid \mu, h\right) \operatorname{Pr}(h \mid \mu)+\operatorname{Pr}\left(\left\{a_{t}\right\} \mid \mu, l\right) \operatorname{Pr}(l \mid \mu) \\
& =\operatorname{Pr}\left(\left\{a_{t}\right\} \mid h\right) \operatorname{Pr}(h \mid \mu)+\operatorname{Pr}\left(\left\{a_{t}\right\} \mid l\right) \operatorname{Pr}(l \mid \mu) \\
& =\left[\operatorname{Pr}\left(\left\{a_{t}\right\} \mid h\right)-\operatorname{Pr}\left(\left\{a_{t}\right\} \mid l\right)\right] \operatorname{Pr}(h \mid \mu)+\operatorname{Pr}\left(\left\{a_{t}\right\} \mid l\right) \tag{13}
\end{align*}
$$

where for the second equality we used the fact that $\mu$ is redundant for $\mathcal{A}$ 's inference about $\mathcal{E}$ 's action when conditioning on the quality of the project. Since in any equilibrium, the evaluator uses a cutoff strategy, we have $\left\{a_{t}\right\}=\left\{\sigma: \sigma \geq \sigma_{t s}\right\}$, and thus $\operatorname{Pr}\left(\left\{a_{t}\right\} \mid h\right)-\operatorname{Pr}\left(\left\{a_{t}\right\} \mid l\right)=\operatorname{Pr}(\sigma \geq$ $\left.\sigma_{t s} \mid h\right)-\operatorname{Pr}\left(\sigma \geq \sigma_{I s} \mid l\right)=F^{l}\left(\sigma_{t s}\right)-F^{h}\left(\sigma_{t s}\right)$. The monotone likelihood ratio property implies first order stochastic dominance, and thus $F^{l}\left(\sigma_{t s}\right)-F^{h}\left(\sigma_{t s}\right)>0$. On the other hand, by Bayes' Rule we have

$$
\begin{equation*}
\operatorname{Pr}(h \mid \mu)=\frac{g^{h}(\mu) \pi}{g^{h}(\mu) \pi+g^{l}(\mu)(1-\pi)}=\frac{\frac{g^{h}(\mu)}{g^{( }(\mu)} \frac{\pi}{1-\pi}}{\frac{g^{h}(\mu)}{g^{l}(\mu)} \frac{\pi}{1-\pi}+1} \tag{14}
\end{equation*}
$$

which is increasing in $\frac{g^{h}(\mu)}{g^{\iota}(\mu)}$, and thus increasing in $\mu$ since $\frac{d}{d \mu}\left[\frac{g^{h}(\mu)}{g^{l}(\mu)}\right]>0$. Thus, $\frac{d}{d \mu} \operatorname{Pr}\left(\left\{a_{t}\right\} \mid \mu\right)>0$.
Now, $b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right)-c_{\mathrm{A}} \geq \max \left\{0, b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right)-c_{\mathrm{B}}\right\}$ if and only if $\operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right) \geq \frac{c_{\mathrm{A}}}{b_{\mathrm{A}}}$ and $b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right) \geq c_{\mathrm{A}}-c_{\mathrm{B}}$. Since in any equilibrium, $\left\{a_{t}\right\}=\left\{\sigma: \sigma \geq \sigma_{t s}\right\}$, it follows that

$$
\begin{gather*}
\frac{\partial}{\partial \mu}\left[b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right)\right]= \\
=\frac{\partial}{\partial \mu}\left[b_{\mathrm{A}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid \mu\right)\right] \\
=\frac{\partial}{\partial \mu}\left\{\begin{array}{l}
b_{\mathrm{A}}\left[\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid h, \mu\right) \operatorname{Pr}(h \mid \mu)+\operatorname{Pr}\left(\sigma_{\mathrm{A}} \geq \sigma_{\mathrm{A}} \mid l, \mu\right) \operatorname{Pr}(l \mid \mu)\right] \\
-b_{\mathrm{B}}\left[\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid h, \mu\right) \operatorname{Pr}(h \mid \mu)+\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid l, \mu\right) \operatorname{Pr}(l \mid \mu)\right]
\end{array}\right\} \\
=\frac{\partial}{\partial \mu}\left\{\begin{array}{l}
b_{\mathrm{A}}\left[\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid h\right) \operatorname{Pr}(h \mid \mu)+\operatorname{Pr}\left(\sigma_{\mathrm{A}} \geq \sigma_{\mathrm{A}} \mid l\right) \operatorname{Pr}(l \mid \mu)\right] \\
-b_{\mathrm{B}}\left[\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid h\right) \operatorname{Pr}(h \mid \mu)+\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid l\right) \operatorname{Pr}(l \mid \mu)\right]
\end{array}\right\} \\
=\frac{\partial}{\partial \mu}\left\{\begin{array}{c}
b_{\mathrm{A}}\left[\left\{\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid h\right)-\operatorname{Pr}\left(\sigma_{\mathrm{A}} \geq \sigma_{\mathrm{A} s} \mid l\right)\right\} \operatorname{Pr}(h \mid \mu)\right]+\operatorname{Pr}\left(\sigma_{\mathrm{A}} \geq \sigma_{\mathrm{A} s} \mid l\right) \\
-b_{\mathrm{B}}\left[\left\{\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid h\right)-\operatorname{Pr}\left(\sigma_{\mathrm{B}} \geq \sigma_{\mathrm{B} s} \mid l\right)\right\} \operatorname{Pr}(h \mid \mu)\right]+\operatorname{Pr}\left(\sigma_{\mathrm{B}} \geq \sigma_{\mathrm{B} s} \mid l\right)
\end{array}\right\} \\
=\left\{b_{\mathrm{A}}\left[\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid h\right)-\operatorname{Pr}\left(\sigma_{\mathrm{A}} \geq \sigma_{\mathrm{A} s} \mid l\right)\right]-b_{\mathrm{B}}\left[\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid h\right)-\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid l\right)\right]\right\} \frac{\partial}{\partial \mu}[\operatorname{Pr}(h \mid \mu)] \\
=\left\{b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]\right\} \frac{\partial}{\partial \mu}[\operatorname{Pr}(h \mid \mu)] \tag{15}
\end{gather*}
$$

where for the third equality we used again $\operatorname{Pr}\left(\left\{a_{t}\right\} \mid \mu, q\right)=\operatorname{Pr}\left(\left\{a_{t}\right\} \mid q\right)$. Since $\frac{\partial}{\partial \mu}[\operatorname{Pr}(h \mid \mu)]>0$, it follows that $\frac{\partial}{\partial \mu}\left[b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right)\right]$ has the same sign for all values of $\mu$, i.e., the sign of $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]$.

Let $\mu^{\prime}$ be the solution to $b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu^{\prime}\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu^{\prime}\right)=c_{\mathrm{A}}-c_{\mathrm{B}}, \mu^{\prime \prime}$ be the solution to $\operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu^{\prime \prime}\right)=\frac{c_{\mathrm{A}}}{b_{\mathrm{A}}}$ and $\mu^{\prime \prime \prime}$ be the solution to $\operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu^{\prime \prime \prime}\right)=\frac{c_{\mathrm{B}}}{b_{\mathrm{B}}}$, and assume for the time being that all these solutions are interior in $[0,1]$. We have two cases to consider. (i) Assume $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]>0$. Then, $\mathcal{A}$ will submit the project to tier A for $\mu \in\left[\mu^{\prime}, 1\right] \cap\left[\mu^{\prime \prime}, 1\right]$, to tier B for $\mu \in \mu \in\left[0, \mu^{\prime}\right] \cap\left[\mu^{\prime \prime \prime}, 1\right]$, and will not submit the project for the rest of the values of $\mu$. (ii) Assume $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]<0$. Then, $\mathcal{A}$ will submit the project to tier B for $\mu \in\left[\mu^{\prime}, 1\right] \cap\left[\mu^{\prime \prime \prime}, 1\right]$, to tier A for $\mu \in\left[0, \mu^{\prime}\right] \cap\left[\mu^{\prime \prime}, 1\right]$, and will not submit the project for the rest of the values of $\mu$. In either case, the set of values of $\mu$ for which $\mathcal{A}$ submits the project to each evaluator is connected. The analysis for the cases when the solutions to the equations that define $\mu^{\prime}, \mu^{\prime \prime}$ and $\mu^{\prime \prime \prime}$ are not interior is similar and leads to the same salient conclusions. For instance, if $b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu^{\prime}\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu^{\prime}\right)>c_{\mathrm{A}}-c_{\mathrm{B}}$ for all $\mu \in[0,1]$, but $\mu^{\prime \prime}$ is interior, then $\mathcal{A}$ will submit the project to tier A for $\mu \in\left[\mu^{\prime \prime}, 1\right]$ and will not submit the project for the rest of the values of $\mu$.

## Appendix A2. Proof of Lemma 3.1

We show that if $\sigma_{\mathrm{B} s} \geq \sigma_{\mathrm{A} s}$, then condition (3) is never satisfied, i.e., that $b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid \mu\right)-$ $c_{\mathrm{B}}<\max \left\{0, b_{\mathrm{A}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-c_{\mathrm{A}}\right\}$. To this end, since $\sigma_{\mathrm{B} s} \geq \sigma_{\mathrm{A} s}$ implies $\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right) \geq$ $\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{Bs}} \mid \mu\right)$, it is enough to show that

$$
b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{As}} \mid \mu\right)-c_{\mathrm{B}}<\max \left\{0, b_{\mathrm{A}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-c_{\mathrm{A}}\right\}
$$

To this aim, we will argue that whenever $b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-c_{\mathrm{B}} \geq b_{\mathrm{A}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-c_{\mathrm{A}}$, it must be that $b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-c_{\mathrm{B}}<0$, which will complete the argument. Since $b_{\mathrm{A}}>b_{\mathrm{B}}$, by assumption $1(i)$, this is equivalent to showing that

$$
\operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right) \leq \frac{c_{\mathrm{A}}-c_{\mathrm{B}}}{b_{\mathrm{A}}-b_{\mathrm{B}}} \text { implies } \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{As}} \mid \mu\right)<\frac{c_{\mathrm{B}}}{b_{\mathrm{B}}}
$$

To show this implication, it is enough to show that $\frac{c_{A}-c_{B}}{b_{A}-b_{B}}<\frac{c_{B}}{b_{B}}$. Rearranging this last condition, we conclude that it is satisfied whenever assumption $1(i i)$ is satisfied, so the proof of the lemma is complete.

## Appendix A3.

From (2) and (3), it follows that $\mathcal{A}$ 's best response function will exhibit positive assortative matching, i.e., $\mathcal{A}$ will submit to tier A for higher values of $\mu$, if and only if

$$
\frac{\partial}{\partial \mu}\left[b_{\mathrm{A}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid \mu\right)\right]>0
$$

Lemma 5.1 provides conditions under which this is satisfied.

Lemma 5.1 Let $\widetilde{\sigma}_{1}$ be the solution to $\frac{f^{l}\left(\widetilde{c}_{1}\right)}{f^{h}\left(\tilde{\sigma}_{1}\right)}=1$, and let $\widetilde{\sigma}_{2}$ be the solution to $\frac{b_{\mathrm{A}}}{b_{\mathrm{B}}}=\frac{F^{l}\left(\widetilde{\alpha}_{1}\right)-F^{h}\left(\widetilde{\sigma}_{1}\right)}{F^{l}\left(\tilde{\sigma}_{2}\right)-F^{h}\left(\widetilde{\sigma}_{2}\right)}$ on $\left[\widetilde{\sigma}_{1}, 1\right]$. (i) If $\sigma_{\mathrm{A} s} \leq \widetilde{\sigma}_{2}$, then $\frac{\partial}{\partial \mu}\left[b_{\mathrm{A}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid \mu\right)\right]>0$ for any $\sigma_{\mathrm{B} s}<\sigma_{\mathrm{A} s}$. (ii) If $\sigma_{\mathrm{A} s}>\widetilde{\sigma}_{2}$, then there exists a neighborhood $\mathcal{N}_{\sigma_{\mathrm{A} s}}$ of $\widetilde{\sigma}_{1}$, such that when $\sigma_{\mathrm{B} s} \in \mathcal{N}_{\sigma_{\mathrm{A} s}}$, we have $\frac{\partial}{\partial \mu}\left[b_{\mathrm{A}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid \mu\right)\right]<0$.

Proof. From (15), we have that when evaluators employ cutoff strategies $\sigma_{\mathrm{A} s}$ and $\sigma_{\mathrm{B} s}$,

$$
\begin{aligned}
\frac{\partial}{\partial \mu}\left[b_{\mathrm{A}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{A} s} \mid \mu\right)-\right. & \left.b_{\mathrm{B}} \operatorname{Pr}\left(\sigma \geq \sigma_{\mathrm{B} s} \mid \mu\right)\right]= \\
& =\left\{b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]\right\} \frac{\partial}{\partial \mu}[\operatorname{Pr}(h \mid \mu)]
\end{aligned}
$$

with $\frac{\partial}{\partial \mu}[\operatorname{Pr}(h \mid \mu)]>0$.
Note that $\frac{d}{d \sigma}\left[F^{l}(\sigma)-F^{h}(\sigma)\right]=0 \Longleftrightarrow f^{l}(\sigma)-f^{h}(\sigma)=0 \Longleftrightarrow \sigma=\widetilde{\sigma}_{1}$. Moreover, we have

$$
\frac{d}{d \sigma}\left[F^{l}(\sigma)-F^{h}(\sigma)\right]>0 \Longleftrightarrow \frac{f^{l}(\sigma)}{f^{h}(\sigma)}>1
$$

Therefore, since $\frac{f^{l}\left(\widetilde{\alpha}_{1}\right)}{f^{h}\left(\widetilde{\sigma}_{1}\right)}=1$ (by the definition of $\widetilde{\sigma}_{1}$ ) and $\frac{d}{d \sigma}\left[\frac{f^{l}(\sigma)}{f^{h}(\sigma)}\right]<0$ (from assumption 2(ii)), we have that

$$
\begin{equation*}
\frac{d}{d \sigma}\left[F^{l}(\sigma)-F^{h}(\sigma)\right]>0 \Longleftrightarrow \sigma<\widetilde{\sigma}_{1} \tag{16}
\end{equation*}
$$

Now, if $\sigma_{\mathrm{A} s}<\widetilde{\sigma}_{2}$, where, by its definition, $\widetilde{\sigma}_{2}$ is the solution to $\frac{b_{\mathrm{A}}}{b_{\mathrm{B}}}=\frac{F^{l}\left(\widetilde{\sigma}_{1}\right)-F^{h}\left(\widetilde{\sigma}_{1}\right)}{F^{l}\left(\tilde{\sigma}_{2}\right)-F^{h}\left(\widetilde{\sigma}_{2}\right)}$, then $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]>0$ for any $\sigma_{\mathrm{B} s}<\sigma_{\mathrm{A} s}$. To see this, assume first that $\sigma_{\mathrm{A} s}>\widetilde{\sigma}_{1}$. Then, since $\widetilde{\sigma}_{2}>\sigma_{\mathrm{A} s}>\widetilde{\sigma}_{1}$, by (16) $F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right) \geq F^{l}\left(\widetilde{\sigma}_{2}\right)-F^{h}\left(\widetilde{\sigma}_{2}\right)=$ $\frac{b_{\mathrm{B}}}{b_{\mathrm{A}}}\left[F^{l}\left(\widetilde{\sigma}_{1}\right)-F^{h}\left(\widetilde{\sigma}_{1}\right)\right]$. Thus, $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[F^{l}\left(\widetilde{\sigma}_{1}\right)-F^{h}\left(\widetilde{\sigma}_{1}\right)\right]>0$. Since $F^{l}\left(\sigma_{\mathrm{B} s}\right)-$ $F^{h}\left(\sigma_{\mathrm{B} s}\right) \leq F^{l}\left(\widetilde{\sigma}_{1}\right)-F^{h}\left(\widetilde{\sigma}_{1}\right)$ by the definition of $\widetilde{\sigma}_{1}$, it follows that indeed $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-$
$b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]>0$ for any $\sigma_{\mathrm{B} s}$. On the other hand, if $\sigma_{\mathrm{A} s}<\widetilde{\sigma}_{1}$ then $F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)>$ $F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)$ by (16) and the fact that $\sigma_{\mathrm{A} s}>\sigma_{\mathrm{B} s}$. Since $b_{\mathrm{A}}>b_{\mathrm{B}}$, it follows again that $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]>0$ for any $\sigma_{\mathrm{B} s}<\sigma_{\mathrm{A} s}$.

On the other hand, if $\sigma_{\mathrm{A} s}>\widetilde{\sigma}_{2}$ and $\sigma_{\mathrm{B} s}$ is sufficiently close to $\widetilde{\sigma}_{1}$, then $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-$ $b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]<0$.

To understand the lemma, note first that $\widetilde{\sigma}_{1}$ is the point at which the difference $F^{l}(\cdot)-F^{h}(\cdot)$ is maximized, whereas when $\sigma_{\mathrm{A} s}$ is sufficiently high, the difference $F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)$ is small. Therefore, when $\sigma_{\mathrm{A} s}$ is high and $\sigma_{\mathrm{B} s}$ is close to $\widetilde{\sigma}_{1}$, the probability that a high-quality project is identified as such from the evaluator's signal is higher when submitting it to tier B (meaning that $\mathcal{E}$ will observe a signal $\sigma \geq \sigma_{\mathrm{B} s}$ with high probability, conditional on $h$, and with a low probability, conditional on $l$ ). The likelihood of a high-quality project is increasing in the signal $\mu$. Thus, given the low probability of acceptance at tier A , when $\mathcal{A}$ has a higher signal, he is more likely to submit the project to tier B in order to have it identified as being of high quality and accepted. On the other hand, if $\frac{b_{\mathrm{A}}}{c_{\mathrm{A}}}$ is high enough, the expected payoff from submitting the project to tier A may be positive even when $\mu$ is small and $\sigma_{\mathrm{A} s}$ is high. Therefore, when $\mathcal{A}$ has a low signal he prefers submitting the project to tier A rather than not submitting it at all. On the other hand, $\mathcal{A}$ 's strategy of submitting to tier B for high signals and to tier A for lower signals, $\mathcal{E}$ 's best response is precisely to adopt a high $\sigma_{\mathrm{A} s}$ and a moderate $\sigma_{\mathrm{B} s}$. Therefore, negative assortative matching may occur in equilibrium.

## Appendix A4. Proof of Lemma 3.2

Employing (14) in (13), it follows that

$$
\begin{equation*}
\operatorname{Pr}\left(\left\{a_{t}\right\} \mid \mu\right)=\frac{\frac{g^{h}(\mu)}{g^{l}(\mu)} \frac{\pi}{1-\pi}}{\frac{g^{h}(\mu)}{g^{l}(\mu)} \frac{\pi}{1-\pi}+1}\left[1-F^{h}\left(\sigma_{t s}\right)\right]+\frac{1}{\frac{g^{h}(\mu)}{g^{\prime}(\mu)} \frac{\pi}{1-\pi}+1}\left[1-F^{l}\left(\sigma_{t s}\right)\right] \tag{17}
\end{equation*}
$$

From (2), under the equilibrium regularity that we assume throughout, we have then that given $\sigma_{\mathrm{A} s}$ and $\sigma_{\mathrm{B} s}, \mathcal{A}$ submits a project to tier A if and only if

$$
\begin{aligned}
& b_{\mathrm{A}} \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right)-c_{\mathrm{A}} \geq b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right)-c_{\mathrm{B}} \Longleftrightarrow \\
& b_{\mathrm{A}}\left\{\frac{g^{h}(\mu)}{g^{l}(\mu)} \frac{\pi}{1-\pi}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]\right.\left.+\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]\right\}-b_{\mathrm{B}}\left\{\frac{g^{h}(\mu)}{g^{l}(\mu)} \frac{\pi}{1-\pi}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]+\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]\right\} \geq \\
& \geq\left(c_{\mathrm{A}}-c_{\mathrm{B}}\right)\left[\frac{g^{h}(\mu)}{g^{l}(\mu)} \frac{\pi}{1-\pi}+1\right] \Longleftrightarrow \\
& \frac{g^{h}(\mu)}{g^{l}(\mu)} \frac{\pi}{1-\pi}\left\{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}\right\} \geq \\
& \geq b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}
\end{aligned}
$$

The last inequality implies that $\mathcal{A}$ employs a cutoff $\mu_{\mathrm{A} s}$ defined by (4) provided that $b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-$ $b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}>0$ and $b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}$. These conditions are not satisfied generically, but they are necessary conditions for the regular equilibrium under consideration. To see this, note first that from the argument in appendix A1, a necessary and sufficient condition for positive assortative matching is that $b_{\mathrm{A}}\left[F^{l}\left(\sigma_{\mathrm{A} s}\right)-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-$ $b_{\mathrm{B}}\left[F^{l}\left(\sigma_{\mathrm{B} s}\right)-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]>0$, which implies by direct computation that

$$
\begin{equation*}
b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]>b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right] \tag{18}
\end{equation*}
$$

Now, we have three cases to consider. (i) If $c_{\mathrm{A}}-c_{\mathrm{B}}<b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]$, then it immediately follows that $b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}<0$, but also that $b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}>0$ by using (18). So $\mathcal{A}$ will never submit a project to tier B , which is something that we precluded by the regularity assumption. (ii) If $c_{\mathrm{A}}-c_{\mathrm{B}}>b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]$, then it immediately follows that $b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-$ $b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}<0$, but also that $b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}>0$ by using (18). So $\mathcal{A}$ will never submit a project to tier A , which is again something that is precluded by the regularity assumption. (iii) Finally, the case when $b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]>$ $c_{\mathrm{A}}-c_{\mathrm{B}}>b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]-b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]$ corresponds to the case where $\mathcal{A}$ submits a project to tier A if and only if (4) is satisfied.

When (4) is not satisfied, $\mathcal{A}$ submits a project to tier B if and only if $b_{\mathrm{B}} \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right)-c_{\mathrm{B}}$, which by straightforward computations using (17), implies $\mathcal{A}$ employs a cutoff $\mu_{\mathrm{B} s}$ defined by (5).

## Appendix A5. Proof of Lemma 3.3

For the evaluator, from (1) and (12) it follows that $\mathcal{E}$ will accept a project submitted to tier $t$ if and only if

$$
\frac{\operatorname{Pr}\left(\left\{s_{t}\right\} \mid h\right) \frac{f^{h}(\sigma)}{f^{l}(\sigma)} \frac{\pi}{1-\pi}}{\operatorname{Pr}\left(\left\{s_{t}\right\} \mid h\right) \frac{f^{h}(\sigma)}{f^{l}(\sigma)} \frac{\pi}{1-\pi}+\operatorname{Pr}\left(\left\{s_{t}\right\} \mid l\right)} \geq \frac{L_{t}}{1+L_{t}} \Longleftrightarrow \frac{f^{h}(\sigma)}{f^{l}(\sigma)} \frac{\pi}{1-\pi} \frac{\operatorname{Pr}\left(\left\{s_{t}\right\} \mid h\right)}{\operatorname{Pr}\left(\left\{s_{t}\right\} \mid l\right)} \geq L_{t}
$$

Thus, given the cutoffs $\mu_{\mathrm{A} s}$ and $\mu_{\mathrm{B} s}$ employed by $\mathcal{A}, \mathcal{E}$ will accept a project submitted to tier A with quality signal $\sigma$ if and only if

$$
\frac{f^{h}(\sigma)}{f^{l}(\sigma)} \frac{\pi}{1-\pi} \frac{1-G^{h}\left(\mu_{\mathrm{A} s}\right)}{1-G^{l}\left(\mu_{\mathrm{A} s}\right)} \geq L_{\mathrm{A}}
$$

and $\mathcal{E}$ will accept a project submitted to tier B with quality signal $\sigma$ if and only if

$$
\frac{f^{h}(\sigma)}{f^{l}(\sigma)} \frac{\pi}{1-\pi} \frac{G^{h}\left(\mu_{\mathrm{A} s}\right)-G^{h}\left(\mu_{\mathrm{B} s}\right)}{G^{l}\left(\mu_{\mathrm{As} s}\right)-G^{l}\left(\mu_{\mathrm{B} s}\right)} \geq L_{\mathrm{B}}
$$

Therefore, indeed, the two evaluators employ cutoff strategies with cutoffs defined by (6) and (7). This completes the proof of the lemma.

## Appendix A6. Proof of Lemma 3.4

Parts (i) and (ii) are immediate. The proof of part (iii) is identical to the corresponding proof from the case of a unique evaluator presented in Barbos (2012). For part (iv), since $\frac{f^{h}(\sigma)}{f^{l}(\sigma)}$ is increasing in $\sigma$, from (7) it follows that $\bar{\sigma}_{\mathrm{B}}(\cdot)$ is decreasing in $\mu_{\mathrm{A} s}$ if and only if

$$
\begin{equation*}
\frac{d}{d \mu_{\mathrm{A} s}}\left[\frac{G^{l}\left(\mu_{\mathrm{A} s}\right)-G^{l}\left(\mu_{\mathrm{B} s}\right)}{G^{h}\left(\mu_{\mathrm{A} s}\right)-G^{h}\left(\mu_{\mathrm{B} s}\right)}\right]<0 \Longleftrightarrow \frac{g^{h}\left(\mu_{\mathrm{A} s}\right)}{g^{l}\left(\mu_{\mathrm{A} s}\right)}>\frac{G^{h}\left(\mu_{\mathrm{A} s}\right)-G^{h}\left(\mu_{\mathrm{B} s}\right)}{G^{l}\left(\mu_{\mathrm{A} s}\right)-G^{l}\left(\mu_{\mathrm{B} s}\right)} \tag{19}
\end{equation*}
$$

where we used the fact that $\mu_{\mathrm{A} s}>\mu_{\mathrm{B} s}$. Similarly, $\bar{\sigma}_{\mathrm{B}}(\cdot)$ is decreasing in $\mu_{\mathrm{B} s}$ if and only if

$$
\begin{equation*}
\frac{d}{d \mu_{\mathrm{B} s}}\left[\frac{G^{l}\left(\mu_{\mathrm{A} s}\right)-G^{l}\left(\mu_{\mathrm{B} s}\right)}{G^{h}\left(\mu_{\mathrm{A} s}\right)-G^{h}\left(\mu_{\mathrm{B} s}\right)}\right]<0 \Longleftrightarrow \frac{G^{h}\left(\mu_{\mathrm{A} s}\right)-G^{h}\left(\mu_{\mathrm{B} s}\right)}{G^{l}\left(\mu_{\mathrm{A} s}\right)-G^{l}\left(\mu_{\mathrm{B} s}\right)}>\frac{g^{h}\left(\mu_{\mathrm{B} s}\right)}{g^{l}\left(\mu_{\mathrm{B} s}\right)} \tag{20}
\end{equation*}
$$

We will show that (19) and (20) are satisfied under assumption 2(ii). Since $\frac{g^{h}(u)}{g^{l}(u)}$ is increasing in $u$, it follows that for $u \in\left[\mu_{\mathrm{B} s}, \mu_{\mathrm{A} s}\right)$, we have $\frac{g^{h}\left(\mu_{\mathrm{A} s}\right)}{g^{l}\left(\mu_{\mathrm{A} s}\right)}>\frac{g^{h}(u)}{g^{l}(u)}$, and thus $g^{h}\left(\mu_{\mathrm{A} s}\right) g^{l}(u)>$
$g^{l}\left(\mu_{\mathrm{A} s}\right) g^{h}(u)$. Integrating this last inequality with respect to $u$ between $\mu_{\mathrm{B} s}$ and $\mu_{\mathrm{A} s}$, we obtain

$$
g^{h}\left(\mu_{\mathrm{A} s}\right)\left[G^{l}\left(\mu_{\mathrm{A} s}\right)-G^{l}\left(\mu_{\mathrm{B} s}\right)\right]>g^{l}\left(\mu_{\mathrm{A} s}\right)\left[G^{h}\left(\mu_{\mathrm{A} s}\right)-G^{h}\left(\mu_{\mathrm{B} s}\right)\right]
$$

which immediately then implies (19). On the other hand, $\frac{g^{h}\left(\mu_{\mathrm{Bs}}\right)}{g^{l}\left(\mu_{\mathrm{B} s}\right)}<\frac{g^{h}(u)}{g^{l}(u)}$ for $u \in\left(\mu_{\mathrm{B} s}, \mu_{\mathrm{A} s}\right]$, implies $g^{h}\left(\mu_{\mathrm{B} s}\right) g^{l}(u)<g^{l}\left(\mu_{\mathrm{B} s}\right) g^{h}(u)$, which integrated with respect to $u$ between $\mu_{\mathrm{B} s}$ and $\mu_{\mathrm{A} s}$, implies

$$
g^{h}\left(\mu_{\mathrm{B} s}\right)\left[G^{l}\left(\mu_{\mathrm{A} s}\right)-G^{l}\left(\mu_{\mathrm{B} s}\right)\right]<g^{l}\left(\mu_{\mathrm{B} s}\right)\left[G^{h}\left(\mu_{\mathrm{A} s}\right)-G^{h}\left(\mu_{\mathrm{B} s}\right)\right]
$$

which implies (20). This completes the proof of the lemma.

## Appendix A7. Proof of Proposition 7

From Barbos (2013), with only one tier of evaluation, the equilibrium is given by

$$
\begin{align*}
\frac{\pi}{1-\pi} \frac{g^{h}\left(\mu_{t}^{1 *}\right)}{g^{l}\left(\mu_{t}^{1 *}\right)} & =\frac{c_{t}-b_{t}\left[1-F^{l}\left(\sigma_{t}^{1 *}\right)\right]}{b_{t}\left[1-F^{h}\left(\sigma_{t}^{1 *}\right)\right]-c_{t}}  \tag{21}\\
\frac{\pi}{1-\pi} \frac{f^{h}\left(\sigma_{t}^{1 *}\right)}{f^{l}\left(\sigma_{t}^{1 *}\right)} \frac{1-G^{h}\left(\mu_{t}^{1 *}\right)}{1-G^{l}\left(\mu_{t}^{1 *}\right)} & =L_{t} \tag{22}
\end{align*}
$$

where $\left(\mu_{t}^{1 *}, \sigma_{t}^{1 *}\right)$ denotes the equilibrium strategies of the game in which $\mathcal{E}$ only offers tier $t \in\{A, \mathrm{~B}\}$.
Consider first the case when the initial tier is A , and then tier B is introduced. Assume by contradiction that $\mu_{\mathrm{A}}^{2 *} \leq \mu_{\mathrm{A}}^{1 *}$. Then

$$
\frac{\pi}{1-\pi} \frac{f^{h}\left(\sigma_{\mathrm{A}}^{1 *}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{1 *}\right)}=L_{\mathrm{A}} \frac{1-G^{l}\left(\mu_{\mathrm{A}}^{1 *}\right)}{1-G^{h}\left(\mu_{\mathrm{A}}^{1 *}\right)} \leq L_{\mathrm{A}} \frac{1-G^{l}\left(\mu_{\mathrm{A}}^{2 *}\right)}{1-G^{h}\left(\mu_{\mathrm{A}}^{2 *}\right)}=\frac{\pi}{1-\pi} \frac{f^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{2 *}\right)}
$$

where the first equality follows from (22) with $t=\mathrm{A}$, the second equality follows from (6), and the inequality from $\mu_{\mathrm{A}}^{2 *} \leq \mu_{\mathrm{A}}^{1 *}$ and the fact that $\frac{d}{d \mu}\left[\frac{1-G^{h}(\mu)}{1-G^{l}(\mu)}\right]>0$. To see this last fact, let $\mu_{\mathrm{A} s}=1$ in equation (20). Thus, $\frac{f^{h}\left(\sigma_{\mathrm{A}}^{1 *}\right)}{f^{l}\left(\sigma_{\mathrm{A}}^{1 *}\right)} \leq \frac{f^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)}{f^{\prime}\left(\sigma_{\mathrm{A}}^{2 *}\right)}$, so by assumption $2(i i)$, we have that $\sigma_{\mathrm{A}}^{2 *} \geq \sigma_{\mathrm{A}}^{1 *}$. Therefore, from (21), $\frac{\pi}{1-\pi} \frac{g^{h}\left(\mu_{\mathrm{A}}^{1 *}\right)}{g^{l}\left(\mu_{\mathrm{A}}^{\Lambda_{\mathrm{A}}}\right)}=\frac{c_{\mathrm{A}}-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A}}^{1 *}\right)\right]}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}^{1 *}\right)\right]-c_{\mathrm{A}}} \leq \frac{c_{\mathrm{A}}-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]-c_{\mathrm{A}}}$. On the other hand, from (4), we have $\frac{\pi}{1-\pi} \frac{g^{h}\left(\mu_{A^{2}}^{2 *}\right)}{g^{( }\left(\mu_{\mathrm{A}}^{2 *}\right)}=\frac{b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}}$. We show next that $\frac{\pi}{1-\pi} \frac{g^{h}\left(\mu_{\mathrm{A}}^{2 *}\right)}{g^{l}\left(\mu_{\mathrm{A}}^{2 *}\right)}>\frac{c_{\mathrm{A}}-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]-c_{\mathrm{A}}}$, which would then imply that $\frac{g^{h}\left(\mu_{\AA}^{2 *}\right)}{g^{l}\left(\mu_{\mathrm{A}}^{*}\right)}>\frac{g^{h}\left(\mu_{\mathrm{A}}^{1 *}\right)}{g^{l}\left(\mu_{\mathrm{A}}^{1 *}\right)}$, and thus that $\mu_{\mathrm{A}}^{2 *}>\mu_{\mathrm{A}}^{1 *}$ contradicting the initial
assumption. Thus, note that

$$
\begin{aligned}
\frac{b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}} & >\frac{c_{\mathrm{A}}-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]-c_{\mathrm{A}}} \Longleftrightarrow \\
\frac{b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]-c_{\mathrm{B}}}{c_{\mathrm{B}}-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]} & >\frac{c_{\mathrm{A}}-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]-c_{\mathrm{A}}}
\end{aligned}
$$

But this last inequality follows from the fact that $\mu_{\mathrm{A}}^{2 *}>\mu_{\mathrm{B}}^{2 *}$ implies from (4) and (5) that

$$
\frac{b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}}{b_{\mathrm{A}}\left[1-F^{h}\left(\sigma_{\mathrm{A}}^{2 *}\right)\right]-b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}}>\frac{c_{\mathrm{B}}-b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]}{b_{\mathrm{B}}\left[1-F^{h}\left(\sigma_{\mathrm{B}}^{2 *}\right)\right]-c_{\mathrm{B}}}
$$

Therefore, indeed $\mu_{\mathrm{A}}^{2 *}>\mu_{\mathrm{A}}^{1 *}$, which then from (22) and (6) immediately also implies that $\sigma_{\mathrm{A}}^{2 *}<\sigma_{\mathrm{A}}^{1 *}$.
For the second part of the proof, consider the case when the initial tier is B and then tier A is introduced, and assume by contradiction that $\sigma_{\mathrm{B}}^{2 *} \leq \sigma_{\mathrm{B}}^{1 *}$. From (21) and (5), this implies that $\mu_{\mathrm{B}}^{2 *} \leq \mu_{\mathrm{B}}^{1 *}$. Therefore, from (22) it follows that $\frac{\pi}{1-\pi} \frac{f^{h}\left(\sigma_{\mathrm{B}}^{1 *}\right)}{f^{l}\left(\sigma_{\mathrm{B}}^{1 *}\right)}=L_{\mathrm{B}} \frac{1-G^{l}\left(\mu_{\mathrm{B}}^{1 *}\right)}{1-G^{h}\left(\mu_{\mathrm{B}}^{1 *)}\right)} \leq L_{\mathrm{B}} \frac{1-G^{l}\left(\mu_{\mathrm{B}}^{2 *}\right)}{1-G^{h}\left(\mu_{\mathrm{B}}^{*}\right)}$. Therefore, to complete the contradiction argument, it would be enough to show that $\frac{\pi}{1-\pi} \frac{f^{h}\left(\sigma_{B}^{2 *}\right)}{f^{l}\left(\sigma_{\mathrm{B}}^{2 *}\right)}>$ $L_{\mathrm{B}} \frac{1-G^{l}\left(\mu_{\mathrm{B}}^{2 *}\right)}{1-G^{h}\left(\mu_{\mathrm{B}}^{2 *}\right)}$, because this would immediately imply $\sigma_{\mathrm{B}}^{2 *}>\sigma_{\mathrm{B}}^{1 *}$. But from (7) we have $\frac{\pi}{1-\pi} \frac{f^{h}\left(\sigma_{\mathrm{B}}^{2 *}\right)}{f^{l}\left(\sigma_{\mathrm{B}}^{2 *}\right)}=$ $L_{\mathrm{B}} \frac{G^{l}\left(\mu_{\mathrm{A}}^{2 *}\right)-G^{l}\left(\mu_{\mathrm{B}^{2}}^{2 *}\right)}{G^{h}\left(\mu_{\mathrm{A}}^{2 *}\right)-G^{h}\left(\mu_{\mathrm{B}}^{2 *}\right)}$, so it suffices to show that $\frac{G^{l}\left(\mu_{\left.\mathrm{A}^{2 *}\right)}^{G^{h}\left(G^{l}\left(\mu_{\mathrm{B}}^{2 *}\right)\right.} \mathrm{G}^{2 *}\right)}{G_{\mathrm{B}}\left(\mu_{\mathrm{B}}^{2 *}\right)}>\frac{1-G^{l}\left(\mu_{\mathrm{B}^{2 *}}\right)}{1-G^{h}\left(\mu_{\mathrm{B}}^{*}\right)}$. This is true from (19). Therefore, indeed $\sigma_{\mathrm{B}}^{2 *}>\sigma_{\mathrm{B}}^{1 *}$, which from (21) and (5) also implies that $\mu_{\mathrm{B}}^{2 *}>\mu_{\mathrm{B}}^{1 *}$.

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[^0]:    *I would like to thank two anonymous referees for very helpful comments. All errors are mine.
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[^1]:    ${ }^{1}$ The article is available at http://www.nydailynews.com/opinion/casey-anthony-trial-case-overzealous-prosecution-death-penalty-bar-high-article-1.160804. See also, for instance, a Los Angeles Times article by a law attorney http://articles.latimes.com/2011/jul/09/opinion/la-oe-shapiro-caylee-anthony-20110709
    ${ }^{2}$ Time magazine called this case the "social media trial of the century" http://www.time.com/time/nation/article/0,8599,2077969,00.html. As of 2013, a Google search for Casey Anthony yields more results than the search for the U.S. Senate Majority Leader at the time.
    ${ }^{3}$ In fact, avoiding some of these costs is also the reason behind prosecutors' willingness to settle in many trials.

[^2]:    ${ }^{4}$ The jury's signal distribution, conditional on the guilt of the defendant, may be an (unmodelled) function of the optimal persuasion strategies adopted by the prosecution and defense teams.
    ${ }^{5}$ The submission fee may be a payment toward the evaluator or a third entity, or it may take a non-monetary form, such as a cost incurred by the agent in terms of time or resources spent on preparing the application or in terms of time by which the evaluator's decision is delayed.
    ${ }^{6}$ In certain situations, an unsuccessful high-end high-price product can be reintroduced as a lower-end item; in others, a competitor may fill in the spot. For instance, the marketing of Apple personal computers as such expensive high-end items in the 1980s allowed Microsoft OS based PCs to dominate the market for the next 25 years.

[^3]:    ${ }^{7}$ Selling the car as scrap or donating it would correspond to not submitting the project.
    ${ }^{8}$ The optimal fees are not unboudedly high because in these models, the evaluators need to accept a minimum number of articles. In our paper, we discard this requirement on the evaluator so as to capture situations of project screening beyond that of the academic articles evaluation examined in those papers.
    ${ }^{9}$ See also Azar (2007) and the references therein. Cotton (2013) distinguishes between monetary costs and time delays and shows that when authors of academic articles are heterogenous, the optimal fee structure implies a combination of these monetary and non-monetary fees. Boleslavsky and Cotton (2013) study a model in which an evaluator has to select one of several competing proposals of unknown quality, and investigate the effect of the limited capacity of the evaluator to accept proposals on the incentives of the proposers to produce information.

[^4]:    ${ }^{10}$ A possible example of this type products are those that are not sold through the usual retail outlets, but through teleshopping programs at arguably high prices given their value to users.

[^5]:    ${ }^{11}$ Heintzelman and Nocetti (2009) also discuss the case of two-sided incomplete information, but in their framework

[^6]:    the model becomes intractable under this assumption.
    ${ }^{12}$ The analysis does not change in a meaningful way if we allow the agent's payoff to also depend on the quality of the project by having him prefer that an accepted project is of high quality. See section 3 for the discussion. Also, the analysis also does not change if we allow that the evaluator be also concerned with the quality of projects that he rejects. Finally, since the submission fee may often take a non-monetary form, we do not include it in the evaluator's payoff. This is without too much loss of generality for the ensuing results. These simplifying modelling specifications are also adopted elsewhere in the literature (see for instance, Cotton (2013)).
    ${ }^{13}$ In line with the motivating example from the introduction, we thus restrict attention to the analysis of those situations when resubmission of a rejected project to a different tier is not possible.

[^7]:    ${ }^{14}$ This condition is reminiscent of the supermodularity condition, which since Becker (1973) is known to be necessary and sufficient for positive assortative matching in the equilibrium allocation of many applications.

[^8]:    ${ }^{15} \mathcal{A}$ 's best response function may involve a corner solution. In particular, for a fixed value of $\sigma_{t s}$, if $\frac{b_{t}}{c_{t}}$ is small enough for some $t \in\{\mathrm{~A}, \mathrm{~B}\}, \mathcal{A}$ never submits to tier $t$. Moreover, when $\frac{b_{\mathrm{A}}}{c_{\mathrm{A}}}$ is much higher than $\frac{b_{\mathrm{B}}}{c_{\mathrm{B}}}$, tier B receives

[^9]:    no submissions. To focus our analysis on developing intuition rather than solving for corner solutions, we restrict attention to the case of interior equilibria.
    ${ }^{16}$ We can model a situation in which $\mathcal{A}$ also prefers that an accepted project is of high quality, by having $\mathcal{A}$ receive an additional benefit $\delta_{t}$ under this contingency. In this case, equation (4) becomes $\left[b_{\mathrm{A}}+\delta_{\mathrm{A}} \operatorname{Pr}\left(h \mid\left\{a_{\mathrm{A}}\right\}, \mu\right)\right] \operatorname{Pr}\left(\left\{a_{\mathrm{A}}\right\} \mid \mu\right)-$ $c_{\mathrm{A}} \geq \max \left\{\left[b_{\mathrm{B}}+\delta_{\mathrm{B}} \operatorname{Pr}\left(h \mid\left\{a_{\mathrm{B}}\right\}, \mu\right)\right] \operatorname{Pr}\left(\left\{a_{\mathrm{B}}\right\} \mid \mu\right)-c_{\mathrm{B}}, 0\right\}$, which after some calculations can be rewritten as $\frac{\pi}{1-\pi} \frac{g^{h}\left(\bar{\mu}_{\mathrm{A}}\right)}{g^{l}\left(\bar{\mu}_{\mathrm{A}}\right)}=$ $\frac{b_{\mathrm{B}}\left[1-F^{l}\left(\sigma_{\mathrm{B} s}\right)\right]-b_{\mathrm{A}}\left[1-F^{l}\left(\sigma_{\mathrm{A} s}\right)\right]+c_{\mathrm{A}}-c_{\mathrm{B}}}{\left(b_{\mathrm{A}}+\delta_{\mathrm{A}}\right)\left[1-F^{h}\left(\sigma_{\mathrm{A} s}\right)\right]-\left(b_{\mathrm{B}}+\delta_{\mathrm{B}}\right)\left[1-F^{h}\left(\sigma_{\mathrm{B} s}\right)\right]+c_{\mathrm{B}}-c_{\mathrm{A}}}$. Equations (3) and (5) are altered in a similar way. The ensuing analysis and results are qualitatively similar to the case when $\delta_{\mathrm{A}}=\delta_{\mathrm{B}}=0$.

[^10]:    ${ }^{17}$ Similarly, for instance, in panel (b), $\bar{\mu}_{\mathrm{B}}^{\mathrm{o}}\left(\sigma_{\mathrm{B} s}\right)$ represents the best response functions $\bar{\mu}_{\mathrm{B}}\left(\sigma_{\mathrm{A}}^{*}, \sigma_{\mathrm{Bs}}\right)$ and $\bar{\mu}_{\mathrm{B}}\left(\sigma_{\mathrm{A}}^{* \prime}, \sigma_{\mathrm{Bs}}\right)$, while $\bar{\sigma}_{\mathrm{B}}^{\mathrm{o}}\left(\mu_{\mathrm{B} s}\right)$ and $\bar{\sigma}_{\mathrm{B}}^{\mathrm{z}}\left(\mu_{\mathrm{B} s}\right)$ represent the best response functions $\bar{\sigma}_{\mathrm{B}}\left(\mu_{\mathrm{A}}^{*}, \mu_{\mathrm{B} s}\right)$ and $\bar{\sigma}_{\mathrm{B}}\left(\mu_{\mathrm{A}}^{* \prime}, \mu_{\mathrm{B} s}\right)$, respectively.

[^11]:    ${ }^{18}$ For instance, both an increase in $b_{\mathrm{A}}$ and a decrease in $c_{\mathrm{A}}$ lead to a decrease in the numerator and an increase in the denominator in the right hand side of (4), which are the only places where the two parameters appear in the four equations (4)-(7) that define the equilibrium of the game. Therefore, the sign of the effect on the equilibrium strategies of either of the two changes in $b_{\mathrm{A}}$ and $c_{\mathrm{A}}$ is the same.

[^12]:    ${ }^{19}$ The expected quality of the projects that are implemented is $l+(h-l) \operatorname{Pr}\left(h \mid \mu \geq \mu^{*}, \sigma \geq \sigma^{*}\right)$.

[^13]:    ${ }^{20}$ As mentioned earlier, an increase in $c_{\mathrm{A}}$ is qualitatively similar to a decrease in $b_{\mathrm{A}}$.

[^14]:    ${ }^{21}$ Thus, for instance, $\bar{\mu}_{\mathrm{A}}^{\mathrm{O}}\left(\sigma_{\mathrm{A} s}\right)$ represents the best response function $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B}}^{*}\right)$, as defined by (4), when the submission fee to tier A is $c_{\mathrm{A}}$, while $\bar{\mu}_{\mathrm{A}}^{\mathrm{x}}\left(\sigma_{\mathrm{A} s}\right)$ represents $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B}}^{*}\right)$ when the submission fee in (4) is $c_{\mathrm{A}}^{\prime}$. On the other hand, $\bar{\mu}_{\mathrm{A}}^{\mathbf{z}}\left(\sigma_{\mathrm{A} s}\right)$, which is defined below, represents $\bar{\mu}_{\mathrm{A}}\left(\sigma_{\mathrm{A} s}, \sigma_{\mathrm{B}}^{* \prime}\right)$ when the submission fee to tier A is $c_{\mathrm{A}}^{\prime}$. Finally, $\bar{\sigma}_{\mathrm{A}}^{\mathrm{O}}\left(\mu_{\mathrm{A} s}\right)$ represents $\bar{\sigma}_{\mathrm{A}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B}}^{*}\right)$ when the submission fee to tier A is $c_{\mathrm{A}}$ or $c_{\mathrm{A}}^{\prime}$, but also $\bar{\sigma}_{\mathrm{A}}\left(\mu_{\mathrm{A} s}, \mu_{\mathrm{B}}^{* \prime}\right)$ when the fee to tier A is $c_{\mathrm{A}}$ or $c_{\mathrm{A}}^{\prime}$. Note also that, for instance in panel (c) the values $\mu_{\mathrm{A}}^{*}$ and $\sigma_{\mathrm{B}}^{*}$ from the initial equilibrium $\xi$ are at the intersection of the curves $\bar{\mu}_{\mathrm{A}}^{\mathrm{o}}\left(\sigma_{\mathrm{B} s}\right)$ and $\bar{\sigma}_{\mathrm{B}}^{\mathrm{O}}\left(\mu_{\mathrm{A} s}\right)$, while the corresponding values from the equilibrium $\xi^{\prime}$ are at the intersection of the curves $\bar{\mu}_{\mathrm{A}}^{\mathrm{z}}\left(\sigma_{\mathrm{B} s}\right)$ and $\bar{\sigma}_{\mathrm{B}}^{\mathrm{Z}}\left(\mu_{\mathrm{A} s}\right)$.

[^15]:    ${ }^{22}$ To see this, note the following in a $\left(\mu_{\mathrm{A} s}, \sigma_{\mathrm{B} s}\right)$ panel. First, from (4), $d c_{\mathrm{B}}>0$ implies $\bar{\mu}_{\mathrm{A}}^{\mathrm{o}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathrm{x}}\left(\sigma_{\mathrm{B} s}\right)$. Second, also from lemma 3.4(i), d $\sigma_{\mathrm{A}}^{*}<0$ implies $\bar{\mu}_{\mathrm{A}}^{\mathrm{x}}\left(\sigma_{\mathrm{B} s}\right) \succ \bar{\mu}_{\mathrm{A}}^{\mathrm{Z}}\left(\sigma_{\mathrm{B} s}\right)$. Third, from lemma 3.4(iv), d $\mu_{\mathrm{B}}^{*}<0$ implies $\bar{\sigma}_{\mathrm{B}}^{\mathrm{Z}}\left(\mu_{\mathrm{A} s}\right) \succ \bar{\sigma}_{\mathrm{B}}^{\mathrm{o}}\left(\mu_{\mathrm{As} s}\right)$. It is then straightforward to see that if $\bar{\sigma}_{\mathrm{B}}^{\mathrm{o}}\left(\mu_{\mathrm{A} s}\right)$ is steeper than $\bar{\mu}_{\mathrm{A}}^{\mathrm{o}}\left(\sigma_{\mathrm{B} s}\right)$ at the intersection point, it must be that $d \mu_{\mathrm{A}}^{*}<0$.

[^16]:    ${ }^{23}$ As an example, assume $\operatorname{Pr}(h \mid \mu)=\mu$, and that $\mu$ is distributed uniformly on $\{0.1 ; 0.2 ; 0.6 ; 0.7\}$. Also, assume that initially $\mu_{\mathrm{B}}^{*}=0.05$ and $\mu_{\mathrm{A}}^{*}=0.65$, while after the increase in $c_{\mathrm{B}}, \mu_{\mathrm{B}}^{* \prime}=0.15$ and $\mu_{\mathrm{A}}^{* \prime}=0.55$. Then, in the initial equilibrium, the average value of $\mu$ for projects submitted to tier B is $(0.1+0.2+0.6) / 3=0.3$, while for tier A is 0.7. After in increase in $c_{\mathrm{B}}$, in the new equilibrium, the former average is 0.2 , while the latter is 0.65 .
    ${ }^{24}$ Note that unlike some of the other papers from the literature, we do not calculate the optimal values of these payoffs parameters, but only elicit the effect of a change in them on the expected quality of projects that are implemented by the evaluator in the two tiers. In fact, in a variety of situations, these costs can be adjusted only at the margin. For instance, while laws can be passed to make a prosecutor's job of collecting evidence less costly, this may be possible only up to a limited extent.

[^17]:    ${ }^{25}$ Note that $\operatorname{Pr}\left(\mu_{\mathrm{A}}^{*} \geq \mu \geq \mu_{\mathrm{B}}^{*} \mid q\right)=G^{q}\left(\mu_{\mathrm{A}}^{*}\right)-G^{q}\left(\mu_{\mathrm{B}}^{*}\right)$, and that one can use $(7)$ to compute $\frac{G^{h}\left(\mu_{\mathrm{A}}^{*}\right)-G^{h}\left(\mu_{\mathrm{B}}^{*}\right)}{G^{l}\left(\mu_{\mathrm{A}}^{*}\right)-G^{l}\left(\mu_{\mathrm{B}}^{*}\right)}$.

[^18]:    ${ }^{26}$ See Barbos (2013) for an alternative interpretation of the conditions in proposition 5 in terms of the elasticities of the conditional hazard rates.

[^19]:    ${ }^{27}$ The payoff characteristics of each tier are assumed identical accross games. Thus, for instance, $b_{\mathrm{A}}, c_{\mathrm{A}}$ and $L_{\mathrm{A}}$ are the same in the game in which only tier A exists and in the game with two tiers of evaluation, A and B.

