Risky Investments with Limited Commitment*

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Abstract

Over the last three decades there has been a dramatic increase in the concentration of income at the very top of the distribution. This increase in income inequality has been especially notable in the compensation of financial executives where it has often been associated with greater risk-taking using more complex financial instruments. Parallel to this trend, organizational forms in the financial sector have been transformed with the traditional partnership form of organization replaced by public companies that compete for managerial talent and have weaker forms of commitment between investors and managers. In this paper we describe the link between commitment, risk taking and competition for managers. We emphasize the increase in competition for human talents that followed domestic and international liberalization of financial markets and its interplay with different degrees of contract enforcement, representing different organizational forms. Because of the limited enforcement of contracts, the increase in competition raises the managerial incentives to undertake risky investment. Although this may have a positive effect on economic growth, the equilibrium outcome is not efficient and generates greater risk-taking and income inequality.

1 Introduction

The compensation of financial executives rose to very high levels in the past two decades, a fact that has been documented by Phillipon and Resheff (2009). Clementi and Cooley (2009) show that between 1980 and 2007 the average compensation levels in the financial sector increased from parity with other sectors of the economy to 181% of other sectors. At the same time, the contribution of the financial sector to corporate profits increased sharply reaching 41% of total corporate profits in 2002 and the finance industry accounted for 8.2% of GDP in 2006. Employment in Financial services increased by 69% from 1980 to its peak in late 2006. This period of rapid growth in importance and compensation in the financial sector, was accompanied by rapid innovation in financial services, growth

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in intermediation and an enormous build up of both household and corporate debt. It ended with the largest crisis of the financial system since the nineteen thirties.

Many observers have blamed compensation practices for encouraging the build-up of risks in the financial system and ultimately for the crisis.[1] Compensation in the financial sector (and elsewhere) tends to be highly convex, meaning that the rewards for good outcomes are very generous but the consequences of bad outcomes are not that dire. This convexity may be a problem because, absent other constraints, managers have a built in incentive to pursue riskier strategies if they face asymmetric rewards.

One common explanation for the convexity of compensation is that this is necessary to induce risk averse managers to make investment decisions that are in the interests of their shareholders. This explanation requires an arbitrary assumption that investors are less risk averse than managers. One can also appeal to the limited liability or built in insurance feature of most employment contracts. Either feature induces some degree of convexity but does not explain why the risk accumulated by the financial sector has increased over time. In this paper we argue that increased competition together with the lack of commitment are the key to understanding compensation arrangements in financial firms. This also explains why compensation has become more unequal overtime.

We study a model where investors compete for and hire managers to run investment projects, each investor-manager pair representing a financial firm. A key feature of the model is that production depends on the human capital of the manager which can be enhanced, within the firm, with costly investment. This human capital accumulation can be understood as acquiring new skills by engaging in risky financial innovations (e.g. implementing new financial instruments which may or may not have positive returns). Since part of the accumulated human capital can be transferred outside the firm by the manager, there is a conflict of interest between the investor and the manager. In this environment, the investment desired by the investor is typically smaller than the investment desired by the manager because the cost is incurred by the firm while the benefits are shared. This implies that, if the investor cannot control the investment policy, the manager has an incentive to deviate from the optimal policy simply because it does not internalize the full cost of the investment. This is a hold-up problem that can result in excessive risk-taking by risk-averse agents. The goal of this paper is to characterize the investment policy that results from the (constrained) optimal contract, under different forms of contractual enforcement, or commitment, and to study the implications for economic growth, welfare and inequality. The incentive problem in the choice of the optimal investment can only be solved when both parties commit to a long term contract. However, the assumption of full commitment requires very strong behavioral assumptions. It requires that the manager never chooses to leave the firm even if the outside option becomes significantly bigger. It also requires that any promises made by the investor to the manager are not renegotiated ex-post. Once we relax these assumptions, the actual investment deviates from the optimal investment with important implications for the compensation structure, the growth rate of the economy, risk-taking and inequality.

[1] A study by McKinsey and Co. of the financial meltdown at UBS highlights the internal compensation practices as a contributing factor in the buildup up of risk on the balance sheet. Those risks ultimately resulted in staggering losses for UBS.

We consider different environments that differ in their degree of contractual enforcement (commitment). Long-term contracts specify a sequence of investments and consumptions. We distinguish between different forms of commitment to the investment and output-share decisions. In the benchmark case of full-commitment, both agents commit to an investment and consumption plan, meeting their initial participation constraints; investment decisions are not affected by the degree of competition for managers services. In our economies full commitment to the investment plan is equivalent to the investor controlling the investment decisions; therefore, we consider the one-sided limited enforcement case, where the investor controls the investment decisions, but the manager may quit if a better offer is available. In this case, consumption shares depend on the past history of binding default values, as in recursive contracts' models. Binding default values may distort investment decisions and creates a role for competition, since outside values rise with greater competition.

When the investment decision is controlled by the manager, the aforementioned 'hold-up' problem may occur. It can be prevented by introducing incentive-compatibility constraints within the contract, as in standard dynamic principal-agent problems. Incentive-compatibility constraints are based on rewards and punishments; however, with limted-enforcement, there may be little scope for credible (i.e. subject to enforcement constraints) punishments and, therefore fulfilling incentive-compatibility constraints may require a high degree of commitment from the investor. It is equivalent to the investor not only matching outside offers, but also matching *potential outside offers* for skills the manager has yet to acquire. Because this is extreme, we also consider the case where the investor only matches *realized outside offers*, that is, offers to the existing level of managerial skills. This economy is characterized by higher risk-taking than the economy with investor's control of investment decisions and limited enforcement, although as long-term contracts unfold, managerial distortions tend to disappear, since the manager's output share keeps increasing.

Next we characterize the contract with managerial control of investments, and limited enforcement and incentive constraints; in this economy, the hold-up problem is resolved by the investor giving up a large share of the output to the manager from the beginning. However, this contract may not be feasible when there is no full-commitment by the investor. As said, this limited-commitment can be because the investor cannot commit to more than matching *realized outside offers*, it can also be because there is limited enforcement on the part of the investor; for example, the investor may find it profitable to breach the contract and hire a new manager. Finally, in a more extreme form, it can be because the investor has no commitment to fulfill the long-term contract. We show that in an environment with *double-sided lack of commitment*, firms over-invest in risky projects. Although the overall economy experiences higher growth, this is not efficient[2].

An important result, which is at the core of this paper, is that higher competition, captured by the transferability of human capital, increases risky innovations and leads to more volatile compensation at the firm level. This observation is important because,

[2]This draft is preliminary mainly in that we only compute three of these environments for a specific form of preferences; see Section 4.

as we argue in the next section of the paper, the widespread shift in organizational form within the financial industry, from partnerships and closed ownership merchant banks to public companies, that began in the 1970's and continued for the next 30 years, changed the nature of commitment relationships within firms and increased the competition for managers. At the macro level this generates greater dispersion of income. If we think that the process of domestic and international financial liberalization that have taken place during the last thirty years had the effect of increasing the degree of competition for human capital, then our paper provides an explanation for the dramatic changes observed in the compensation structure of the financial sector and the overall increase in income inequality.

The organization of the paper is as follows. In the next Section 2 we relate our paper to the existing literature on optimal contracts within the firm. We also describe how the recent evolution in the organizational structure of financial institutions may have changed the contractual incentives within the firm. In Section 3 we describe the theoretical environment and in Section 4 we characterize the optimal contract under several assumptions about commitment. In Section 5 we provide a numerical characterization of the equilibrium and relates its properties to the empirical facts that motivate this paper. Section 6 concludes.

2 Relation to the literature

The heart of the modern theory of executive compensation is the conflict of interest that characterizes the relationship between executives and their ultimate employers, i.e. the shareholders. The conflict arises because executives are called on to take actions that are not contractible in advance and that have differential effects on their own welfare and on the welfare of the ultimate owners, the shareholders. These issues and their impact on the very structure of organizations have been studied extensively.[3] The conclusion from most of this literature is that incomplete contracts have implications for the capital structure of firms. They influence the ideal ownership of the firms assets, the control of the surplus, and the compensation of the managers of the firms assets.

Historically it was common for investment firms to be organized as partnerships. Many argued that this was a preferred form of organization because in a partnership, the managers and investors were the same and when risks were taken it was the partners own assets that were at risk. This was largely consistent with the view that emerges from the incomplete contracting literature of Grossman and Hart (1986) and Hart and Moore (1990) that more efficient organizational forms are those where the agents who control the allocation of the surplus from investments own more of the assets.

Until 1970 the New York Stock Exchange prohibited member firms from being public companies. When this rule was relaxed there was huge rush to go public and partnerships virtually disappeared. Merrill Lynch went public in 1971, followed by Bear Stearns in 1984, Morgan Stanley in 1985, Lehman Brothers in 1994 and Goldman Sachs in 1999. Other venerable investment banks were taken public and either absorbed by commercial

[3]This literature is vast, beginning with Grossman and Hart (1986).

banks, or converted to bank holding companies. The same evolution occurred elsewhere, for example in Britain where the closed ownership Merchant Banks virtually disappeared.

The partnership form and its customs had some important implications for compensation. The capital in a partnership and the ownership shares are typically relatively illiquid so it was difficult for partners to liquidate their ownership positions and move to other firm. Also important is the process of becoming a partner. In the typical firm, new professionals are hired as associates and after a certain trial period, they are either chosen to be partners or released. In this environment separation is viewed as a signal of inferior performance, thus affecting the external option of a financial professional. Becoming a partner, on the other hand, represented a firm commitment to continued employment on the part of the other partners. Thus the change in organizational form was really significant for the nature of contracts and competition in the financial sector.

Another features of financial firms that is worth noting is that the market does not value highly the large complex financial institutions that characterize world financial markets today. The ratio of market value to book value of assets for the 10 largest U.S. and European financial firms is 0.58 and for the largest U.S. firms is 0.50. In contrast the ratio of market value to book value for the total market ETF is 1.2 and averages 0.71 over the long run. This may well be a reflection of compensation practices in the modern financial sector for reasons we will cite below.

The basic framework that is used to study most executive compensation is adapted from the model of dynamic moral hazard by Spear and Srivastava (1987).[4] It posits that shareholders, or their representatives, choose the compensation package that maximizes their ex-ante expected value while delivering to the manager a given ex-ante expected utility, which we denote as $\underline{v}$. The model is silent about the determinants of this utility. In order to enhance its descriptive power, it must be coupled with a theory of the determination of $\underline{v}$.

The standard Principle Agent problem assumes that an employment contract is chosen to maximize the residual rents to the investors (shareholders/managers), conditional on delivering the promised utility to managers, subject to appropriate participation constraints, and assuming that the investor can fully commit. The optimal allocations specified by such a contract can be implemented through a series of state contingent cash payments. However, the assumption of full commitment is problematic. Employers can often renege on compensation contracts—easily in the case of Chapter 11 bankruptcy—and compensation related litigation has increased significantly over time. Clementi, Cooley and Wang (2006) describe how stock grants can be used as a device to overcome the problems of limited commitment where cash compensation is concerned. This is consistent with the finding of Clementi and Cooley (2009) that the use of stock grants and other forms of deferred compensation has increased over time, particularly in the financial sector. This does not, however, take account of strategic behavior on the part of managers and the important role of their outside options.

Aside from that, the standard model is silent on the issue of limited commitment.

[4]Among the models in this class see, for example, Wang (1997), Quadrini (2004), Clementi and Hopenhayn (2006).

It assumes that firms and managers commit to the policies specified by the contract and it ignores or takes as exogenous the outside option of the manager. An important consequence of the demise of the partnership form is that financial managers are no longer constrained by the limited liquidity of the portion of their wealth that is tied to their firm. In a sense the partnership form of organization was also a way of solving the commitment problem by insuring that the managers had a greater ownership share of the assets as prescribed by Hart and Moore (1990).

Until the onset of the financial crisis, turnover of managers was increasing and competition for skilled financial managers was very robust. This is completely consistent with the increase in earnings in the financial sector and the increase in employment. External labor market conditions are very important for compensation within a firm. The important element that is missing in understanding the impact of this environment, is the role of limited commitment on the part of both investors and managers. The nature of contracts in environments with limited commitment has been studied in Cooley, Marimon and Quadrini (2004) and Marimon and Quadrini (2011).

In a competitive setting managers have a strategic interest in increasing the value of their outside option, even if this could be detrimental to investors. We show in this paper that even if investors are risk neutral, this strategic behavior in a dynamic setting with limited commitment can lead to greater risk taking and higher compensation for managers. Importantly, there are features of this environment that cannot be fixed by redesigning the contract. The importance of limited commitment and endogenous outside options will be made clear in the following sections.

3 The model

We consider a firm that consists of a partnership between an investor who provides financial capital and a manager with human capital h_t who has the know-how to operate the firm. The central decision of the firm, regulated by a contract between the investor and the manager, is the choice of risky projects. The income generated by the firm in period t is equal to the human capital of the manager h_t. Human capital can be increased over time by investing in risky projects. The main choice of the firm is given by a variable $\lambda_t \in [0,1]$ which denotes the scale of the innovation project. Given the choice of λ_t, the firm incurs the investment cost $\kappa\lambda_t h_t$ and the next period human capital evolves according to

$$h_{t+1} = (1 + \lambda_t \varepsilon_{t+1}) h_t,$$

where $\varepsilon_{t+1} \in \{0, \bar{\varepsilon}\}$ is a stochastic variable. The probability of the good outcome $\bar{\varepsilon}$ is denoted by p. The stochastic nature of ε_{t+1} makes the innovation project risky.

Agents have preferences over consumption represented by utility functions $u(c)$ and $U(c)$ for the manager and the investor, respectively. They satisfy $u' > 0$, $u'' < 0$ and $U' > 0$, $U'' \leq 0$. We allow for different discounting, δ for the investor and β for the manager.

To use a compact notation, we define $y(\lambda_t) = 1 - \kappa\lambda_t$ the income per unit of human capital, net of the investment cost and $g(\lambda_t, \varepsilon_{t+1}) = (1 + \lambda_t \varepsilon_{t+1})$ the gross growth rate of human capital. The net income and the evolution of human capital can be written as

$$\begin{aligned} Y_t &= y(\lambda_t)h_t, \\ h_{t+1} &= g(\lambda_t, \varepsilon_{t+1})h_t. \end{aligned}$$

A nice property of these functions is that they are linear in h_t, a property that will be convenient for the analytical characterization of the contract. In particular, when there is an optimal stationary investment plan $\lambda^* \in [0,1]$, human capital and output are expected to grow at the constant rate

$$E_t\left(\frac{y_{t+1}-y_t}{y_t}\right) = E_t\left(\frac{h_{t+1}-h_t}{h_t}\right) = p\bar{\varepsilon}\lambda^*,$$

where p is the probability that the innovation project succeeds ($\varepsilon_{t+1} = \bar{\varepsilon}$).

Managers have the option to quit the firm and sign a new contract with another firm. However, only a fraction ρ of human capital can be transferred to the new firm. The parameter $\rho \in [0,1]$ provides a measure of *competition* in the market for managerial capital, in the sense that higher values of ρ correspond to more competitive economies because managers have the ability to transfer a higher share of their human capital.

We will consider alternative regimes characterized by different degrees of commitment to the partnership decisions and invoke different assumptions about the *control* of the investment choice.

3.1 Full commitment by managers and investors

As a benchmark, we consider the case where both investors and the managers can commit to a contract from period zero. The partnership requires to solve a standard contractual problem, which takes the form

$$\max_{\{C_t,\lambda_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U\Big(y(\lambda_t)h_t - C_t\Big)$$

s.t.

$$h_{t+1} = g(\lambda_t, \varepsilon_{t+1})h_t \text{ and } h_0 \text{ given,} \tag{1}$$

$$(1-\beta)\, E_0 \sum_{t=0}^{\infty} \beta^t U(y(\lambda_t)h_t - C_t) \geq \overline{V}, \tag{2}$$

$$(1-\beta)\, E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \geq v(\rho h_0), \tag{3}$$

where $u(c)$ and $U(c)$ are the utility functions the manager and the investor, respectively. They satisfy $u' > 0$, $u'' < 0$ and $U' > 0$, $U'' \leq 0$. Furthermore, $\overline{V}$ and $v(\rho h_0)$ are the initial reservation values for the investor and the manager, respectively.

Since the problem is convex, we can formulate it as a Planner's problem, in which both agents are assigned weights so as to satisfy their participation constraints. In particular, normalizing the weight to the investor to one, manager's weight, μ_0, can be set to satisfy (2) with equality.

$$\max_{\{C_t,\lambda_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \Big[U\Big(y(\lambda_t)h_t - C_t\Big) + \mu_0 u(C_t)\Big]$$
$$\text{s.t.}$$
$$\text{and } h_{t+1} = g(\lambda_t, \varepsilon_{t+1})h_t; \text{ and } h_0 \text{ given.}$$

The problem for $\{C_t,\, \lambda_t\}_{t=0}^{\infty}$ has a dynamic programming formulation:

$$V(h) = \max_{C,\lambda}\{(1-\beta)\left(U(y(\lambda)h - C) + \mu u(C)\right) + \beta EV(h')\}$$
$$\text{s.t.} h' = g(\lambda, \varepsilon')h,$$

with optimal policies $C^* = C^{FC}(h), \lambda^* = \lambda^{FC}(h)$, which, as the value function V, implicitly depend on μ. The first order condition with respect to C results in the familiar intrapersonal condition,

$$\frac{U'\left(y(\lambda)h - C\right)}{u'(C)} = \mu. \tag{4}$$

With respect to the investment decision, λ,assuming differntiability of V_μ, the FOC is:

$$-\left(1-\beta\right)U'\left(y(\lambda)h - C\right)\kappa + \beta EV'(g(\lambda,\varepsilon')h)g_\lambda(\lambda,\varepsilon') - \eta \leq 0 \tag{5}$$

where $\eta h \geq 0$ is the Lagrange multiplier for the constraint $\lambda \leq 1$; i.e. $\eta h\left(1-\lambda\right) = 0$. In particular, $\lambda \in (0,1)$ whenever

$$-\left(1-\beta\right)U'\left(y(\lambda)h - C\right)\kappa + \beta p\overline{\varepsilon}V'((1+\lambda\overline{\varepsilon})h) = 0 \tag{6}$$

and $\lambda = 0$ whenever

$$-\left(1-\beta\right)U'\left(h - C\right)\kappa + \beta p\overline{\varepsilon}V'(h) \leq 0. \tag{7}$$

By the envelope theorem:

$$V'(h) = \left(1-\beta\right)U'\left(y(\lambda)h - C\right)y(\lambda) + \beta EV'(h')g(\lambda,\varepsilon'),$$

and (7) can be written as

$$U'\left(y(\lambda)h - C\right)\kappa = \beta p\overline{\varepsilon}\left[U'\left(y(\lambda')(1+\lambda\overline{\varepsilon})h - C'\right)y(\lambda') + \beta EV'((1+\lambda\overline{\varepsilon})h')g(\lambda',\varepsilon'')\right].$$

Therefore, if $\lambda = 0$, $V'(h')g(\lambda,\varepsilon') = V'(h)$,

$$V'(h) = U'\left(h - C\right),$$

and (7) simplifies to

$$\left(1-\beta\right)\kappa \geq \beta p\overline{\varepsilon}. \tag{8}$$

Denote the benefit-cost ratio by $B = \frac{\beta p\overline{\varepsilon}}{(1-\beta)\kappa}$, and notice that, for any set of parameters $(p, \overline{\varepsilon}, \kappa)$, there is a $\overline{\beta} \in (0,1)$, such that $\frac{\overline{\beta} p\overline{\varepsilon}}{(1-\overline{\beta})\kappa} = 1$ and, therefore, for any $\beta \in (0, \overline{\beta})$, $B < 1$, and for any $\beta \in \left(\overline{\beta}, 1\right)$, $B > 1$.

Lemma 1 *If $B < 1$ the optimal investment plan is the stationary investment plan of zero growth, $\lambda^{FC}(h) = 0$, for all h. While, if $B > 1$ the optimal investment plan is risky: $\lambda^{FC}(h) > 0$.*

It is important to notice that, given h, the investment decision does not depend on μ or ρ. In other words, since the degree of competition ρ does not appear in (5). it follows that *with two-sided full commitment, the degree of competition ρ only plays a role in determining the initial wealth distribution within the partnership, given by μ, but does not affect investment decisions.*

A special, and more tractable, case is when the investor is risk-neutral and the manager's preferences are represented by a log utility; in this case, the optimal investment is independent of h; i,e, $\lambda^{FC}(h) = \lambda^*$. We discuss and comute this case in the next section.

3.2 Limited enforcement with λ_t controlled by investors

We now consider departures from the partnership with full commitment. We first assume that manager is free to quit the partnership after observing the value of the realized investment. For the moment, we continue to assume that the managers commit to implement the agreed optimal investment λ_t. This is equivalent to assuming that the investor has control of the investment λ_t. The contract must satisfy the inter-temporal participation constraints of the manager. The optimization problem involves the additional particpation constraints:

$$(1-\beta)\,\beta E_{t+1}\sum_{n=0}^{\infty}\beta^n\left(u(C_{t+1+n}\right) \geq \beta v(\rho h_{t+1}), \quad t \geq 0 \tag{9}$$

We can now write the optimization problem in lagrangian form with γ_t the multiplier associated with the enforcement constraint in period t. Following Marcet and Marimon (2011), after some re-arrangement, the contractual problem can be written as

$$\min_{\{\gamma_t\}_{t=0}^{\infty}} \max_{\{C_t,\lambda_t\}_{t=0}^{\infty}} \quad E_0\sum_{t=0}^{\infty}\beta^t\left\{(1-\beta)\left[U\Big(y(\lambda_t)h_t - C_t\Big) + (\mu_t+\gamma_t)u(C_t)\right] - \gamma_t v(\rho h_t)\right\}$$
$$\mu_{t+1} = \Big(\mu_t + \gamma_t\Big), \text{ given } \mu_0$$
$$h_{t+1} = g(\lambda_t, \varepsilon_{t+1})h_t, \text{ given } h_0.$$

Notice that without loss of generality $\mu_0 = 0$ since $\mu_1 = \mu_0 + \gamma_0^*$ guarantees that the manager's participation constraint in period zero, (3), is satisfied. The correspondig value function satisfies the Bellman saddle-point equation:

$$W(h,\mu) = \min_{\gamma(\varepsilon')}\max_{C,\lambda}\left\{(1-\beta)\left[U((y(\lambda)h - C) + (\mu+\gamma)\,u(C)\right] - \gamma v(\rho h)) + \beta E W(h',\mu')\right\}$$
$$\text{s.t.} \quad \mu' = \mu + \gamma, \quad h' = g(\lambda, \varepsilon')h.$$

with optimal policies $C^* = C^{LC}(h,\mu), \lambda^* = \lambda^{LC}(h,\mu)$ and $\gamma^*(h,\mu)$. The optimal consumption is given by

$$\frac{U'\left(y(\lambda)h - C\right)}{u'(C)} = \mu + \gamma^*, \tag{10}$$

instead of – and being equal to, when $\gamma^* = 0$, – (4). In other words, $C^{LC}(h, \mu)$ has state variables (h, μ) but it also a 'best reply' to the current Lagrange multiplier γ^*, which determines the evolution of μ.

With respect to the investment decision, λ,the first order condition is

$$-\left(1 - \beta\right) U'\left(y(\lambda)h - C\right) \kappa + \beta E\left[W_h(g(\lambda, \varepsilon')h, \mu')g_\lambda(\lambda, \varepsilon')\right] - \eta \leq 0 \tag{11}$$

In particular, $\lambda \in (0, 1)$ whenever

$$-\left(1 - \beta\right) U'\left(y(\lambda)h - C\right) \kappa + \beta p\bar{\varepsilon} W_h((1 + \lambda\bar{\varepsilon})h, \mu') = 0 \tag{12}$$

and $\lambda = 0$ whenever

$$-\left(1 - \beta\right) U'\left(h - C\right) \kappa + \beta p\bar{\varepsilon} W_h(h, \mu') \leq 0. \tag{13}$$

By the Envelope theorem,

$$W_h(h, \mu) = \left(1 - \beta\right) U'\left(y(\lambda)h - C\right) y(\lambda) - \gamma^* \rho v'(\rho h) + \beta E W_h(h', \mu') g(\lambda, \varepsilon');$$

therefore, if $\lambda = 0$

$$W_h(h, \mu) = \left(1 - \beta\right) U'\left(h - C\right) - \gamma^* \rho v'(\rho h) + \beta W_h(h, \mu^{*\prime});$$

if, in addition, $\gamma^* = 0$, we recover the full-commitment condition (8). However, if $\gamma^* > 0$ (13) becomes

$$\left(1 - \beta\right) \kappa \geq \beta p\bar{\varepsilon} - \beta(1 - \beta)^{-1} p\bar{\varepsilon} \left[\gamma^* \rho v'(\rho h) - \beta \left(W_h(h, \mu^{*\prime}) - W_h(h, \mu)\right)\right] / U'\left(h - C\right). \tag{14}$$

This last inequality shows that when enforcement constraints are binding there are three three effects which are not present in (8): *i)* the marginal value of increasing the outside value next period $(\gamma^* \rho v'(\rho h))$; *ii)* the competitiveness effect measured by ρ, provided $\rho v'(\rho h) \neq v'(h)$, and *iii)* the effect of increasing the Pareto weight of the manager in W_h. These effects are also present when $\lambda > 0$. So, if we compare investment decisions without limited enforcement to those with full enforcement: the first two effects tend to reduce λ, when $\lambda > 0$, while the third tends to increase it[5]. In other words, the first two effects make the partnership more cautious, while the latter effect makes it more prompt to take risks and it is more likely to be predominant when the manager has (has achieved) a high Pareto weight in the partnership.

[5]To see this, notice that if $\lambda^{LC}(h, \mu^{*\prime}) = 0$, then $\frac{W_h(h, \mu^{*\prime}) - W_h(h, \mu)}{U'\left(h - C^{LC}(h, \mu)\right)}$ in the above inequality becomes $\frac{U'\left(h - C^{LC}(h, \mu^{*\prime})\right)}{U'\left(h - C^{LC}(h, \mu)\right)} - 1 > 0$. Notice that this third effect is not present when W is independent of h.

Proposition 2 *Assume $v'(h) > 0$ and $v'(h) \searrow 0$ as $h \nearrow \infty$. With limited enforcement and investment decisions controlled by the investor, the investment choice is distorted whenever $\gamma^* > 0$. This distortion increases with ρ, provided that $hv'(h) \neq 1$. However, $\gamma_t^* \searrow 0$ as $t \nearrow \infty$ and, therefore, the distortion asymptotically disappears.*

We make assumptions on v' which almost imply the result that distortions tend to dissapear; however, when we endogeneize v, these assumptions become properties of the endogeneous value functions, to be shown. In particular, the value function of the partnership takes the form $W(h, \mu) = \widehat{\omega}(h, \mu) + \mu\omega(h, \mu)$, where ω is the value function of the manager satisfyies the following recursive equation:

$$\omega(h, \mu) = (1 - \beta)\, u(C^*) + \beta E\omega(h'^*, \mu'^*)].$$

We will take $v(\rho h) = \omega(\rho h, \mu_0)$, where μ_0 is the initial Pareto weight in starting a new partnership with ρh human capital.

3.3 Limited enforcement with λ_t controlled by managers

The partnership problem with limited enforcement can be decomposed in two parts. On the one hand, the investment decision, which is given by $\lambda^{LC}(h, \mu)$ when the investor controls the investment decisions and, on the other hand, the manager Pareto weights, $\mu'^* = (\mu + \gamma^*)$, which determine the division of output within the partnership and, in the case where the investor controls the investment decisions, results in $C^{LC}(h, \mu)$. If managers control the investment process, they will take into account that riskier investments will increase their outside value. In principle, this imposes further incentive-compatibility constraints on the optimal contract. In addition to making sure that the manager does not quit, the compensation structure must also guarantee that the manager implement the recommended investment $\lambda^{LC}(h, \mu)$. This corresponds to modeling the partnership as a dynamic *Principal Agent problem:*

$$\max_{\{C_t, \lambda_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \Big[U\Big(y(\lambda_t)h_t - C_t\Big) + \mu_0 u(C_t) \Big]$$

$$\text{s.t. } t \geq 0 \quad E_t \sum_{n=0}^{\infty} \beta^n \left(u(C_{t+n}) \geq v(\rho h_t), \right. \tag{15}$$

$$E_t \left[\beta \sum_{n=0}^{\infty} \beta^n \left(u(C_{t+1+n}) \mid \varepsilon_{t+1}\right.\right] \geq E_t \beta v(\rho g(\widehat{\lambda}_t, \varepsilon_{t+1}) h_t), \quad \text{for all } \widehat{\lambda}_t \in [0, 1] \tag{16}$$

$$\text{and } h_{t+1} = g(\lambda_t, \varepsilon_{t+1}) h_t; \text{ and } h_0 \text{ given.}$$

Notice that the intertemporal *enforcement constraints* (15) incorporate (3) and (9), while (16) are the intertemporal *incentive-compatibility constraints.* We will also require that the contract specifies punishments and rewards which satisfy (15). In any case, when the manager controls invesment decisions she takes the partnership contract as given and, in doing so, takes into consideration how investment decision affect the the evolution of her Pareto weights; that is, she takes into account the fact that the investor must match outside offers in order to maintain the productive partnership and that threats that do not satisfy her participation constraints are not credibe.

Introducing the *incentive-compatibility costraints* (16) within the partnership contract avoids the hold-up problem of having the manager manipulating investments by only taking into account her prefences, but we would like to be more explicit about how the manager internalizes the cost of distoritng investments and how the investor comits to satisfy the above intertemporal constraints. We start by considering a partnership contract *without incentive-compatibility costraints*. There are three reasons to do so: first, it maybe that only 'realized' outside offers are credible, which is consistent with the common practice of making counteroffers when outside offers are made and not when the manager simply says that may receive outside offers; second, incentive compatibility constraints require a higher level of commitment of the investor, since he has commit not only to realized, but also to potential, outside offers; third, even if the investor is committed to match potential offers, it may well be that other investors – say, all but him – in the sector are not, in which case to obtain the outside value of the manager we need to know what she will get when investors only commit to match 'realized offers' and, therefore, guaranteeing that incentive-compatibility punishments and rewards are properly subject to participation constraints[6].

4 An optimal contract

We now specialize the model to particular preferences which allows us to derive a more precise characterization of the equilibrium. In particular we assume that investors have linear preferences and managers have logarithmic utility. Since human capital grows on average over time, it will be convenient to normalize all growing variables so that they become stationary in levels.

Values of managers and investors: Denote by Q_t the manager's value of the contract which is given by

$$Q_t = E_t \sum_{j=0}^{\infty} \beta^j \log(C_{t+j}).$$

This can be written recursively as

$$Q_t = \log(C_t) + \beta E_t Q_{t+1}.$$

Let $\tilde{\beta} = 1/(1-\beta)$. If we subtract $\tilde{\beta}\log(h_t)$ on both sides and then we add and subtract $\beta\tilde{\beta}E_t\log(h_{t+1})$ on the right hand side we obtain

$$Q_t - \tilde{\beta}\log(h_t) \quad = \quad \log(C_t) - \tilde{\beta}\log(h_t) + \beta\tilde{\beta}E_t\log(h_{t+1}) + \beta E_t\left[Q_{t+1} - \tilde{\beta}\log(h_{t+1})\right].$$

[6]We will analytically discuss these two cases in more detail in the next draft.

If we define $C_t = c_t h_t$ and use this expression to eliminate C_t we obtain

$$Q_t - \tilde{\beta}\log(h_t) \quad = \quad \log(c_t) + \beta\tilde{\beta}E_t \log\left(\frac{h_{t+1}}{h_t}\right) + \beta E_t\left[Q_{t+1} - \tilde{\beta}\log(h_{t+1})\right].$$

This can be rewritten more compactly as

$$q_t = \log(c_t) + \beta E_t\left[\tilde{\beta}\log\Big(g(\lambda_t, \varepsilon_{t+1})\Big) + q_{t+1}\right]. \tag{17}$$

The variables c_t and $q_t = Q_t - \tilde{\beta}\log(h_t)$ are, respectively, the normalized consumption and manager's value.

We now turn to the value for the investor V_t, which is equal to

$$V_t = E_t \sum_{j=0}^{\infty} \delta^j \Big[y(\lambda_{t+j})h_{t+j} - C_{t+j}\Big].$$

Notice that we allow for different rates of time preferences between managers and investors (e.g. $\beta < \delta$), but as discussed in the previous section, our main results do not depend on this difference. The value of the investor can be written recursively as

$$V_t = E_t\Big[y(\lambda_t)h_t - C_t\Big] + \delta E_t V_{t+1},$$

and normalized to

$$v_t = y(\lambda_t) - c_t + \delta E_t g(\lambda_t, \varepsilon_{t+1})v_{t+1}, \tag{18}$$

where $v_t = V_t/h_t$ is the normalized investor's value.

Manager's outside value: The manager has the option of quitting and sign a contract with a new firm. However, only some of the human capital can be transferred to the new firm.

Let $\bar{Q}_{t+1}(\rho h_{t+1})$ be the value of quitting at the beginning of $t+1$. This is a function of the human capital that the manager can transfer to the new firm, that is, ρh_{t+1}.

We can normalize the value of quitting in the same way we normalized Q_t, that is,

$$\bar{q}_{t+1} = \bar{Q}_{t+1}(\rho h_{t+1}) - \tilde{\beta}\log(\rho h_{t+1}).$$

This is the normalized value that the manager obtains from the new contract. If there is free entry with competition for managers, this value is determined by the breaking-even condition for the new investor, that is, $V_{t+1}(\rho h_{t+1}) = 0$. Since $V_{t+1}(\rho h_{t+1}) = v_{t+1}\rho h_{t+1}$, the normalized value of quitting $\bar{q}_{t+1}$ is determined by the condition $v_{t+1} = 0$.

From now on we assume that there is competition for managers through free entry. Thanks to the linearity of the model with respect to h, $\bar{q}_{t+1}$ is independent of time. This should be the case because h rescales the model proportionally but leaves the normalized value of q that satisfies the break-even condition for the new entrant unchanged. Therefore, we can write $\bar{q}_{t+1} = \bar{q}$.

Enforcement and incentive-compatibility: We specify the contractual problem as maximizing the value for the investor subject to participation, enforcement, and incentive-compatibility constraints. We use a recursive approach that is made possible by the normalization described above (so that the model is stationary in level). Let's define first the enforcement constraint which guarantees that the manager does not quit the firm. This requires

$$Q_{t+1}(h_{t+1}) \geq \bar{Q}_{t+1}(\rho h_{t+1}).$$

Subtracting $\tilde{\beta}\log(h_{t+1})$ on both sides and then adding and subtracting $\tilde{\beta}\log(\rho h_{t+1})$ on the right-hand-side, the enforcement constraint can be written in normalized form as

$$q_{t+1}(\varepsilon_{t+1}) \geq w(\rho, \bar{q}),$$

where $w(\rho, \bar{q}) = \bar{q} + \tilde{\beta}\log(\rho)$. Therefore, the normalized outside value does not depend on h_{t+1}, and therefore, on λ. Intuitively, λ does affect the non-normalized outside value $\bar{Q}_t$. However, it also affects the value of not quitting Q_{t+1}. Since the impacts on $\bar{Q}_{t+1}$ and Q_{t+1} are additive and of the same magnitude (thanks to log-utility), they cancel out.

We can now consider the incentive compatibility constraint,

$$E_t Q_{t+1}\Big(g(\lambda_t, \varepsilon_{t+1})h_t\Big) \geq E_t\bar{Q}_{t+1}\Big(\rho g(\hat{\lambda}_t, \varepsilon_{t+1})h_t\Big), \qquad \text{for all } \hat{\lambda}_t \in [0, 1],$$

where we made explicit that the human capital at $t+1$ depends on the choice of λ_t if the manager does not deviate from the recommended policy and $\hat{\lambda}_t$ if the manager deviates.

Following the same procedure used to normalize the enforcement constraint we obtain

$$E_t q_{t+1}(\varepsilon_{t+1}) \geq w(\rho, \bar{q}) + \tilde{\beta}E_t \log\left(\frac{g(\hat{\lambda}_t, \varepsilon_{t+1})}{g(\lambda_t, \varepsilon_{t+1})}\right),$$

where $w(\rho, \bar{q})$ was defined above as $w(\rho, \bar{q}) = \bar{q} + \tilde{\beta}\log(\rho)$.

From this equation we can see that ρ enters additively in the right-hand-side of the incentive-compatibility constraint. Therefore, the choice of $\hat{\lambda}_t$ is not affected by ρ.

Alternative assumption about mobility: We now propose an alternative assumption that introduces dependence of $\hat{\lambda}_t$ from the degree of human capital mobility. Suppose that, if the manager quits, with probability η he/she will be able to transfer ρh_{t+1} to the new firm. However, with probability $1-\eta$ he/she can transfer only ρh_t. Therefore, with some probability the manager cannot transfer any of the newly created human capital. The parameter η also captures the degree of human capital mobility. The role of ρ is the same as before. The parameter η, instead, plays a different role as we will see.

Let's reconsider the enforcement constraint which now becomes

$$Q_{t+1}(h_{t+1}) \geq \eta\bar{Q}_{t+1}(\rho h_{t+1}) + (1-\eta)\bar{Q}_{t+1}(\rho h_t).$$

Subtracting $\tilde{\beta}\log(h_{t+1})$ on both sides and then adding and subtracting $\eta\tilde{\beta}\log(\rho h_{t+1}) + (1-\eta)\tilde{\beta}\log(\rho h_t)$ on the right-hand-side, the enforcement constraint can be written in normalized form as

$$q_{t+1}(\varepsilon_{t+1}) \geq w(\rho, \bar{q}) - (1-\eta)\tilde{\beta}\log\Big(g(\lambda_t, \varepsilon_{t+1})\Big),$$

where the term $w(\rho, \bar{q}) = \bar{q} + \tilde{\beta}\log(\rho)$ is as before.

Differently from the previous formulation, the right-hand-side depends on λ_t (provided that $\eta < 1$). Thus, increasing λ_t is less appealing for the investor when the enforcement constraint is binding because this must be associated with higher compensation for the manager.

Let's look now at the incentive compatibility constraint,

$$E_t Q_{t+1}\Big(g(\lambda_t, \varepsilon_{t+1})h_t\Big) \geq \eta E_t \bar{Q}_{t+1}\Big(g(\hat{\lambda}_t, \varepsilon_{t+1})\rho h_t\Big) + (1-\eta)\bar{Q}_{t+1}(\rho h_t),$$

which must be satisfied for all $\hat{\lambda}_t \in [0, 1]$. This can be normalized to

$$E_t q_{t+1}(\varepsilon_{t+1}) \geq w(\rho, \bar{q}) + \tilde{\beta} E_t \log\left(\frac{g(\hat{\lambda}_t, \varepsilon_{t+1})^{\eta}}{g(\lambda_t, \varepsilon_{t+1})}\right).$$

In this new formulation the parameter η acts as a multiplicative factor to the growth rate of h. Higher is η and higher is the incentive for the manager to raise $\hat{\lambda}_t$ in order to increase the outside value.

To see this more clearly, suppose that innovations require effort from the manager. The utility of the manager takes the form

$$\log(C_t) + \alpha \log(1 - \lambda_t).$$

With this specification, the incentive-compatibility constraint becomes

$$\beta E_t q_{t+1}(\varepsilon_{t+1}) \geq \beta w(\rho, \bar{q}) + \alpha \log\left(\frac{1-\hat{\lambda}_t}{1-\lambda_t}\right) + \beta\tilde{\beta} E_t \log\left(\frac{g(\hat{\lambda}_t, \varepsilon_{t+1})^{\eta}}{g(\lambda_t, \varepsilon_{t+1})}\right).$$

The strategic behavior of the manager is captured by the optimal choice of $\hat{\lambda}_t$. This maximizes the right-hand-side of the incentive-compatibility constraint and it is characterized by the first order condition

$$E_t\left[\frac{(1-\hat{\lambda}_t)g_{\hat{\lambda}}(\hat{\lambda}_t, \varepsilon_{t+1})}{g(\hat{\lambda}_t, \varepsilon_{t+1})}\right] \leq \frac{(1-\beta)\alpha}{\eta\beta\tilde{\beta}}, \tag{19}$$

which is satisfied with the inequality sign if $\hat{\lambda}_t = 0$. We do not have to worry about the upper bound on $\hat{\lambda}_t$ since the cost of choosing $\hat{\lambda}_t = 1$ is infinity.

The left-hand-side term is strictly decreasing in $\hat{\lambda}_t$, implying that the strategic innovation choice $\hat{\lambda}_t$ is increasing in the mobility parameter η. Instead, the parameter ρ does not affect the choice of $\hat{\lambda}_t$. Notice that this specification embeds the baseline specification, which is obtained when $\alpha = 0$ and $\eta = 1$. In this case the optimal 'deviation' choice of $\hat{\lambda}_t$ is 1.

4.1 Contract with investor's commitment

We can now specify the contractual problem recursively when the investor commits to the contract. This maximizes the investors' value subject to the promised-keeping and incentive-compatibility constraints. In detrended form this can be written as

$$v(q) = \max_{\lambda,c,q(\varepsilon)} \left\{ y(\lambda) - c + \delta E g(\lambda,\varepsilon) v\Big(q(\varepsilon)\Big) \right\} \tag{20}$$

subject to

$$q = \log(c) + \alpha\log(1-\lambda) + \beta E\left[\tilde{\beta}\log\Big(g(\lambda,\varepsilon)\Big) + q(\varepsilon)\right] \tag{21}$$

$$\beta E q(\varepsilon) \geq \beta w(\rho,\bar{q}) + \alpha\log\left(\frac{1-\hat{\lambda}}{1-\lambda}\right) + \beta\tilde{\beta}E\log\left(\frac{g(\hat{\lambda},\varepsilon)^{\eta}}{g(\lambda,\varepsilon)}\right) \tag{22}$$

$$q(\varepsilon) \geq w(\rho,\bar{q}) - (1-\eta)\tilde{\beta}\log\Big(g(\lambda,\varepsilon)\Big), \qquad \text{for all } \varepsilon, \tag{23}$$

where ε denotes next period realization of the shock.

The problem is subject to three constraints. The first is the promise-keeping constraint; the second is the incentive-compatibility constraint; the third is the enforcement constraint. The incentive-compatibility constraint insures that the manager chooses the λ recommended by the investor. The variable $\hat{\lambda}$ is the optimal (manipulation) value of λ chosen by the manager in case he/she deviates from the recommended policy. This solves equation (19) derived above. The enforcement constraint insures that the manager does not quit at the end of the period.

The first order conditions to Problem (20), derived in Appendix A, are

$$\kappa + \frac{\alpha(\gamma+\chi)}{1-\lambda} = \left[\delta v\Big(q(\bar{\varepsilon})\Big) + \left(\frac{\beta\tilde{\beta}}{1+\lambda\bar{\varepsilon}}\right)\Big(\gamma+\chi+(1-\eta)\mu(\bar{\varepsilon})\Big)\right]p\bar{\varepsilon} \tag{24}$$

$$c = \gamma \tag{25}$$

$$\left(\frac{\delta}{\beta}\right) g(\lambda,\varepsilon)\gamma(\varepsilon) = \gamma + \chi + \mu(\varepsilon) \tag{26}$$

This general problem can be specialized to three different environments:

1. **Perfect enforcement**: In this case we solve Problem (20) without constraints (22) and (23). Both parties commit to the contract and, therefore, we do not have to verify whether the manager has an advantage to quit in future period (enforcement constraint) and choose a value of λ different from the one recommended by the optimal contract (incentive-compatibility).

2. **Limited enforcement with λ controlled by the investor**: In this case we solve Problem (20) without constraint (22). The manager does not commit to stay with the firm. So we need to impose the enforcement constraint (23). However, he/she cannot choose λ different from the value recommended by the optimal contract. Thus, we do not have to impose the incentive compatibility constraint (22).

3. **Limited enforcement with λ controlled by the manager**: In this case we solve Problem (20) with all constraints. The manager does not commit to stay with the firm. So we need to impose the enforcement constraint (23). Furthermore, he/she has control of λ and can choose a value different from the one recommended by the investor. Therefore, we have to impose the incentive compatibility constraint (22).

The first order conditions derived above can be easily specialized to the three environments considered above. For example, the environment with full commitment is obtained by imposing χ and $\mu(\varepsilon)$ equal to zero. The environment with limited commitment where the investor controls λ is obtained by imposing $\chi = 0$.

4.2 Contract with double-sided lack of commitment

In the previous section we have considered the case in which the investor commits to the contract. The enforcement and incentive problems were on the side of the manager who could quit and choose an investment level different from the one chosen by the investor. Now we also allow for the limited commitment of the investor. What this implies is that, if the utility promised to the manager is bigger than the outside value, the investor renegotiates down his/her promise.

This case can still be characterized using a specialized version of Problem (20). First, since the manager's utility is always equal to the outside value, the enforcement constraint (23) is always satisfied with equality. Second, and as a consequence of the renegotiation, the manager always chooses a value of λ that maximizes his outside value net of the dis-utility from effort, that is $\hat{\lambda}$. This is the value that solves equation (19). Thus, the optimal contract with double-sided lack of commitment solves

$$v(q) = \max_{c,q(\varepsilon)} \left\{ y(\hat{\lambda}) - c + \delta E g(\hat{\lambda}, \varepsilon) v\Big(q(\varepsilon)\Big) \right\} \tag{27}$$

subject to

$$q = \log(c) + \alpha \log(1 - \hat{\lambda}) + \beta E \left[\tilde{\beta} \log \Big(g(\hat{\lambda}, \varepsilon) \Big) + q(\varepsilon) \right] \tag{28}$$

$$q(\varepsilon) = w(\rho, \bar{q}) - (1 - \eta) \tilde{\beta} \log \Big(g(\hat{\lambda}, \varepsilon) \Big), \qquad \text{for all } \varepsilon, \tag{29}$$

5 Numerical example

We show here the properties of the optimal contract under three environments:

1. **Limited enforcement with λ controlled by the investor:** The investor commits while the manager commits to the choice of λ but not to stay. The contract must satisfy the enforcement constraint for the manager but not the incentive-compatibility constraint for the choice of λ.

2. **Limited enforcement with λ controlled by the manager:** The investor commits while the manager does not commit neither to the choice of λ nor to stay with the firm. The contract must satisfy the enforcement and incentive-compatibility constraints.

3. **Double-sided lack of commitment:** Neither the investor nor the manager commit to the contract. This implies that the manager always receive the outside value since values exceeding the outside option will be renegotiated by the investor.

We assign the following parameter values: $\delta = 0.96$, $\beta = 0.94$, $\bar{\varepsilon} = 0.03$, $p = 0.5$, $\kappa = 0.012$, $\rho = 0.95$, $\alpha = 0.05$, $\eta = 0.5$. The computational procedure is described in Appendix B.

Figure 1 plots the values of next period (normalized) continuation utilities, $q(\varepsilon)$, as a function of current period (normalized) utility, q. Each panel refers to one of the three environments described above. The dotted vertical line is $\bar{q}$, that is, the initial (normalized) utility received from a new contract.

The first thing we observe is that the structure of the contract in the first two environments where the investor commits are very similar. The normalized promised utility received under $\varepsilon = \bar{\varepsilon}$, is lower than the promised utility when the shock is low, $\varepsilon = 0$. Notice, however, that these are normalized utilities. The next period non-normalized utility is higher under $\varepsilon = \bar{\varepsilon}$. The fact that $q(\bar{\varepsilon}) < q(0)$ simply means that the non-normalized utility grows less than the growth in h_{t+1}, which is a consequence of the insurance provided by the optimal contract. However, as the continuation utility reaches a minimum and the lower bound on q binds, the contracts can no longer provide insurance and consumption grows at the same rate as h. In terms of normalized utilities this means that $q(\bar{\varepsilon}) = q(0)$.

In the third environment without commitment from the investor, the manager always gets the outside value. Therefore, whatever the initial q, it jumps immediately to the outside value starting in the next period. Then it fluctuates between these two values over time.

Figure 2 plots the innovation variable λ. While for the first two environments λ increases with the promised utility, in the environment with double-sided lack of commitment, λ is independent of q. More importantly, the innovation is higher in this environment. This is because λ is chosen by the manager to maximize the outside value and it does not take into account the innovation cost $\kappa\lambda$.

5.1 The mobility of managers.

We now look at the central issue of the paper which is the increase in competition captured by the higher degree of human capital mobility and how this affects inequality.

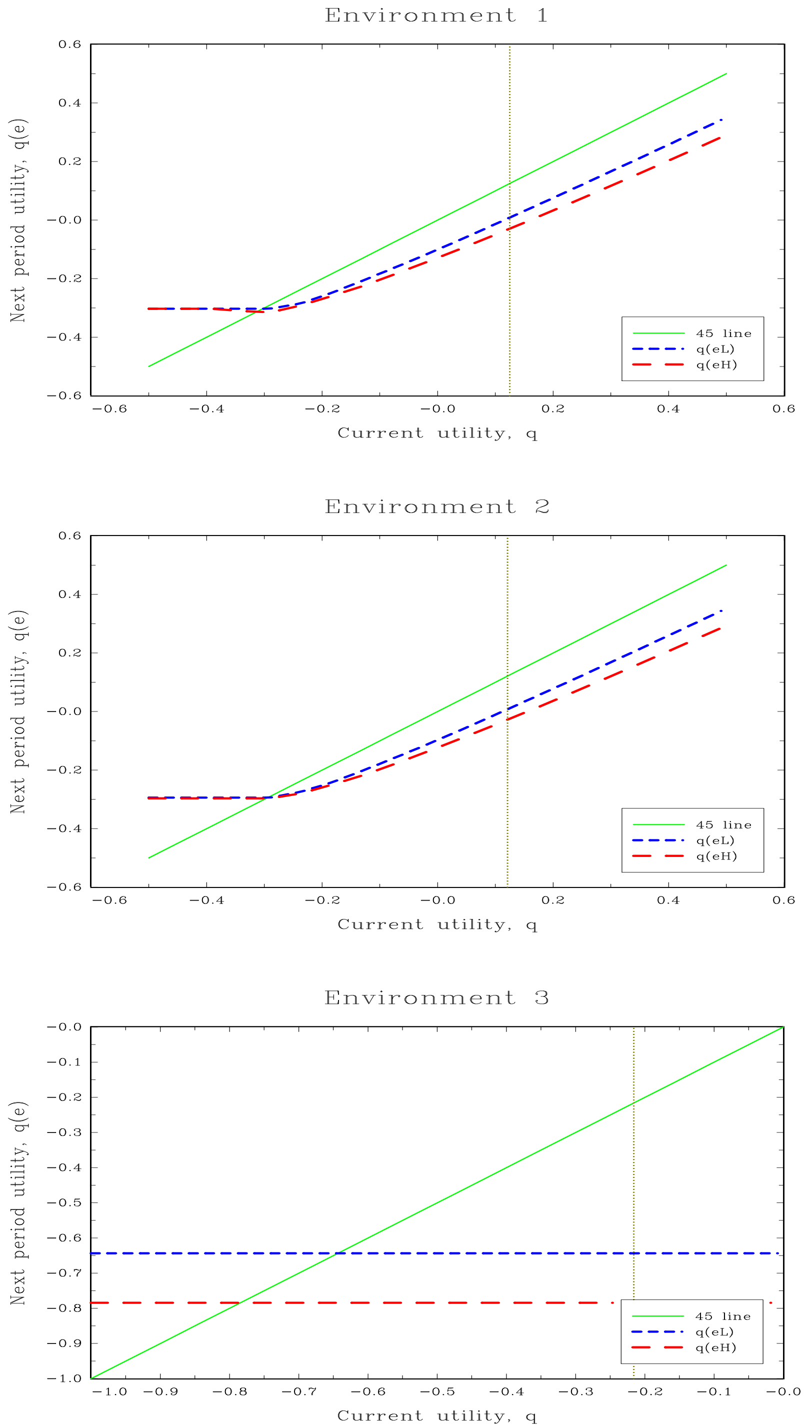

Figure 1: Continuation utilities in optimal contracts.

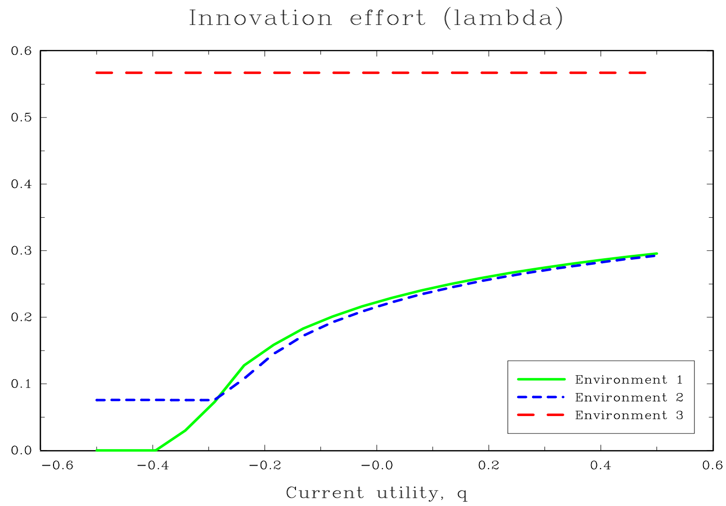

Figure 2: Innovation as a function of q.

In the model mobility is formalized by two parameters, ρ and η. We now show how these two parameters affect the choice of λ.

Figure 3 plots the value of λ as a function of q in each of the three environments and for alternative values of the mobility parameters. Focusing on the first two environments where the investor commits, we observe that higher mobility (either in the form of higher η or higher ρ) reduces innovation. The reason is because higher mobility increases the outside value and a larger share of the surplus needs to be shared with the manager. The opposite result is obtained in the third environment with double-sided lack of commitment. In this case ρ does not affect the innovation λ. However, higher values of η do lead to more innovation. Therefore, the limited commitment from both parties is crucial to have a positive relation between mobility and risk-taking.

5.2 Mobility and inequality

As we have seen above, the environment with double-sided lack of commitment (environment 3) generates a positive relation between mobility and risk-taking. We now want to examine the relation between mobility, captured by the parameter η, and inequality.

Since there is growth in the model, we can focus on the log of normalized human capital $\tilde{h}_t = \log(h_t) - E\log(h_t)$, that is, the log of human capital minus its mean at time t. Then an index of inequality can be defined as the standard deviation of $\tilde{h}_t$, that is,

$$\text{Inequality index} \equiv \text{Std}(\tilde{h}_t)$$

Individual human capital evolves according to $h_{t+1} = h_t g(\lambda, \varepsilon_{t+1})$. Since all managers choose the same $\lambda = \hat{\lambda}$, the expected growth Eg_{t+1} is the same across managers. Taking logs we obtain

$$\log(h_{t+1}) = \log(h_t) + \log(g(\hat{\lambda}, \varepsilon_{t+1})). \tag{30}$$

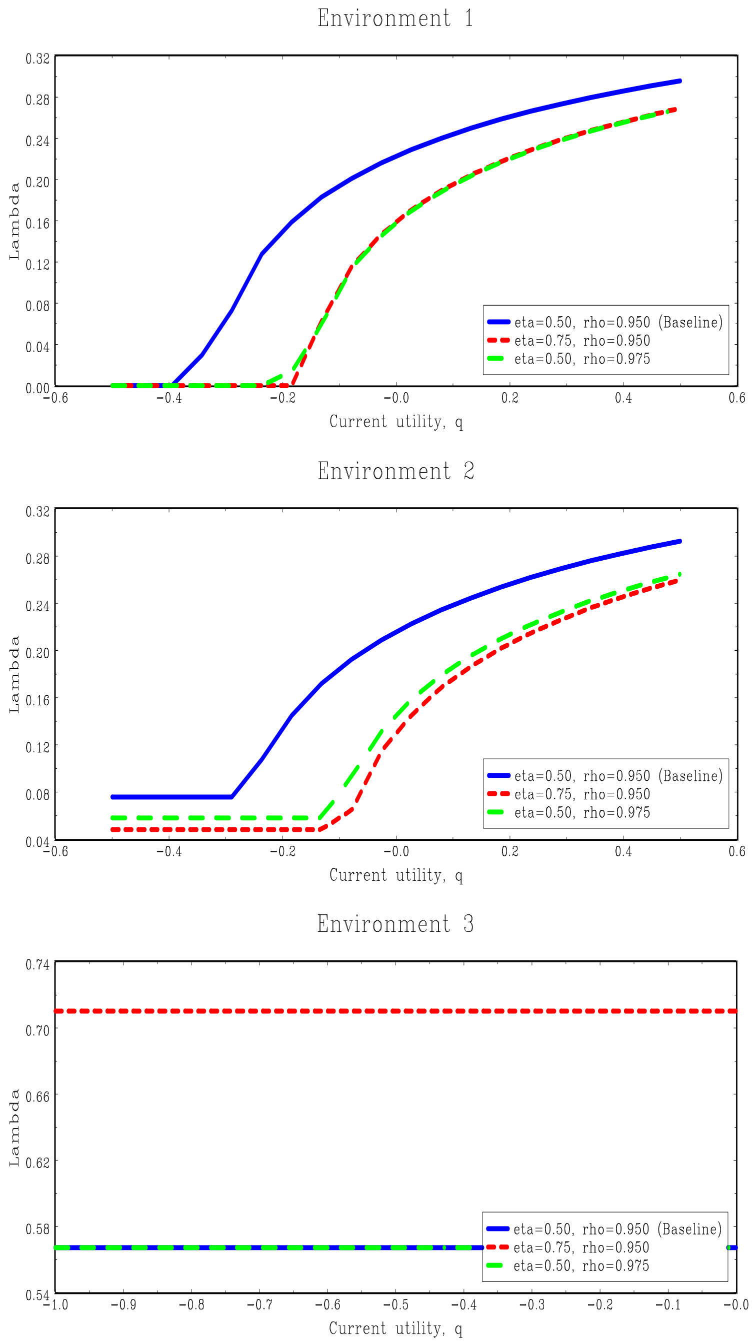

Figure 3: Innovation as a function of q.

Taking advantage of the fact that the expected growth rate is the same across managers, the unconditional mean of $\log(h_{t+1})$ is

$$E\log(h_t) = t \cdot E\log(g(\hat{\lambda}, \varepsilon)).$$

Subtracting the unconditional mean to both sides of equation (30) we obtain

$$\tilde{h}_{t+1} = \tilde{h}_t + \tilde{g}_{t+1},$$

where $\tilde{g}_{t+1} = \log(g(\hat{\lambda}, \varepsilon)) - E\log(g(\hat{\lambda}, \varepsilon))$. Notice that the unconditional mean of $\tilde{h}_t$ is zero since $E\tilde{g}_{t+1} = 0$.

Using the above expression we can compute

$$\text{Std}(\tilde{h}_t) = \sqrt{t} \cdot \text{Std}(\tilde{g}),$$

where we dropped the time subscript in $\tilde{g}$ since it is stationary. We can then see that the index of inequality goes to infinity as we increase t. This can be corrected by assuming entry and exit.

Suppose that in period t there is a unit mass of managers who survive to $t+1$ with probability $1-\phi$. The mass ϕ of exiting managers is replaced by new managers who are endowed with normalized human capital $\tilde{h}_{t+1} = 0$. Appendix C shows that

$$\text{Std}(\tilde{h}) = \sqrt{\frac{1-\phi}{\phi}} \cdot \text{Std}(\tilde{g}). \tag{31}$$

Let's focus now on the standard deviation of growth:

$$\begin{aligned}
\text{Std}(\tilde{g}) &= \text{Std}\Big(\log(g(\hat{\lambda}, \varepsilon)) - E\log(g(\hat{\lambda}, \varepsilon))\Big) \\
&= \text{Std}\Big(\log(g(\hat{\lambda}, \varepsilon))\Big) \\
&= \text{Std}\Big(\log(1 + \hat{\lambda}\varepsilon)\Big) \\
&\approx \text{Std}(\hat{\lambda}\varepsilon) \\
&= \hat{\lambda}\sigma_\varepsilon,
\end{aligned}$$

where σ_ε is the standard deviation of the shock ε. Thus, the final formula for the inequality index is:

$$\text{Std}(\tilde{h}) \approx \hat{\lambda}\sigma_\varepsilon \sqrt{\frac{(1-\phi)}{\phi}}$$

This shows that inequality increases with $\hat{\lambda}$. Thus, greater is the innovation effort and greater is the degree of inequality. The next step is to link $\hat{\lambda}$ to the degree of mobility η. We have seen that in the environment without commitment from both the investor and the manager (environment 3), $\hat{\lambda}$ is increasing in η. Thus we get the result that higher mobility is associated with higher growth but also greater inequality.

6 Conclusion

The fact that inequality has increased over time, especially in anglo saxon countries, is well documented. The increase in inequality has been especially steep for managerial occupations in financial industries. In this paper we propose one possible explanation for this change. We emphasize the increase in competition for human talents that followed domestic and international liberalization of financial markets. In an industry where the enforcement of contractual relations is limited, the increase in competition raises the managerial incentives to undertake risky investments. Although this may have a positive effect on economic growth, the equilibrium outcome is not efficient and generates greater income inequality. The higher competition for managerial talents seems consistent with the evidence that managerial turnover has also increased during the last thirty years.

Appendix

A First order conditions for Problem (20)

Let γ and $\mu(\varepsilon)$ be the lagrange multipliers associated with the promise-keeping constraint and the enforcement constraint. Then the lagrangian can be written as

$$\begin{aligned} v(q) &= y(\lambda) - c + \sum_{\varepsilon}\left[\delta g(\lambda,\varepsilon) v\Big(q(\varepsilon)\Big)\right] p(\varepsilon) \\ &+ \gamma\left\{\log(c) + \alpha\log(1-\lambda) + \beta\sum_{\varepsilon}\left[\tilde{\beta}\log\Big(g(\lambda,\varepsilon)\Big) + q(\varepsilon)\right]p(\varepsilon) - q\right\} \\ &+ \chi\sum_{\varepsilon}\left[\beta q(\varepsilon) - \beta w(\rho,\bar{q}) - \alpha\log\left(\frac{1-\hat{\lambda}}{1-\lambda}\right) - \beta\tilde{\beta}\log\left(\frac{g(\hat{\lambda},\varepsilon)^{\eta}}{g(\lambda,\varepsilon)}\right)\right] p(\varepsilon) \\ &+ \beta\sum_{\varepsilon}\left[q(\varepsilon) - w(\rho,\bar{q}) + (1-\eta)\tilde{\beta}\log\Big(g(\lambda,\varepsilon)\Big)\right]\mu(\varepsilon)p(\varepsilon) \end{aligned}$$

The first order conditions are

$$\lambda: \quad y_{\lambda}(\lambda) + \sum_{\varepsilon}\left[\delta g_{\lambda}(\lambda,\varepsilon) v\Big(q(\varepsilon)\Big) + \beta\tilde{\beta}\left(\frac{g_{\lambda}(\lambda,\varepsilon)}{g(\lambda,\varepsilon)}\right)\Big(\gamma + \chi + (1-\eta)\mu(\varepsilon)\Big)\right]p(\varepsilon) - \frac{\alpha(\gamma+\chi)}{1-\lambda} = 0$$

$$c: \quad -1 + \frac{\gamma}{c} = 0$$

$$q(\varepsilon): \quad \delta g(\lambda,\varepsilon) v_q(q(\varepsilon)) + \beta(\gamma + \chi + \mu(\varepsilon)) = 0$$

and the envelope condition $v_q(q) = -\gamma$. Substituting in the first order conditions and using the functional forms of $y(\lambda)$ and $g(\lambda,\varepsilon)$ we obtain equations (24)-(26).

B Numerical solution for Problem (20)

We describe here the solution procedure for exogenous values of the outside option $w(\rho,\eta)$. The iterative procedure is based on the guesses for two functions

$$\gamma = \psi(q)$$

$$v = \Psi(q).$$

The first function returns the multiplier γ (derivative of investor's value) as a function of the promised utility. The second function gives us the investor value v also as a function of the promised utility.

Given the functions $\psi(q)$ and $\Psi(q)$, we can solve the system

$$\left[\delta v\Big(q(\bar{\varepsilon})\Big) + \left(\frac{\beta\tilde{\beta}}{1+\lambda\bar{\varepsilon}}\right)\Big(\gamma + \chi + (1-\eta)\mu(\bar{\varepsilon})\Big)\right]p\bar{\varepsilon} = \kappa + \frac{\alpha(\gamma+\chi)}{1-\lambda}$$

$$c = \gamma \tag{32}$$

$$\left(\frac{\delta}{\beta}\right) g(\lambda,\varepsilon)\psi(q(\varepsilon)) = \gamma + \chi + \mu(\varepsilon) \tag{33}$$

$$v = -c + \sum_{\varepsilon}\Big[y(\lambda,\varepsilon) + \delta g(\lambda,\varepsilon)\Psi\Big(q(\varepsilon)\Big)\Big]p(\varepsilon) \tag{34}$$

$$q = \log(c) + \alpha\log(1-\lambda) + \beta\sum_{\varepsilon}\Big[\tilde{\beta}\log\Big(g(\lambda,\varepsilon)\Big) + q(\varepsilon)\Big]p(\varepsilon) \tag{35}$$

$$\chi\sum_{\varepsilon}\left[\beta q(\varepsilon) - \beta w(\rho,\eta) - \alpha\log\left(\frac{1-\hat{\lambda}}{1-\lambda}\right) - \beta\tilde{\beta}\log\left(\frac{g(\hat{\lambda},\varepsilon)^{\eta}}{g(\lambda,\varepsilon)}\right)\right]p(\varepsilon) = 0 \tag{36}$$

$$\mu(\varepsilon)\Big[q(\varepsilon) - w(\rho,\eta) + (1-\eta)\tilde{\beta}\log\Big(g(\lambda,\varepsilon)\Big)\Big] = 0 \tag{37}$$

The first three equations are the first order conditions with respect to λ, c, $q(\varepsilon)$, respectively. Equation (34) defines the value for the investor and equation (35) is the promise-keeping constraint. Equations (36) and (37) formalize the Kuhn-Tucker conditions for the incentive-compatibility and enforcement constraints.

Notice that equations (33) and (37) must be satisfied for all values of ε which can take two values. Therefore, we have a system of 9 equations in 9 unknowns: λ, c, v, γ, χ, $q(\varepsilon)$, $\mu(\varepsilon)$. Once we have solved for the unknowns we can update the functions $\Psi(q)$ and $\psi(q)$ using the solutions for v and γ.

C Derivation of equation (31)

Suppose that we start in period zero with all managers having the same log-normalized human capital $\tilde{h}_0 = 0$. Therefore, in period zero we have

$$\text{Var}_0(\tilde{h}) = 0$$

In period 1, a fraction ϕ of managers are new entrants with $\tilde{h} = 0$. Thus, the standard deviation of their log-normalized human capital is zero. The remaining fraction $1-\phi$ are incumbents who survived from the previous period. The standard deviation of their log-normalized human capital is $\text{Var}(\tilde{g})$ and the standard deviation for the whole population in period 1 is

$$\text{Var}_1(\tilde{h}) = \phi\cdot 0 + (1-\phi)\cdot\text{Var}(\tilde{g})$$

Moving to period 2 we have three cohorts: a mass ϕ of new entrants whose $\text{Var}(\tilde{h}) = 0$; a mass $1-\phi$ of one-year old managers with $\text{Std}(\tilde{h}) = \text{Var}(\tilde{g})$; a mass $(1-\phi)^2$ of two-year old manager with $\text{Std}(\tilde{h}) = 2\text{Var}(\tilde{g})$. Therefore,

$$\text{Var}_2(\tilde{h}) = \phi \cdot 0 + (1-\phi) \cdot \text{Var}(\tilde{g}) + (1-\phi)^2 \cdot 2\text{Var}(\tilde{g})$$

At time t this will become

$$\text{Var}_t(\tilde{h}) = \phi \cdot 0 + (1-\phi) \cdot \text{Var}(\tilde{g}) + (1-\phi)^2 \cdot 2\text{Var}(\tilde{g}) + ... + (1-\phi)^t \cdot t\text{Var}(\tilde{g}).$$

In the limit with $t \to \infty$ this becomes

$$\text{Var}(\tilde{h}) = \left(\frac{1-\phi}{\phi}\right) \cdot \text{Var}(\tilde{g}),$$

or, equivalently,

$$\text{Std}(\tilde{h}) = \sqrt{\frac{1-\phi}{\phi}} \cdot \text{Std}(\tilde{g}).$$

■

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