Self Governance in Networked Relationships

Felipe Balmaceda

University of Chile*

Juan F. Escobar

University of Chile†

February 4, 2011
Comments very welcome

Abstract

This paper studies the problem of self governance in a model in which information flows are governed by the community structure. Players receive information only from their own social contacts and may act upon after receiving news about opportunistic behavior. We explore the social structures leading to self governance and emphasize how the matching technology shapes the architecture of such structures. Conditions under which self governance emerges only in cohesive communities are derived.

JEL classification numbers: D85, C73, L14, D82

Keywords: repeated games, social networks, self governance, information transmission

*Center for Applied Economics, Department of Industrial Engineering, University of Chile, Republica 701, Santiago, Chile, E-mail: `fbalmace@dii.uchile.cl`

†Center for Applied Economics, Department of Industrial Engineering, University of Chile, Republica 701, Santiago, Chile, E-mail: `jescobar@dii.uchile.cl`

1 Introduction

The use of self governance as a mechanism to facilitate efficient economic transactions has been widely recognized and documented by economists (Milgrom, North, and Weingast 1990, Greif 1993, Dixit 2006), political scientists (Ostrom 1990, Fearon and Laitin 1996), sociologists (Coleman 1990, Raub and Weesie 1990), and legal scholars (Bernstein 1992). As illustrated by Greif's (2006) historical accounts, close knit communities can align their members' incentives by employing implicit, community based sanctions that punish behavior deemed unacceptable or opportunistic. These implicit sanctions include ostracism and exclusion from trade and social networks, and their deterrence power is enhanced when information flows are rich and multilateral punishments can credible operate.

Implicit mechanisms of misconduct deterrence are also at work in modern industrial societies. Uzzi (1996) analyzes the apparel industry in New York and shows how firms repeatedly dealing with a single partner in a fully embedded relationship are more likely to survive. Cooperation and trust also allow firms to organize their procurement relationships, even when the threat of hold up can destroy any relationship specific joint value (Williamson 1979). A nice example of relationship specific value creation comes from Japanese's keiretsu (see McMillan 1995). In this system procuring firms are organized to exchange information about mischievous actions, and contracts are renewed only when implicit contractual terms are followed.[1]

This paper studies, at a theoretical level, which characteristics of a community are important for the emergence of cooperation and trust as a self enforcing phenomenon. We view the community structure as a key determinant of information flows concerning improper behaviors and, as such, it constraints feasible social norms and the extent to which efficient economic transactions can be attained. As highlighted by Milgrom, North, and Weingast (1990), Greif (1993), Bernstein (1992) and Dixit (2006), in cohesive communities information flows are rich and thus self governance is a feasible outcome. While one may be tempted to conjecture that cohesiveness is necessary for self governance, our results only partially confirm such presumption.

We consider an infinitely repeated game played by N investors and one agent. At each round $t \geq 1$, one out of the N investors is selected and plays a trust game with the agent. More specifically, the investor decides whether or not to invest; if he invests, then the agent chooses whether to cooperate or to defect. The socially desirable outcome is obtained when the investor

[1]These business practices not only impact the national supply chains, but can also affect the patterns of international trade (Rauch 2001, Head, Ries, and Spencer 2004).

invests and the agent cooperates. As it is well known, when the relationship lasts one period only, the agent will not cooperate (i.e, holds-up the investor). This temptation may be moderated by the existence of community sanctions governed by a social network of investors G. We assume that if the agent misbehaves when facing investor i, then i and all his connections in G (i.e., all those who are linked to i) become aware of that and act upon by refusing to invest in all subsequent rounds. This information does not disseminate through the network any further or, in other words, only victims of mischievous actions complain to their neighbors. We focus on *sustainable* networks, loosely defined as social networks of investors in which it is in the agent's interest to cooperate in all encounters.

An important determinant of the architecture of social networks conducive to efficient transactions is the technology with which investment opportunities arise. We explore how two polar matching technologies shape the architecture of sustainable networks. More specifically, we study a random matching model, where the identity of the selected investor is randomly and uniformly determined across time, and a directed matching model, where the identity of the selected investor is determined by the agent. The random matching model captures situations in which, while investors are ex-ante identical, ex-post one of them –the randomly selected one– has a technological advantage that results in surplus creation only when a partnership between him and the agent is successfully realized. In contrast, in the directed matching model, from the agent's viewpoint investors are substitutes in terms of value creation. While these distinctions are immaterial in a complete information model, they play a key role in determining the architecture of sustainable networks when information is incomplete.

Our model yields several novel insights. When matching is directed, the set of sustainable networks coincides with the set of communities in which all their members are connected. More technically, the set of sustainable networks equals the set of complete components, though the size of the network is undetermined. This result is due to the fact that if two community members are unconnected, a deviation against one of them entails no continuation value loss to the agent as the cheated agent can be easily substituted in the continuation game. This property of sustainable networks provides a particularly strong force towards cohesiveness –a theme we discuss ion detail below–.

We also explore the architecture of efficient networks, defined as networks that maximize the total sum of players' benefits minus the total costs of maintaining links. Because in the directed matching model investors are subsitutable, the total return of the network is independent of the network size, while the costs are strictly increasing on the number of links. As a result, efficient networks are bilateral relationships in which the agent interacts repeatedly with a single investor. This result resonates well with Uzzi's (1996) empirical findings on the stability and durability of fully embedded relationships.

When matching is random, the architecture of sustainable networks is much richer than that when matching is directed. We first show that the size of the neighborhood of any member of a sustainable network must be sufficiently high. Otherwise, defecting against a barely connected investor does not entail a sufficiently significant loss of future trading opportunities, even though he and his connections cannot be subsequently substituted. This lower bound on the size of each neighborhood belonging to a sustainable network is, in general, not a sufficient condition. The reason is that upon a deviation, there might be opportunities to deviate against remaining uninformed investors. Yet, we identify conditions under which these deviating opportunities are unattractive, and under which finding the stability of a social network reduces to checking the number of connections each investor has.

We also identify conditions in the random matching model under which sustainable networks exhibit certain degree of cohesiveness. This follows observing that if two unconnected investors have many connections in common, a deviation against one of them leaves the other barely connected in the after-deviation network and, as a result, it is in the agent's interest to deviate once more. This makes the first deviation attractive and ruling out these double deviations imposes upper bounds on the numbers of paths of length 2 that unconnected investors must have in a sustainable network. We derive lower bounds on the local clustering coefficient of members of sustainable networks as a function of the size of their neighborhoods. Moreover, there are restrictions on the game implying that the set efficient networks coincides with the set of networks formed by complete components of a given size.

These results show that cohesiveness may be necessary for self governance as it creates local common knowledge that deters opportunistic behaviors. Our results thus complement Coleman's (1988) insights on the importance of closure for the emergence of cooperative behavior. Closure has been found to be important in other social settings, notably in coordination games (e.g. Chwe 2000). Some empirical studies have related closure to community size (Allcott, Karlan, Mobius, Rosenblat, and Szeidl 2007). Our work provides a new rationale for the importance of network closure, and the mechanisms at work emerge from a full fledged dynamic model of incomplete information.

We apply our framework to study the interaction between formal and informal institutions of exchange. In particular, we explore how the existence of an anonymous market through which standardized transactions can be successfully realized limits the feasibility of socially desirable relationship-based trade. While there are many channels through which markets and relationships may interact, our imperfect monitoring setting is particularly appropriate to explore a new dimension of this interaction, namely, that the existence of market standards may make harder to monitor unfaithful behavior by providing opportunities to defect by mimicking market outcomes. We modify our baseline model to incorporate the possibility of market-based trade and show how markets can severely limit the feasibility of relationships.

Our work connects to the repeated games literature with imperfect monitoring (Abreu, Pearce, and Stacchetti 1990, Kandori 2002). In particular, Kandori (1992), Ellison (1994), Harrington (1995), and Okuno-Fujiwara and Postlewaite (1995) explore how extreme assumptions on the information flows shape players' incentives to cooperate in large communities in which agents interact infrequently.[2] We contribute to this literature by providing a new model of a large community in which information flows are not governed by how trading opportunities arise –which in some of these paper makes feasible the use of the so called contagious strategies– but by how the social structure determines the flows of soft information.

There has been some interest in embedding repeated game models into social networks, and early antecedents come from work by Raub and Weesie (1990), Bendor and Mookherjee (1990), and Ahn and Suominen (2001).[3] Ahn and Suominen (2001) is perhaps more closely related to our work as they study a set up in which the social network is randomly and independently drawn across time. In particular, the question of how the social structure and the matching technology limit the emergence of cooperative behaviors cannot be attacked in their model. From a technical perspective, their assumption about a community-wide observed randomization device allows simplifications that our model does not.

[2] Several papers have extended this line of research. Ghosh and Ray (1996) study a repeated game model with adverse selection in which players build relationships starting small (as in Watson 2002) and decide whether to continue the relationship at the end of each round. Takahashi (2010) proves a folk theorem for a community enforcement game with first order information.

[3] Bhaskar (1998) studies an OLG model in which each player only observes the play of the previous cohort. His model can be interpreted as a game in which both the order of moves and the information flows are governed by a social network G, the set of nodes of the network being the naturals and the set of directed links being all pairs of the form $(t, t+1)$, with t a natural numbers. Nodes in the network are seen as players, links are seen as determining the flows of information. While Bhaskar (1998) does not push the social network interpretation of his results, his paper connects to the current social network literature as it explores how particular networks of information flows constraint the outcomes of his game.

More recently, Balmaceda (2006), Lippert and Spagnolo (2010) and Ali and Miller (2010) study repeated prisoners' dilemmas with information asymmetries played on networks. Our model is distinctive in several aspects. First, the incentive problem here studied is one sided; as a result, our work immediately connects to the hold up literature, as initiated by (Williamson 1979), and to Greif's (1993) agency relations problems. Second, this paper explores how feasible social norms are determined when the only source of information flows is given by communication, and communication itself is the result of the community structure; in contrast, Balmaceda (2006) and Lippert and Spagnolo (2010) study models in which information can be transmitted through both actions and communication, while in Ali and Miller (2010) the network creates trading opportunities and information about unfaithful behavior is transmitted through actions. In Ali and Miller's (2010) model, the mechanism by which the network helps to sustain cooperation is rather different from ours and it is based on the contagious rationale (as in Kandori 1992). Yet, they, as we do, show that cooperative networks tend to be highly clustered. Third, we explore how alternative matching models, interpreted as different technologies with which surplus can be created, impact the architecture of sustainable and efficient network; previous work has ignored this important aspect.[4]

The paper is organized as follows. Section 2 introduces and discusses the model and presents some definitions. Section 3 illustrates some of our results using a simple three-node model. Section 4 presents necessary and sufficient conditions for stability and derive conditions under which sustainable networks are cohesive. Section 5 presents variations and applications of our main results. In particular, Section 5.1 presents our results on the impact of markets on networked relationships. Section 6 presents some concluding comments. Supporting arguments and proofs are relegated to the Appendix.

[4]Mihm, Toth, and Lang (2009) and Kinateder (2010) establish folk theorems for games played on networks. Our focus is different as we fix the discount factor and derive properties of sustainable networks. For completeness, we present a folk theorem for the random matching model in the Appendix. There are also models of repeated games on networks with complete information. Haag and Lagunoff (2006) study a repeated game model in which each individual chooses a single action that determines the payoff with all his links (for example, the action of turning on the porch light at night affects all street neighbors, and each individual makes just one choice, to turn the light on or off). They focus on a central planner's choice of a neighborhood design. Jackson, Rodriguez-Barraquer, and Tan (2010) study a complete information repeated game and nails down the architecture of robust renegotiation-proof social networks. Bloch, Genicot, and Ray (2008) and Karlan, Mobius, Rosenblat, and Szeidl (2009) study models of informal insurance.

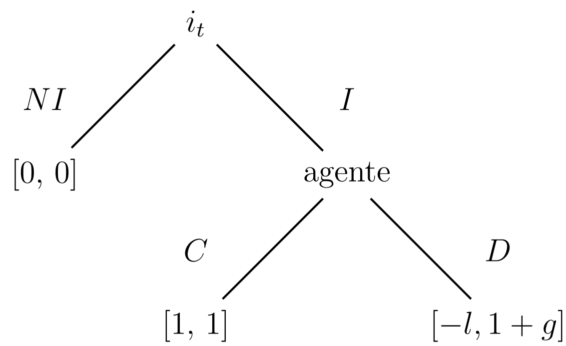


Figure 1: A trust game.

2 The Model

2.1 The Game

We study a repeated game model in discrete time between $N+1$ players, with $N \geq 2$. There are N investors and one agent, hereinafter player 0. At each round $t = 1, 2, \ldots$ the agent faces an investor $i_t \in \{1, \ldots, N\}$ and play a trust game. The way in which the investor i_t is selected to play the trust game may be random or directed. Each round of interaction generates information that is spread through a social network G of investors. Below we describe the details of the game.

Payoffs The investor selected at round t, i_t, and the agent play the trust game represented in Figure 1, where $l, g > 0$. Action I (resp. NI) stands for invest (resp. do not invest), while C (resp. D) stands for cooperate (resp. defect). We observe that the only Nash equilibrium of this trust game results in the inefficient payoff profile $(0, 0)$.

We assume that investors who are not selected to play the trust game obtain a period payoff equal to 0. As usually assumed in the repeated game literature, all players $\{0, 1, \ldots, N\}$ discount period payoffs with a discount rate $\delta \in]0, 1[$.

Matching Technology We will consider two different ways in which the investor is selected in each round.

Random Matching (RM) The identity of the selected investor, i_t, is uniform and independently drawn from $\{1, \ldots, N\}$ across time.

Directed Matching (DM) The investor i_t is selected by the principal from $\{1, \ldots, N\}$ at the beginning of each round $t \geq 1$.

These two matching technologies capture different ways in which surplus can be produced. In the RM model, the only way in which surplus can be created is through a partnership between the agent and a randomly determined investor i_t; this can be seen as a project in which the technology owned by i_t cannot be substituted by that owned by other investors. In contrast, in the DM model, all investors have access to the same technology and therefore, from the agent's point of view, all investors are exactly alike in terms of the possibilities of developing the joint project.[5]

Information Flows and Histories To describe the monitoring technology, we will introduce a social network $G = (N^G, E^G)$ of investors, where $N^G \subseteq \{1, \ldots, N\}$ is a set of nodes and E^G is a set of links between the $|N^G|$ nodes. We denote by $N^G(i) = \{j \in N^G \mid ij \in E^G\}$ the set of i's neighbors in G and $\bar{N}^G(i) = N^G(i) \cup \{i\}$. For each $i \in N^G$, we denote by $G \setminus \bar{N}^G(i)$ the network consisting of nodes $N^G \setminus \bar{N}^G(i)$ and links $\{jk \in E^G \mid \text{ there is no } l \in \bar{N}^G(i) \text{ with } l = j \text{ or } l = k\}$.

Information flows are as follows. Let $i_t \in \{1, \ldots, N\}$ be the investor chosen at round t. If the chosen investor i_t picks I, then all investors $j \in \bar{N}^G(i_t)$ become informed of that and observe whether the agent chooses C or D. Otherwise, no information is transmitted. We observe that from the perspective of an investor who is not selected, whether one of his neighbors was matched to the agent and decided NI cannot be distinguished from whether his neighbors were not selected. Recall is perfect and the network (and actually the details of the game) is common knowledge.

This completes the description of our dynamic game of incomplete information in which information flows are determined by G. A history, h_i^t, for investor $i \in \{1, \ldots, N\}$ consists of the rounds at which he has faced the agent, what the outcomes of those stage games have been, and the observations he has received from each of his neighbors in G who have been selected and have chosen I. A history h^t for the agent consists of the whole history in the game as, by construction, he has been involved in all the stage games that have taken place.

In our model, the social network plays the most minimal role in information transmission one could think of: all what an investor conveys to his neighbors is whether the agent has

[5] Results about intermediate matching technologies are available upon request.

misbehaved after he invested. In particular, an investor aware of a mischievous action in an encounter involving one of his neighbors does not spread the news any further. Of course, even if he could, his continuation payoffs are not determined by how continuation play evolves and thus it is not in his interest to further the news.[6] Thus, the assumptions on the information transmission technology can be seen as the result of voluntary communication in a larger game.[7]

Alternatively, we could think that the social network is not a network of information transmission, but a network in which players observe the play of their neighbors. For example, if we interpret the model as a game between a monopolist and a set of consumers (as discussed below), it seems conceivable that a consumer could observe the quality of the good bought by their neighbors, but is not able to observe or become informed of the trading experiences of distant connections. The network of information transmission can be though of as a network of observational learning.[8]

We think our assumptions on the information transmission technology are of interest in other settings too. It is possible to model a richer information flow protocol in which news about deviations travel $n \geq 1$ steps at once by elaborating on our results. Now, if we were to allow for richer information flows, it would seem appropriate to assume that as information travels farther away from its initial source –the cheated investor– its quality deteriorates. While one may have a relatively accurate assessment of the interactions in which one's friends have been involved, such estimate is likely to be distorted when assessing friends of friends' social interactions. Our sustainable networks (defined in the next sub-section) can be seen as social arrangements in which distant news are of low quality and thus ignored; sustainable networks are thus robust to all those information distortions.

2.2 Sustainable Networks

Before discussing which networks are conducive to efficient trade, we introduce a class of stationary strategies. We define a profile of strategies $\sigma^G = (\sigma_i^G)_{i=1}^N$ for investors $i \in \{1, \ldots, N\}$

[6]This logic rests on the assumption that the agent cannot observe what players communicate. In a model in which communication is voluntary, the agent could condition his behavior on whether an investor communicates his trading experiences and by manipulating the messages an investor aware of some defection could strictly benefit by omitting some news. Lippert and Spagnolo (2010) are concerned with issues of this sort.

[7]While the equilibrium of the larger game may seem arbitrary in that only the victim complains, we think it is an appropriate approximation as victims of opportunistic behavior tend to have a natural tendency to complaining.

[8]While a player who has observed a deviation could be tempted to communicate the deviation to uninformed players, it is easy to see that it is not in his interest to do so.

as follows. Investors belonging to the network N^G employ trigger strategies; that is, given a history h_i^t, investor $i \in N^G$ plays NI unless h_i^t is the null history or h_i^t shows that the outcome in each of the preceding stage games in which i has been involved was (I, C) and he has not received any information showing the play of (I, D). If $i \notin N^G$, then play NI. These strategies are a natural generalization of the well-known trigger strategies to our social network setting (Mailath and Samuelson 2006). A player belonging to the social network trusts, unless the agent has misbehaved when facing him or one of his neighbors at some previous round.

Take now $\mathrm{BR}^{G,m}(\sigma)$ as the set of sequential best replies in behavior strategies for the agent, when the network is G, the matching technology is $m \in \{RM, DM\}$ and the investors' strategies are given by $\sigma = (\sigma_i)_{i=1}^N$.[9] Take $\sigma_0 \in \mathrm{BR}^{G,m}(\sigma^G)$, where σ^G is the trigger strategy for investors $i \in \{1, \ldots, N\}$ defined above. Such σ_0 induces a probability distribution over the sequence of (random) networks $(G^t)_{t \geq 1}$ formed by investors who are willing to invest at the beginning of round t (so that $G^1 = G$). Denote such probability distribution over the sequence of networks as $\mathbb{P}[\cdot \mid m, G, \sigma_0]$. We now introduce our definition of *sustainable* networks.

Definition 1 *Take a network G and fix a matching technology $m \in \{RM, DM\}$. We say that the network G is* sustainable *if there exists $\sigma_0 \in BR^{G,m}(\sigma^G)$ such that*

$$\mathbb{P}[G^t = G \mid m, G, \sigma_0] = 1.$$

When the condition above holds, we will also say that G is σ_0-sustainable.

Our definition of sustainable networks imposes two key requirements on the social network G and the agent's strategy σ_0. On the one hand, given the matching technology m and information flows G, it must be sequentially rational for the agent to comply with σ_0. On the other hand, on the equilibrium path, σ_0 must leave the initial network G unchanged or, in other words, the model is stationary in the sense that all possible cheating opportunities have been exhausted and the network is kept unchanged as time passes by. Thus, sustainable networks can be seen as the result of a long run process, where the agent behaves in a sequentially rational way and there is no more room for on path unfaithful behavior (the Appendix formally explores this interpretation.

Sustainability is a relatively mild requirement and stronger restrictions on strategies could seem appropriate. For example, instead of requiring $\sigma_0 \in \mathrm{BR}^{G,m}(\sigma^G)$, one may want to impose a

[9] What kind of functions the set of best replies contains depends on the matching technology.

sequential equilibrium restriction on (σ_0, σ^G) (Kreps and Wilson 1982). Our results obviously apply when more demanding requirements are imposed, but in the Appendix we show the sequential equilibrium requirement turns out to be immaterial (see Lemma 2).

We will also study efficient networks, defined as networks that maximize the sum of players' payoffs. Given a matching technology m and a σ_0-sustainable network G, we denote the expected total payoff accruing to player $i \in \{0, \ldots, N\}$ by $u_i(G, m, \sigma_0)$. We assume that links are costly and that player $i \in \{1, \ldots, N\}$ must incur a cost of $c_i(|N^G(i)|)$, with $c_i \colon \mathbb{N} \to \mathbb{R}_+$ strictly increasing and $c_i(0) = 0$, to maintain the $N^G(i)$ connections.

Definition 2 *Given $m \in \{RM, DM\}$, a σ_0-sustainable network G is* efficient *if*

$$(G, \sigma_0) \in \arg\max \Big\{ \sum_{i=0}^{N} u_i(G', m, \sigma_0') - \sum_{i=1}^{N} c_i\big(|N^{G'}(i)|\big) \mid G' \textit{ is } \sigma_0'\textit{-sustainable} \Big\} \tag{2.1}$$

A network is efficient if it maximizes the sum of players' payoffs, which consists of the benefits obtained in the repeated game minus the costs of forming the links. See Jackson (2008, Chapter 6) for additional discussion on efficiency concepts in games of strategic network formation.

We also consider sustainable networks for which no link can be removed without impairing its sustainability.

Definition 3 *A sustainable network G is* minimally sustainable *if for all $ij \in E^G$ the network $G' = (N^G, E^G \setminus \{ij\})$, obtained by deleting link ij, is not sustainable.*

The idea behind the definition of minimally sustainable networks is that the network should solve the social dilemma of enforcing proper behavior, but if any link is deleted, then the resulting network is not sustainable. Minimal sustainability can be justified on several grounds. Efficient networks are minimally stable. So are Nash equilibrium networks of the network formation game in which investors publicly announce whether they want to be part of the network, which links they want to form, a link ij is formed iff both i and j announce link ij, and the payoffs to i are

$$U_i(G, m) = \begin{cases} u_i(G, m, \sigma_0(G)) - c_i(|N^G(i)|) & \text{if } G \text{ is sustainable,} \\ -c_i(|N^G(i)|) & \text{if not} \end{cases}$$

where $\sigma_0(G)$ is such that G is $\sigma_0(G)$-sustainable and for all $i \in N^G$, $u_i(G, m, \sigma_0(G)) > 0$.[10] We thus see minimal sustainability as a mild restriction when links are instrumentally formed either through centralized or decentralized mechanisms.

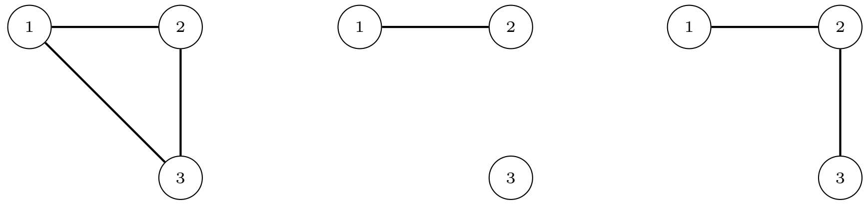
2.3 Interpretations

Below we provide some interpretations of our model.

Specific investments and hold up Investors $i \in \{1, \dots, N\}$ are suppliers, the agent is a firm. Value can be created through a partnership between a supplier and the firm. To create the value, the supplier i_t must make a specific investment (play I) and the firm may hold up the investment (play D) or share the returns (Klein, Crawford, and Alchian 1978, Williamson 1979, Grossman and Hart 1986). The network represents the business relations among the suppliers, and each of the suppliers can communicate to all its business partners whether its investment was held up. The technology among the suppliers may be substitutable or not (matching may be directed or random). Applying the model to the automobile industry –where specific investments and hold up are a theme of detailed study– the firm may be a car producer while suppliers may procure car components. When all suppliers supply the same component, say mufflers, matching is likely to be directed as no supplier has a significant technological advantage over the others to make the specific investment. Alternatively, it could also be the case that suppliers produce different components, say one of them produces mufflers, another one produces wheels, a third one produces filters, etc. In this case, while suppliers are ex-ante identical, ex-post only one of them will be able to carry out the specific investment and, thus, the matching technology is likely to be random.

Maghribi traders Each investor $i \in \{1, \dots, N\}$ is a merchant, who may need an agent to handle their overseas operations. A merchant may hire or not the agent (play I or NI), and the agent, if hired, may or may not embezzle the merchant's goods (play C or D). The social network represents all the social ties between merchants facilitating information exchange. Greif (1993) studies a coalition between a group of Medieval merchants, the Maghribis, that allowed full and rich information flows among their members. A coalition of merchants can be seen as a complete network, in which all potential merchants can fully exchange information. We introduce

[10] An immediate implication of Proposition 1 is that such selection always exists.



(a) Network G^1 (b) Network G^2 (c) Network G^3

Figure 2: When $\alpha \leq 1$, the complete information network G^1 is sustainable regardless of the matching technology. Network G^2 is sustainable when (i) $m = DM$ and $\alpha < 1$ or (ii) $m = RM$ and $\alpha < \frac{2}{3}$. Network G^3 cannot be sustainable when $m = DM$. When $m = RM$, G^3 is sustainable when $\alpha \in]\frac{1}{3}, \frac{2}{3}[$ and δ is low enough.

a friction in Greif's (1993) model by embedding his framework into a social network model of imperfect information transmission.

Experience goods The agent is a monopolist who can produce a good of high or low quality (play C or D). Investors $i \in \{1, \ldots, N\}$ are clients who can buy or not buy the good (play I or NI). In this game, the client can verify the quality of the good purchased only after he has made the (unrecoverable) payment. In most applications matching is likely to be random: While the monopolist could approach clients to offer its good, clients are likely to realize purchases independently across time according to their needs. A social network can be seen as modeling all the alternative sources of information on the monopolist's past performance, ranging from online feedback systems and media coverage to plain word of mouth. Dellarocas (2003) discusses how consumers can exchange their opinions and reactions about firms through the internet. In this context, our social network model can accommodate overlapping internet platforms where some users participate in two or more forums and a user who have learnt the monopolist misbehaved in one platform does not spread the news to other platforms in which he participates.

3 An Illustrative Example

We consider a model with $N = 3$ investors and, for concreteness, let us assume δ and g are such that $\alpha := \frac{1-\delta}{\delta} g \in]\frac{1}{3}, \frac{2}{3}[$. We will study the sustainability properties of each of the three networks in Figure 2.

Take first the complete information network G^1. After an improper action of the agent, all investors become aware of that and act upon in all subsequent rounds. Thus, regardless of the matching technology m, the agent's continuation value after defection is equal to 0. The network will be sustainable iff

$$(1-\delta)+\delta \geq (1-\delta)(1+g)$$

which can be equivalently written as $\alpha \leq 1$, a condition that we assume here and in the rest of the paper. In the complete information network G^1, whether the matching process is random or directed is irrelevant to determine the sustainability of the social arrangement.

Consider now network G^2, which consists of two nodes –investors 1 and 2– that are linked. Network G^2 is not a complete information network as investor 3 does not become aware of how play evolves when investors 1 and 2 are selected. Consider first the case of random matching, in which investors are unsubstitutable. Observe that, by construction of the trigger strategies, investor 3, who is not part of the network, never invests. Only principals 1 and 2 are part of the network and thus only them are expected to invest. If the agent cooperates in all encounters with 1 and 2, then his period payoffs conditional on facing either 1 or 2 equals $(1-\delta)+\delta\frac{2}{3}$. If he deviates when facing either principal 1 or principal 2, then both of them will refuse to trade in all subsequent rounds and the agent's normalized total payoff would equal $(1-\delta)(1+g)$. This implies that the sustainability of G^1 is equivalent to

$$(1-\delta)+\delta\frac{2}{3} \geq (1-\delta)(1+g).$$

This condition holds under our assumption that $\alpha < \frac{2}{3}$ and thus G^2 is sustainable when matching is random.

But network G^2 is also sustainable when matching is directed. Indeed, when $m = DM$, it is optimal for the agent to select either investor 1 or investor 2 and to cooperate after they invest. Such a strategy yields a normalized total payoff equal to 1, while the best deviation entails a current payoff of $(1-\delta)(1+g)$ followed by a continuation payoff of 0. Such a deviation cannot be optimal as $\alpha \leq 1$ and, therefore, G^2 is sustainable when $m = DM$.

We observe that determining the sustainability of networks G^1 and G^2 amounts to comparing α to a given threshold. The parameter α fully captures the severity of the incentive problem in models in which trading investors are fully connected and will play a key role throughout the paper.

Let us finally explore the sustainability properties of network G^3, in which all investors have at least one link but investors 1 and 3 are not connected. Social network G^3 is of interest as all investors are willing to invest, yet there are no information flows between investors 1 and 3. Consider first the model with directed matching. Observe that if the network is sustainable, then the normalized total payoff to the agent equals 1. Yet, the strategy of choosing investor 1 and defecting together with the continuation strategy of selecting investor 3 –who does not become aware of the deviation against 1– and cooperating yields a payoff equal to $(1-\delta)(1+g)+\delta > 1$. When matching is directed, network G^3 is not sustainable regardless of α. This arises because network incompleteness allows the agent to deviate against one of the community members bearing no loss in continuation values. This makes such a deviation attractive and breaks down the sustainability of the social arrangement.

The incentives the agent faces in network G^3 are different when matching is random. For G^3 to be sustainable, it must be that it is in the agent's interest to choose the high action in all encounters. Now, if the agent deviates when facing principal 1, then principal 2 becomes informed of that and both agents refuse trade in all subsequent encounters. Principal 3, who is not connected to 1, does not become aware of the deviation against principal 1 and is still willing to trade when a subsequent trading opportunity arises. When such trading opportunity with principal 3 arises, however, the agent choose the low action as $\frac{1-\delta}{\delta}g > \frac{1}{3}$. Thus, when $m = RM$, network G^3 is sustainable iff

$$1 \geq (1-\delta)(1+g) + \sum_{t=1}^{\infty} \delta^t (\frac{2}{3})^{t-1} \frac{1}{3}(1-\delta)(1+g).$$

The incentive constraint ensuring the sustainability of G^3 can be written as $\alpha \leq \frac{2-\delta}{3-\delta}$. Keeping the severity of the incentive problem $\alpha \in]\frac{1}{3}, \frac{2}{3}[$ fixed, we observe that there exists $\bar{\delta}$ such that the social network G^3 is sustainable iff $\delta \geq \bar{\delta}$.[11] To get an intuition why this result holds, note that after a deviation against 1 the agent is loosing all subsequent trading opportunities arising from 1 and 2. However, in contrast to network G^2, now the deviation against 1 creates one more deviating opportunity as investor 3 is still willing to invest, since he is not informed about the agent's deviation against investor 1. If the agent's objective puts sufficiently high weight on future payoffs, that second deviation will cause the breakdown of incentives to cooperate when facing investors 1. The double deviation is unattractive only when δ is sufficiently low.

[11] Observe that network G^3 is formed as a tree union of the network G^2 and the network formed by nodes 2 and 3 and a link between 2 and 3. The fact that G^3 need not be sustainable contrasts with Jackson, Rodriguez-Barraquer, and Tan's (2010) results.

In sum, the matching technology plays a key role determining the architecture of sustainable networks. When investors are perfectly substitutable, as is the case when matching is directed, deterring opportunistic behavior is possible only in complete networks, and the set of sustainable networks coincide with the set of complete components. The size of the sustainable network is undetermined, but all investors willing to invest must be connected. sustainable networks are thus fully clustered. In contrast, when matching is random, deviating against one of the members of the network entails losses in continuation values to the agent. Such losses may or may not be significant enough to deter the deviation. When the network is incomplete, the possibility to double deviate may break down the incentives of the agent to cooperate. In the random matching model, sustainability imposes lower bounds on the minimum number of connections each investor must have as a function of α, yet whether sustainable networks will exhibit cohesiveness properties will depend on the parameters of the game and the density properties of the network.

4 Analysis

We now go back to the general model presented in Section 2. In this section we explore properties of sustainable networks and discuss how the parameters of the game (including the matching technology) shape the architecture of sustainable networks. Section 4.1 studies the directed matching model, while Sections 4.2 and 4.3 are devoted to the study of the random matching model. In Section 4.4, we discuss the results.

Consider a complete information model, in which the social network G consists of all nodes $N^G = \{1, \dots, N\}$ and all links $E^G = \{ij \mid i, j \in N^G\}$. Regardless of the matching technology, the network is sustainable iff

$$(1 - \delta) + \delta \geq (1 - \delta)(1 + g)$$

or, equivalently, $\alpha := \frac{1-\delta}{\delta} g \leq 1$. This condition is assumed throughout the paper.

Assumption 1 (Nontrivial model) $\alpha \leq 1$.

Subsequent analysis will show that when this assumption does not hold, the set of sustainable networks is empty. Without loss, instead of using δ and g, we will use α and g or α and δ as parameters of the model to state most results.

Recall that a network G is a complete network if all its members are linked: $\bar{N}^G(i) = N^G$ for all $i \in N^G$. A component of a network G is a subnetwork G' consisting of nodes $N^{G'} \subseteq N^G$

and links $E^{G'} \subseteq E^G$ such that any two nodes in $N^{G'}$ are connected through a path in G' and if $i \in N^{G'}$ and $ij \in E^G$, then $ij \in E^{G'}$. Consult Jackson (2008) for additional definitions.

4.1 Directed Matching Model: Characterization

The following proposition characterizes sustainable networks in the directed matching model.

Proposition 1 *Suppose matching is directed. Then,*

G is sustainable if and only if G is a complete network.

Proof. If G is a complete network, then any $\sigma_0 \in BR^{G,m}(\sigma^G)$ is such that the outcome of the game has only members of N^G chosen, the chosen investors invest, and the agent cooperates. Thus $\mathbb{P}[G^t = G, \forall t \mid m, G, \sigma_0] = 1$ and G is sustainable.

Take now a nonempty sustainable network G and assume it is not a complete network. Take $\sigma_0 \in \mathrm{BR}^{G,m}(\sigma^G)$ such that on path the network is kept unchanged. Then, the discounted sum of normalized payoffs for the agent at the beginning of $t = 1$ is equal to 1. Since G is not a complete network, there exist $i, j \in N^G$ such that $j \notin \bar{N}^G(i)$. Consider the strategy $\bar{\sigma}_0$ for the agent: at $t = 1$ choose $i_1 = i$ and defect, in the continuation game starting at $t = 2$ play a best reply given the network $G \setminus \bar{N}^G(i)$. Note that $j \in G \setminus \bar{N}^G(i)$ and thus the normalized continuation value starting at $t = 2$ is at least 1. It follows that $\bar{\sigma}_0$ yields a normalized payoff to the agent greater than or equal to $(1-\delta)(1+g)+\delta > 1$ and thus σ_0 cannot be a sequential best reply. This implies that G is not sustainable. ■

This simple result captures a key mechanism arising in a social network of information transmission. When the agent behaves unfaithfully against i, then only i's connections become informed of that, while investors who are not directly connected to i remain willing to invest. Because matching is directed, this defection entails no loss in continuation values. That makes cooperation unattractive for the agent in the first place and thus a deviating opportunity arises.

Corollary 1 *Suppose matching is directed. Then, the following hold:*

i. A network G is efficient iff $|N^G| = 1$;

ii. A network G is minimally sustainable iff G is sustainable.

The first part of the corollary shows that in a random matching model, efficiency nails down the set of sustainable networks to one-node networks. In other words, the efficient organization is a bilateral relation, where the agent and a single isolated investor interact throughout the whole game. This is driven by the fact that, since investors are perfectly substitutable, two or more nodes produce the same total expected payoffs in the repeated game as a single node, yet a network with two or more nodes must be a complete network and therefore it costs strictly more than a one-node network. The second part of the corollary shows that minimal sustainability does not narrow down the set of sustainable networks. This is due to the fact that if a network is sustainable and a link is removed, then the resulting network will not be complete and thus will not be sustainable.

4.2 Random Matching Model: Degree Properties

Let us start the analysis of the random matching model by considering a complete network formed by $\kappa \geq 1$ investors. It will be in the agent's interest to cooperate when matched to one of the κ investors if and only if $(1-\delta)+\delta\frac{\kappa}{N} \geq (1-\delta)(1+g)$. This condition can be equivalently written as $\kappa \geq \kappa^*$, where $\kappa^* = \lceil \alpha N \rceil$. We will assume that $\kappa^* > \alpha N$. Under Assumption 1, $\kappa^* \leq N$ and thus the set of sustainable networks is nonempty as it contains at least all the complete networks of κ^* or more nodes. We will be interested in more general networks (not necessarily complete networks) but the threshold κ^* will be important in subsequent analysis.

We assume that a bilateral relation between the agent and a single investor is not sustainable.

Assumption 2 $\kappa^* \geq 2$.

Many results in this and the following section apply when this assumption is dropped, but this assumption is natural and simplifies the exposition.

Before presenting the main substantive results, we observe that the agents' incentive problem can be formulated as a dynamic programming problem in which the state variable is the identity of the current trading partner i_t and the network of investors who are still willing to trade. We denote the normalized expected sum of discounted per-period payoffs to the agent when facing i and given the network G by $v(i, G)$, and explore its main properties in the Appendix. This formulation allows us to manipulate the agent's incentive constraints to derive necessary and sufficient conditions for sustainability.

The following result presents a necessary and a sufficient condition for sustainability in terms of the sizes of the neighborhoods of each of the network members.

Proposition 2 *Suppose matching is random. Then, the following hold:*

i. If G is sustainable, then $|\bar{N}^G(i)| \geq \kappa^*$ for all $i \in N^G$;

ii. If $|\bar{N}^G(i)| \geq \frac{g}{1+g}|N^G| + \frac{1}{1+g}\alpha N$ for all $i \in N^G$, then G is sustainable.

The first part of the proposition establishes that in any sustainable network G, each player has a number of links at least equal to the threshold κ^*. If this necessary condition fails, the after-deviation reduction in continuation values due to the loss of investors willing to invest is small and therefore, the agent prefers to defect. As shown in Section 3, the converse need not hold. In the example, $\kappa^* = \lceil \alpha N \rceil = 2$, but in network G^3 all of the nodes have at least one neighbor, yet G^3 is not sustainable when δ is high enough.

The second part of the proposition presents a partial converse by showing that sufficiently dense networks are always sustainable. This is driven by the fact that in dense networks a deviation entails losses in trading opportunities that are large compared to what can be gained by exploiting subsequent deviating opportunities. A simple comparative statics result follows: The larger the network size $|N^G|$, the larger the threshold guaranteeing sustainability of networks with closed degrees above that threshold. In particular, keeping $\alpha = \frac{1-\delta}{\delta}g$ fixed, the bound is informative when g and δ are sufficiently small.

A simple, but important corollary follows.

Corollary 2 *Suppose matching is random. Let G be such that $g(N^G - \kappa^*) \leq \kappa^* - \alpha N$. Then, the following hold:*

i. G is sustainable iff $|\bar{N}^G(i)| \geq \kappa^*$ for all $i \in N^G$.

ii. G is minimally sustainable iff $\min_{k \in \bar{N}^G(i)} |\bar{N}^G(k)| = \kappa^*$ for all $i \in N^G$;

This result completely characterizes sustainable and minimally sustainable networks when, keeping the intensity of the incentive problem α fixed, the agent is impatient and the defection gains are low (δ and g are low). The result shows that sustainability only imposes restrictions on the closed degrees of the network members when g and δ are low enough. As illustrated in Section 3, this is so because when there are two or more network members who are not connected, and thus the network allows two or more deviations, the potential gains of future deviations are low when the agent is impatient and g is low. Under the conditions of the proposition, establishing the sustainability of a network reduces to counting the number of connections each member has.

When the agent is impatient, any link between two nodes, each of them having κ^* or more connections, can be removed without impairing sustainability. Thus, as established by the second part of the proposition, minimally sustainable networks can be easily found by checking that all neighborhoods have a member attaining the lower bound κ^* on the closed degree. Note that, in contrast to the directed matching model, there are sustainable networks that are not minimally sustainable.

4.3 Random Matching Model: Cohesiveness Properties and Efficient Networks

The results in the previous subsection show that when the agent is impatient, sustainability is purely a restriction on the number of connections each member has. We now explore situations in which the agent is patient and show conditions under which sustainable networks have cohesiveness properties, to be formerly defined later on.

Let $M^G(i)$ be the set of nodes within distance 2 of investor i, formally defined as

$$M^G(i) = \Big\{ j \in N^G \mid \text{ there exists } k \in N^G \setminus \{i,j\} \text{ with } ik, kj \in E^G \Big\}.$$

The following result provides an upper bound on the number of common connections between an investor i and any subset Q of investors indirectly connected through a path of length 2 to i.

Proposition 3 *Suppose matching is random. Let G be sustainable and $i \in N^G$. Then, for any subset $Q \subseteq M^G(i)$*

$$\sum_{j \in Q} |\{k \in N^G \mid ik, kj \in E^G\}| \leq$$
$$\Big(\frac{N - \delta N}{\delta} + |Q|\Big)\big(|\bar{N}^G(i)| - \alpha N\big) + \sum_{j \in Q}\big(|\bar{N}^G(j)| - \alpha N\big).$$

To see the logic behind this result, note the following three important facts: (i) after the agent has defected when facing $i \in N^G$, players in $Q \subset M^G(i)$ are still willing to invest since they are not informed of the agent's deviation; (ii) defecting when facing investors in Q is more attractive the lower the number of neighbor investors $j \in Q$ have in the resulting after-deviation network $G \setminus \bar{N}^G(i)$; and (iii) if G is sustainable, the continuation value after defecting against i is low enough. The second observation implies that, after defecting against i, the value of continuation play $\frac{1}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i))$ is bounded below by a nondecreasing function of the total number of neighbors in $G \setminus \bar{N}^G(i)$ of nodes $j \in Q$, and the third observation implies an upper bound for $\frac{1}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i))$ as a function of the parameters of the model. It then follows that the number of common paths between investor i and any subset $Q \subseteq M^G(i)$ cannot be too large, otherwise the agent can exploit the lack of information of players in Q to double deviate, first against i, then against players in Q.

We now investigate some corollaries to Proposition 3, each of them establishing cohesiveness properties for sustainable networks.[12] Recall that for each network G and $i \in N^G$, the *individual clustering coefficient for node i* is defined as

$$\textsc{Cluster}^G(i) = 2\frac{|\{jk \in E^G \mid j \neq k, j,k \in N^G(i)\}|}{|N^G(i)|(|N^G(i)| - 1)}.$$

The clustering coefficient for node i measures the number of links between i's neighbors as a fraction of the maximum number of potential links. It gives a measure of how cohesive network G is around i. While there are many measures of cohesiveness, the local clustering coefficient is easy to interpret and has been used in several studies (Jackson and Rogers 2005, Jackson and Rogers 2007).

Corollary 3 *Suppose matching is random. Let G be a sustainable network such that for some $\beta \in]0,1[$, $\bar{N}^G(i) = \beta N$, for all $i \in N^G$. Then,*

$$\textsc{Cluster}^G(i) \geq 1 - \frac{\alpha/g + 2(1-\beta)}{(\beta - \frac{1}{N})(\beta - \frac{1}{N})}(\beta - \alpha),$$

for all $i \in N^G$.

An investor $i \in N^G$ in a sustainable network has a bounded number of connections within

[12] An immediate implication from Proposition 3 is that when two investors have a sufficiently large number of connections in common, then those investors must be connected. This can be seen as a version of the so called *small world* phenomenon, which states that most of one's connections are themselves connected.

distance 2 and thus, most of i's neighbors must have connections within i's neighborhoods. This implies a lower bound on the number of links between members of $N^G(i)$ and thus a lower bound on the clustering coefficient. When the nodes have a degree close to αN, it is harder to deter the previously described double deviation and, as a result, the bound on the clustering coefficient is tighter. On the other hand, the lower bound on the clustering coefficient becomes less tight as the network becomes more dense and g falls. Indeed, as we have already seen in Corollary 3, any sufficiently dense network is sustainable and thus a sustainable network need not be clustered.

We finally investigate the architecture of efficient networks in the random matching model. To simplify the exposition, we assume that the costs of forming links are symmetric across players and equal to $c\colon \mathbb{N} \to \mathbb{R}_+$ (which, as discussed in Section 2, is assumed strictly increasing with $c(0) = 0$).

Corollary 4 *Suppose matching is random. Then, the following hold:*

i. If $N(1-\delta)c(\kappa^* - 1) > 2$, then the only efficient network is the empty network.

ii. Suppose that

a. $N(1-\delta)c(\kappa^* - 1) < 2$;

b. $\frac{N}{\kappa^*} \in \mathbb{N}$;

c. $(\kappa^* - \alpha N)\left(N\frac{1-\delta}{\delta} + 2\right) < 1$;

Then, a network G is efficient iff it consists of $\frac{N}{\kappa^*}$ complete components, each of them having κ^* nodes.

This corollary provides conditions under which the architecture of efficient networks can be fully characterized. When the gains from cooperation are sufficiently low compared to the costs of maintaining links, any nonempty network is dominated by the empty one. On the other hand, when the costs of forming links are sufficiently low, the optimal organization consists of separate complete components of size κ^*, provided such organization is feasible and, more crucially, the agent is sufficiently patient. The result follows since efficiency pushes the number of links to the threshold κ^*, organizing several separate complete components of κ^* members is feasible, and any other organization in which all the closed degrees equal κ^* must be given by separate complete components, as otherwise the resulting network cannot be sustainable when the agent is patient.

4.4 Discussion and Summary

We have presented a set of results relating the parameters of the game to topological properties of sustainable networks. One of the properties sustainable networks have is their cohesiveness. When matching is directed, the cohesiveness of sustainable networks is particularly stark: sustainable networks are complete networks in which all network members are linked. In contrast, when matching is random the cohesiveness of sustainable networks depends on: (i) the network density; and (ii) holding the complete information intensity of the incentive problem α fixed, on players' patience. Propositions 1 and 3 identify conditions under which some level of common knowledge of part of the history of the game is essential for attaining self governance.

However, in the random matching model, self governance can be attained even in barely cohesive communities. Indeed, as Proposition 2 shows, any sufficiently dense network is stable and, when players are impatient, the stability of a social arrangement is purely a matter of degree. A sufficiently dense network is always cohesive, and a consequence of Proposition 2 and Corollary 3 is that, for regular networks, the local clustering coefficient of sustainable networks, as a function of their degree, is bounded below by a U-shaped function when matching is random and players are patient.

Efficient networks exhibit sharper clustering properties. In the directed matching model, Corollary 1 shows that the set of efficient networks reduces to bilateral relationships in which the agent repeatedly trades with a single investor. When matching is directed, Corollary 4 shows conditions under which efficient networks are either empty or formed by complete components. These results identify game theoretical forces that favor the formation of clustered and embedded relationships.

These results identify conditions under which cohesiveness is a crucial ingredient to attain efficient economic transactions. However, we have also found limits to the natural presumption that cohesiveness is necessary for self governance. Case studies show how cohesive communities trade by means of community based sanctions (Milgrom, North, and Weingast 1990, Greif 1993, Bernstein 1992), yet anecdotic evidence suggests there is a fair amount of trade even in the absence of perfect information dissemination.

Our results have testable implications about the architecture of sustainable networks as a function of the fundamentals of the transaction. In particular,

i. When matching is directed networks must be complete;

ii. When matching is random, the agent is impatient and defection gains are low (δ and g are low), networks are sustainable as long as each agent has a sufficiently high degree (i.e., $|\bar{N}^G(i)| \geq \kappa^*$);

iii. When matching is random, either the agent is patient or defection gains are low or both, networks are sustainable as long as each agent's clustering coefficient exceeds a lower bound.

5 Applications and Variations

5.1 Markets and Networked Relationships

The implicit mechanism of misconduct deterrence studied in this paper is one among many alternatives to organize exchange. In this section, we add to the model in the previous section an anonymous market, and study how its existence limits the possibility of self-governance or arm's length relationships.

There are several channels through which formal institutions and informal relationships can interact. Attanasio and Rios-Rull (2000) show that, in a model with risk-sharing through long-run relationships, government insurance of aggregate shocks can worsen private sharing of idiosyncratic shocks by tightening of incentive compatibility constraints, with the resulting drop in welfare. Spagnolo (2002) shows how cooperation through long-run relationships can be hurt by an improvement in the functioning of financial markets. Kranton (1996) studies the tension between effective search markets and bilateral cooperation and shows that either of these can drive the other out of existence. Either outcome could be highly inefficient, and the model is intended as a metaphor for the diverse forms of economic interaction one finds in developing countries. We add to this list the idea that the existence of an anonymous market through which standardized goods can be traded could hinder the detection of opportunistic behavior in relationship-based exchange. In a nutshell, a seemingly innocuous market transaction may actually hide an unfaithful transaction in which one of the parties did not play according to the implicit agreement.

There is suggestive evidence of some of these effects. Bertrand (2004) provides evidence of weakening long-term relationships in US firms that have been hit by tough import competition. In those firms, the wage paid to workers is more responsive to fluctuations in labor demand than

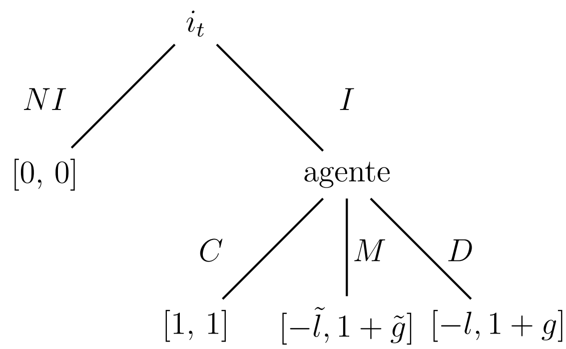


Figure 3: A trust game.

in firms less affected by imports. Cox, Jimenez, and Okrasa (1997) find a breakdown in intra-family risk-sharing transfers in Poland as the market economy became pervasive. Anecdotal evidence show a broad erosion of the importance of long-term relationships in Japan as the economy became more open.

To be more concrete, consider the experience good interpretation of our model (Section 2.3), assuming that players who are not selected in a round can buy a standardized version of the good through an anonymous market. Such transaction yields a per-period payoff equals to 0. The selected player i_t may choose NI and buy the standardized good through the market, or may choose I and engage in a partnership with the agent, paying upfront a sufficiently high amount of money with the expectation of receiving an upgraded version of the experience good (in which case the per-period payoff to i_t equals 1). As in the model discussed in the previous section, the agent may choose to deliver a low-quality good, which is observed by all of i_t's neighbors, but now we also allow the agent to deliver the standardized version of the good. When neighbors only observe the quality of the goods delivered to an investor $i \in \{1, \ldots, N\}$, but not the channel through which the good was acquired (partnership vs. anonymous market), this induces a new and potentially attractive deviation opportunity to the agent.

We formalize these ideas by assuming that the investor selected at round t, i_t, and the agent play a modified trust game:

We assume that $\tilde{l} > 0$ and $\tilde{g} > 0$ so that the only equilibrium of the stage game yields, once again, the inefficient payoff vector $[0, 0]$. We assume that if the selected investor i_t plays I and the agent subsequently chooses M, then i_t's neighbors cannot distinguish that outcome from an anonymous market transaction, which provides the same payoff as the play of NI. As in the previous section, an investor cannot distinguish whether one of his neighbors was not selected from whether the neighbor was selected, but chose NI. All what an investor can learn about his neighbors is whether they invested and had a fully satisfactory experience (in which the agent played C) or a totally unsatisfactory experience (in which the agent played D).

We extend trigger strategies by assuming that a player belonging to the social network G will invest, unless the agent has behaved unfaithfully when facing him or one of his neighbors. We will say that a social network G is *sustainable with partially observable deviations* if there is a sequential best reply σ_0 for the agent to the trigger strategies used by investors such that the agent plays C in all encounters occurring on the path of play. This definition is analogous to Definition 1, with the added twist that in this model there are more deviations as the agent may choose to deviate playing either M or D.

Proposition 4 *The following hold:*

i. If G is sustainable with partially observable deviations, then G is sustainable.

ii. Suppose that $m = DM$ and let G be a nonempty network. Then,

$$G \text{ is sustainable with partially observable deviations iff } |N^G| = 1.$$

iii. Suppose that $m = RM$ and $\frac{1-\delta}{\delta}\tilde{g}N > 1$. Then,

$$G \text{ is sustainable with partially observable deviations iff } N^G = \emptyset.$$

The first part of the proposition shows that adding the possibility of deviating in a partially observable way restricts the set of social networks conducive to efficient trade. Thus, the way we have introduced anonymous markets shrinks the set of sustainable networks. In the previous section, if a network G is not sustainable, then no best reply for the agent leaves the network unchanged. However, a best reply that leaves network G unchanged in the model with partially observable deviations can also be seen as a best reply leaving the network unchanged in the model of the previous section. The result then follows since in the model with partially observable deviations, more deviations must be deterred.

The second part of the proposition shows that when matching is directed, a network is conducive to efficient trade iff the network consists of a single node. Any network consisting of two or more nodes opens up the possibility for the agent to deviate playing M against one of them, without incurring in continuation value losses. The only way in which such deviation can be deterred is by having a network consisting of a single trading investor.

Finally, the third part shows that when $\frac{1-\delta}{\delta}\tilde{g}N$ is above one, then it is always optimal to deviate by playing M and, as a result, the only sustainable network is the empty one. Regardless of the network architecture, deviating playing M entails losses of trading opportunities with a single agent and such deviation cannot be deterred if the severity of the incentive problem associated to that deviation, $\tilde{\alpha} = \frac{1-\delta}{\delta}\tilde{g}N$, is large enough. Observe that we already assumed that in the random matching model, $\kappa^* = \lceil \frac{1-\delta}{\delta} gN \rceil \geq 2$. Thus, when the temptation to deviate playing M is similar to the temptation to deviate playing C, the only sustainable network with partially observable deviations is the empty one.

5.2 Examples

This section provides a few examples of sustainable networks. The following property turns out to be useful in some applications.

Proposition 5 *Suppose that matching is random and let G be such that $\bar{N}^G(i) \geq \kappa^*$ and $G \setminus \bar{N}^G(i)$ is sustainable, for all $i \in N^G$. Then, G is sustainable.*

This proposition shows that a network is sustainable provided each investor i has at least κ^* links and deleting any node i and all the investors connected to i results in a smaller sustainable network. Such a network G can also be seen as robust in the sense that an off-path deviation against any $i \in N^G$ does not lead the agent to commit additional mischievous actions.

Stars We now derive necessary and sufficient conditions for a star to be sustainable, and explore how our general results compare to these conditions. This exercise can also be seen as a generalization of the study of the incomplete network G^3 in Section 3.

Consider the model studied in Section 2. Let G be a star consisting of $k+1 \geq N$ nodes, with $k \geq 2$. Recall that G is a star if there exists a node $i \in N^G$ such that for every node $j \in N^G$, $N^G(j) = \{i\}$. The question we ask is whether network G is sustainable.

There are simple conditions ruling out the sustainability of the star. When matching is directed, Proposition 1 implies that only complete networks can be sustainable and as result, the star, being an incomplete network, cannot be sustainable. When matching is random, Proposition 2 implies that members of sustainable networks must have closed degrees greater than or equal to κ^*. Therefore, when matching is random and $\kappa^* \geq 3$, the star cannot be sustainable. On the other hand, when matching is random and $\kappa^* = 1$, the star is sustainable as can be seen from by applying Proposition 5.

The only case left to study is when matching is random and $\kappa^* = 2$.

Proposition 6 *Suppose that matching is random, $\kappa^* = 2$, and let $\bar{k} = 1 + \frac{\kappa^* - \alpha N}{\alpha N - 1}\Big(N(1-\delta) + \delta\Big)$. Then,*

$$G \textit{ is sustainable iff } k \leq \bar{k}.$$

This result shows that the star will be sustainable if and only if its size is sufficiently low. If the network is too large, then there are attractive opportunities to double deviate by first defecting against a peripheral investor and then taking advantage of uninformed investors. This implies that sustainable stars cannot have too many members.

We can also use our general results to derive necessary and sufficient conditions for sustainability. Corollary 2 implies that if $k \leq 1 + \frac{\kappa^* - \alpha N}{g}$, then the star is sustainable. Proposition 3 allows us to derive the following necessary condition for sustainability when $\kappa^* - \alpha N < \frac{1}{2}$: $k \leq 1 + \frac{N(1-\delta)}{\delta} \frac{\kappa^* - \alpha N}{1 - 2(\kappa^* - \alpha N)}$. These bounds are not tight, but they restrict the parameters of the game in a meaningful way and have qualitative implications consistent with the sharp characterization of Proposition 6.

Unions of Complete Components We now restrict attention to the random matching model. Complete networks consisting of κ^* or more nodes are always sustainable, regardless of the matching technology. The question we ask is whether unions of complete components can result in an sustainable network.

Let G^1 and G^2 be two disjoint complete components, with $N^{G^n} \geq \kappa^*$ for $n = 1, 2$. Take the graph $G = (N^G, V^G)$, with $N^G = N^{G^1} \cup N^{G^2}$ and $V^G = V^{G^1} \cup V^{G^2} \cup \{lm\}$, where $l \in N^{G^1}$ and $m \in N^{G^2}$. Network G is the union of networks G^1 and G^2, and a bridge connecting both of them.

Proposition 7 *The following hold:*

i. If $|N^{G^n}| \geq \kappa^* + 1$ *for* $n = 1, 2$, *then* G *is sustainable.*

ii. If $|N^{G^n}| = \kappa^*$ *for some* n *and* $\kappa^* - \alpha N < \frac{1}{N\frac{1-\delta}{\delta}+2}$, *then* G *is not sustainable.*

It is also of interest to study whether tree unions of networks can be sustainable. Assume that G^1 and G^2 have a single node in common and let G be the tree union of G^1 and G^2 defined as $N^G = N^{G^1} \cup N^{G^2}$ and $E^G = E^{G^1} \cup E^{G^2}$. The three union operation has been shown to result in equilibrium networks (or, using our terminology, sustainable networks) in a repeated game model of complete information (Jackson, Rodriguez-Barraquer, and Tan 2010). As the following Proposition shows, this need not be the case in our model of incomplete information.

Proposition 8 *The following hold:*

i. If $|N^{G^n}| \geq \kappa^* + 1$ *for* $n = 1, 2$, *then* G *is sustainable*

ii. If $|N^{G^n}| = \kappa^*$ *for* $n = 1, 2$ *and* $(\kappa^* - \alpha N) < \frac{\kappa^*-1}{N\frac{1-\delta}{\delta}+1}$, *then* G *is not sustainable.*

As already illustrated in Section 3, tree unions of cliques of a minimal size κ^* need not be sustainable as, after a deviation against one of the network members, it may still be possible to exploit further deviating opportunities. The same mechanism is at work in the general model, and this contrasts with the model of Jackson, Rodriguez-Barraquer, and Tan (2010).

6 Concluding Remarks

This paper studies the problem of self governance in a model in which information flows are governed by the community structure. Our results show that the way in which trading opportunities arise is a crucial determinant of the architecture of social networks conducive to efficient trade. How easily it is to substitute potential trading partners has a nontrivial effect on the architecture of social networks conducive to efficient trade. In particular, when matching is directed and investors are easily substitutable, efficient trade is attainable only in complete networks. In contrast, when matching is random and investors cannot be substituted to produce surplus, there are conditions under which self-governance can emerge even in barely cohesive communities. Efficient networks maximize the total surplus created and we have identified conditions under which their architecture reduces to one or more complete components. Our model yields testable implications relating the fundamentals of the game to the architecture of social networks and can be accommodated to explore the interaction between formal and informal institutions.

Several variations seem worth exploring. First, whether our framework could be extended to study two-sided incentive problems (Kandori 1992) seems an interesting question. Second, the social network G need not be known to the agent. For example, in transactions of experience goods, a firm is unlikely to know the social ties a client has.[13] Third, one could study alternative matching technologies in which investors could be selected randomly according to a Markov process of recognition. It seems important to understand how this process (which can be seen as a network of emerging trading opportunities) restricts sustainable networks.

[13] Fainmesser (2010) studies a repeated game model in which the social network is not common knowledge.

A Dynamic Programming Formulation of the RM Model

In this appendix, we study the agent's optimization problem in the random matching model. Suppose the agent is facing $i \in \{1, \ldots, N\}$ at the first round of play $t = 1$. If $i \notin N^G$, then i plays NI. This implies that regardless of player 0's action no information is transmitted and thus it is in player 0's interest to choose C. If $i \in N^G$, player i chooses I. If the agent chooses D, then his period payoff will be $1 + g$. In the next round $t = 2$, player i's neighbors get informed and, as mandated by the strategy profile $(\sigma_j^G)_{j \in \bar{N}^G(i)}$, they do not to trade. Thus, none of the links involving nodes in $\bar{N}^G(i)$ transmits any information. Thus, at round $t = 2$, the agent is effectively facing a network in which all nodes in $\bar{N}^G(i)$ and their links have been removed. In other words, the problem the producer will face is similar to the one faced at $t = 1$, but with the smaller network $G \setminus \bar{N}^G(i)$ replacing G. When facing investor i at $t = 1$, the agent can also choose the high action, get a period payoff of 1, and keep the network G unchanged.

We can then formulate the decision problem faced by agent as a dynamic programming problem. Denoting by $v(i, G')$ the expected discounted sum of normalized payoffs when the agent faces investor i, given a network G', it follows that

$$v(i, G') = \begin{cases} \max\left\{(1-\delta) + \frac{\delta}{N}\sum_{j=1}^{N} v(j, G'), (1-\delta)(1+g) + \frac{\delta}{N}\sum_{j=1}^{N} v(j, G' \setminus \bar{N}^{G'}(i))\right\} & \text{if } i \in N^{G'} \\ \frac{\delta}{N}\sum_{j=1}^{N} v(j, G') & \text{if not.} \end{cases} \tag{A1}$$

Equation (A1) is a Bellman equation and standard arguments show the existence and uniqueness of the value function $(v(i, G'))_{i,G'}$ (Stokey and Lucas 1989).

It will be in general hard to find closed form solutions to continuation values. Yet, the following lemma provides bounds that will be useful in the sequel.

Lemma 1 *The following statements hold:*

i. For all G, $\sum_{j=1}^{N} v(j, G) \geq |N^G|$. The equality holds provided G is sustainable.

ii. For all G, $\sum_{j=1}^{N} v(j, G) \leq (1 + g)|N^G|$.

Proof. Let us prove the first part of the lemma. By definition, for all $i \in N^G$,

$$v(i, G) \geq (1-\delta) + \frac{\delta}{N}\sum_{k=1}^{N} v(k, G) \tag{A2}$$

while for $i \notin G$, $v(i,G) = \frac{\delta}{N}\sum_{k=1}^N v(k,G)$. Adding up and solving for $\sum_{k=1}^N v(k,G)$, it follows that $\sum_{k=1}^N v(k,G) \geq |N^G|$. Note also that if G is sustainable then (A2) holds with equality, this completes the first part of the lemma.

To prove the second part, note that the statement holds when $N^G = \emptyset$. Assume that the statement holds for all networks G' with $|N^{G'}| \leq n-1$ and let us prove the result when G is such that $|N^G| = n$. Note first that for all $j \in N^G$

$$v(j,G) \leq (1-\delta)(1+g) + \frac{\delta}{N}\max\Big\{(1+g)\Big(|N^G| - |\bar{N}^G(j)|\Big), \sum_{k=1}^N v(k,G)\Big\}$$

where we use that $N^{G\setminus\bar{N}^G(i)} \leq n-1$. Let $P \subseteq N^G$ be the set of all j such that $(1+g)\Big(|N^G| - |\bar{N}^G(j)|\Big) \geq \sum_{k=1}^N v(k,G)$. If P is empty, the result follows immediately. If not, we deduce that

$$\sum_{j=1}^N v(j,G) \leq (1-\delta)(1+g)|N^G| + \frac{\delta}{N}\Big\{(1+g)\sum_{j\in P}\Big(|N^G| - |\bar{N}^G(j)|\Big) + (N-|P|)\sum_{k=1}^N v(k,G)\Big\}$$

and thus

$$\begin{aligned}\sum_{j=1}^N v(j,G) \leq& \frac{(1-\delta)(1+g)|N^G|}{1-\delta(\frac{N-|P|}{N})} + \frac{\delta(1+g)}{1-\delta(\frac{N-|P|}{N})}\sum_{j\in P}\frac{|N^G| - |\bar{N}^G(j)|}{N} \\ \leq& \frac{(1-\delta)(1+g)|N^G|}{1-\delta(\frac{N-|P|}{N})} + \frac{\delta(1+g)}{1-\delta(\frac{N-|P|}{N})}|P|\frac{|N^G|}{N} \\ \leq& (1+g)|N^G|\Big\{\frac{(1-\delta)+\delta\frac{|P|}{N}}{1-\delta(\frac{N-|P|}{N})}\Big\} \\ =& (1+g)|N^G|\end{aligned}$$

which proves the result. ■

This lemma provides upper and lower bounds on the agent's continuation values for any network G. The lower bound states that the agent's continuation value is at least what he gets if he complies in all possible encounters when facing members of the network G, while the upper bound says that the continuation value cannot be greater than what the agent could get if he systematically chooses the low action in all encounters, but he does not loose any trading opportunity.

B Proofs of Section 4.2

Proof of Proposition 2. *Part (i)* Since G is sustainable, $\sum_{j=1}^{N} v(j,G) = |N^G|$. For $i \in N^G$,

$$\begin{aligned}(1-\delta)+\delta\frac{1}{N}|N^G| &\geq (1-\delta)(1+g)+\delta\frac{1}{N}\sum_{j=1} v(j, G\setminus \bar{N}^G(i))\\ &\geq (1-\delta)(1+g)+\delta\frac{|G\setminus \bar{N}^G(i)|}{N}\\ &= (1-\delta)(1+g)+\delta\frac{|N^G|-|N^G(i)|}{N},\end{aligned}$$

where the first inequality is by definition of sustainability and the second one follows from the lemma above. This in turn implies $(1-\delta)+\delta\frac{|\bar{N}^G(i)|}{N} \geq (1-\delta)(1+g)$. By definition of κ^*, $|\bar{N}^G(i)| \geq \kappa^*$.

Part (ii) Assume that G is not sustainable. Then, there exists $i \in N^G$ such that

$$(1-\delta)(1+g)+\frac{\delta}{N}\sum_{k=1}^{N} v(k, G\setminus \bar{N}^G(i)) > (1-\delta)+\frac{\delta}{N}\sum_{k=1}^{N} v(k,G).$$

Using both parts of Lemma 1, it follows that

$$(1-\delta)(1+g)+(1+g)\frac{\delta}{N}\big(|N^G|-|\bar{N}^G(i)|\big) > (1-\delta)+\delta\frac{|N^G|}{N}.$$

The result follows rearranging terms. ■

Proof of Corollary 2. *Part (i)* If G is sustainable, then $\bar{N}^G(i) \geq \kappa^*$ for all $i \in N^G$. Conversely, if $\bar{N}^G(i) \geq \kappa^*$ for all $i \in N^G$, then

$$\bar{N}^G(i) \geq \kappa^* \geq \kappa^* + \frac{g}{1+g}(|N^G|-\kappa^*) - \frac{1}{1+g}(\kappa^* - \frac{1-\delta}{\delta}gN) = \frac{g}{1+g}|N^G| + \frac{1}{1+g}\frac{1-\delta}{\delta}gN$$

and the sustainability of G is deduced from Proposition 2.

Part (ii) If G is minimally sustainable, then $i \in N^G$, $\min_{k\in \bar{N}^G(i)}|\bar{N}^G(k)| \geq \kappa^*$ from Part (i). Moreover, Proposition 2, there must be $k \in \bar{N}^G(i)$ such that

$$|\bar{N}^G(k)| < \kappa^* + 1 + \frac{1}{1+g}\Big(g(|N^G|-\kappa^*) + (\frac{1-\delta}{\delta}gN - \kappa^*)\Big) \leq \kappa^* + 1$$

and thus $|\bar{N}^G(k)| = \kappa^*$. Conversely, let G be such that for all $i \in N^G$, $\min_{k\in \bar{N}^G(i)}|\bar{N}^G(k)| = \kappa^*$.

Then, Part (i) implies that G is sustainable. To see G is minimally sustainable, note that no link can be removed without impairing sustainability as a consequence of Proposition 2. ■

C Proofs of Section 4.3

Proof of Proposition 3. Let us partition the set of nodes as

$$\{1,\dots,N\} = Q \cup \Big[N^G \setminus (Q \cup \bar{N}^G(i))\Big] \cup \Big[(\{1,\dots,N\} \setminus N^G) \cup \bar{N}^G(i)\Big].$$

First note that for all $j \in Q$

$$\begin{aligned} v(j, G \setminus \bar{N}^G(i)) &\geq (1-\delta)(1+g) + \frac{\delta}{N}\Big(\sum_{k=1}^{N} v(k, [G \setminus \bar{N}^G(i)] \setminus \bar{N}^{G \setminus \bar{N}^G(i)}(j))\Big) \\ &\geq (1-\delta)(1+g) + \frac{\delta}{N}\Big(|\bar{N}^G| - |\bar{N}^G(i)| - |\bar{N}^{G \setminus \bar{N}^G(i)}(j)|\Big) \end{aligned}$$

while for $j \in N^G \setminus (Q \cup \bar{N}^G(i))$

$$v(j, G \setminus \bar{N}^G(i)) \geq (1-\delta) + \frac{\delta}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i))$$

and for $j \in (\{1,\dots,N\} \setminus N^G) \cup \bar{N}^G(i)$

$$v(j, G \setminus \bar{N}^G(i)) = \frac{\delta}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i)).$$

It then follows that

$$\begin{aligned} \sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i)) \geq |Q|\Big((1-\delta)(1+g) + \frac{\delta}{N}(|N^G| - |\bar{N}^G(i)|)\Big) - \frac{\delta}{N}\sum_{j \in Q}|\bar{N}^{G \setminus \bar{N}^G(i)}(k)| \\ +\big(|N^G| - |Q| - |\bar{N}^G(i)|\big)\Big((1-\delta) + \frac{\delta}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i)\Big) \\ +\big(N - |N^G| + |\bar{N}^G(i)|\big)\frac{\delta}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i)) \end{aligned}$$

and solving for $\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i))$

$$\frac{1}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i)) \geq \frac{1}{N(1-\delta)+\delta|Q|}\Big\{(1-\delta)(|N^G| - |\bar{N}^G(i)|) + (1-\delta)g|Q| + \frac{\delta}{N}|Q||N^G| - \frac{\delta}{N}\sum_{j\in Q}\big(|\bar{N}^{G\setminus\bar{N}^G(i)}(j)| + |\bar{N}^G(i)|\big)\Big\}.$$

Since G is sustainable, $(1-\delta) + \delta\frac{|N^G|}{N} \geq (1-\delta)(1+g) + \frac{\delta}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i))$. Plugging in the inequality above for $\frac{1}{N}\sum_{k=1}^{N} v(k, G \setminus \bar{N}^G(i))$, we deduce that

$$\frac{N(1-\delta)}{N(1-\delta)+\delta|Q|}\Big\{2g|Q| - |\bar{N}^G(i)| + \frac{1-\delta}{\delta}gN\Big\} \leq \frac{\delta}{N(1-\delta)+\delta|Q|}\sum_{j\in Q}\big(|\bar{N}^{G\setminus\bar{N}^G(i)}(j)| + |\bar{N}^G(i)|\big).$$

Now, note that for all $j \in Q \subseteq M^G(i)$, $|\bar{N}^{G\setminus\bar{N}^G(i)}(j)| = |\bar{N}^G(j)| - |\{k \in N^G \mid ik, kj \in E^G\}|$ and arrange terms to obtain the desired results. ■

Proof of Corollary 3. Take a sustainable network G and $i \in N^G$. Proposition 3 implies that

$$\begin{aligned}\sum_{k\in N^G(i)} |N^G(k) \setminus \bar{N}^G(i)| &= \sum_{j\in M^G(i)} |\{k \in N^G \mid ik, kj \in E^G\}| \\ &\leq \Big(\frac{N(1-\delta)}{\delta} + 2|M^G(i)|\Big)\Big(K^G(i) - \frac{1-\delta}{\delta}gN\Big)\end{aligned}$$

where $K^G(i) = \max\{|\bar{N}^G(j)| \mid j \in \bar{N}^G(i) \cup M^G(i)\}$. Thus, the number of links between nodes in $N^G(i)$ is at least

$$\begin{aligned}&\frac{1}{2}\Big\{\sum_{k\in N^G(i)}\Big(|N^G(k)| - 1\Big) - \sum_{k\in N^G(i)} |N^G(k) \setminus \bar{N}^G(i)|\Big\} \\ &\geq \frac{1}{2}\Big\{\sum_{k\in N^G(i)}\Big(|N^G(k)| - 1\Big) - \Big(\frac{N(1-\delta)}{\delta} + 2|M^G(i)|\Big)\Big(K^G(i) - \frac{1-\delta}{\delta}gN\Big)\Big\}\end{aligned}$$

and thus the clustering coefficient of node i can be bounded below by

$$\textsc{Cluster}^G(i) \geq \frac{\sum_{k\in N^G(i)}\Big(|N^G(k)| - 1\Big) - \Big(\frac{N(1-\delta)}{\delta} + 2|M^G(i)|\Big)\Big(K^G(i) - \frac{1-\delta}{\delta}gN\Big)}{|N^G(i)|(|N^G(i)| - 1)}. \qquad \text{(C1)}$$

Since for all $i \in N^G$, $\bar{N}^G(i) = \beta N$. Then, (C1) reduces to

$$\textsc{Cluster}^G(i) \geq 1 - \frac{\alpha/g + 2(1-\beta)}{(\beta - \frac{1}{N})(\beta - \frac{1}{N})}(\beta - \alpha).$$

■

Proof of Corollary 4. *Part (i)* Suppose that a nonempty network G is efficient. Proposition 2 implies that for all $i \in N^G$, $|N^G(i)| \geq \kappa^* - 1$ and thus $\sum_{i=1}^N c(|N^G(i)|) \geq |N^G| c(\kappa^* - 1)$. On the other hand, the sum across all players of total expected payoffs in the repeated game equals $\frac{|N^G|}{N}\frac{2}{1-\delta} < |N^G| c(\kappa^* - 1) \leq \sum_{i=1}^N c(|N^G(i)|)$ and therefore the empty network, yielding a total expected payoff of 0, strictly dominates G.

Part (ii) Consider a sustainable network G such that $|N^G| = m\kappa^* + n$, where $m, n \geq 1$ and $n \leq \kappa^* - 1$. At least one component of network G has $M \geq \kappa^* + 1$ nodes. Condition 3 in the statement of the result together with Proposition 3 imply there exists a node i in such component such that $\bar{N}^G(i) \geq \kappa^* + 1$. Since G is sustainable, all remaining nodes must have closed degree at least equal to κ^*. The objective function defining efficient networks evaluated at G is at most

$$\frac{m\kappa^* + n}{N}\frac{2}{1-\delta} - c(\kappa^*) - (m\kappa^* + n)c(\kappa^* - 1). \tag{C2}$$

Form now a new network $\bar{G}$ with $|N^{\bar{G}}| = (m+1)\kappa^*$ consisting of $m+1$ complete components, each having κ^* nodes. Such network can always be formed, as a result of Condition 2, and is sustainable. The objective function (2.1) at $\bar{G}$ equals

$$\frac{(m+1)\kappa^*}{N}\frac{2}{1-\delta} - (m+1)\kappa^* c(\kappa^* - 1).$$

The difference between the term above and (C2) equals

$$(\kappa^* - n)\Big(\frac{2}{1-\delta}\frac{1}{N} - c(\kappa^* - 1)\Big) + \Big(c(\kappa^*) - c(\kappa^* - 1)\Big),$$

which is strictly positive. It then follows that any efficient network G is such that $\frac{|N^G|}{\kappa^*} \in \mathbb{N}$. For any such network size $|N^G|$, a new application of Proposition 3 and Condition 3 implies that the (unique) least total number of links is attained when the $|N^G|$ nodes are arranged in $\frac{|N^G|}{\kappa^*}$ components. Condition 1 implies that the value of each component is strictly positive and the result follows. ■

D Proofs of Section 5

Proof of Proposition 5. Since $G \setminus \bar{N}^G(i)$ is sustainable, $\sum_{j=1}^N v(j, G \setminus \bar{N}^G(i)) = |G \setminus \bar{N}^G(i)| = |N^G| - |\bar{N}^G(i)|$. sustainability of G requires for $i \in N^G$ that

$$(1-\delta) + \frac{\delta}{N}\sum_{j=1} v(j,G) \geq (1-\delta)(1+g) + \frac{\delta}{N}\sum_{j=1} v(j, G \setminus \bar{N}^G(i))$$
$$= (1-\delta)(1+g) + \delta\frac{|N^G| - |N^G(i)|}{N}.$$

Because of Lemma 1, the inequality is satisfied if $(1-\delta) + \frac{\delta}{N}|N^G| \geq (1-\delta)(1+g) + \frac{\delta}{N}(|N^G| - |\bar{N}^G(i)|)$ or equivalently, $|\bar{N}^G(i)| \geq \frac{1-\delta}{\delta}Ng$. But $|\bar{N}^G(i)| \geq \kappa^*$, and thus the result follows. ■

Proof of Proposition 6. The social network G is sustainable iff

$$(1-\delta) + \delta\frac{k+1}{N} \geq (1-\delta)(1+g) + \delta v_{k-1}$$

where v_{k-1} is the ex ante continuation value accruing to the agent when he is facing a set of $k-1$ unconnected nodes. Now, note that $(v_k)_{k=0}^N$ satisfies the following recursion

$$v_k = \frac{k}{N}\Big((1-\delta)(1+g) + \delta v_{k-1}\Big) + \frac{N-k}{N}\delta v_k$$

with $v_0 = 0$. The recursion can be written as $v_k = \frac{k}{N(1-\delta)+\delta k}\Big((1-\delta)(1+g) + \delta v_{k-1}\Big)$ and is solved by

$$v_k = k\frac{(1-\delta)(1+g)}{N(1-\delta)+\delta}.$$

Thus the star G composed of $k+1$ nodes is sustainable iff

$$\frac{1}{N}(2 - \frac{1-\delta}{\delta}gN) \geq (k-1)\Big(\frac{(1-\delta)(1+g)}{N(1-\delta)+\delta} - \frac{1}{N}\Big) \tag{D1}$$

which can be equivalently written as

$$k \leq 1 + \frac{\kappa^* - \frac{1-\delta}{\delta}gN}{\frac{1-\delta}{\delta}gN - 1}\Big(N(1-\delta)+\delta\Big)$$

which proves the result. ■

Proof of Proposition 7. The first statement follows from Proposition 5 by noting that for all $i \in N^G$, $\bar{N}^G(i) \geq \kappa^* + 1$ and $G \setminus \bar{N}^G(i) \geq \kappa^*$ is always stable. The second part follows by applying Proposition 3 to a node in N^{G^n} that is not a bridge. ■

Proof of Proposition 8. The first part follows from Proposition 5. The second part follows by taking any node which is not in $N^{G^1} \cap N^{G^2}$ and noting that a necessary condition for stability, as implies by Proposition 3, is

$$\kappa^* - 1 \leq (N\frac{1-\delta}{\delta} + 2)(\kappa^* - \alpha N).$$

The result follows by noting this necessary condition is violated. ■

E Additional Results

E.1 Sustainable Networks and Sequential Equilibrium

The following result connects sustainable networks to sequential equilibria.

Lemma 2 *Let G be a σ_0-sustainable network. Then, (σ^0, σ^G) is a sequential equilibrium of the game.*

Proof. Suppose first that matching is random, fix σ_0 and σ_G and take any consistent system of beliefs. When $i \in N^G$ knows the agent has played D when facing him or one of his neighbors, it is common knowledge between the agent and i that the agent will play D against i in all subsequent rounds. Thus, it is in player i's interest to play NI. It follows that the prescribed strategies are sequentially rational. When matching is directed, the result follows noting that G is a complete network. Therefore, if an investor is selected off-path, the investor knows the continuation strategy of the agent and, given that continuation strategy it is optimal to play as mandated by the trigger strategy. ■

It is perhaps of interest to contrast the above construction when matching is random to Kandori's (1992). In our random matching model, after player i_t has observed some off path behavior, regardless of his belief about when the deviation occurred, it is optimal for him to play NI in all subsequent rounds. In Kandori's (1992) model, after an off-path observation, a player's optimal continuation strategy will, in general, depend on his belief about when the off-path phase was triggered. That makes Kandori's (1992) construction much more involved.

E.2 A Folk Theorem

In contrast to most work in repeated games (e.g., Fudenberg and Maskin 1986, Fudenberg, Levine, and Maskin 1994), we do not restrict attention to δ arbitrarily close to 1. On the contrary, we fix players' preferences and ask what properties *sustainable* networks exhibit. Just for the record, though, we offer a folk-theorem-like result.

Proposition 9 *Suppose matching is random. Fix $g > 0$, $N \in \mathbb{N}$ and a social network G. Then, there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, G is a sustainable network.*

Proof. Let $\bar{\delta} = \frac{g}{g+\frac{1}{N}}$. Fix $\delta > \bar{\delta}$ and note that $(1-\delta) + \frac{\delta}{N} \geq (1-\delta)(1+g)$. This means that $\kappa^* = 1$ and thus any network with an empty set of links is sustainable. For each $n \geq 0$, let $\mathbb{G}^n = \{G = (N^G, E^G) \mid |N^G| \leq n\}$. We have already argued that any $G \in \mathbb{G}^0$ is sustainable. Assume that all networks in $\mathbb{G}^{n-1}$ are sustainable. Let $G \in \mathbb{G}^n$. Then, for all $i \in N^G$, $\bar{N}^G(i) \geq 1 = \kappa^*$. Moreover, if $i \in N^G$, then $G \backslash \bar{N}^G(i) \in \mathbb{G}^{n-1}$ is sustainable. Proposition 5 implies that G is sustainable. ■

Our game model is a private monitoring game –a player $i \in \{1, \ldots, N\}$ cannot observe all players' actions and how continuation play evolves is not common knowledge–. Though our game is extremely simple, existent general folk theorems (Mailath and Samuelson 2006) do not apply.

References

Abreu, D., D. Pearce, and E. Stacchetti (1990): "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica*, 58(5), 1041–1063.

Ahn, I., and M. Suominen (2001): "Word-of-Mouth Communication and Community Enforcement," *International Economic Review*, 42, 399–415.

Ali, S., and D. Miller (2010): "Cooperation and Collective Enforcement in Networked Societies," Discussion paper.

Allcott, H., D. Karlan, M. Mobius, T. Rosenblat, and A. Szeidl (2007): "Community Size and Network Closure," *The American Economic Review: Papers and Proceedings*, pp. 80–85.

Attanasio, O., and J.-V. Rios-Rull (2000): "Consumption smoothing in island economies: Can public insurance reduce welfare?," *European Economic Review*, 44(7), 1225–1258.

Balmaceda, F. (2006): "Endogenous Cooperative Networks: Social Capital and the Small-World Property," *http://ssrn.com/abstract=930053*.

Bendor, J., and D. Mookherjee (1990): "Norms, Third Party Sanctions, and Cooperation," *Journal of Law, Economics, and Organization*, 6(1), 33.

Bernstein, L. (1992): "Opting Out of the Legal System: Extralegal Contractual Relations in the Diamond Industry," *The Journal of Legal Studies*, 21(1), 115–157.

Bertrand, M. (2004): "From the Invisible Handshake to the Invisible Hand? How Import Competition Changes the Employment Relationship," *Journal of Labor Economics*, 22(4), pp. 723–765.

Bhaskar, V. (1998): "Informational Constraints and the Overlapping Generations Model: Folk and Anti-Folk Theorems," *Review of Economic Studies*, 65(1), 135–149.

Bloch, F., G. Genicot, and D. Ray (2008): "Informal Insurance in Social Networks," *Journal of Economic Theory*, 143(1), 36–58.

Chwe, M. (2000): "Communication and Coordination in Social Networks," *Review of Economic Studies*, 67(1), 1–16.

Coleman, J. (1988): "Social Capital in the Creation of Human Capital," *American Journal of Sociology*, 94(1), 95–120.

——— (1990): *Foundations of Social Theory*. Belknap Press.

Cox, D., E. Jimenez, and W. Okrasa (1997): "Family Safety Nets and Economic Transition: A Study of Worker Households in Poland," *Review of Income and Wealth*, 43(2), 191–209.

Dellarocas, C. (2003): "The Digitization of Word of Mouth: Promise and Challenges of Online Feedback Mechanisms," *Management Science*, 49(10), 1407–1424.

Dixit, A. (2006): *Lawlessness and Economics. Alternative Modes of Governance*. Oxford University Press.

Ellison, G. (1994): "Cooperation in the Prisoner's Dilemma with Anonymous Random Matching," *The Review of Economic Studies*, 61(3), 567–588.

Fainmesser, I. (2010): "Community Structure and Market Outcomes: A Repeated Games in Networks Approach," Discussion paper, Brown University.

Fearon, J., and D. Laitin (1996): "Explaining Interethnic Cooperation," *American Political Science Review*, pp. 715–735.

Fudenberg, D., D. Levine, and E. Maskin (1994): "The Folk Theorem with Imperfect Public Information," *Econometrica*, 62(5), 997–997.

Fudenberg, D., and E. Maskin (1986): "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information," *Econometrica*, 54(3), 533–554.

Ghosh, P., and D. Ray (1996): "Cooperation in Community Interaction without Information Flows," *The Review of Economic Studies*, 63(3), 491–519.

Greif, A. (1993): "Contract Enforceability and Economic Institutions in Early Trade: The Maghribi Traders' Coalition," *The American Economic Review*, 83(3), 525–548.

——— (2006): *Institutions and the Path to the Modern Economy: Lessons from Medieval Trade*. Cambridge University Press.

Grossman, S., and O. Hart (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *The Journal of Political Economy*, 94(4), 691–719.

Haag, M., and R. Lagunoff (2006): "Social Norms, Local Interaction and Neighborhood Planning," *International Economic Review*, 47(1), 265–296.

Harrington, J. (1995): "Cooperation in a One-Shot Prisoners' Dilemma*," *Games and Economic Behavior*, 8(2), 364–377.

Head, K., J. Ries, and B. Spencer (2004): "Vertical Networks and US Auto Parts Exports: Is Japan Different?," *Journal of Economics & Management Strategy*, 13(1), 37–67.

Jackson, M. (2008): *Social and Economic Networks*. Princeton University Press.

Jackson, M., T. Rodriguez-Barraquer, and X. Tan (2010): "Social Capital and Social Quilts: Network Patterns of Favor Exchange," Discussion paper, Stanford University.

Jackson, M., and B. Rogers (2005): "The Economics of Small Worlds," *Journal of the European Economic Association*, 3(2-3), 617–627.

——— (2007): "Meeting Strangers and Friends of Friends: How Random Are Social Networks?," *American Economic Review*, 97(3), 890–915.

Kandori, M. (1992): "Social Norms and Community Enforcement," *Review of Economic Studies*, 59(1), 63–80.

——— (2002): "Introduction to Repeated Games with Private Monitoring* 1," *Journal of Economic Theory*, 102(1), 1–15.

Karlan, D., M. Mobius, T. Rosenblat, and A. Szeidl (2009): "Trust and Social Collateral," *Quarterly Journal of Economics*, 124(3), 13071–361.

Kinateder, M. (2010): "The Repeated Prisoner's Dilemma in a Network," Discussion paper, Universidad de Navarra.

Klein, B., R. Crawford, and A. Alchian (1978): "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process," *Journal of Law and Economics*, 21(2), 297–326.

Kranton, R. (1996): "Reciprocal Exchange: A Self Sustaining System," *The American Economic Review*, 86(4), 830–851.

Kreps, D., and R. Wilson (1982): "Sequential Equilibria," *Econometrica*, 50(4), 863–894.

Lippert, S., and G. Spagnolo (2010): "Networks of Relations and Word of Mouth Communication," *Games and Economic Behavior*.

Mailath, G., and L. Samuelson (2006): *Repeated Games and Reputations*. Oxford University Press, Oxford.

McMillan, J. (1995): "Reorganizing Vertical Supply Relationships," in *Trends in Business Organization: Do Participation and Cooperation Increase Competitiveness*, pp. 203–222. Tubingen:JCB Mohr.

Mihm, M., R. Toth, and C. Lang (2009): "What Goes Around Comes Around: A Theory of Indirect Reciprocity in Networks," Working papers, Cornell University, Center for Analytic Economics.

Milgrom, P., D. North, and B. Weingast (1990): "The Role of Institutions in the Revival of Trade: The Law Merchant, Private Judges, and the Champagne Fairs," *Economics and Politics*, 2(1), 1–23.

Okuno-Fujiwara, M., and A. Postlewaite (1995): "Social Norms and Random Matching Games," *Games and Economic Behavior*, 9(1), 79–109.

Ostrom, E. (1990): *Governing the Commons: The Evolution of Institutions for Collective Action*. Cambridge University Press.

Raub, W., and J. Weesie (1990): "Reputation and Efficiency in Social Interactions: An Example of Network Effects," *American Journal of Sociology*, 96(3), 626–654.

Rauch, J. (2001): "Business and Social Networks in International Trade," *Journal of Economic Literature*, 39(4), 1177–1203.

Spagnolo, G. (2002): "Globalization and Cooperative Relations," CEPR Discussion Papers 3522, C.E.P.R. Discussion Papers.

Stokey, N., and E. Lucas, R. with Prescott (1989): *Recursive Methods in Economic Dynamics*. Harvard University Press, Cambridge.

Takahashi, S. (2010): "Community Enforcement when Players Observe Partners' Past Play," *Journal of Economic Theory*, 145(1), 42–62.

Uzzi, B. (1996): "The Sources and Consequences of Embeddedness for the Economic Performance of Organizations: The Network Effect," *American Sociological Review*, 61(4), 674–698.

Watson, J. (2002): "Starting Small and Commitment," *Games and Economic Behavior*, 38(1), 176–199.

Williamson, O. (1979): " Transaction-Cost Economics: The Governance of Contractual Relations," *Journal of Law and Economics*, 22(2), 233–261.