# Reputation and Accountability in Repeated Elections* 

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#### Abstract

This paper studies a model of infinitely repeated elections in which voters attempt simultaneously to select competent politicians and to provide them with incentives to exert costly effort. Voters are unable to incentivize effort if they base their reelection decisions only on incumbent reputation. However, equilibria in which voters use reputation-dependent performance cutoffs (RDC) to make reelections decisions exist and support positive effort. In these equilibria, politicians' effort is decreasing in reputation, and expected performance is decreasing in tenure. Like the equilibria in Ferejohn 1986, RDC equilibria rely on voters being indifferent between reelecting incumbents and electing challengers. I show that this voter-indifference condition is closely related to weak renegotiation-proofness (Farrell and Maskin 1989).


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## 1. Introduction

How can electoral competition prevent incumbent politicians from using their position to extract rents at the electorate's expense? Political agency theory, exemplified by the work of Key (1966), Barro (1973), and Ferejohn (1986), has stressed that voters provide incentives for incumbents to work in their interest by conditioning reelection on performance. I refer to this as the sanctioning role of elections.

This is not the only way in which voters may use elections to get better government. Given that there are likely to be considerable differences among politicians, voters may use elections to their benefit by attempting to retain only those politicians who are most willing or able to work in their interest. I refer to this as the selection role of elections.

The relative importance of these roles is a theoretical question which will have different answers depending on the institutional environment (e.g. are there term limits?) and the type of heterogeneity among politicians which is considered (e.g. do they differ in honesty or competence?). The main goal of this paper is to contribute to the understanding of the twin roles of elections, how voters may optimally and credibly balance them, and the implications of this balance for voter behavior and political careers through time.

Most work in this area has concluded that voters will focus exclusively on selection. In an influential article, Fearon (1999, p. 77) states that "when it comes time to vote it makes sense for the electorate to focus completely on the question of type: which candidate is more likely to be principled and share the public's preferences?" Similar claims are made by Besley (2006) and others (see the related literature section below) whose models predict that, when making reelection decisions, voters will consider only the incumbent's probability of being a 'good' type. In contrast to most previous related work, I find that sanctioning must be a central component of voter behavior in any equilibrium in which rentextraction is limited.

I study an infinite-horizon model of repeated elections with no term limits in which politicians differ in their competence. There are two types of politician: H (high) and L (low). H-types are competent: by exerting costly effort they can improve the expected utility of voters. L-types, on the other hand, are incompetent: they do not have the ability to improve outcomes, or it is prohibitively costly for them to do so. If H-types are believed to exert some effort, the voters' beliefs about the likelihood of an incumbent being an H-type will evolve along with his observed performance. I refer to these beliefs as a politician's reputation.

As in most infinitely repeated games, the set of equilibria of this model is large. In fact, any pure reelection strategy may be supported as part of an equilibrium (see Proposition 1). The fact that arbitrary behavior on the part of voters can be derived as a prediction of this model highlights the importance of equilibrium selection. Appealing to Markov perfection, using incumbent reputation as the state variable, is a popular and intuitive approach. Furthermore, in a Markov perfect equilibrium the voters' reelection decision depends only on the incumbent's reputation so that these equilibria can be interpreted as those in which voters focus only on selection. In the first of this paper's main results (Proposition 2), I show that there exists no Markov perfect equilibrium in which incumbents ever exert positive levels of effort.

If focusing only on selection leads to low payoffs for voters, one might conjecture that they will focus only on sanctioning instead. Indeed, there are equilibria in which voters use performance standards to make reelection decisions without being responsive to an incumbent's reputation, and which yield positive payoffs for the voters (these are described in Section 3.2). However, in these equilibria the voters' continuation payoffs vary systematically with the incumbent's reputation, and the voters will be expected to throw incumbents out of office who would normally outperform challengers. That is, the voters face a commitment problem which undermines the credibility of their reelection strategy. Formally, these equilibria are not weakly renegotiation-proof (WRP, Farrell and Maskin 1989).

In this model, WRP is qualifiedly equivalent to the condition that the vot-
ers' expected payoffs be constant across incumbent reputations (see Claim 1 and Proposition 3 for details). If this is the case, voters face no commitment problem when making reelection decisions because they will be indifferent between having the incumbent or a randomly-selected challenger in office.

It is instructive to note the similarity with the equilibria in a seminal work on political agency, Ferejohn 1986, in which voters commit to a reelection rule based on a fixed performance standard. In Ferejohn 1986 this voter indifference condition arises automatically from the assumption that politicians are identical. Thus, if one considers this assumption to be too strong, one may worry about the robustness of the proposed equilibria. However, I find that voter indifference has an important theoretical justification in a model with heterogeneous politicians. In this sense, my results provide fresh perspective on, and microfoundations for, the equilibria of Ferejohn 1986.

Building on these insights, I go on to prove existence of a class of WRP equilibria in which H-types are incentivized to exert positive effort (Theorem 1). In these equilibria, incumbents are reelected only if their observed performance exceeds a cutoff which varies with the incumbent's reputation at the beginning of his term. Crucially, these performance cutoffs vary in such a way as to make it incentive compatible for politicians to exert just enough effort to leave voters indifferent between reelecting the incumbent and electing an inexperienced challenger, thus making the voters' value function constant across reputations. I refer to this class of equilibria as equilibria in reputation-dependent performance cutoffs (RDC). I view RDC equilibria as a natural generalization of the equilibria in Ferejohn 1986 because they rely on the same basic insights. First, performance cutoffs are an plausible and effective way to provide incentives. Second, voters can credibly commit to using these strategies if they are indifferent between reelecting an incumbent and electing a challenger.

An implication of voter indifference is that politicians are able to appropriate the benefits of increases in their reputation by exerting lower effort. This highlights a tension between the selection and incentivizing roles of elections. Voters
could do better by committing to a reelection rule which optimally incentivized incumbents. However, such a commitment is not WRP and, thus, not credible. RDC strategies reconcile this tension in a way that is as simple as possible, while passing a stringent credibility test and providing politicians with incentives to exert costly effort.

Because, in a model with heterogeneous politicians, selection will play some role in explaining voter behavior, one may reasonably expect that a veteran politician who has developed a reputation for being of a certain type will be treated differently by voters than a first-termer with no record. This, in turn, suggests that a model of electoral control which simultaneously contemplates selection and sanctioning will help us understand the dynamics of political careers. That is, there is likely to be an interplay between an incumbent's reputation, tenure, and behavior, and the standards to which he is held by voters. In RDC equilibria, politicians of high reputation exert lower effort. Also, in expectation, reputation is positively related to tenure so that, for a given politician, tenure is negatively related to performance (see Claim 3).

The paper proceeds as follows. In the following subsection, I discuss related work and its relationship to this paper. In section 2, I describe the model and its assumptions. Section 3 addresses the problem of multiplicity of equilibria, and uses some simple equilibria of the model to motivate equilibrium selection criteria. In section 4, I define RDC equilibrium and prove its existence. In section 4.1, I look at what RDC equilibria can tell us about the career dynamics of politicians. In section 4.2 I present a simplified version of the model which clarifies the mechanics of RDC equilibrium and allows me to address questions of reelection rates and efficiency which I am unable to answer in the general model. Section 5 concludes.

### 1.1. Related Literature

There is a growing number of papers which study the selection and incentivizing roles of elections in a unified framework. Much of this work builds on work by Holmström (1999) on career concerns, with the relationship most directly apparent
in Persson and Tabellini (2000, ch. 4.5). Notable contributions include Reed (1994), Banks and Sundaram (1998), Fearon (1999), Berganza (2000), Ashworth (2005), and Besley (2006, ch. 3.3). Each of these works studies a model in which voters consider both the selection and incentivizing roles of elections and politicians face term limits. Additionally, several papers have applied similar models to the study of subjects such as transfers to special interest groups (Coate and Morris 1995 and Lohmann 1998), the incumbency advantage (Ashworth and Bueno de Mesquita 2006), constituency service (Ashworth and Bueno de Mesquita 2008), and CEO activism (Dominguez-Martinez, Swank, and Visser 2008) to name a few.

There are two important differences between the models cited in the previous paragraph and this paper. First, imposing term limits means that last period behavior is easily solved for, and reelection rules are derived by backward induction. In this paper there are no term limits, so voters and politicians face a dynamic problem at every stage. The second is the type of politician heterogeneity studied. In the papers above, voters are assumed to benefit from having a high type in office even if the he will exert no effort. In this paper, high types differ from low types only in their ability to induce outcomes preferred by the voters. However, improving outcomes is costly to high types so, in the absence of electoral incentives, average performance is equal for high and low types. I feel that this is a more natural way of modeling differences in ability or competence, while the alternative approach is best suited to modeling differences in honesty or alignment of preferences.

In a closely related paper, Banks and Sundaram (1993) study the selection and sanctioning roles of elections in a fully dynamic framework with no term limits. This paper differs from their's mainly in terms of equilibrium selection, although this leads to important differences in predictions made about political careers and in insights gained into the interplay of the two roles of elections. I discuss the differences in greater detail in section 3.2.

Duggan (2000) and Banks and Duggan (2006) study a model of repeated elec-
tions in which politicians differ in their spatial policy preferences. Voters use the incentive of reelection to induce politicians to temper their policy choices while in office. However, because there is no uncertainty in the execution of policy and strategies are stationary, there is no evolution of beliefs about the incumbent's preferences beyond their first period in office.

Meirowitz (2007) proposes a model of repeated elections in which two longlived parties, differing in their policy preferences and valence, compete in elections each period. Voters are uncertain about the set of feasible policies rather than about the parties' characteristics or the policy choices made. He shows that, while electoral control is impossible if voters are constrained to using pure strategies, perfect control is possible in mixed strategies. If mixed strategies are to be used, each party must provide the same expected utility to the voter when in office. This leads to a voter indifference condition analogous to the one emphasized in this paper.

Smart and Sturm (2006) present a model of repeated elections in which incumbents' actions are publicly observable, but the underlying state of the world which determines which action is good for the voters is observed only by the incumbent. In this context, they prove that the best Markov perfect equilibrium in the absence of term limits involves all politicians taking the same action regardless of the state of the world. They go on to argue that imposing term limits may help voters by decreasing the incentives for politicians to conform. Their result on the limits of Markov perfect equilibria are in the spirit of the first main result of this paper. The existence of RDC equilibria in my model suggests that allowing voters to condition on more information than Markov perfection allows, the most recent realization of voter utility in particular, may be an alternative way to increase their expected payoffs.

This paper is related to the literature on dynamic principal-agent interactions outside of the political sphere. The approach taken here differs from that taken in much that literature in two main dimensions. First, this paper focuses on the use of a retention rule rather than a compensation contract as an incentivizing
mechanism. Second, in most of the literature on principal-agent relationships the principal is assumed to be a Stackelberg first-mover, leaving the agent only his reservation utility. In this paper, I look at Nash equilibria which admit the possibility that the gains from interaction may be shared. Indeed, in the RDC equilibria which I focus on, the agent reaps all of the benefits from increases in his reputation and enjoys utility strictly greater than his reservation value.

Mailath and Samuelson 2001 and Hörner 2002 study related models of reputation formation in which firms attempt to convince consumers that they are competent. Consumers are willing to pay more for a competent firm's products only if they expect the firm to exert high effort. In a result reminiscent my Proposition 2, Mailath and Samuelson show that, with persistent types, Markov perfect equilibria cannot support high effort. They share my skepticism of trigger strategy equilibria but, rather than using a renegotiation-proofness argument to make this point, they argue instead for a restriction to markovian strategies. They show that effort can be incentivized if a firm's type changes with some positive probability every period so that reputation cannot become 'too good'. Hörner studies a similar model in which many firms compete with each other for marketshare while developing a reputation for competence. He shows that, even with persistent types, effort can be incentivized if firms believe that customers will leave them for a competitor after a bad outcome. Intriguingly, his equilibria involve customers being left indifferent among firms of varying reputations as high reputation firms charge higher prices. However, this is assumed as an equilibrium condition and supported by appropriate beliefs off the equilibrium path. While the equilibria in Hörner's model are Markov perfect, this relies on having only two possible outcomes so that reputations correspond to particular histories of play. Having continuous outcomes would make this partition impossible and make clear that his equilibria have a similar strategic complexity to the RDC equilibria studied in this paper.

## 2. The Model

I study a discrete-time, infinite horizon model of a democratic society. In order to focus on the problems of selecting competent politicians and providing them with incentives to perform well, I abstract from ideological differences in the electorate. Instead, I model citizens as a single, infinitely-lived representative voter.

### 2.1. Preferences, Timing, and Information in the Stage Game

Each period, indexed by $t \in\{1,2, \ldots\}$, the voter must select a politician to carry out a task. There is an infinite set $P$ of potential politicians from which challengers are randomly drawn. Each politician is infinitely-lived and may serve for as many periods, or terms, in office as the voter asks him to. Once replaced by a challenger, however, a politician cannot return to office.

In order to consider differences in competence, I assume that politicians are one of two types: high (H) or low (L). H-types may choose to exert some level of effort $a \in \mathbb{R}_{+}=[0, \infty)$. This effort impacts, but does not perfectly determine, the voter's stage-game utility $r \in \mathbb{R}_{+}$. Specifically, $r=a+\varepsilon$ where $\varepsilon$ is a zero-mean noise term with distribution function $F$ and density $f$. Thus, effort is related to voter utility via a conditional distribution function $F(r \mid a)$ with density $f(r \mid a)$. Note that $E(r \mid a)=\int_{-\infty}^{\infty} r f(r \mid a) d r=a+\int_{-\infty}^{\infty} \varepsilon f(\varepsilon) d \varepsilon=a$.

H-types receive per-period utility $u(a)$ when in office, and 0 otherwise. The utility function $u(a)$ is twice continuously differentiable and strictly concave. Effort is costly so that $u$ is weakly decreasing in $a$ with $u^{\prime}(0)=0$ and $u^{\prime}(a)<0$ $\forall a>0$. I also assume that $u(a)>0$ for all $a \in[0, \bar{a})$ and some $\bar{a}>0$.

L-type politicians, on the other hand, are unable ${ }^{1}$ to affect the distribution of $r$ so that it is always $F(r \mid 0)$ when an L-type is in office. They receive a payoff $u_{L}>0$ when in office and 0 otherwise, so that they are always willing to serve if

[^1]elected. Because L-types are always willing to hold office but cannot make choices which influence payoffs in this game, I will focus on the behavior of H-types.

As is standard in games with imperfect monitoring (Abreu, Pearce, and Stachetti 1990), I assume that the distribution of results has full support: $f(r \mid a)>0$ for all $r$ and $a$. This guarantees that effort levels can never be perfectly inferred by observing results. I also make the following assumptions for analytical convenience. First, that $f(r \mid a)$ is twice continuously differentiable in both arguments. Second, $f(r \mid a)$ is symmetric around its mean.

Because the evolution of beliefs about incumbent types is central in this paper, it is useful to make assumptions ensuring that good results are more likely when effort is high. Thus, I assume that $f(r \mid a)$ satisfies the monotone likelihood ratio property (Milgrom 1981): $\frac{f(x \mid a)}{f\left(x \mid a^{\prime}\right)}>\frac{f(y \mid a)}{f\left(y \mid a^{\prime}\right)}$ whenever $x>y$ and $a>a^{\prime}$.

In order to guarantee that the politician's objective function is concave so that I may work with first order conditions, I make the following joint assumption on $u(a)$ and $f(r \mid a):-u^{\prime \prime}(a)>\max _{Q} \int f_{a a}(r \mid a) Q(r) d r$ where $Q$ is any function $Q: \mathbb{R} \rightarrow\left[u(0), \frac{u(0)}{1-\delta}\right]$ and $f_{a a}(r \mid a)$ is the second derivative of $f(r \mid a)$ with respect to $a$. If $f(r \mid a)$ is the density of the normal distribution with mean $a$ and variance 1 , $-u^{\prime \prime}(a)>0.4839\left(\frac{\delta}{1-\delta} u(0)\right)$ for all $a$ is a sufficient condition.

A politician's type is the private information of the politician. The voter assigns a probability $\mu_{j}$ of being an H-type to politician $j$. I call $\mu_{j}$ politician $j$ 's reputation. For ease of notation, when referring to the incumbent's reputation I drop the subscript $j$. Note that the expected stage-game payoff to the voter when an H-type incumbent exerts effort $a$ is $\mu a$, so that reputation is payoff relevant.

The proportion of H -types among the set of potential politicians $P$ is $\mu_{0}$. Because new politicians are selected randomly from this set, $\mu_{0}$ will also be the reputation of any politician at the beginning of his first term.

### 2.2. Histories, Strategies, and the Repeated Game

At time $t$, the voter and all politicians will have information about who has been in office and what rewards the voter has received in all previous periods, $1,2, \ldots, t-1$.

## The Stage Game



I call this information a t-history and label it $h_{t}$. Let $H$ denote the set of all possible t-histories.

A reelection strategy is a measurable function $\sigma: H \rightarrow[0,1]$ denoting the probability with which the voter will reelect the incumbent, conditional on all currently available information.

Similarly, an effort strategy for H-type politician i is a measurable function $\alpha_{i}: H \rightarrow \mathbb{R}_{+}$denoting the effort which a given politician will exert conditional on being in office and on all currently available information. To focus on differences in type, I assume that all H-types play the same strategy $\alpha_{i}=\alpha$. Note also that I am ruling out effort strategies which condition current actions on an incumbent's past effort choices, which are the incumbent's private information. This is because I am looking only at perfect public equilibria which restrict strategies to depend on public histories.

A belief function $\hat{\mu}: H \rightarrow[0,1]^{\infty}$ is a measurable function specifying the probability with which the voter believes each politician in $P$ to be an H-type. For any politician $i$ who has never previously held office, $\hat{\mu}_{i}=\mu_{0}$ regardless of the relevant t-history $\left(h_{t}\right)$. In equilibrium, beliefs about a politician's type evolve according to Bayes' rule:

$$
\hat{\mu}_{j}\left(h_{t+1}\right)=\frac{\hat{\mu}_{j}\left(h_{t}\right) f\left(r_{t+1} \mid \alpha\left(h_{t}\right)\right)}{\hat{\mu}_{j}\left(h_{t}\right) f\left(r_{t+1} \mid \alpha\left(h_{t}\right)\right)+\left(1-\hat{\mu}_{j}\left(h_{t}\right)\right) f\left(r_{t+1} \mid 0\right)}
$$

Because the distribution $f$ satisfies the monotone likelihood ratio property, $\hat{\mu}_{j}\left(h_{t}\right)$ is strictly increasing in $r_{t}$. For ease of notation, in what follows I drop the subscript when referring to beliefs about the incumbent so that $\hat{\mu}\left(h_{t}\right)$ denotes the probability that the incumbent at time $t$ is an H-type.

It is important to note that different histories can lead to the same incumbent reputation. I can group these together to define a coarser partition of the set of all histories as follows: if $\hat{\mu}\left(h^{1}\right)=\hat{\mu}\left(h^{2}\right)$ for $h^{1}, h^{2} \in H$ then $h^{1}, h^{2} \subset \hat{h} \in \hat{H}$. Note that $h^{1}$ and $h^{2}$ need not be of the same length. I will refer to this as a markovian partition of histories and I will use this definition of $\hat{H}$ in the following section to define Markov perfection.

Given a strategy profile ( $\sigma, \alpha$ ) and beliefs $\hat{\mu}$, the voter can compute his expected future payoffs at $h_{t}$. Keeping in mind that $\sigma, \alpha$ and $\hat{\mu}$ denote functions while $\sigma\left(h_{t}\right)$, $\alpha\left(h_{t}\right)$ and $\hat{\mu}\left(h_{t}\right)$ are particular values, I write $V\left(\sigma, \alpha, \hat{\mu} ; h_{t}\right)$ for the voter's value function. Letting $h_{t+1}(r)$ denote the $\mathrm{t}+1$-history reached from $h_{t}$ after a result $r$ is observed, it may be defined recursively:

$$
V\left(\sigma, \alpha, \hat{\mu} ; h_{t}\right)=\hat{\mu}\left(h_{t}\right) \alpha\left(h_{t}\right)+\delta \int_{-\infty}^{\infty} V\left(\sigma, \alpha, \hat{\mu} ; h_{t+1}(r)\right) f\left(r \mid \alpha\left(h_{t}\right)\right) d r
$$

Where $\delta \in(0,1)$ is a discount factor common to the voter and all politicians. Note that I do not explicitly write the reelection probability $\sigma\left(h_{t+1}(r)\right)$ here. Instead, $h_{t+1}(r)$ captures whether the incumbent is reelected or a challenger with reputation $\mu_{0}$ is elected.

Similarly, I denote the value function of an incumbent H-type politician $Q\left(\sigma, \alpha, \hat{\mu} ; h_{t}\right)$. It may be defined recursively as:

$$
Q\left(\sigma, \alpha, \hat{\mu} ; h_{t}\right)=u\left(\alpha\left(h_{t}\right)\right)+\delta \int_{-\infty}^{\infty} \sigma\left(h_{t+1}(r)\right) Q\left(\sigma, \alpha, \hat{\mu} ; h_{t+1}(r)\right) f\left(r \mid \alpha\left(h_{t}\right)\right) d r
$$

Note that I explicitly write the reelection probability $\sigma\left(h_{t+1}(r)\right)$ in the politician's value function to highlight that the reelection decision determines whether an incumbent will receive $Q\left(\sigma, \alpha, \hat{\mu} ; h_{t+1}(r)\right)$ the following period, or 0 if he is not reelected.

Definition 1. A perfect public equilibrium is a strategy profile ( $\sigma^{*}, \alpha^{*}$ ) and a belief function $\hat{\mu}$ such that:

1. $V\left(\sigma^{*}, \alpha^{*}, \hat{\mu} ; h_{t}\right) \geq V\left(\sigma^{\prime}, \alpha^{*}, \hat{\mu} ; h_{t}\right)$ for all $\sigma^{\prime}$ and $h_{t}$.
2. $Q\left(\sigma^{*}, \alpha^{*}, \hat{\mu} ; h_{t}\right) \geq Q\left(\sigma^{*}, \alpha^{\prime}, \hat{\mu} ; h_{t}\right)$ for all $\alpha^{\prime}$ and $h_{t}$.
3. $\hat{\mu}$ evolves according to Bayes' rule ${ }^{2}$ using the strategies $\left(\sigma^{*}, \alpha^{*}\right)$.

In what follows, I will use the term equilibrium to refer to perfect public equilibria.

## 3. Equilibrium Selection

As in most infinitely repeated games, I expect there to be a large set of equilibria. In this section, I discuss the problem of the multiplicity of equilibria and some possibilities for narrowing my focus to those equilibria which are most appealing. I begin with the following result which starkly outlines the problem of multiplicity. Then, I proceed by describing several classes of equilibria of this model and using them to motivate equilibrium refinements.

[^2]Figure 2.1: Summary of Important Notation
r
Voter's stage game utility. Politician's effort.
$u(a) \quad$ Politician's stage game utility.
$f(r \mid a) \quad$ pdf of $r$ given $a$.
$V \quad$ Voter's value function.
Q Politician's value function.
$\sigma \quad$ Voter's reelection strategy.
a Politician's effort strategy.
$\mu \quad$ Incumbent's reputation.

Proposition 1. Any pure reelection strategy $\sigma$ can be supported as part of an equilibrium.

To see that this is true, I first identify the equilibrium with the lowest payoffs for all players in the equilibrium set, which I call an equilibrium in grim strategies. Suppose H-type politicians always choose $a=0$. Then, the voter is left indifferent among all politicians and may choose any reelection rule. In particular, it is a best response for him never to reelect a politician, regardless of his performance. This reelection strategy makes $a=0$ a best response.

Next, I note that this equilibrium may be used as part of other equilibria as a credible punishment to the voter for not following a prescribed reelection strategy. Because the voter's expected payoff can never be worse than 0 , the following is an equilibrium for any pure reelection strategy $\sigma$ : the voter plays $\sigma$ on the equilibrium path while politicians play a best response to $\sigma$. If the voter ever deviates from $\sigma$, equilibrium play switches to grim strategies.

One may object to the equilibria above by arguing that it is implausible that all politicians in $P$ will coordinate on playing grim strategies in the continuation game. Since the physical environment is identical each time a politician is elected
to his first term, it seems natural to focus on equilibria in which strategies are the same every time the voter begins a fresh relationship with a politician. This, of course, implies that the value of the outside option for the politician is constant through all histories. In a sense, this is a stationarity condition which I will call challenger-stationarity. Because it is sufficient for my purposes and a weaker condition, I define challenger-stationarity in terms of the value of electing a challenger rather than the continuation strategies played.

Definition 2. An equilibrium satisfies challenger-stationarity if the value of electing a challenger is history-independent.

### 3.1. Selection-Only Equilibria

One restriction on the set of equilibria that is particularly appealing is to consider reelection strategies which are a function only of beliefs about the incumbent's type. In other words, our hypothesis is that voters focus only on the selection role of elections. Considering this possibility has the added benefit of providing some insight into the trade-offs between the sanctioning and selection roles of elections, and contributing to the understanding of the implications of the heterogeneity of politicians to voter behavior. As I discussed in the literature review above, previous work on related models has tended to predict that voters will use a simple reputation cutoff as a reelection rule ${ }^{3}$.

In the current set-up, in which only beliefs about the incumbent's type affect the set of feasible payoffs, incumbent reputation is the natural state space for one to use when considering an appeal to Markov perfection (Maskin and Tirole 2001). Thus, we have a convenient equivalence of the selection-only and the Markov perfection restrictions.

[^3]Markov perfection is one of the most common equilibrium refinements used in applied theory. Indeed, related work on repeated elections by Duggan (2000), Banks and Duggan (2006), and Meirowitz (2007), as well as related work on reputation games such as Mailath and Samuelson (2001) and Hörner (2002) has focused on Markov perfect equilibria. Standard arguments for Markov perfection stress the simplicity of markovian strategies. Additionally, Banks and Sundaram (1993, p. 310) end their article by asking whether 'interesting' equilibria which are stationary in reputation exist. Proposition 2 below answers in the negative, at least for this slightly simpler setting.

Definition 3. An equilibrium is Markov perfect if its strategies ( $\sigma, \alpha$ ) are measurable with respect to the markovian partition $\hat{H}$.

Existence is easily verified as equilibrium in grim strategies provides a trivial example of a Markov perfect equilibrium. However, the following result makes clear that the markovian criterion is too strict to allow for the voter to effectively incentivize H -type politicians.

Proposition 2. There is no Markov perfect equilibrium with positive value for the voter $(V>0)$.

A full proof is provided in the Appendix (Section 6.2). To develop some of the intuition behind the proof, suppose that politicians of all reputations provide effort of at least $\hat{a}>0$ in equilibrium. If reelection strategies depend only on reputation, the politician's ex-ante value of acquiring a reputation $\mu$ is $\sigma(\mu) Q(\mu)=\hat{Q}(\mu)$. Because posterior reputation is increasing in performance, in order to provide incentives for effort the function $\hat{Q}(\mu)$ must be increasing in reputation. As a politician's reputation nears 1 , the change in his reputation for a fixed but wide set of outcomes $(r)$ approaches 0 . Therefore, the politician's value function $\hat{Q}$ must increase at least a fixed amount (itself dependent on $\hat{a}$ ) in each of an infinity of ever smaller intervals. However, I know that $\hat{Q}$ is bounded above by the value of
holding office forever while exerting zero effort: $\frac{u(0)}{1-\delta}$. Therefore, providing incentives for effort at least $\hat{a}$ for all reputations is infeasible. Conversely, if politicians of reputation at least $\mu$ do not provide effort, it is not worthwhile for the voter to reelect them. This in turn, means that politicians should avoid ending up with a reputation higher than $\mu$, and they can only do this by providing lower effort, leading to an unraveling of incentives for incumbents of all reputations.

The result, and its proof, echo Proposition 2 in Mailath and Samuelson 2001 (henceforth M-S). There are, however, important differences. For instance, the relation between prices and reputation assumed in M-S gives the agent builtin incentives to improve his reputation which are absent in the current setting. Interestingly, the presence of continuous noise (and, thus, continuous outcomes) in my model rules out the type of partition of reputation space which makes mixed strategy equilibria with positive effort possible in M-S.

### 3.2. Sanctioning-Only Equilibria

If focusing only on selection leads to low payoffs for voters, one might wonder if focusing only on sanctioning instead is a feasible way for voters to obtain more favorable outcomes. In a closely related paper, Banks and Sundaram (1993) describe equilibria of the repeated elections game ${ }^{4}$ in which voters use performance standards to incentivize incumbents to exert effort and in which reelection strategies do not vary with incumbent reputation. In these equilibria, politicians are held to a fixed performance standard $r^{*}$. When this performance standard is not met, the politician is not reelected.

Because incumbents face a constant standard for reelection, their effort choices are constant through time. To see this, consider a competent incumbent's problem:
$\max \left\{u(a)+\delta\left(1-F\left(r^{*} \mid a\right)\right) Q\right\}$
Note that I write the value of being in office as a constant $Q$ because, given

[^4]this reelection strategy, the history of play will not have any effect on future play. Clearly, the solution to this problem is some constant level of effort $a^{*}$ which leads to a constant probability of reelection $1-F\left(r^{*} \mid a^{*}\right)$. Thus, these equilibria predict no career dynamics of interest.

Furthermore, as an incumbent's reputation $(\mu)$ increases, so does his expected future performance: $\mu a^{*}$ in each period in which the incumbent in question holds office. This raises the question of why voters are willing to dismiss incumbents who have developed good reputations, given that they would normally outperform a randomly-drawn challenger. The answer is that this strategy is enforced through the following trigger strategy: after a politician has missed his performance target once, he never expects to be reelected again and will therefore never again exert effort.

A serious criticism of these equilibria, in my view, is that after a politician with high reputation misses a performance target, both the voter and the politician would benefit from agreeing to keep the politician in office and continue play as if the incumbent had not violated the voter's performance standard. Therefore, the punishment prescribed by the equilibrium is not credible. More formally, the equilibria are not weakly renegotiation-proof (Farrell and Maskin 1989) ${ }^{5}$. There is a feasible continuation equilibrium whose payoffs strictly Pareto dominate those specified as following the history in question.

### 3.3. Renegotiation-Proofness and Voter-Indifference

Weak renegotiation-proofness (WRP) not only helps us rule out trigger strategy equilibria like those discussed in the previous subsection, it also provides theoreti-

[^5]cal clues to how more attractive equilibria may be constructed. In this subsection I formally define WRP, and I study its implications for this paper's model.

Farrell and Maskin's definition of WRP equilibrium is as follows: an equilibrium strategy profile $\varphi$ is WRP if there do not exist continuation equilibria $\varphi^{1}$ and $\varphi^{2}$ of $\varphi$ such that $\varphi^{1}$ strictly Pareto dominates $\varphi^{2}$ (i.e. payoffs under $\varphi^{1}$ are strictly greater for both players than under $\varphi^{2}$ ). To adapt the definition of WRP to the current game, I must take into account that the politician's reputation is payoff relevant, so that continuation payoffs when the politician's reputation is $\mu$ may not be feasible when his reputation is $\mu^{\prime} \neq \mu$. The following definition formalizes this notion.

Definition 4. An equilibrium is weakly renegotiation-proof (WRP) if, for any two histories $h^{1}, h^{2} \in H$ leading to a reputation $\mu$, i.e. $h^{1}, h^{2} \subset \hat{h} \in \hat{H}, V\left(h^{1}\right)>$ $V\left(h^{2}\right)$ implies $Q\left(h^{1}\right) \leq Q\left(h^{2}\right)$ (and therefore $Q\left(h^{1}\right)>Q\left(h^{2}\right)$ implies $V\left(h^{1}\right) \leq$ $\left.V\left(h^{2}\right)\right)$.

Clearly, any Markov perfect equilibrium is WRP.
Because the strategies I will consider in the rest of the paper depend only on reputation at the beginning of the term and current performance, I henceforth drop the notation emphasizing the dependence of the voter's and the politicians' value functions $V$ and $Q$ on the entire history of play and strategy profiles. Instead, I emphasize their dependence on incumbent reputation by writing $V(\mu)$ and $Q(\mu)$.

In order to find equilibria in which the voter provides incentives for H-types to provide positive effort but that are WRP and depend on history in the simplest way possible, I look to the structure of the equilibria in the baseline models of political agency. In my view, this has the added virtue of providing some continuity in the modeling and understanding of electoral incentives. The seminal work of Ferejohn 1986 makes two important observations:

- Performance cutoffs are effective means of providing incentives to politicians.
- Voter indifference over incumbents and replacements can be exploited to sustain equilibria with performance cutoffs.

In this model, politicians differ only in their perceived probability of being an H-type - their reputation. An incumbent's reputation will evolve as his record of performance grows and, once he has served at least one term, it will never (with probability zero) be exactly the same as that of a challenger $\left(\mu_{0}\right)$. Therefore, for the voter to be kept indifferent between reelecting an incumbent and electing a challenger, it must be that politicians of different reputations provide the same expected utility to the voter: $V(\mu)=V\left(\mu_{0}\right)$, at least for $\mu>\mu_{0}$. Therefore, I speak of voter indifference and a constant voter value function interchangeably.

Definition 5. An equilibrium satisfies voter indifference if $V(\mu)=V\left(\mu_{0}\right)$ for all $\mu$ which are reelected with positive probability.

Intuitively, this voter indifference condition can be seen as a formalization of the often-voiced sentiment: "One politician is as bad as another." This does not mean that there are no differences in competence among politicians, but that they all exploit the system in their favor to the point where expected performance is constant across politicians.

In addition to the connection to earlier models of political accountability, the voter indifference condition is connected in the current model to WRP. Clearly, voter indifference implies WRP since continuation payoffs are the same for the voter after any history of play, ruling out Pareto improvements.

Claim 1. Any equilibrium satisfying voter indifference is weakly renegotiationproof (WRP).

The following Proposition goes some way toward establishing the reverse implication: that WRP implies voter indifference. Specifically, the indifference condition will hold for a set of reputations of positive measure, and strategies outside of this set will be "uninteresting". In order to do so, I assume that the effort strategies of newly elected politicians do not depend on prior history (i.e. equilibria are challenger-stationary, see Definition 2). This seems natural in the current context where each time a politician is elected for the first time, the continuation game looks identical to the start of the game at time 0 .

Proposition 3. In any equilibrium satisfying weak renegotiation-proofness (WRP) and challenger-stationarity the following conditions hold:

- There is a subset of reputation space of strictly positive measure $S \subset[0,1]$ such that, for any $\mu \in S$, if $\hat{\mu}(h)=\mu$ then $V(h)=V\left(\mu_{0}\right)$.
- For any $\mu \in S^{C}=[0,1] \backslash S$, if $h^{1}, h^{2} \subset \hat{h}(\mu)$ then, either $\sigma\left(h^{1}\right)=\sigma\left(h^{2}\right)=1$ or $\sigma\left(h^{1}\right)=\sigma\left(h^{2}\right)=0$. That is, strategies in the complement of $S$ are markovian and degenerate.

Proof. If an equilibrium does not provide positive value for the voter, then the voter's value function is constant at 0 and the conditions above are trivially satisfied. Thus, in what follows, I look at equilibria in which $V\left(\mu_{0}\right)>0$.

Consider any reputation $\mu$ such that one can find histories $h^{1}$ and $h^{2}$ satisfying $\hat{\mu}\left(h^{1}\right)=\hat{\mu}\left(h^{2}\right)=\mu, \sigma\left(h^{1}\right)=1$ and $\sigma\left(h^{2}\right)=0$ (or strategies are mixed but may lead to reelection after $h^{1}$ and dismissal after $h^{2}$ ). Then WRP implies that, because $Q\left(h^{1}\right)>Q\left(h^{2}\right), V\left(h^{1}\right) \leq V\left(h^{2}\right)$. Also, because it is a best response to reelect after $h^{1}, V\left(h^{1}\right) \geq V\left(\mu_{0}\right)$. Because it is a best response not to reelect after $h^{2}$, $V\left(h^{2}\right)=V\left(\mu_{0}\right)$. From this I conclude that $V\left(h^{1}\right)=V\left(h^{2}\right)=V\left(\mu_{0}\right)$.

This leaves reputation levels at which incumbents are always reelected or always thrown out of office. However, any reelection strategy leading to this sort of behavior over almost all reputations is an essentially Markovian reelection strategy. By the generalization of Proposition 2 in the appendix, this contradicts the premise that the equilibrium in question provides positive value for the voter.

As it relates to the model of Ferejohn 1986, the relationship between WRP and voter indifference solidifies the microfoundations of equilibria in performance cutoffs. Even if one allows for heterogeneity among politicians, there is an intuitively appealing equilibrium refinement (WRP) which leads back to voter indifference. Thus, its use as a commitment device is both credible and focal.

## 4. Equilibria in Reputation-Dependent Performance Cutoffs (RDC)

For incumbent behavior to satisfy voter indifference, we must use performance cutoffs which adjust to the incumbent's reputation. Otherwise, expected results will vary systematically with reputation as in Banks and Sundaram 1993's equilibria.

In order to keep the voter indifferent between incumbents and replacements $\left(V(\mu)=V\left(\mu_{0}\right)\right)$, it must be that
$V(\mu)=\mu \alpha(\mu)+\delta \int_{-\infty}^{\infty}\left[\sigma(\hat{\mu}(r, \mu)) V(\hat{\mu}(r, \mu))+(1-\sigma(\hat{\mu}(r, \mu))) V\left(\mu_{0}\right)\right] f(r \mid \alpha(\mu)) d r$
Solving for $\alpha(\mu)$ and substituting $V(\mu)=V\left(\mu_{0}\right)=V$, we find that $V=$ $\mu \alpha(\mu)+\delta V$. Solving for the incumbent's effort strategy: $\alpha(\mu)=\frac{V(1-\delta)}{\mu}$. Denoting $v=V(1-\delta)$, I write the identity for effort levels which keep the voter indifferent among politicians as:

$$
\alpha(\mu)=\frac{v}{\mu}
$$

I refer to $v$ as the value to the voter of an effort profile $\alpha(\mu)$. Note that $\alpha^{\prime}(\mu)=-\frac{v}{\mu^{2}}$ so that effort is decreasing in reputation. Clearly, any equilibrium with positive value to the voter $(v>0)$ will involve a lowest reputation politician which will ever be elected, since $\alpha(\mu) \rightarrow \infty$ as $\mu \rightarrow 0$. I denote this lowest reelectable reputation $\dot{\mu}$.

Because effort strategies $\alpha(\mu)$ keep the voter indifferent among reelection strategies, if there exists a performance cutoff function $r(\mu):[0,1] \rightarrow \mathbb{R}$ which makes $\alpha(\mu)$ a best response, this will be an equilibrium.

Definition 6. An equilibrium in reputation-dependent performance cutoffs (RDC) with value $v$ is an equilibrium in which:

- Politicians follow an effort strategy $\alpha\left(\mu_{t}\right)=\frac{v}{\mu_{t}}$.
- The voter follows a reelection strategy $\sigma\left(r_{t}, \mu_{t-1}\right)=\left\{\begin{array}{c}1 \text { if } r_{t} \geq r\left(\mu_{t-1}\right) \\ 0 \text { otherwise }\end{array}\right.$

Such strategies are Markovian if we take $\left(\mu_{t-1}, \mu_{t}\right)$ rather than $\mu_{t}$ as the state variable. This condition is equivalent to constraining strategies to depend only on reputation and performance: $\left(r_{t}, \mu_{t}\right)$ or $\left(\mu_{t-1}, r_{t}\right)$. These equilibria are also WRP. The following Theorem states the existence of equilibria in reputation-dependent cutoff strategies.

Theorem 1. There exists a class of equilibria in reputation-dependent performance cutoffs (RDC) in which the voters use a reputation-dependent performance cutoff as their reelection strategy, are indifferent among politicians of all reputations above some threshold $\stackrel{\mu}{\mu}$, and receive strictly positive expected utility.

The proof (in Section 6.1) proceeds as follows: let $Q(\mu)$ be any bounded and well-behaved candidate for the politician's value function. If I have chosen $v$ carefully, it will be obtainable under $Q(\mu)$ in an RDC equilibrium since $Q(\mu)$ is bounded below by $u(0)$, making it worthwhile for an incumbent to exert some effort in order to make his reelection more likely. I then define an operator $T(Q)(\mu)=u(\alpha(\mu))+\delta \int_{r_{Q}(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r \mid \alpha(\mu)) d r$ where $r_{Q}(\mu)$ is a reputationdependent performance cutoff implementing $v$. A fixed point of $T$ will be a value function $Q$ with associated cutoff function $r_{Q}(\mu)$ implementing an effort strategy $\alpha(\mu)=\frac{v}{\mu}$. Because this effort strategy leaves the voter indifferent between reelecting the incumbent or not, the cutoff function describes a reelection strategy which is a best response. Therefore, once I check sufficient conditions for a fixed point of $T$, I have found an RDC equilibrium.

### 4.1. Career Dynamics and Comparative Statics

In this Section, I describe some properties of RDC equilibria with positive value $v$ for the voter. I begin with a definition:

Definition 7. Let $\hat{t}(t)$ be the date at which the date-t incumbent was first elected to office. An incumbent's tenure at time $t$ is the total number of periods he has served in office: $\tau(t)=t-\hat{t}(t)$.

In any RDC equilibrium, expected reputation increases with tenure. This is easy to see by the following argument. Because, in equilibrium, the voter correctly anticipates the incumbent's behavior as a function of his type, the expected reputation of the incumbent after serving a term is the same as his reputation at the beginning of the term. In other words, a given incumbent's reputation is a martingale: $E_{t}\left(\mu_{i, t+1}\right)=\mu_{i, t}$. However, those incumbents who end the term with the lowest reputation will be thrown out of office, leading us to conclude that expected reputation will increase every time an incumbent is reelected.

Claim 2. In any $R D C$ equilibrium with positive value to the voter:

1. An incumbent's expected level of reputation conditional on tenure is strictly increasing in tenure: $E(\mu \mid \tau)>E\left(\mu \mid \tau^{\prime}\right)$ whenever $\tau>\tau^{\prime}$.
2. Politicians will fully reveal their type if they can stay in office forever: $\lim _{\tau \rightarrow \infty} E(\mu \mid \tau, H)=1$ and $\lim _{\tau \rightarrow \infty} E(\mu \mid \tau, L)=0$.
3. However, any incumbent will be thrown out of office in finite time: $\lim _{t \rightarrow \infty} \lim _{K \rightarrow \infty} \operatorname{Pr}(\tau(t) \geq$ $K)=0$.

Proof. The first part of the claim is argued in the text above.
To see that incumbents fully reveal their type if they can stay in office forever, recall that in an RDC equilibrium with value $\frac{a}{1-\delta}>0$, any competent incumbent exerts effort weakly greater than $a>0$ each period. Consider the following test: the voter calculates a sample average of an incumbent's performances $\bar{r}_{n}=$ $\frac{1}{n} \sum_{t=1}^{n} r_{t}$. For all $n, E\left(\bar{r}_{n} \mid H\right) \geq a$. Also, $\operatorname{Var}\left(\bar{r}_{n}\right)=\frac{1}{n^{2}} n v=\frac{v}{n}$ where $v$ is the variance of $\varepsilon$. Thus, $\lim _{n \rightarrow \infty} \operatorname{Var}\left(\bar{r}_{n}\right)=0$. Therefore, the probability that $\bar{r}_{n}<a$ goes to zero as the number of observations ( $n$ ) grows arbitrarily large. In this way, the voter may perfectly infer an incumbent's type, given enough observations of his performance in office.

To see that any incumbent is thrown out of office in finite time with probability one, observe first that competent incumbents always survive with a higher
probability than incompetent incumbents. This is the case because competent incumbents exert effort so that the distribution of outcomes when they are in office first order stochastically dominates that of incompetent incumbents. In order to incentivize positive effort, there must be a subset of outcomes of positive mass on which the incumbent is not reelected for almost every $\mu$ since, otherwise, we would have an absorbing state in which no effort is provided. Because the set of reputations is compact, there is a maximum probability of reelection $p<1$. Therefore, the probability that any given incumbent survives at least $n$ periods in office is bounded above by $p^{n-1}$. Clearly, $\lim _{n \rightarrow \infty} p^{n-1}=0$.

A straight-forward implication of RDC equilibrium is that effort decreases with reputation. This, along with Claim 1, implies a negative relationship between expected performance and tenure for a given politician (though not across politicians).

Claim 3. For a given politician, expected effort and performance are negatively related to tenure.

Proof. Our variables of interest are $E\left(r_{t} \mid \tau(t), i\right)$ and $E\left(a_{t} \mid \tau(t), i\right)$. We note, first, that $E\left(r_{t} \mid \tau(t), i\right)=E\left(a_{t}+\varepsilon_{t} \mid \tau(t), i\right)=E\left(a_{t} \mid \tau(t), i\right)$.

A given politician $i$ is either competent or incompetent. If he is incompetent, then his expected effort and performance are zero, regardless of his tenure: $E\left(a_{t} \mid \tau(t), i\right)=0 \forall \tau$.

If $i$ is competent, then his expected effort and performance will be decreasing in his reputation: $a_{t}=\frac{v}{\mu_{t}}$. By Claim 1, on average, reputation is increasing in tenure. Thus, $i^{\prime} s$ expected effort and performance will be decreasing in his tenure.

This is a prediction which has been emphasized by others, including Banks and Sundaram (1998) and Ashworth (2005), though their derivation relies on last-period effects. As Ashworth (2005) points out, the prediction fits well with the negative correlation between tenure and personal constituent services in the U.S. House of Representatives examined in Cain, Ferejohn and Fiorina (1990).

In a study of the U.S. senate, Levitt (1996) finds some evidence of a positive correlation between ideological shirking and tenure.

Recent work by Galasso, et al. (2009) finds a negative relationship between tenure and attendance in the Italian legislature. Attendance may be interpreted as an observation of effort in this context if I reinterpret the model to fit Italy's parliamentary system. In this case, politicians are directly accountable to their party rather than to the voters. One might imagine that parties face a similar retention problem to that faced by voters in democracies with direct representation, and thus may use RDC strategies.

An important question we may ask is whether heterogeneity among politicians is good or bad for voters. Clearly, if there are only incompetent politicians, the voter will have expected utility of zero. By Theorem 1 the voter can achieve strictly positive expected utility in an RDC equilibrium whenever $\mu_{0}>0$, so that moving from a state of the world with only incompetent politicians to one in which some may be competent is good for the voter. A world in which all politicians are competent takes us back to the environment of Ferejohn 1986 in which voters can achieve positive utility. The following proposition shows that adding the possibility of incompetent politicians is never good for voters.

Claim 4. The highest expected payoff to the voter in an RDC equilibrium is weakly increasing in the proportion of high types $\left(\mu_{0}\right)$ in the set of politicians $P$, and is strictly increasing for some $\mu_{0}$.

Proof. Given an RDC equilibrium under $\mu_{0}$, the same strategies may be used when new politicians are more likely to be H-types $\left(\mu_{0}^{\prime}>\mu_{0}\right)$. This implies that the highest achievable voter utility is weakly increasing in $\mu_{0}$. However, we know that as $\mu_{0} \rightarrow 0$, so do the feasible payoff levels for voters since $\frac{a}{\mu_{0}} \rightarrow \infty$. Therefore, for any RDC equilibrium with positive value to the voter, there is a $\mu_{0}$ at which this value is not feasible so that there is a $\mu_{0}$ at which the highest expected payoff to the voter is strictly increasing.

### 4.2. An Example with Perfect Monitoring: Reelection Rates and Efficiency

The analysis has thus far omitted any predictions for reelection rates and any analysis of the costs of the voters' inability to commit to using trigger strategies. This is because of the complexity of the general model presented above. In this subsection, I present a simple example which preserves the logic and main features of the general model, while allowing me to advance informed, if speculative, answers to these important questions.

Before proceeding with the example, let me explain the difficulty in answering these questions in the general model. The equilibria constructed in the proof of Theorem 1 use performance cutoffs which are above the expected performance of high types (i.e. $r(\mu)-\alpha(\mu)>0$ ), and therefore politicians are always reelected with probability strictly less than $\frac{1}{2}$. Because of the relatively low reelection probability, the politician's value function is lower than it would be in an equilibrium with higher reelection rates, and therefore the highest level of implementable effort would likely be higher in this alternative scenario. Generally, I would expect similar equilibria using cutoffs below expected performance to exist and guarantee reelection rates strictly higher than $\frac{1}{2}$. However, moving performance cutoffs below expected performance allows for the possibility that you may be reelected when your reputation has decreased, and thus that a politician will be reelected even if it is infeasible for him to be incentivized to provide the required effort to keep indifference. Whether this takes place will depend on the slope of $r(\mu)$, which in turn depends on the shape of $Q(\mu)$, which is an endogenous object.

The model in this subsection is a special case of the general model described in section 2. There is an infinite set of potential politicians $P$ from which voters randomly draw when they wish to replace an incumbent. A proportion $1-\mu_{0}$ are incompetent and will provide utility $r=0$ to voters. A proportion $\mu_{0}$ are competent and can improve voter utility by exerting costly effort: $r=a$. Competent incumbents receive utility $U-a^{\rho}(U>0, \rho>1)$ each period they are in office.

Note that there is no noise term in the expression linking effort to voter utility,
so that effort is perfectly and publicly observable. This implies that any positive performance by an incumbent will reveal him to be competent with probability 1. Because of this, learning happens all at once and there are only two reputation levels which an incumbent will have on the equilibrium path. Let 1 denote the state in which the incumbent is known to be competent, and 0 the state in which the incumbent is believed to be competent with probability $\mu_{0}$. Henceforth, subscripts $i \in\{0,1\}$ will denote values which variables or functions take in state i ; for instance $Q_{1}$ is the incumbent's value of being in office in state 1.

Once the incumbent is known to be competent, voters can elicit effort $a=r^{*}$ by playing the following reelection strategy:

$$
\sigma=\left\{\begin{array}{c}
1 \text { if } r \geq r^{*} \\
0 \text { otherwise }
\end{array}\right.
$$

if the following incentive constraint is satisfied:

$$
Q_{1}=U-a^{\rho}+\delta Q_{1} \geq U
$$

Given these reelection strategies, competent incumbents will always be reelected with probability one if they exert the equilibrium level of effort. Conversely, any lower level of effort will lead to the incumbent being thrown out of office, so that the most attractive deviation for a competent incumbent is to $a=0$. This is why the incentive constraint above compares the utility of exerting the required amount of effort and staying in office with the utility of exerting no effort and not surviving the next election.

For any level of supportable effort $a$, the incumbent's state 1 value function is $Q_{1}=\frac{U-a^{\rho}}{1-\delta}$. Thus, the maximum level of effort which can be supported is:

$$
\bar{a}=(\delta U)^{\frac{1}{\rho}} .
$$

Intuitively, this increasing in $\delta$ and $U$ and decreasing in $\rho$ but, interestingly,
unresponsive to $\mu_{0}$. Thus, the state 1 value function for the voter is $\bar{V}_{1}=\frac{\bar{a}}{1-\delta}$.
In state 0 , before the incumbent's type has been revealed, the voter can implement effort $a$ using the same type of reelection strategies as above as long as the following incentive constraint is satisfied:

$$
Q_{0}=U-a^{\rho}+\delta Q_{1} \geq U
$$

If the voters elicit the same level of effort in both states, then $Q_{0}=Q_{1}$. Thus, the maximum level of effort that can be incentivized from a competent incumbent when his type is unknown is also $\bar{a}$. Thus, the state 0 value function for the voter in this equilibrium is $\bar{V}_{0}=\frac{\bar{a} \mu_{0}}{(1-\delta)\left(1-\delta\left(1-\mu_{0}\right)\right)}$.

However, as is argued in section 3 , these strategies are not weakly renegotiationproof if the expected value to the voter of being in state 1 is greater than his expected value of being in state 0 , which is clearly the case when the same level of effort is elicited in either case:

$$
\begin{aligned}
& V_{1}=\frac{a}{1-\delta}>\mu_{0} a+\delta\left(\mu_{0} \frac{a}{1-\delta}+\left(1-\mu_{0}\right) V_{0}\right)=V_{0} \\
& V_{0}=\frac{\mu_{0} a}{(1-\delta)\left(1-\delta\left(1-\mu_{0}\right)\right)}
\end{aligned}
$$

To see this, consider the off-equilibrium outcome in which a competent incumbent (whether we are in state 0 or 1 does not matter here) exerts effort $0<a^{\prime}<a$. He is now revealed to be competent and could provide the voter with continuation utility $\frac{a}{1-\delta}$ if the voter and the incumbent agreed to continue play as if expectations of performance had been met. Because $\frac{a}{1-\delta}>\frac{\mu_{0} a}{(1-\delta)(1-\delta(1-\mu))}$ and $Q_{1}>0$, this agreement would be strictly beneficial to both parties.

In order to make the threat of electing a challenger credible, more effort should be elicited of unknown incumbents than of known competent incumbents. Specifically, we can derive the following relation:

$$
V=\frac{a_{1}}{1-\delta}=\mu_{0} a_{0}+\delta V \Rightarrow a_{0}=\frac{a_{1}}{\mu_{0}}
$$

The state 0 incentive constraint is now:

$$
Q_{0}=U-a_{0}^{\rho}+\delta Q_{1}=U-\left(\frac{a_{1}}{\mu_{0}}\right)^{\rho}+\frac{\delta}{1-\delta}\left(U-a_{1}^{\rho}\right) \geq U
$$

The highest state-1 level of effort which can be supported of incumbents who are known to be competent is:

$$
a^{R D C}=\frac{(\delta U)^{\frac{1}{\rho}} \mu_{0}}{\left(1-\delta+\delta \mu_{0}^{\rho}\right)^{\frac{1}{\rho}}}
$$

This increasing in $\delta, \mu_{0}$, and $U$, and decreasing in $\rho$. In an RDC equilibrium, the voter's state 1 and state 0 value function is the same: $V_{1}^{R D C}=V_{0}^{R D C}=$ $V^{R D C}=\frac{a^{R D C}}{1-\delta}$.

Note that there is a stark incumbency advantage exhibited in both the equilibria discussed above. The ex-ante probability of reelection in state 1 , that is for incumbents who have survived one term and therefore revealed themselves to be competent, is 1 . The ex-ante probability of reelection following an incumbent's first term is $\mu_{0}$. This incumbency advantage arises entirely out of a selection effect as in Zaller (1998) or Ashworth (2005). Politicians who have survived in office are better equipped to survive future elections than a randomly chosen challenger.

Claim 5. Reelection rates are increasing in tenure. Incumbents in their first term win reelection with probability $\mu_{0}$, while incumbents who have been in office longer are reelected with probability 1.

Although there is no difference in reelection rates, RDC equilibria predict that incumbents will work less once their type is revealed, whereas trigger strategy equilibria predict no variation in effort or performance levels over a given politician's career. To measure the loss voters suffer because of their inability to commit to trigger strategy equilibria, we can look at the ratio of the voters' value functions at the best trigger strategy and RDC equilibria respectively:

$$
\frac{\bar{V}_{0}}{V^{R D C}}=\frac{\frac{\overline{1-\delta)}\left(\frac{\overline{1} \mu_{0}}{}\left(1-\mu_{0}\right)\right)}{a^{R D C}}}{1-\delta}=\frac{\bar{a}}{a^{R D C}} \frac{\mu_{0}}{\left(1-\delta\left(1-\mu_{0}\right)\right)}=\frac{(\delta U)^{\frac{1}{\rho}}\left(1-\delta+\delta \mu_{0}^{\rho}\right)^{\frac{1}{\rho}}}{(\delta U)^{\frac{1}{\rho}} \mu_{0}} \frac{\mu_{0}}{\left(1-\delta\left(1-\mu_{0}\right)\right)}=\frac{\left(1-\delta\left(1-\mu_{0}^{\rho}\right)\right)^{\frac{1}{\rho}}}{\left(1-\delta\left(1-\mu_{0}\right)\right)}
$$

The following Claim points out two implications of the expression above. While I choose to compare the voter's welfare in state 0 , it is straightforward to show that the same results hold when comparing state 1 value functions.

Claim 6. The efficiency loss due to the voters' commitment problem vanishes as voters and politicians become arbitrarily patient or impatient, or as the adverse selection problem vanishes:

- $\lim _{\delta \rightarrow 1} \frac{\bar{V}_{0}}{V^{R D C}}=1$ and $\lim _{\delta \rightarrow 0} \frac{\bar{V}_{0}}{V^{R D C}}=1$
- $\lim _{\mu_{0} \rightarrow 1} \frac{\bar{V}_{0}}{V^{R D C}}=1$

Proof. The limit results can be derived from inspection of the expression for $\frac{\bar{V}}{V^{R D C}}$ :

$$
\begin{aligned}
& \lim _{\delta \rightarrow 1} \frac{\bar{V}_{0}}{V^{R D C}}=\lim _{\delta \rightarrow 1} \frac{\left(1-\delta\left(1-\mu_{0}^{\rho}\right)\right)^{\frac{1}{\rho}}}{\left(1-\delta\left(1-\mu_{0}\right)\right)}=\frac{\mu_{0}^{\rho}}{\mu_{0}}=1 \\
& \lim _{\delta \rightarrow 0} \frac{\bar{V}_{0}}{V^{R D C}}=\lim _{\delta \rightarrow 0} \frac{\left(1-\delta\left(1-\mu_{0}^{\rho}\right)\right)^{\frac{1}{\rho}}}{\left(1-\delta\left(1-\mu_{0}\right)\right)}=\frac{1^{\frac{1}{\rho}}}{1}=1 \\
& \lim _{\mu_{0} \rightarrow 1} \frac{\bar{V}_{0}}{V^{R D C}}=\lim _{\mu \rightarrow 1} \frac{\left(1-\delta\left(1-\mu_{0}^{\rho}\right)\right)^{\frac{1}{\rho}}}{\left(1-\delta\left(1-\mu_{0}\right)\right)}=\frac{1^{\frac{1}{\rho}}}{1}=1 .
\end{aligned}
$$

As $\mu_{0} \rightarrow 1$, the model approaches a traditional political agency model with only one type of politicians. In these models there is no commitment problem for the voter because voter indifference arises from the fact that all politicians are identical, so there is no difference between RDC strategies and trigger strategies. This should be true in the general model as well.

As politicians become arbitrarily patient, future payoffs which are realized once their types are known, become more important in the incumbents' incentive constraints. Thus, it becomes easier to incentivize very high effort early on with the prospect of many future periods of reduced effort, accounting for the convergence in the voters' value function. The same forces would be at work in the general model. However, the noisiness of the general model puts a limit on the amount of learning that is expected in equilibrium, as politicians will be thrown out of office before full learning has taken place even if they are exerting high levels of effort.

Thus, I would not expect full convergence of trigger strategy and RDC payoffs unless we also take $\varepsilon \rightarrow 0$. The following figures illustrate the result above for given parameter values.



## 5. Conclusions

The aim of this paper has been to improve the general understanding of the dual role of elections: selecting competent politicians and incentivizing them to exert costly effort to the benefit of the electorate. In particular, I have focused on the potential interaction between a politician's reputation, the voter's willingness to replace him with a lesser known candidate, and the politician's performance. I have done so in the context of a simple model of repeated elections without term limits which does not assume that competence is desirable to the voter even in the absence of incentivizing mechanisms.

As in many infinitely repeated games, the problem of equilibrium selection takes center stage. However, attention paid to this issue has been rewarded in unexpected ways. I have shed light on the question of whether voters can effectively incentivize politicians by simply conditioning reelection on reputation. The answer is no (Proposition 2), at least in the setting I study. I have uncovered an interesting relationship between weak renegotiation-proofness and the condition that the voter be left indifferent among politicians of different reputations and, therefore, between reelecting an incumbent and electing an inexperienced challenger (Claim 1 and Proposition 3). This has given us fresh perspective on a seminal work in political agency (Ferejohn 1986) and increased confidence in its underlying logic. Finally, I have considered some of the virtues and limitations of the large set of equilibria in trigger strategies.

My exploration of the equilibrium set and its refinements led me to generalize the equilibria of Ferejohn 1986 to a model with heterogeneous politicians (RDC equilibria, section 4). The use of voter indifference to support performance cutoffs which, in turn, allow the voter to incentivize effort from politicians is consistent with several intuitively appealing equilibrium refinements. Additionally, after establishing existence (Theorem 1), I go on to explore the predictions of the model for political careers. The results presented in section 4.1 replicate those derived in similar models with term limits and in which incumbent type directly affects voter
utility. That they continue to hold when there are no term limits and politicians differ only in competence should encourage researchers to look for evidence of these career dynamics in contexts such as the U.S. Congress and understand them as a consequence of political agency.

I conclude with some thoughts on the direction of future research. Different institutions, as well as different types of heterogeneity among politicians, lead models of political agency to make different predictions about voter behavior and political careers. The literature is nearing an understanding of the relation between assumptions and predictions. A natural next step is to endogenize candidate entry in order to better understand the sources and magnitude of differences among politicians. Ultimately, the predictions of these models should be verified empirically.

## 6. Appendix

For easy reference in the proofs that follow, I rewrite and label the assumptions on the density function $f(r \mid a)$ discussed in Section 2.1.

Full support: $f(r \mid a)>0$ for all $r$ and $a$.
$f(r \mid a)$ is twice continuously differentiable in both arguments.

Monotone Likelihood Ratio Property (MLRP): $\frac{f(x \mid a)}{f\left(x \mid a^{\prime}\right)}>\frac{f(y \mid a)}{f\left(y \mid a^{\prime}\right)}$ whenever $x>y$ and $a>a \prime$.

Symmetry: $f(r \mid a)$ is symmetric around its mean.

Strict Concavity: $-u^{\prime \prime}(a)>\max _{Q} \int f_{a a}(r \mid a) Q(r) d r$ for $Q: \mathbb{R} \rightarrow\left[u(0), \frac{u(0)}{1-\delta}\right]$.

A straightforward but useful implication of assuming the independence of noise from effort levels $(r=a+\varepsilon)$ is that $f(r \mid a)=f(r-a \mid 0)$. Another way of stating this is:

$$
\begin{equation*}
f(r \mid a)=f(r+k \mid a+k) \text { for any } k \in \mathbb{R} \tag{NI}
\end{equation*}
$$

It is useful to note that A3. and NI imply that $f(r \mid a)$ is log-concave in $r$ (see Bagnoli and Bergstrom 2005 for some implications). I use this fact in the proof of Lemma 2 below.

Lemma 1. If $f(r \mid a)$ is twice continuously differentiable and it satisfies the monotone likelihood ratio property and immutability, it is log-concave in $r$.

Proof. Let $x \prime>x$ and $y \prime>y$.
A density function satisfies the monotone likelihood ratio property if:
$\frac{f\left(x^{\prime}, y^{\prime}\right)}{f\left(x^{\prime}, y\right)}>\frac{f\left(x, y^{\prime}\right)}{f(x, y)}$
Taking logs on both sides:
$\ln f\left(x^{\prime} \mid y^{\prime}\right)-\ln f\left(x^{\prime} \mid y\right)>\ln f\left(x \mid y^{\prime}\right)-\ln f(x \mid y)$
If $f$ is twice continuously differentiable, $\ln f\left(x^{\prime} \mid y^{\prime}\right)-\ln f\left(x^{\prime} \mid y\right) \approx \frac{\partial \ln f(x \mid y)}{\partial y}\left(y^{\prime}-y\right)$ when $\left(y^{\prime}-y\right)$ is small. Thus, the inequality above implies:
$\frac{\partial \ln f\left(x^{\prime} \mid y\right)}{\partial y}>\frac{\partial \ln f(x \mid y)}{\partial y}$
Because this most hold for all $x \prime>x$, this is equivalent to $\frac{\partial^{2} \ln f(x \mid y)}{\partial y \partial x}>0$.
Immutability states that $f(x \mid y)=f(x-y \mid 0)$.
Therefore, it must be that $\frac{\partial^{2} \ln f(x \mid y)}{\partial y \partial x}=-\frac{\partial^{2} \ln f(x-y \mid 0)}{\partial x^{2}}>0$. Since this holds for all $x$ and $y$, I conclude that $f$ is $\log$ concave.

In what follows, $f_{a}(r \mid a)=\frac{\partial f(r \mid a)}{\partial a}, f_{a a}(r \mid a)=\frac{\partial^{2} f(r \mid a)}{\partial a^{2}}$ and $\hat{\mu}_{2}(r, \mu)=\frac{\partial \hat{\mu}(r, \mu)}{\partial \mu}$.

### 6.1. Existence of RDC equilibria - proof of Theorem 1

I proceed by determining reputation-dependent performance cutoffs which implement effort levels which make the voter's expected utility constant across reputations. Once I have done this, I define an operator which, for any well-behaved candidate value function for the incumbent, determines performance cutoffs and a new candidate value function. A fixed point of this operator gives us an incumbent value function and associated performance cutoffs. Because, at every reputation point, the voter is indifferent between reelecting the incumbent and electing a challenger, using these performance cutoffs as a reelection strategy is sequentially rational for the voter. Thus, the following four elements describe an equilibrium: value functions for the politician and the voter, effort strategies which keep the voter's value function constant, and reelection strategies which use the derived
performance cutoffs to make reelection decisions. In order to guarantee the existence of a fixed point, I must check that the conditions for Schauder's fixed point theorem hold. I do so in a series of Lemmas.

When facing a reputation-dependent performance cutoff, an H-type politician with reputation $\mu$ solves the problem:

$$
\max _{a}\left\{u(a)+\delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r \mid a) d r\right\}
$$

To implement performance $v$ (or effort strategy $\alpha(\mu)=\frac{v}{\mu}$ ) with a reputationdependent cutoff $r(\mu)$ the politician's first order condition (FOC) must be satisfied at $\alpha(\mu)$ :

$$
u^{\prime}(\alpha(\mu))+\delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f_{a}(r \mid \alpha(\mu)) d r=0
$$

The FOC uniquely determines the incumbent's action since, by assumption A5.,

$$
u^{\prime \prime}(\alpha(\mu))+\delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f_{a a}(r \mid \alpha(\mu)) d r<0
$$

The FOC must hold at every reputation point $\mu$ so that the derivative of the F.O.C. with respect to $\mu$ must be 0 :
$u^{\prime \prime}(\alpha(\mu)) \alpha^{\prime}(\mu)-\delta r^{\prime}(\mu) Q(\hat{\mu}(r(\mu), \mu)) f_{a}(r(\mu) \mid \alpha(\mu))+\delta \int_{r(\mu)}^{\infty} Q^{\prime}(\hat{\mu}(r, \mu)) \hat{\mu}_{2}(r, \mu) f_{a}(r \mid \alpha(\mu))+$ $Q(\hat{\mu}(r, \mu)) f_{a a}(r \mid \alpha(\mu)) \alpha^{\prime}(\mu) d r=0$

Solving for $r^{\prime}(\mu)$ :

$$
\begin{equation*}
r^{\prime}(\mu)=\frac{\frac{1}{\delta} u^{\prime \prime}(\alpha(\mu)) \alpha^{\prime}(\mu)+\int_{r(\mu)}^{\infty} Q^{\prime}(\hat{\mu}(r, \mu)) \hat{\mu}_{2}(r, \mu) f_{a}(r \mid \alpha(\mu))+Q(\hat{\mu}(r, \mu)) f_{a a}(r \mid \alpha(\mu)) \alpha^{\prime}(\mu) d r}{f_{a}(r(\mu) \mid \alpha(\mu)) Q(\hat{\mu}(r(\mu), \mu))} \tag{6.1}
\end{equation*}
$$

The Fundamental Theorem of Differential Equations guarantees the existence of a function $r(\mu)$ satisfying the equation above as long as the first order condition
is feasible and I can bound $r(\mu)$ away from the point where $f_{a}(r(\mu) \mid \alpha(\mu))=0$ (for symmetric distributions, this point is $\alpha(\mu)$ ), since the RHS of the expression above is continuous and the domain of $r(\mu)$ is compact.

Before presenting a proof of existence of these equilibria, I select a feasible value for the voter: $v>0$. For analytical convenience, I focus on cutoffs where $r(\mu)-\alpha(\mu)>0$ and $f_{a}(r(\mu) \mid \alpha(\mu))>0$.

A lower bound for the value of holding office is $\bar{Q}=u(0)$. To emphasize its dependence on $v$, I write $\alpha(\mu, v)=\frac{v}{\mu}$ for the incumbent's effort strategy. Using this lower bound as a hypothetical constant value function and invoking the immutability assumption NI:

$$
-u^{\prime}(\alpha(\mu, v))=\delta \int_{r(\mu)}^{\infty} \bar{Q} f_{a}(r \mid \alpha(\mu, v)) d r=\delta \bar{Q} f(r(\mu) \mid \alpha(\mu, v))
$$

Clearly, this equality cannot hold for $v$ large enough. However, as $v \rightarrow 0$, $\alpha(\mu, v) \rightarrow 0$ and therefore $u^{\prime}(\alpha(\mu, v)) \rightarrow 0$. However, $\bar{Q}>0$, so that the equation must hold for appropriate $r(\mu)$ for $v$ low enough (but still strictly positive). Indeed, I can guarantee that a strictly positive $v$ may be sustained as above even if I restrict attention to cutoffs satisfying $r(\mu)-\alpha(\mu)>L$ for any given lower bound $L$. This will be useful when proving Lemma 2.

I now present the fixed point problem, referring to the derivations above as they become useful.

Definition 8. Let $C([0,1])$ be the space of bounded, continuous functions $f$ : $[0,1] \rightarrow \mathbb{R}$.

Let $\hat{C} \subset C([0,1])$ be the restriction of this space to functions with $K$-bounded first derivative and codomain $\left[u(0), \frac{u(0)}{(1-\delta)}\right]$.

It is clear that $\hat{C}$ is non-empty, bounded, closed, and convex.
Definition 9. The operator $T: \hat{C} \rightarrow \hat{C}$ is:

$$
T(Q)(\mu)=u(\alpha(\mu))+\delta \int_{r_{Q}(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r \mid \alpha(\mu)) d r
$$

A fixed point of this operator will define a value function for the politician in a reputation-dependent cutoff equilibrium. To prove the existence of a fixed point, I will use Schauder's fixed point theorem. Schauder's theorem is a generalization of Brouwer's fixed point theorem to infinite-dimensional spaces. For a proof, see Lusternik and Sobolev (1974).

Theorem 1 (Schauder's Fixed Point Theorem). Let $X$ be a bounded subset of $\mathbb{R}^{m}$, and let $C(X)$ be the space of bounded continuous functions on $X$, with the sup norm. Let $F$ be nonempty, closed, bounded and convex. If the mapping $T: F \rightarrow F$ is continuous and the family $T(F)$ is equicontinuous, then $T$ has a fixed point in $F$.

I must first verify that $T$ maps $\hat{C}$ to $\hat{C}$.
That $T(Q)$ is continuously differentiable in $\mu$ is immediate from the differentiability of $f, Q, \alpha(\mu)$, and $r_{Q}(\mu)$.

That $T(Q)$ has a $K$-bounded derivative is verified in the following Lemma.
It will be useful in proving the Lemma to note that the first derivative with respect to $\mu$ of the Bayesian updating function is:

$$
\frac{\partial \hat{\mu}(r, \mu)}{\partial \mu}=\hat{\mu}_{2}(r, \mu)=\frac{f(r \mid \alpha(\mu)) f(r \mid 0)+\mu(1-\mu) \alpha^{\prime}(\mu) f_{a}(r \mid \alpha(\mu)) f(r \mid 0)}{(\mu f(r \mid \alpha(\mu))+(1-\mu) f(r \mid 0))^{2}}
$$

It is useful to note that $\hat{\mu}_{2}(r, \mu) \rightarrow 1$ as $\alpha(\mu) \rightarrow 0$. The second term in the numerator converges to zero since $f_{a}(r \mid \alpha(\mu)) f(r \mid 0)$ is uniformly bounded above.

Lemma 2. For any continuously differentiable function $Q$ with absolutely $K$ bounded first derivative, $\left|\frac{\partial T(Q)}{\partial \mu}\right|<K$ for any $\mu \in\left[\mu_{0}, 1\right]$ and small enough $v>0$.

Proof. $\frac{\partial T(Q)}{\partial \mu}=\frac{\partial\left[u(\alpha(\mu))+\delta \int_{r(\mu)}^{\infty} Q(\hat{\mu}(r, \mu)) f(r \mid \alpha(\mu)) d r\right]}{\partial \mu}=$
$u^{\prime}(\alpha(\mu)) \alpha^{\prime}(\mu)+\delta \int_{r(\mu)}^{\infty} \alpha^{\prime}(\mu) Q(\hat{\mu}(r, \mu)) f_{a}(r \mid \alpha(\mu)) d r+\delta \int_{r(\mu)}^{\infty} Q^{\prime}(\hat{\mu}(r, \mu)) \hat{\mu}_{2}(r, \mu) f(r \mid \alpha(\mu)) d r-$ $\delta r^{\prime}(\mu) Q(\hat{\mu}(r(\mu), \mu)) f(r(\mu) \mid \alpha(\mu))$

The first two terms add up to zero by the politician's F.O.C. Substituting equation 6.1 into the fourth term:
$\delta \int_{r(\mu)}^{\infty} Q^{\prime}(\hat{\mu}(r, \mu)) \hat{\mu}_{2}(r, \mu) f(r \mid \alpha(\mu)) d r$
$-\frac{f(r(\mu) \mid \alpha(\mu))}{f_{a}(r(\mu) \mid \alpha(\mu))}\left(u^{\prime \prime}(\alpha(\mu)) \alpha^{\prime}(\mu)+\delta \int_{r(\mu)}^{\infty} Q^{\prime}(\hat{\mu}(r, \mu)) \hat{\mu}_{2}(r, \mu) f_{a}(r \mid \alpha(\mu))+Q(\hat{\mu}(r, \mu)) f_{a a}(r \mid \alpha(\mu)) \alpha^{\prime}(\mu) d r\right.$
I first consider the terms which include $Q^{\prime}$. Combining them gives:
$\left|\int_{r(\mu)}^{\infty} \hat{\mu}_{2}(r, \mu) Q^{\prime}(\hat{\mu}(r, \mu))\left(f(r \mid \alpha(\mu))-\frac{f(r(\mu) \mid \alpha(\mu))}{f_{a}(r(\mu) \mid \alpha(\mu))} f_{a}(r \mid \alpha(\mu))\right) d r\right|$
$<K\left|\int_{r(\mu)}^{\infty} \hat{\mu}_{2}(r, \mu)\left(f(r \mid \alpha(\mu))-\frac{f(r(\mu) \mid \alpha(\mu))}{f_{a}(r(\mu) \mid \alpha(\mu))} f_{a}(r \mid \alpha(\mu))\right) d r\right|$
$<K\left|\frac{f(r(\mu) \mid \alpha(\mu))}{f_{a}(r(\mu) \mid \alpha(\mu))} \int_{r(\mu)}^{\infty} \hat{\mu}_{2}(r, \mu) f_{a}(r \mid \alpha(\mu)) d r\right|$
Where I use Lemma 1 to derive both inequalities as it guarantees that the terms involving $f(r \mid \alpha(\mu))$ and $f_{a}(r \mid \alpha(\mu))$ will not change sign.

Because $\lim _{a \rightarrow 0} \hat{\mu}_{2}(r, \mu)=1$ and using Lemma 1 again I can say that the last expression is finite for any $r(\mu)>\alpha(\mu)$ and low enough $\alpha(\mu)$. Therefore, $\lim _{r(\mu) \rightarrow \infty} \frac{f(r(\mu) \mid \alpha(\mu))}{f_{a}(r(\mu) \mid \alpha(\mu))} \int_{r(\mu)}^{\infty} \hat{\mu}_{2}(r, \mu) f_{a}(r \mid \alpha(\mu)) d r=0$.

As argued in the text preceding the Lemma, since $Q$ is bounded below, I may choose $r(\mu)$ as large as I like while still supporting positive effort. In particular, I may choose $r(\mu)$, and thus $\alpha(\mu)$, so that the following inequality holds for all $\mu>\mu_{0}$ :
$\delta \int_{r(\mu)}^{\infty} \hat{\mu}_{2}(r, \mu) f(r \mid \alpha(\mu)) d r-\frac{f(r(\mu) \mid \alpha(\mu))}{f_{a}(r(\mu) \mid \alpha(\mu))} \delta \int_{r(\mu)}^{\infty} \hat{\mu}_{2}(r, \mu) f_{a}(r \mid \alpha(\mu)) d r<.9$
Therefore, $\delta \int_{r(\mu)}^{\infty}\left[Q^{\prime}(\hat{\mu}(r, \mu)) f(r \mid \alpha(\mu))-Q^{\prime}(\hat{\mu}(r, \mu)) \frac{f(r(\mu) \mid \alpha(\mu))}{f_{a}(r(\mu) \mid \alpha(\mu))} f_{a}(r \mid \alpha(\mu))\right] \hat{\mu}_{2}(r, \mu) d r<$ . 9 K .

The maximum value $Q$ can take is the value of exerting minimum effort (0) and holding office forever: $\frac{u(0)}{1-\delta}$. Thus, because
$\left|\int_{r(\mu)}^{\infty} f_{a a}(r \mid \alpha(\mu)) d r\right|<B$ for some $B>0$ I can conclude that
$\left|\int_{r(\mu)}^{\infty} \alpha^{\prime}(\mu) Q(\hat{\mu}(r, \mu)) f_{a a}(r \mid \alpha(\mu)) d r\right|<B \frac{v}{\mu_{0}^{2}} \frac{u(0)}{1-\delta}$ where $v$ is determined by the choice of $r(\mu)$ made above.

Similarly, by assumption $\left|u^{\prime \prime}(\alpha(\mu))\right|<\infty$. I may focus on a closed interval $a \in\left[0, \frac{v}{\mu_{0}}\right]$ so that the second derivative is uniformly bounded above:
$\left|u^{\prime \prime}(\alpha(\mu))\right|<U$ for some $U>0$.

Using these bounds, I have that the absolute value of the derivative above is bounded by:
$0.9 K+\frac{f(r(\mu) \mid \alpha(\mu))}{f_{a}(r(\mu) \mid \alpha(\mu))} \frac{v}{\mu_{0}^{2}}\left(U+B \frac{u(0)}{1-\delta}\right)<K$
The first term is strictly less than $K$. The second term does not depend on $K$, so that choosing $K$ high enough makes it strictly less than $0.1 K$.

Having a bounded derivative also ensures that the class of functions $T(\hat{C})$ is equicontinuous. A class of functions is equicontinuous if, given $\varepsilon>0$, there is a $\delta>0$ such that $|f(x)-f(y)|<\varepsilon$ whenever $|x-y|<\delta$ for any $x$ in the domain of $f$ and any $f \in T(\hat{C})$.

Lemma 3. Let $T(\hat{C})$ be a class of bounded, continuous and differentiable functions with a uniformly bounded derivative. Then $T(\hat{C})$ is equicontinuous.

Proof. For any $f \in T(\hat{C}), \frac{|f(x)-f(y)|}{|x-y|} \approx f^{\prime}(x)$. Because $\left|f^{\prime}(x)\right|<B,|f(x)-f(y)|<$ $B|x-y|$.

Because the bound $B$ on the derivative is the same for all $f \in T(\hat{C})$, if we choose $x$ and $y$ such that $|x-y|<\frac{\varepsilon}{B},|f(x)-f(y)|<\varepsilon$ for any $f \in T(\hat{C})$. Therefore $T(\hat{C})$ is equicontinuous.

Next I verify that the operator $T$ is continuous.

Lemma 4. The operator $T$ is continuous.
Proof. Let $\left\{Q_{i}\right\}_{i \in \mathbb{N}} \subset \hat{C}$ be a sequence of functions converging to $Q$ in the sup norm.

Then, for any $\beta>0 \exists j \in \mathbb{N}$ such that $\forall i>j,\left\|Q_{i}-Q\right\|<\beta$.

$$
\left(T\left(Q_{i}\right)-T(Q)\right)(\mu)=\delta \int_{r_{Q}(\mu)}^{\infty}\left[Q_{i}(\hat{\mu}(r, \mu))-Q(\hat{\mu}(r, \mu))\right] f(r \mid \alpha(\mu)) d r
$$

$$
+\delta \int_{r_{Q_{i}}(\mu)}^{r} r_{Q}(\mu) Q_{i}(\hat{\mu}(r, \mu)) f(r \mid \alpha(\mu)) d r
$$

if $r_{Q_{i}}(\mu)>r_{Q}(\mu)$. For the reverse case, an identical argument may be used.
The first term converges to zero by definition of $Q_{i}$.
The second term converges to zero because $r_{Q_{i}}(\mu) \rightarrow r_{Q}(\mu)$. To see this, consider the following equality derived from the politician's F.O.C.:

$$
\int_{r_{Q}(\mu)}^{\infty}\left[Q_{i}(\hat{\mu}(r, \mu))-Q(\hat{\mu}(r, \mu))\right] f_{a}(r \mid \alpha(\mu)) d r=\int_{r_{Q_{i}}(\mu)}^{r_{Q}(\mu)} Q_{i}(\hat{\mu}(r, \mu)) f_{a}(r \mid \alpha(\mu)) d r
$$

Again, the term on the LHS converges to zero by convergence of $Q_{i}$. Hence the RHS must also converge to zero. However, because $r_{Q_{i}}(\mu)>\alpha(\mu)$ and $Q_{i}(\hat{\mu}(r, \mu)) \geq u(0)$, the terms inside the integral are bounded away from zero. Therefore, it must be that $r_{Q_{i}}(\mu) \rightarrow r_{Q}(\mu)$.

I have now established that $\left\|T\left(Q_{i}\right)-T(Q)\right\| \rightarrow 0$ so that $T$ is a continuous operator.

I may now apply Schauder's FPT to find a value function and a reputationdependent cutoff function $r(\mu)$ implementing effort strategy $\alpha(\mu, v)$.

This completes the proof of existence.

### 6.2. Impossibility of Markov perfect equilibria with positive effort proof of Proposition 2

In this section I present a proof of a slightly more general version of Proposition 2. Specifically, I generalize the statement to include strategies which are Markovian with probability 1.

Definition 10. An equilibrium is essentially Markov perfect if strategies ( $\sigma, \alpha$ ) are measurable with respect to the Markovian partition for a set of reputations $M \subset[0,1]$ of Lebesgue measure 1.

Note that any Markovian strategy is also essentially Markovian. Although the distinction is not of interest in and of itself, I make it here as it is useful in establishing Proposition 3 in Section 4. The extension does not significantly complicate the proof since it requires only that we note that non-Markovian strategies which are played with probability 0 do not affect the strategic calculus of players involved.

Proposition 4. There is no essentially Markov perfect equilibrium with positive value for the voter.

In what follows, for ease of exposition I write $\hat{Q}(\hat{\mu}(r, \mu))$ for $\sigma(\hat{\mu}(r, \mu)) Q(\hat{\mu}(r, \mu))$.
The proof proceeds as follows. First, I consider the case in which effort is bounded below for some interval $[m, 1$ ) of reputations and $\hat{Q}$ is weakly monotonic. This leads me to conclude that $\hat{Q}$ is unbounded, a contradiction.

Then, I generalize the result in several ways. First, if $\hat{Q}$ is not weakly monotonic, I show that one may look at a moving average of $\hat{Q}$ and that repeated application of the moving average operator leads to a function which is monotonic or approximately constant over an interval $[z, 1)$, and thus to the same contradiction as above.

Once this is done, I am left with the possibility that effort is not bounded below. However, I show that, if positive effort is ever incentivized, politicians with high reputation must be reelected with positive probability and, if that is the case, there must be politicians of arbitrarily high reputation who exert effort above some fixed lower bound. Therefore, I am able to complete the argument by showing that these minimum conditions are enough to lead to the conclusion that $\hat{Q}$ is unbounded. Thus, there can be no Markov perfect equilibrium supporting positive effort if the politician's payoffs are bounded.

Consider first the case in which there is a lower bound $b>0$ on the effort exerted by politicians with reputation in $[x, 1)$. Using the politician's FOC, I know that his value function must satisfy

$$
\delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r, \mu)) f_{a}(r \mid \alpha(\mu)) d r \geq-u^{\prime}(b)=B>0
$$

By NI I can rewrite $\hat{Q}(\hat{\mu}(r, \mu)) f_{a}(r \mid \alpha(\mu))$ as $\left.\hat{Q}(\hat{\mu}(r+\alpha(\mu), \mu)) f_{a}(r \mid 0)\right)$.
Because $\int_{0}^{\infty} f_{a}(r \mid 0) d r<\infty$, I can find a value $r^{*} \in \mathbb{R}_{+}$such that
$\delta \int_{-r^{*}}^{r^{*}} \hat{Q}(\hat{\mu}(r+\alpha(\mu), \mu)) f_{a}(r \mid 0) d r \geq \delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r+\alpha(\mu), \mu)) f_{a}(r \mid 0) d r-\varepsilon$
for some fixed $\varepsilon \in\left(0, \frac{B}{2}\right)$.
Suppose $\hat{Q}$ is weakly monotonic. If $\hat{Q}$ is weakly decreasing, the integrals above will be weakly negative since, by A4., $\delta \int_{-\infty}^{\infty} \hat{Q}(\hat{\mu}(r+\alpha(\mu), \mu)) f_{a}(r \mid 0) d r=$ $\delta \int_{0}^{\infty}[\hat{Q}(\hat{\mu}(r+\alpha(\mu), \mu))-\hat{Q}(\hat{\mu}(-r+\alpha(\mu), \mu))] f_{a}(r \mid 0) d r<0$. Thus, the F.O.C.
will not be satisfied. Suppose $\hat{Q}$ is weakly increasing. By the monotone likelihood ratio property (A3.), I know that there is a unique point at which $f_{a}(r \mid 0)=0$ with the derivative being negative to the left and positive to the right of that point. Because $f(r \mid 0)$ is symmetric (A4.), this point is 0 . Then,
$\delta \int_{-r^{*}}^{r^{*}} \hat{Q}(\hat{\mu}(r+\alpha(\mu), \mu)) f_{a}(r \mid 0) d r$
$\leq \delta \int_{0}^{r^{*}} \hat{Q}\left(\hat{\mu}\left(r^{*}+\alpha(\mu), \mu\right)\right) f_{a}(r \mid 0) d r+\delta \int_{-r^{*}}^{0} \hat{Q}\left(\hat{\mu}\left(-r^{*}+\alpha(\mu), \mu\right)\right) f_{a}(r \mid 0) d r$
$\leq \delta\left[\hat{Q}\left(\hat{\mu}\left(r^{*}+\alpha(\mu), \mu\right)\right)-\hat{Q}\left(\hat{\mu}\left(-r^{*}+\alpha(\mu), \mu\right)\right] k\right.$
where $k=\int_{0}^{r^{*}} f_{a}(r \mid 0) d r$.
Therefore, $\hat{Q}\left(\hat{\mu}\left(r^{*}+\alpha(\mu), \mu\right)\right)-\hat{Q}\left(\hat{\mu}\left(-r^{*}+\alpha(\mu), \mu\right) \geq \frac{B}{2 \delta k}>0\right.$ for all $\mu$.
Given $\mu$ and $r^{*}$, there is a $\mu^{\prime}$ such that $\mu=\hat{\mu}\left(-r^{*}+\alpha\left(\mu^{\prime}\right), \mu^{\prime}\right)$. Therefore, $\hat{Q}$ must increase by at least $\frac{B}{2 \delta k}$ over $\left[\hat{\mu}\left(-r^{*}+\alpha\left(\mu^{\prime}\right), \mu^{\prime}\right), \hat{\mu}\left(r^{*}+\alpha\left(\mu^{\prime}\right), \mu^{\prime}\right)\right]$. Because this process can be repeated indefinitely, this implies that $\hat{Q}$ grows without bound, which is a contradiction. Therefore, there can be no Markov reelection strategy leading to a weakly monotonic $\hat{Q}$ over any interval $[x, 1]$ while effort is bounded below by $b>0$.

I am left with the possibility of a $\hat{Q}$ which is non-monotonic over every interval of the form $[x, 1]$. Suppose I have found such a $\hat{Q}$. Then,
$\delta \int_{-r^{*}}^{r^{*}} \hat{Q}(\hat{\mu}(r+\alpha(\mu), \mu)) f_{a}(r \mid 0) d r \geq \frac{B}{2}$ for all $\mu$.
Define $\hat{Q}(x)=\hat{Q}(\hat{\mu}(r+\alpha(x), x))$ for $x \in[m, 1]$. Then,
$\delta \int_{-r^{*}}^{r^{*}} \hat{Q}(x) f_{a}(r \mid 0) d r \geq \frac{B}{2}$
Therefore, $\delta \int_{-r^{*}}^{r^{*}} \frac{1}{\hat{\mu}\left(r^{*}, \mu\right)-\mu} \int_{\mu}^{\hat{\mu}\left(r^{*}, \mu\right)} \hat{Q}(x) d x f_{a}(r \mid 0) d r \geq \frac{B}{2}$
$\frac{1}{\hat{\mu}\left(r^{*}, \mu\right)-\mu} \int_{\mu}^{\hat{\mu}\left(r^{*}, \mu\right)} \hat{Q}(x) d x$ is a moving average of $\hat{Q}$. We may apply this operator repeatedly defining $\hat{Q}_{0}=\hat{Q}$ and $\hat{Q}_{i}(\mu)=\frac{1}{\hat{\mu}\left(r^{*}, \mu\right)-\mu} \int_{\mu}^{\hat{\mu}\left(r^{*}, \mu\right)} \hat{Q}_{i-1}(x) d x$. The following Lemma establishes a basic but useful fact about the moving average operator.

Lemma 5. Given a function $\hat{Q}$, there exists an interval of positive length $[z, 1)$ such that $\hat{Q}_{2}$ is either weakly monotonic or approximately constant on $[z, 1)$.

Proof. After the moving average operator has been applied once, $\hat{Q}_{1}$ is continuous and differentiable with derivative
$\hat{Q}_{1}^{\prime}(\mu)=\frac{\partial\left(\frac{1}{\hat{\mu}\left(r^{*}, \mu\right)-\mu}\right)}{\partial \mu} \int_{\mu}^{\hat{\mu}\left(r^{*}, \mu\right)} \hat{Q}_{0}(x) d x+\frac{1}{\hat{\mu}\left(r^{*}, \mu\right)-\mu}\left(\hat{\mu}_{2}\left(r^{*}, \mu\right) \hat{Q}_{0}\left(\hat{\mu}\left(r^{*}, \mu\right)\right)-\hat{Q}_{0}(\mu)\right)$. Therefore $\hat{Q}_{2}$ is continuously differentiable and
$\hat{Q}_{2}^{\prime}=\frac{\partial\left(\frac{\bar{\mu}\left(r^{*}, \mu\right)-\mu}{}\right)}{\partial \mu} \int_{\mu}^{\hat{\mu}\left(r^{*}, \mu\right)} \hat{Q}_{1}(x) d x+\frac{1}{\hat{\mu}\left(r^{*}, \mu\right)-\mu}\left(\hat{\mu}_{2}\left(r^{*}, \mu\right) \hat{Q}_{1}\left(\hat{\mu}\left(r^{*}, \mu\right)\right)-\hat{Q}_{1}(\mu)\right)$.
$\hat{Q}_{2}^{\prime \prime}=\frac{\partial^{2}\left(\frac{\bar{\mu}\left(r^{*}, \mu\right)-\mu}{}\right)}{\partial \mu^{2}} \int_{\mu}^{\hat{\mu}\left(r^{*}, \mu\right)} \hat{Q}_{1}(x) d x+2 \frac{\partial\left(\frac{\bar{\mu}\left(r^{*}, \mu\right)-\mu}{}\right)}{\partial \mu}\left(\hat{\mu}_{2}\left(r^{*}, \mu\right) \hat{Q}_{1}\left(\hat{\mu}\left(r^{*}, \mu\right)\right)-\hat{Q}_{1}(\mu)\right)$
$+\frac{1}{\hat{\mu}\left(r^{*}, \mu\right)-\mu}\left(\hat{\mu}_{22}\left(r^{*}, \mu\right) \hat{Q}_{1}\left(\hat{\mu}\left(r^{*}, \mu\right)\right)+\hat{\mu}_{2}\left(r^{*}, \mu\right) \hat{Q}_{1}^{\prime}\left(\hat{\mu}\left(r^{*}, \mu\right)\right)-\hat{Q}_{1}^{\prime}(\mu)\right)$.
Because $\hat{Q}$ is bounded, $\hat{Q}_{2}^{\prime}$ and $\hat{Q}_{2}^{\prime \prime}$ are bounded. Let $B>0$ denote the bound on $\hat{Q}_{2}^{\prime \prime}$.

Given an $\varepsilon>0$, there is a $z$ such that if $\left|\hat{Q}_{2}^{\prime}(\mu)\right|>\varepsilon$ for some $\mu \in[z, 1)$ then $\hat{Q}_{2}$ is strictly monotonic over $[z, 1)$. This is because the most $\hat{Q}_{2}^{\prime}$ can change in a distance less than $1-z$ is $B(1-z)<\varepsilon$ for $z$ close enough to 1 . If there is no $\mu \in[z, 1)$ such that $\left|\hat{Q}_{2}^{\prime}(\mu)\right|>\varepsilon$, then $\left\|\hat{Q}_{2}-C\right\|_{\infty}<\eta$ for some constant function $C$ and a $\eta$ which becomes arbitrarily small as $\varepsilon \rightarrow 0$. Thus, $\hat{Q}_{2}$ is approximately constant.

If $\hat{Q}_{2}$ is weakly monotonic over $[z, 1)$, I may now repeat the arguments for weakly monotonic functions on $\hat{Q}_{2}$ starting at the point $z$. Since a bounded $\hat{Q}$ should imply a bounded $\hat{Q}_{2}$, I am once again left with a contradiction. If $\hat{Q}_{2}$ is merely approximately constant, I note that $\delta \int_{-r^{*}}^{r^{*}} C f_{a}(r \mid 0) d r=0$ by symmetry of $f(r \mid 0)(\mathrm{A} 4$.$) and, for \mu$ such that $\left.\hat{\mu}\left(-r^{*}+\alpha(\mu), \mu\right)\right)>z$,
$\left|\int_{-r^{*}}^{r^{*}} \hat{Q}_{2}(\hat{\mu}(r+\alpha(\mu), \mu)) f_{a}(r \mid 0) d r-\int_{-r^{*}}^{r^{*}} C f_{a}(r \mid 0) d r\right|$
$=\left|\int_{-r^{*}}^{r^{*}} \hat{Q}_{2}(\hat{\mu}(r+\alpha(\mu), \mu)) f_{a}(r \mid 0) d r\right|<\frac{B}{2}$ (if $\eta$ is chosen small enough) which contradicts the derived properties of $\hat{Q}$.

Now, I consider the case where there is no lower bound on effort exerted. The following Lemmas provide constraints on what can happen in such a hypothetical equilibrium.

Lemma 6. In any Markov equilibrium with positive value $V\left(\mu_{0}\right)>0$, every interval of the form $[\mu, 1]$ must contain reputation points at which politicians are reelected with strictly positive probability.

Proof. Suppose not. Let $\hat{r}(a)$ denote the outcome which would keep the politician's reputation constant:

$$
\hat{r}(a)=\{r \mid \hat{\mu}(r, \mu)=\mu\}
$$

Note that, using assumption A3., $\hat{r}(a)<a$ (if $r$ is normally distributed $\hat{r}(a)=$ $\left.\frac{a}{2}\right)$.

Then consider the first order condition of a politician with the highest reputation which is reelected with positive probability $\mu$ :

$$
u^{\prime}(\alpha(\mu))+\delta \int_{-\infty}^{\hat{r}(a)} Q(\hat{\mu}(r, \mu)) f_{a}(r \mid \alpha(\mu)) d r<0 \text { for any } \alpha(\mu)
$$

Because $f_{a}(r \mid \alpha(\mu))$ is negative for all values below $\alpha(\mu)$. Therefore, $\alpha(\mu)=0$ and $\mu$ is an absorbing state. Since I assumed $V\left(\mu_{0}\right)>0$, it is not a best response for the voter to reelect a politician with reputation $\mu$, contradicting the definition of $\mu$.

Lemma 7. Consider a Markov perfect equilibrium with positive value for the voter $V\left(\mu_{0}\right)>0$. In every reputation interval of the form $[\mu, 1]$ there must be a subset of positive measure in which politicians exert effort above some fixed lower bound $b>0$.

Proof. Suppose not. Then, choose a lower bound $b<\frac{1}{2} V\left(\mu_{0}\right)$ and let $[\mu, 1]$ be an interval over which effort is bounded above by $b$ almost everywhere. $V$ is bounded above by the constant function $\bar{V}=\frac{\bar{a}}{1-\delta}$ where $u(\bar{a})=0$. Let $k$ satisfy $\sum_{i=0}^{k} \delta^{i} b+\sum_{i=k+1}^{\infty} \delta^{i} \bar{V}<V\left(\mu_{0}\right)$. By Lemma 6 , there must be reputations arbitrarily close to 1 which are reelected with positive probability. Because effort is bounded, I may choose a reputation (call it $\hat{\mu}$ ) which is reelected with positive probability and from which the probability of transitioning out of $[\mu, 1]$ in $k$ periods or fewer (call it $p$ ) is arbitrarily small. In particular, if I choose $p<\frac{V\left(\mu_{0}\right)}{\delta V}$, an upper bound on the value to the voter of having a politician with reputation $\hat{\mu}$ in office
$(V(\hat{\mu}))$ is:

$$
V(\hat{\mu})<(1-p)\left(\sum_{i=0}^{k} \delta^{i} b+\sum_{i=k+1}^{\infty} \delta^{i} \bar{V}\right)+p \delta \bar{V}<V\left(\mu_{0}\right)
$$

If the politician is reelected in each of his first $k$ terms. Note that the probability of transitioning to a point in $[\mu, 1]$ at which effort higher than $b$ is exerted is zero because this may happens only on a subset of measure 0 , and therefore this possibility does not affect the calculation of expected rewards.

If he does not survive $k$ terms, then $V(\hat{\mu})$ is less than:

$$
V(\hat{\mu})<b+\delta V\left(\mu_{0}\right)<V\left(\mu_{0}\right)
$$

Therefore, it is not a best response to reelect a politician when his reputation is $\hat{\mu}$, which contradicts the definition of $\hat{\mu}$.

Given Lemma 7, if I have a weakly monotonic value function I need only to modify the arguments above as follows. Instead of moving to a reputation satisfying $\mu=\hat{\mu}\left(-r^{*}+\alpha\left(\mu^{\prime}\right), \mu^{\prime}\right)$ I move to one satisfying $\alpha\left(\mu^{\prime}\right)>b$ and $\mu<$ $\hat{\mu}\left(-r^{*}+\alpha\left(\mu^{\prime}\right), \mu^{\prime}\right)$. Once again, I conclude that $\hat{Q}$ must increase by at least a fixed amount $\frac{B}{2 \delta k}$ infinitely many times, contradicting its boundedness.

To deal with non-monotonic candidate value functions $\hat{Q}$ I note that, given Lemma 7, repeated application of the moving average operation ensures that the value of all integrals $\delta \int_{-r^{*}}^{r^{*}} \hat{Q}_{i}(\hat{\mu}(r, \mu)) f_{a}(r \mid \alpha(\mu)) d r$ will be positive. Because these are defined on a closed set $\left[\mu^{\prime}, 1\right]$, there exists a minimum value of these integrals. Now, I may apply the same arguments as above: $\hat{Q}_{2}$ includes a weakly monotonic segment $[z, 1)$, and this contradicts of the boundedness of $\hat{Q}$.

Finally, note that in all the arguments above, having a function $Z$ which differs from $\hat{Q}$ only on a set of Lebesgue measure 0 will not change any of the results, because the integrals will yield the same values under both functions. Therefore, it is immediate that the result extends to rule out essentially Markov perfect
equilibria with positive value for the voter.

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[^1]:    ${ }^{1}$ Alternatively, effort is too costly for L-types for it to be worthwhile exerting. $-u_{L}^{\prime}(0)>$ $\frac{\delta}{1-\delta} u_{L}(0) f(0 \mid 0)$ is sufficient for this if I make the same assumptions on $u_{L}$ as on $u$.

[^2]:    ${ }^{2}$ The full support assumption ensures that Bayes' rule is always applicable since all histories are reached with positive density.

[^3]:    ${ }^{3}$ See Reed (1994), Banks and Sundaram (1998), Fearon (1999), Berganza (2000), and Ashworth (2005). In these models with term limits, it is assumed that high types perform better than low types in the last term in which no incentives for effort can be provided. Therefore, incumbents are reelected if their expected type is higher than that of a replacement.

[^4]:    ${ }^{4}$ The model in Banks and Sundaram 1993 includes an arbitrary but finite number of types of politician who differ in their cost of providing effort. This paper's model is a special case with low types having arbitrarily high cost of effort.

[^5]:    ${ }^{5}$ Weak renegotiation-proofness is a condition of internal consistency that makes comparisons between the continuation payoffs of a given equilibrium strategy profile. Competing notions of renegotiation-proofness, such as that advocated by Pearce (1987), call for external consistency so that comparisons are made across equilibria. In particular, Pearce argues that comparisons should be made among the the lowest continuation payoffs of equilibria. Because not reelecting politicians (giving them continuation payoff of zero) is the voter's only effective tool for providing incentives, this approach is unlikely to narrow the set of equilibria in this game.

