A Spatial Model of Common-value Elections: Electoral Mandates, Minor-Party Candidates, and the Signaling Voter's Curse

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Abstract

This paper analyzes a spatial model of common-value elections. Within a continuum of alternatives, one policy is designated as optimal, but citizens observe only private signals of this policy's location. When two candidates compete for office by making binding policy commitments, their platforms converge in equilibrium, as in standard median voter theorems, though with dramatically different welfare implications. When candidates are instead policy motivated, their platforms diverge. If platform commitments are not binding, the winning candidate departs from his platform policy in response to "mandates" conveyed by his margin of victory. This signaling role for voters renders every vote "pivotal", including votes for candidates who are unlikely to win the election. This eliminates the swing voter's curse, but introduces an analogous "signaling voter's curse", causing uninformed citizens to abstain from voting even when voting is costless.

1 Introduction

One of the earliest formal arguments in favor of the democratic institution of majority voting is the Condorcet (1785) jury theorem: if one of two candidates or policy alternatives is better, in some objective sense, for every member of society, then majority opinion in a

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large electorate will almost surely favor the superior alternative, as long as individual voters' policy opinions are based on independent and minimally accurate information. Since public policies inevitably effect different groups of citizens differently, however, existing literature has largely dismissed this information pooling role of elections as applicable only to a few specific voting environments such as juries and small committees. On the other hand, the broad goals of many policies—such as national defense, economic and environmental stability, and eliminating crime, poverty, or corruption—have essentially unanimous appeal; if voters base policy evaluations on societal outcomes such as these, their preferences are likely to be correlated, making the assumption of identical preferences a plausible approximation. Feddersen and Pesendorfer (1996) also point out that in a common-value environment a "swing voter's curse" leads uninformed citizens to abstain from voting, in deference to those with superior information, providing an explanation for otherwise puzzling empirical features of voter participation such as the phenomenon of *roll-off* (i.e. voting in some but not all races on the same ballot) and the correlation between voter participation and information variables, as well as later evidence from McMurray (2010a) that having well-informed peers makes a citizen more likely to abstain.[1]

This paper extends the original Condorcet framework to accommodate a continuum of policy alternatives, rather than just two. As in Condorcet's model, one of these policies is optimal, in the sense that it would maximize utility for every member of the electorate, if implemented. Citizens cannot observe the optimal policy directly, but receive independent private signals that are correlated with the truth. For simplicity, the optimal policy is assumed to be at one of the two extremes of the policy space; because voters are risk-averse, however, they nevertheless support moderate policies to avoid the severe disutility of implementing the wrong policy extreme.[2] Whereas voters in Condorcet's original model share identical information quality, this paper instead follows McMurray (2010b) in assuming a continuum of expertise from which individual signal quality is drawn. The distribution of expertise is assumed to have full support, implying that the distribution of voters' preferred policies has full support as well. Thus, an immediate result of this model is a theoretical foundation for the common assumption that voters' preferences are *single-peaked* over the policy interval. Citizens with the most extreme policy preferences also have the strongest convictions, providing an explanation for the empirical correlation between information and ideology, observed by Palfrey and Poole (1987).

The large menu of policy alternatives in this model gives candidates an active role to play in determining policy outcomes. In the first version of the model, two candidates commit

[1] McMurray (2010) discusses empirical evidence that information influences voting, and explains the puzzles that these findings present for standard models.

[2] Allowing any policy to be optimal leads to similar results.

to campaign policy platforms prior to the election, motivated by a desire to win office. In the unique equilibrium, both adopt platforms at the same moderate policy. This result is reminiscent of the canonical median voter theorems of Black (1948) and Downs (1957), and arises for the same reason: moving away from the center merely concedes votes to a candidate's opponent. The welfare implication of this result, however, differs starkly from previous models. In models of political compromise between competing interests, moderate policies minimize the maximum distance to any voter's ideal point, and so are desirable from a utilitarian perspective.[3] Here, the equilibrium policy outcome is optimal only on the basis of prior information; if any private information were available, a superior policy could be identified.

A second version of this model assumes that candidates seek office in an effort to influence policy outcomes, as other authors have assumed at least as early as Wittman (1977).[4] As in that literature (e.g. Wittman 1983), policy-motivated candidates' platforms diverge in equilibrium. In this case, policy platforms become more extreme because informative voting leads candidates to form more extreme beliefs.[5] Accordingly, divergence from the center actually enhances welfare, again unlike standard models.

With informative voting, the candidate with the superior platform is more likely to win the election by a single vote than to lose by a single vote. Accordingly, an additional vote for that candidate is less likely to be *pivotal* (i.e. change the election outcome) than a vote for his opponent. An uninformed citizen therefore suffers from a *swing voter's curse*, as in Feddersen and Pesendorfer (1996) and McMurray (2010b), and prefers to abstain rather than vote for either candidate, even if voting is costless. Like those models, therefore, this model provides an explanation for empirical phenomena such as roll-off, and the correlation between relative information variables and turnout.

Following the citizen-candidate tradition of Osborne and Slavinski (1996) and Besley and Coate (1997), a third specification of this model assumes that candidate commitments prior to an election are not credible: once elected, a candidate may implement the policy of his choice.[6] For policy-motivated candidates, this means using available information to estimate the location of the optimal policy. Since voting takes place before policy decisions are made, that information may include the total numbers of votes each candidate received. Consistent with the popular notion of an *electoral mandate*, a candidate who wins the election

[3] Though Duggan (2005) notes that this normative conclusion may not be justifiable.

[4] Besley and Case (2003) argue that policy-motivations are necessary for explaining empirical evidence that policy outcomes depend on the identities of political office holders.

[5] Specifically, a candidate bases his platform decision on the belief that he will receive a majority of votes (since otherwise his platform will not be implemented).

[6] Throughout this paper, masculine pronouns refer to candidates and feminine pronouns refer to citizens.

by a larger margin than expected develops more extreme beliefs, and therefore adopts a more extreme policy than his campaign platform. Similarly, a candidate becomes more moderate in response to a narrow victory. Since literally every vote influences the margin of victory, the result that candidates respond to vote totals provides a foundation for the popular mantra that "every vote counts", contrary to standard models in which a vote has no influence unless it creates or breaks a tie.

The result that every vote influences policy outcomes undermines the logic of the swing voter's curse, since a citizen no longer needs to condition her behavior on the unlikely event in which her vote is pivotal. However, a relatively uninformed citizen now suffers from a *signaling voter's curse*, and therefore once again has reason to abstain. The logic behind this result is that voters have heterogeneous information quality. Since the winning candidate cannot observe the underlying quality of each vote, he infers that each vote is of average quality. If a citizen of below-average quality votes, therefore, it will prompt a more drastic policy response than the underlying information merits. For a sufficiently well-informed citizen, too large a policy move may be better than no move at all; for a sufficiently uninformed citizen, however, it is better to abstain from voting. As in Feddersen and Pesendorfer (1996) and McMurray (2010b), then, abstention is strategic, reflecting the effort of uninformed citizens to delegate to those with better information. Abstention is also welfare-improving, as in those models, even though the private information of nonvoters is not utilized.

One standard result in spatial voting models is Duverger's (1954) Law, which essentially states that plurality rule elections foster two strong parties, and discourage the creation of smaller parties. This is because, while a vote for either of two major candidates is already unlikely to be pivotal, voting for a sure loser is less likely still to change the election outcome. The standard model predicts that citizens should not vote for fringe parties, who therefore should have no incentive to make a costly run for office. If a minor party candidate did manage to attract strong support, he would risk splitting votes with the closer of the two major candidates, thereby inadvertently deciding the election in favor of his least-favored opponent. That analysis changes in this setting, however, because eventual policy outcomes depend on each candidate's vote total. Even if he does not win office, then, voting for an extreme candidate pulls policy in the desired direction. This therefore justifies the casting of a "protest vote" for a candidate that is likely to lose, in an effort to send a message to the candidate who wins, in turn providing an incentive for the losing candidate to have run for office in the first place.

In addition to the references above, this model shares much in common Razin's (2003) model of signaling in common-value elections. Most notably, that model demonstrates the possibility of electoral mandates, inferred by the winning candidate from his margin of victory. In that model, however, large margins of victory can also make a winning candidate more moderate, rather than more extreme. Candidates also behave deterministically, rather than strategically, and voters have homogeneous information quality. The possibility of multiple candidates is not considered, and voter abstention is not allowed. Shotts (2006) and Meirowitz and Shotts (2007) consider an alternative role for signaling in elections, which is to influence incumbent politicians' perceptions of re-election prospects.

The remainder of this paper is organized as follows. Section 2 introduces the model, and Section 3 characterizes equilibrium, assuming the various combinations of office or policy motivation, and credible commitments or responsive candidates. Section 4 then introduces the possibility of abstention, and Section **??** analyzes welfare for the above model specifications. Section 6 considers the possibility of multiple candidates, and Section 7 concludes. Proofs of most formal results are presented in the Appendix.

2 The Model

A society consists of N citizens and two candidates, A and B. From an interval $[-1,1]$ of alternatives, a policy must be chosen that will provide a common benefit to every citizen. The optimal policy $Z \in \{-1,1\}$ lies at one of the two extremes of the policy space, but is an unknown state of the world; ex ante, either extreme is equally likely to be optimal (i.e. $\Pr(Z=1) = \Pr(Z=-1) = \frac{1}{2}$). Citizens unanimously prefer policies that are as close as possible to Z: if policy x is implemented, each receives the following utility:

$$u(x,Z) = -(x-Z)^2. \tag{1}$$

Note that $u(x,Z)$ is strictly concave in x, reflecting an assumption that citizens are risk-averse.

On the issue at hand, citizens differ in expertise. Independently from one another (and independent of Z), each citizen is endowed with information quality $Q_i \in [0,1]$, drawn from a common distribution F which, for technical convenience, is assumed to have a differentiable and strictly positive density f. Each citizen also observes a private signal $S_i \in \{-1,1\}$, which is positively correlated with the true state variable Z (but independent of Q_i and, conditional on Z, independent of other citizens' signals). The strength of this correlation varies with expertise; conditional on Z and Q_i, the distribution of S_i is given as follows, where $s, z \in \{-1,1\}$ and $q \in [0,1]$,

$$\Pr(S_i = s | Z = z, Q_i = q) = \frac{1}{2}(1 + zsq). \tag{2}$$

The correlation between S_i and Z (conditional on Q_i), then, is simply

$$corr(S_i, Z|Q_i) = Q_i.$$

Thus, S_i can be interpreted as a citizen's private opinion of Z, and Q_i can be interpreted as the strength of her conviction. To a perfectly informed citizen (i.e. $Q_i = 1$), for instance, S_i reveals Z perfectly; to a perfectly uninformed citizen (i.e. $Q_i = 0$) S_i reveals nothing.

The distribution F of expertise within the population is common knowledge, but Q_i and S_i are observed only privately. Conditional on private information, the posterior distribution of Z is given by the same expression as in (2):

$$\Pr(Z = z | S_i = s, Q_i = q) = \frac{1}{2}(1 + zsq). \tag{3}$$

for $s, z \in \{-1, 1\}$ and $q \in [0, 1]$. A citizen's expectation of the optimal policy, then, is simply $E(Z|S_i, Q_i) = S_i Q_i$. Based on her private information alone, this is the policy that would maximize the expectation of (1). Thus, preferences over policies are single-peaked, as in traditional models. By the assumption that F has full support (i.e. that f is strictly positive), the distribution of citizens' ideal points has full support on the policy interval.

With individual citizens thus informed, candidates propose policy platforms $x_A, x_B \in [0, 1]$ (say $x_A \leq x_B$). Observing these platforms, each citizen then votes (at no cost) for one of the two candidates. For most of Section 3, abstention from voting is not allowed. Section 4 introduces voter abstention, and then Section 6 considers the possibility of additional candidates. Representing voter abstention as a vote for candidate 0, therefore, the set of actions expands from $\{A, B\}$ to $\{A, B, 0\}$ to $\{A, B, C, D, 0\}$. I restrict attention to pure strategies[7] that are also *symmetric*, meaning that citizens of the same type $(q, s) \in [0, 1] \times \{-1, 1\}$ respond to candidate platforms $(x_A, x_B) \in [0, 1]^2$ identically. Let $\Sigma = \left\{\sigma : [0, 1] \times \{-1, 1\} \times [0, 1]^2 \to \{A, B\}\right\}$ denote the set of all such strategies.[8]

Votes are cast simultaneously, and an election winner $W \in \{A, B\}$ is determined by simple majority rule, breaking a tie if necessary by a fair coin toss. Let $y_j \in [0, 1]$ denote the policy that candidate j implements after winning the election (i.e. if $W = j$) and taking office. In Section 3.4, $y_j : \mathbb{Z}_+^2 \to [-1, 1]$ is a strategic choice variable, possibly differing in response to vote totals $(a, b) \in \mathbb{Z}_+^2$. In Sections 3.2 and 3.3, an election winner is instead required to implement his campaign platform $y_j = x_j$.

Following Myerson (1998, 2000), I assume that the precise number N of citizens is unknown, but is commonly known to follow a Poisson distribution with mean μ; together with voter and candidate strategies, the realization of N determines the numbers N_A and N_B of votes for either candidate, which in turn determine the election winner W, and therefore the ultimate policy outcome $Y \in [-1, 1]$.[9] Citizens and *policy-motivated* candidates seek

[7]Mixed strategies could be allowed, but would be used with zero probability in equilibrium.

[8]If abstention is allowed, the set of strategies is instead $\Sigma' = \left\{\sigma : [0, 1] \times \{-1, 1\} \times [0, 1]^2 \to \{A, B, 0\}\right\}$.

[9]As Bade (2006) discusses, one advantage of this assumption is that N_A and N_B are independent. In the numerical examples in Sections 3.4 and 6, N is instead fixed and known.

to maximize the expectation of $u(Y, Z)$, while *office-motivated* candidates seek to maximize the probability of being elected. The analysis in Section 3 seeks to characterize a *perfect symmetric Bayesian equilibrium*, in which citizen and candidate strategies are optimal in each subgame, given the behavior of others.

3 Equilibrium

This section analyzes perfect Bayesian equilibrium behavior for both citizens and candidates, for various specifications of the model outlined above. This analysis proceeds by backward induction: section 3.1 begins with the voting stage-game, assuming that candidate platforms have been fixed. Given voters' behavior, sections 3.2 and 3.3 then allow candidates to choose policy platforms, with the assumption that platform commitments are binding: the winning candidate must implement his platform policy (i.e. $y_j = x_j$ for $j = A, B$). These two sections differ in the assumption of candidate motivations: in Section 3.3, candidates—themselves citizens—seek to maximize (1) by implementing superior policies; in Section 3.2, candidates desire only the perquisites of winning and holding office. Finally, Section 3.4 treats campaign platform promises as non-credible, allowing the winning candidate to implement any policy of his choice. As in Section 3.3, candidates are assumed to be policy-motivated.

For the exposition of results in Section 3, some additional notation is useful. First, let $p_\sigma(j|z)$ denote the probability with which a citizen of type (q, s) votes for candidate $j \in \{A, B, 0\}$ in state $z \in \{-1, 1\}$, according to voting profile σ (which depends implicitly on candidate platforms (x_A, x_B), which are suppressed from notation).

$$p_\sigma(j|z) = \sum_{s=1,-1} \int_{q:\sigma(q,s,x_A,x_B)=j} \frac{1}{2}(1 + zsq) f(q)\, dq. \tag{4}$$

By the decomposition property of Poisson random variables (see Myerson, 1998), the numbers N_A and N_B of A and B votes in state z are independent Poisson random variables with means $\mu p_\sigma(A|z)$ and $\mu p_\sigma(B|z)$. Accordingly, let $\psi_\sigma(a, b|z)$ denote the probability in state z of a particular voting outcome $(N_A, N_B) = (a, b)$.

$$\psi_\sigma(a, b|z) = \frac{e^{-\mu p_\sigma(A|z)}}{a!}[\mu p_\sigma(A|z)]^a \frac{e^{-\mu p_\sigma(B|z)}}{b!}[\mu p_\sigma(B|z)]^b. \tag{5}$$

Of particular interest are events in which one additional vote for either candidate would be *pivotal*, changing the election outcome. Let $\pi_\sigma^0(z)$ denote the probability in state z of an exact tie, and let $\pi_\sigma^A(z)$ and $\pi_\sigma^B(z)$ denote the probabilities in state z with which candidates

A and B, respectively, win the election by exactly one vote:

$$\pi_\sigma^0(z) \equiv \sum_{k=0}^{\infty} \psi_\sigma(k,k|z) \tag{6}$$

$$\pi_\sigma^A(z) \equiv \sum_{k=0}^{\infty} \psi_\sigma(k+1,k|z) \tag{7}$$

$$\pi_\sigma^B(z) \equiv \sum_{k=0}^{\infty} \psi_\sigma(k,k+1|z). \tag{8}$$

A vote for candidate A is pivotal (event piv_A) when either the candidates tie and A loses the tie-breaking coin toss, or A wins the coin toss but loses the election by exactly one vote; a B vote is pivotal (event piv_B) under symmetric circumstances. In state z, therefore, these events occur with the following probabilities:

$$\begin{aligned} \Pr_\sigma(piv_A|z) &= \frac{1}{2}\pi_\sigma^0(z) + \frac{1}{2}\pi_\sigma^B(z) \\ \Pr_\sigma(piv_B|z) &= \frac{1}{2}\pi_\sigma^0(z) + \frac{1}{2}\pi_\sigma^A(z). \end{aligned}$$

By the environmental equivalence property of Poisson games (see Myerson, 1998), an individual citizen from within the game reinterprets N_A and N_B as the numbers of A and B votes cast by her peers; by voting herself, she can add one to either total. Accordingly, $\psi_\sigma(a,b|z)$ is the probability of a particular election outcome should she abstain, and $\Pr_\sigma(piv_A|z)$ and $\Pr_\sigma(piv_B|z)$ are the probabilities with which her own A or B vote, respectively, will be pivotal. From (1), the utility difference in state Z between two policies x_1 and x_2 can be written as

$$\begin{aligned} u(x_2,Z) - u(x_1,Z) &= -(x_2-Z)^2 + (x_1-Z)^2 \\ &= 2(x_2-x_1)(Z-\bar{x}), \end{aligned}$$

where $\bar{x} \equiv \frac{x_1+x_2}{2}$ is the midpoint between the two policies. The sign of this expression depends on which policy is closer to the ideal policy Z: having assumed that $x_A \leq x_B$, the superior policy in state -1 is x_A, while x_B is better in state 1. Accordingly, let P_z denote the probability in state z with which a single vote for the superior candidate is pivotal, and let $\tilde{P}_z$ denote the probability in state z with which a single vote for the inferior candidate is pivotal (where, in both cases, the implicit dependence on the voting strategy σ is suppressed from notation):

$$\begin{aligned} P_1 &= \Pr_\sigma(piv_B|Z=1) \quad & P_{-1} &= \Pr_\sigma(piv_A|Z=-1) \\ \tilde{P}_1 &= \Pr_\sigma(piv_A|Z=1) \quad & \tilde{P}_{-1} &= \Pr_\sigma(piv_B|Z=-1). \end{aligned}$$

3.1 Equilibrium Voting Behavior

If the policy outcomes that will result from electing candidates A and B are determined exogenously, then candidates have no role to play, and the election reduces to a game among citizens only. Citizens seek to implement the superior policy, but which policy outcome is superior depends on the state variable: when $y_A < y_B$, candidate A's policy is superior to candidate B's policy if $Z = -1$ and candidate B's policy is superior if $Z = 1$. A citizen's expectation $E(Z|Q_i, S_i) = Q_i S_i$ of the state depends on her private information; of natural interest, therefore, is a *belief threshold strategy* σ_T, according to which citizens with high expectations vote B and citizens with low expectations vote A.

Definition 1 *The symmetric voting strategy profile σ_T is a belief threshold strategy, with belief threshold $T \in [-1, 1]$, if*

$$\sigma_T(q, s) = \begin{cases} A \text{ if } qs \leq T \\ B \text{ if } qs > T \end{cases}.$$

σ_T is a symmetric belief threshold strategy if $T = 0$.

Lemma 1 states that optimal voter responses to exogenous platforms $y_A < y_B$ can be characterized by a belief threshold, and that an equilibrium belief threshold exists.[10] If platforms $y_B = -y_A$ are symmetric around zero then equilibrium voting is a symmetric belief threshold strategy, meaning that voting is also *sincere* (i.e. $\sigma(q, -1) = A$ and $\sigma(q, 1) = B$).

Lemma 1 *If y_A, y_B are fixed exogenously then the best response to any symmetric voting strategy σ is a belief threshold strategy σ_T. Furthermore, there exists a belief threshold T^* such that σ_{T^*} is a symmetric Bayesian equilibrium. σ_0 is such an equilibrium if and only if policy outcomes $y_B = -y_A$ are symmetric around zero.*

Proof. See Appendix. ■

In this section, candidate platforms are assumed to be fixed. In what follows, candidates instead choose policy outcomes (or at least platforms) to maximize some objective, taking the belief threshold voting behavior of characterizes as given.

3.2 Platform Commitment and Office Motivation

In this and the following section, candidates are assumed to commit credibly to policy platforms x_A and x_B before the election. In other words, the winning candidate must

[10] If $y_A = y_B$ then, trivially, any voting strategy constitutes an equilibrium.

implement his platform (i.e. $y_j = x_j$ for $j = A, B$). Candidates are further assumed to be *office motivated*, each receiving utility 1 if he wins the election and utility 0 if he loses. A candidate's expected utility, then, is merely his probability of winning the election, given voters' strategies and his own and his opponent's platforms.

A citizen prefers the policy platform that is nearest to her private expectation of Z; accordingly, as Lemma 1 states, citizens with high expectations vote for candidate B while citizens with low expectations vote for candidate A. Which candidate receives the larger share of votes, therefore, depends (in expectation) on which platform is closest to a larger fraction of citizens' expectations. Given the symmetry of the model, this is the platform that is closer to the zero policy (i.e. the *ex ante* median of citizens' expectations). In choosing his platform, therefore, an office-motivated candidate seeks to adopt a more moderate position than his opponent. Accordingly, as Theorem 1 now states, the unique perfect symmetric Bayesian equilibrium is such that both candidates adopt the zero policy.

Theorem 1 (Median Voter Theorem) *If policy platform commitments are binding and candidates are office-motivated, and if there is a unique equilibrium threshold $T^*(x_A, x_B)$ in the stage-game associated with every pair $x_A \neq x_B$ of candidate platforms, then (σ^*, x_A^*, x_B^*) is a perfect symmetric Bayesian equilibrium if and only if σ^* is the belief threshold strategy σ_{T^*} and $x_A^* = x_B^* = 0$.*

Proof. See Appendix. ■

The candidate behavior predicted by Theorem 1 closely resembles that predicted by the well-known Median Voter Theorem introduced by Black (1948) and Downs (1952), and for the same reason: efforts to attract large fractions of the electorate push candidates toward one another, and toward the median voter's ideal point. The welfare implications in this model differ dramatically, however, from the implications in a model with fundamental differences in tastes. In that context, the median voter theorem is a positive outcome, representing a compromise between the competing desires of citizens at opposite ends of the preference spectrum; if citizens are risk-averse, the median voter's ideal policy minimizes the maximum disutility experienced by any citizen, and so may maximize a utilitarian social welfare function. In this context, by contrast, citizens unanimously prefer (ex post) a more extreme policy; the zero policy is optimal only when no information is available beyond the common prior. In this setting, then, the median voter theorem represents a complete failure to utilize citizens' private information.

The result that candidate platforms converge in equilibrium has sometimes been viewed as an empirical failing of rational voting models, both because candidate platforms differ in real-world elections, and because if candidates did adopt identical platforms then citizens would have no incentive to vote. As in previous literature, however, the prediction here of platform convergence depends on candidate motivation; in the following section, equilibrium candidate platforms do not coincide.

3.3 Platform Commitment and Policy Motivation

In this section, candidates are assumed to be ordinary citizens, choosing policy platforms to maximize the expectation of (1). The optimal policy for such a candidate is his expectation of the state, conditional on available information. Though candidate j must commit to a policy platform before observing the election outcome, he can condition his expectation on the event $W = j$ in which he wins the election, since only then will his platform policy be implemented. Theorem 2 now states that equilibrium exists, and that equilibrium voting is informative, implying that candidates anticipate learning different pieces of information upon winning the election, and that policy platforms diverge accordingly. In particular, an equilibrium exists in which voting is also sincere.

Theorem 2 (Policy divergence) *If policy platform commitments are binding and candidates are policy-motivated, and if there is a unique equilibrium threshold $T^*(x_A, x_B)$ in the stage-game for every pair $x_A \neq x_B$ of candidate platforms, then (σ^*, x_A^*, x_B^*) is a perfect symmetric Bayesian equilibrium only if σ^* is the belief threshold strategy $\sigma_{T^*(x_A^*, x_B^*)}$ and $x_A^* = \hat{z}_A(\sigma^*, x_A^*, x_B^*) < \hat{z}_B(\sigma^*, x_A^*, x_B^*) = x_B^*$, where $\hat{z}_j(\sigma, x_A, x_B) \equiv E(Z|W = j; \sigma, x_A, x_B)$. Furthermore, such an equilibrium exists, with $T^* = 0$.*

Proof. See Appendix. ■

The reason that candidates adopt different policy platforms in Theorem 2 is that they learn different information from voters: candidate A is more likely to be elected in state -1 and candidate B is more likely to be elected in state 1. Conditional on being elected, therefore, A's expectation of Z is lower than B's. It may be, however, that voting conveys more information than merely the identity of the election winner. For example, with informative voting, if candidate B wins the election in state -1 it will likely be by only a few votes, whereas in state 1 he may win by a landslide. Put differently, the election winner is determined by the sign of the difference $N_B - N_A$ in vote totals for the two candidates; it may be the case that the magnitude $|N_B - N_A|$ of this difference carries informational content as well. If this is the case, a candidate who observes vote totals may wish to deviate from the campaign policy platform that he committed to before the election. In this section, such deviations are prohibited. In real-world elections, however, pre-election commitments may be quite difficult or even impossible to enforce. Accordingly, the next section relaxes the assumption that campaign commitments are binding.

3.4 Responsive Candidates

In this section, as in Section 2, candidates are policy-motivated, seeking to maximize the expectation of (1), just like ordinary citizens. Unlike Section 2, however, a winning candidate is no longer required to implement the platform policy that he adopted before the election.[11] As Lemma 2 now states, this leads a candidate to implement his expectation of Z. Because policy is implemented after vote totals are observed, a candidate conditions his expectations on this information.

Lemma 2 *If candidates are responsive then candidate j's best response to vote totals* $N_A = a$ *and* $N_B = b$*, given the voting profile* σ*, is*

$$y_j^* (a, b; \sigma) = E (Z | N_A = a, N_B = b; \sigma) \equiv \hat{z}_{a,b} (\sigma) .$$

Proof. For any set Ω of information, expected utility

$$Eu (x, Z | \Omega) = \sum_{z=1,-1} - (x - z)^2 \Pr (z | \Omega)$$

is strictly concave in x, and is uniquely maximized at $x^* = E (Z | \Omega)$. Letting $\Omega = \{N_A, N_B\}$ yields the desired result. ■

Because candidates are not required to implement their platform policies, campaign promises lack credibility. Voters no longer choose between candidates' platforms, therefore; instead, they anticipate candidates' choices in the final subgame in which policy is implemented, as given by Lemma 2. If citizens vote according to a belief threshold strategy, as in the equilibrium of Sections 3.1 through 3.3, then the impact on policy of a single vote is described by Lemma 3: essentially, every A vote pushes the winning candidate's expectation to the left, and every B vote pushes his expectation to the right.

Lemma 3 *If* σ_T *is a belief threshold strategy (with* $-1 < T < 1$*) then* $\hat{z}_{a+1,b} (\sigma_T) < \hat{z}_{a,b} (\sigma_T) < \hat{z}_{a,b+1} (\sigma_T)$ *for all* $a, b \in \mathbb{Z}_+$*.*

Proof. See Appendix. ■

When her fellow-citizens vote according to a belief threshold strategy, Lemma 3 implies that a citizen must choose between pushing the winning candidate's expectations—and therefore, by Lemma 2, the ultimate policy outcome—to the left or to the right. An individual whose private expectation of Z is low will therefore vote A, and an individual whose private expectation of Z is high will vote B. In other words, the best response to a belief

[11] Since policy-motivated candidate behavior is optimal from voters' perspective, an office-motivated candidate would wish to mimic the same behavior.

threshold strategy is another belief threshold strategy. Accordingly, Theorem 3 identifies a perfect symmetric Bayesian equilibrium, in which citizens vote according to the sincere belief threshold strategy σ_0, sincerely reporting their private signals, and candidates respond by implementing their expectations of Z, given vote totals, as in Lemma 2. Platform policies do not matter, since platform commitments are not credible.

Theorem 3 (Signaling) *If candidates are responsive then the best voting response to a belief threshold voting strategy is another belief threshold strategy. Furthermore, if* $T^* = 0$ *and* $y_j^*(a, b; \sigma) = \hat{z}_{a,b}(\sigma)$ *is the policy choice described in Lemma 2 for* $j = A, B$*, then* $(x_A^*, x_B^*, \sigma_{T^*}, y_A^*, y_B^*)$ *is a perfect symmetric Bayesian equilibrium for any pair* $x_A^*, x_B^* \in [-1, 1]$ *of policy platforms.*

Proof. See Appendix. ■

In Sections 3.1 through 3.3, as in standard voting models, an individual vote has influence only in the extremely unlikely event that it either makes or breaks a tie. In the equilibrium of Theorem 3, however, voting follows a belief threshold strategy; according to Lemma 3, therefore, every vote influences the ultimate policy outcome, by pushing the policy-maker's expectations one way or the other. Thus, the popular mantra that "every vote counts" in public elections can, in this setting, be taken quite literally.

The assumption that candidate platforms lack credibility implies that platforms have no role to play in the equilibrium of Theorem 3. If candidates nevertheless adopted platforms as in Theorem 2 of Section 3.3, however—that is, based on the expectation of Z conditional only on winning the election—then deviations from pre-election commitments would be predictable in the following way: a candidate who won the election by more than the expected number of votes would implement a more extreme policy than his campaign platform, and a candidate who won the election by fewer than the expected number of votes would implement a more moderate policy than his campaign platform.[12]

Propositions 1 and 2 illustrate the equilibrium identified in Theorem 3 with simple examples, assuming N to be fixed and known, and F to be uniform. The electorate in Proposition 1 consists of only two citizens. In equilibrium, candidates first propose platform policies $x_A^* = -0.5$ and $x_B^* = 0.5$. If both citizens vote for A or for B, the winning candidate then implements -0.8 or 0.8, respectively, instead of the platform policy; if the election is tied, the winning candidate implements 0 instead.

[12] Since equilibrium voting follows a belief threshold strategy whether platform commitments are binding (as in Theorem 2 of Section 3.3) or not (as in Theorem 3 of this section), adopting the policy platform prescribed by Theorem 2 might be optimal in a hybrid model in which candidate platforms are "sticky" (e.g. commitments are binding with some probability, or deviations from platforms are costly), though such a model is beyond the scope of this paper.

Proposition 1 *Let F be a uniform distribution and let $N = 2$. If candidates are responsive then there exists an equilibrium $(x_A^*, x_B^*, \sigma^*, y_A^*, y_B^*)$ such that candidate platforms are $x_B^* = -x_A^* = 0.5$, voting $\sigma^* = \sigma_0$ is sincere, and policy outcomes are as follows: $y_A^*(1,1) = y_B^*(1,1) = 0$ and $y_B^*(0,2) = -y_A^*(2,0) = 0.8$.*

Proof. With sincere voting, a citizen votes B if $S_i = 1$ and votes A otherwise, so vote probabilities reduce from (4) to $p(B|-1) = \int_0^1 \frac{1}{2}(1-q)\,dq = \frac{1}{4}$ and $p(B|1) = \int_0^1 \frac{1}{2}(1+q)\,dq = \frac{3}{4}$, and expectations are given by $\hat{z}_{0,2} = \frac{-\frac{1}{2}(1/4)^2 + \frac{1}{2}(3/4)^2}{\frac{1}{2}(1/4)^2 + \frac{1}{2}(3/4)^2} = 0.8$ and $\hat{z}_{1,1} = \frac{-\frac{1}{2}[2(1/4)(3/4)] + \frac{1}{2}[2(1/4)(3/4)]}{\frac{1}{2}[2(1/4)(3/4)] + \frac{1}{2}[2(1/4)(3/4)]} = 0$. Candidate B can win the election either by receiving both citizens' votes or by receiving one vote and winning the tie-breaking coin toss. Conditional only on winning the election, therefore, his expectation of Z is given by $\hat{z}_B = \frac{-\frac{1}{2}\left[(1/4)^2 + .5*2(1/4)(3/4)\right] + \frac{1}{2}\left[(3/4)^2 + .5*2(1/4)(3/4)\right]}{\frac{1}{2}\left[(1/4)^2 + .5*2(1/4)(3/4)\right] + \frac{1}{2}\left[(3/4)^2 + .5*2(1/4)(3/4)\right]} = 0.5$. Probabilities and expectations for candidate A are determined symmetrically. That $\sigma^* = \sigma_0$, $x_j^* = \hat{z}_j$, and $y_j^*(a,b) = \hat{z}_{a,b}$ together constitute an equilibrium follows from Theorem 3. ■

Proposition 2 illustrates the same basic behavior, but with three citizens instead of two (therefore avoiding the possibility of a tie). In equilibrium, candidates propose platform policies ± 0.57 but then implement ± 0.5 or ± 0.93.

Proposition 2 *Let F be a uniform distribution and let $N = 3$. If candidates are responsive then there exists an equilibrium $(x_A^*, x_B^*, \sigma^*, y_A^*, y_B^*)$ such that candidate platforms are $x_B^* = -x_A^* \approx 0.5652$, voting $\sigma^* = \sigma_0$ is sincere, and policy outcomes are as follows: $y_B^*(1,2) = -y_A^*(2,1) = 0.5$ and $y_B^*(0,3) = -y_A^*(3,0) \approx .9286$.*

Proof. As in Proposition 1, sincere voting produces vote probabilities

$$p(A|-1) = p(B|1) = \frac{3}{4}$$
$$p(A|1) = p(B|-1) = \frac{1}{4}.$$

Upon winning, therefore, candidate B's expectation of Z is either $\hat{z}_{1,2} = \frac{-\frac{1}{2}\left[3(3/4)(1/4)^2\right] + \frac{1}{2}\left[3(1/4)(3/4)^2\right]}{\frac{1}{2}\left[3(3/4)(1/4)^2\right] + \frac{1}{2}\left[3(1/4)(3/4)^2\right]} = 0.5$ or $\hat{z}_{0,3} = \frac{-\frac{1}{2}(1/4)^3 + \frac{1}{2}(3/4)^3}{\frac{1}{2}(1/4)^3 + \frac{1}{2}(3/4)^3} = \frac{26}{28} \approx .9286$. Conditional only on candidate B winning the election, the expectation of Z is given by $\hat{z}_B = \frac{-\frac{1}{2}\left[3(3/4)(1/4)^2 + (1/4)^3\right] + \frac{1}{2}\left[3(1/4)(3/4)^2 + (3/4)^3\right]}{\frac{1}{2}\left[3(3/4)(1/4)^2 + (1/4)^3\right] + \frac{1}{2}\left[3(1/4)(3/4)^2 + (3/4)^3\right]} = \frac{26}{46} \approx 0.5652$. Probabilities and expectations for candidate A are determined symmetrically. That $\sigma^* = \sigma_0$, $x_j^* = \hat{z}_j$, and $y_j^*(a,b) = \hat{z}_{a,b}$ together constitute an equilibrium follows from Theorem 3. ■

Propositions 1 and 2 illustrate basic equilibrium behavior for an electorate with responsive candidates. They also demonstrate how the announcement of an election outcome might alter a candidate's beliefs about the true state of the world. This behavior is consistent with popular assessment of actual candidate behavior in real-world elections. Observers have long noted an empirical tendency for winning candidates to moderate their political stances (relative to campaign platforms) after close-shave elections, and to move toward extreme policy options after landslide victories, interpreting such as "mandates" from voters. In this setting, the notion of a mandate can be interpreted quite literally: large margins of victory communicate strong evidence in favor of extreme policy moves.

4 Abstention

The analysis of Section 3 assumes that every citizen must vote. Most democracies allow abstention, however, and abstention rates tend to be fairly high. In this section, citizens are allowed to vote for either candidate or to abstain from voting. With this modification, Lemma 4 repeats the voting stage-game analysis of Section 3.1, treating policy outcomes $y_A < y_B$ as exogenous. As before, citizens who strongly believe the state to be high or low will have strong preferences for y_B or y_A, respectively. Now, however, a *belief threshold strategy* σ_{T_1,T_2}, can be redefined using two thresholds instead of one, to allow for the possibility that citizens who are almost indifferent between the two policy outcomes prefer to abstain altogether from voting.

Definition 2 *The symmetric voting strategy profile* σ_{T_1,T_2} *is a belief threshold strategy, with belief thresholds* $T_1 \leq T_2$*, if*

$$\sigma_{T_1,T_2}(q,s) = \begin{cases} A \text{ if } qs \leq T_1 \\ 0 \text{ if } T_1 < qs < T_2 \\ B \text{ if } qs \geq T_2 \end{cases}.$$

σ_{T_1,T_2} *is a symmetric belief threshold strategy if* $T_2 = -T_1$*.*

Since voting is costless, and since each citizen's private signal induces a strict preference ordering over the two policy outcomes, it may seem unlikely that allowing abstention will alter equilibrium behavior. However, as Lemma 1 states, $T_1 < T_2$ in equilibrium, implying positive abstention. The logic behind this result is the swing voter's curse (Feddersen and Pesendorfer, 1996): because citizens' opinions are correlated with the truth, and voting is informative, the candidate with the superior policy is more likely to win the election by one vote than to lose by one vote. A vote for the inferior candidate is therefore more likely to be pivotal than is a vote for the superior candidate, so a citizen who is indifferent—or, by continuity, almost indifferent—between voting for the two candidates strictly prefers to abstain.

Lemma 4 (Swing voter's curse) *If policy outcomes $y_A < y_B$ are exogenous and abstention is allowed then the best response to any voting strategy σ is a belief threshold strategy σ_{T_1,T_2}. Such a strategy is a symmetric Bayesian (partial) equilibrium only if $T_1 < T_2$. Furthermore, there exist belief thresholds $T_1^* < T_2^*$ such that $\sigma_{T_1^*,T_2^*}$ is a symmetric Bayesian (partial) equilibrium. If $y_B = -y_A$ then there exists a threshold $T^* > 0$ such that σ_{-T^*,T^*} is a symmetric Bayesian (partial) equilibrium.*

Proof. See Appendix. ■

The last part of Lemma 4 points out that if policy outcomes are symmetric around the zero policy then equilibrium voting behavior may exhibit the same symmetry. In that case, whether a citizen votes or not depends only on her whether her information quality Q_i exceeds the threshold T^*. Like Lemma 1, Lemma 4 characterizes equilibrium responses to exogenous policy outcomes. Proposition 3 now treats the case in which policy outcomes are determined by campaign platforms, which are chosen by policy-motivated (i.e. citizen) candidates before the election. Like Theorem 2, Proposition 3 predicts that campaign platforms will diverge in equilibrium. Partial equilibrium voting behavior is given by Lemma 4, implying positive abstention in equilibrium. As in Lemma 4, equilibrium voting behavior and candidate platforms may be symmetric around the zero policy.

Proposition 3 *If policy platform commitments are binding, candidates are policy-motivated and abstention is allowed, and if there is a unique equilibrium voting response $\sigma^*(x_A, x_B)$ in the stage-game for every pair $x_A \neq x_B$ of candidate platforms, then there exists a threshold T^* such that $(\sigma_{-T^*,T^*}, x_A^*, x_B^*)$ is a perfect Bayesian equilibrium, where $x_B^* = -x_A^* = \hat{z}_B(\sigma_{-T^*,T^*})$.*

Proof. See Appendix. ■

Like Theorem 2, Proposition 3 predicts that candidate platforms will diverge in equilibrium, as candidates learn different information upon winning the election. Other than the fact of winning the election, however, a candidate can infer no information about vote totals because he must choose his platform policy before voting takes place. Once vote totals are announced, a candidate may therefore wish to implement some policy other than his campaign platform. This possibility is allowed in Theorem 4.

By allowing the winning candidate to implement any policy of his choice, Theorem 4 resembles Theorem 3. Like that theorem, Theorem 4 predicts that candidates respond identically to vote totals. This implies, however, that a citizen do not actually care who wins the election, and therefore no longer restricts her attention to the rare case in which her vote changes the identity of the election winner. The logic of the swing voter's curse, therefore, no longer applies.

Since voting is costless and pivotal votes are no longer of concern, it may seem unlikely that citizens will abstain from voting in equilibrium—even a minimally informed citizen's signal, after all, is more likely to be Z than $-Z$. To the contrary, however, Theorem 4 states that equilibrium belief thresholds diverge, implying positive abstention. The logic of this result is as follows: in equilibrium, the winning candidate interprets vote totals as indicative of voters' private information. Each A vote, therefore, lowers his expectation of Z, while each B vote raises his expectation of Z. Since individual signals are correlated with the truth and voting is informative, the winning candidate's policy expectations will likely be pushed in the true direction of Z. An additional vote in the proper direction, therefore, has less marginal impact than an additional vote in the wrong direction. This makes a perfectly uninformed citizen—and, by continuity, a poorly informed citizen—prefer to abstain. Perhaps more intuitively, a perfectly uninformed citizen prefers, given her fellow-citizens' vote totals a and b, to implement the policy $E(Z|a,b;\sigma^*) \equiv \hat{z}_{a,b}$. Since (by Lemma 2) this is precisely the choice made by the winning candidate, the uninformed citizen achieves her optimum by abstaining.[13]

Theorem 4 (Signaling voter's curse) *If candidates are policy-motivated and responsive to vote totals and voter abstention is allowed then the best response to any belief threshold strategy* σ_{T_1,T_2} *is another belief threshold strategy* $\sigma_{T_1^*(\sigma_{T_1,T_2}),T_2^*(\sigma_{T_1,T_2})}$*, where* $T_1^*(\sigma_{T_1,T_2}) < T_2^*(\sigma_{T_1,T_2})$*; furthermore, a threshold* T^* *exists such that* $\left(\sigma_{-T^*,T^*}, x_A, x_B, y^*_{a,b}\right)$ *is a perfect Bayesian equilibrium for any candidate platform pair* (x_A, x_B)*, where* $y^*_{a,b}$ *is the candidate response function described in Lemma 2.*

Proof. See Appendix. ■

Proposition 4 provides a simplistic, but illustrative, example of the equilibrium identified in Theorem 4. Information quality is distributed uniformly, and there are only two citizens. In this simple example, because of the signaling voter's curse, equilibrium voter turnout is only 42%! If campaign platforms reflect candidates' expectations of the state conditional only on winning, as in Section 3.3, then they will diverge (to ± 0.5242), as in Theorem 2. Once election results are known, policy will then adjust, as in Theorem 3. A tie will cause either candidate to moderate his policy choice (to 0), while a slight majority will push his policy choice in the opposite direction (to ± 0.7907), and a large majority will make the policy even more extreme (± 0.9730). More often than not (i.e. with probability 0.63), then, the ultimate policy outcome that is implemented is more extreme than either candidate's campaign platform policy.

[13] By voting, a citizen shifts the policy response to either $\hat{z}_{a+1,b}$ or $\hat{z}_{a,b+1}$.

Proposition 4 *Let $N = 2$ be known and let F be uniform on $[0, 1]$. If candidates are responsive then $(x_A^*, x_B^*, \sigma_{-T^*,T^*}, y_A^*, y_B^*)$ is a perfect Bayesian equilibrium, where $x_B^* = -x_A^* \approx 0.5242$, $y_B^*(0,0) = y_A^*(0,0) = 0$, $y_B^*(0,1) = -y_A^*(1,0) \approx 0.7907$, and $y_B^*(0,2) = -y_A^*(2,0) \approx 0.9730$, and where σ_{-T^*,T^*} is a symmetric belief threshold strategy with $T^* \approx 0.5814$. In this equilibrium, expected turnout is approximately 42%.*

Proof. See Appendix. ■

Proposition 5 next demonstrates that the logic of voter abstention applies even to an "electorate" comprised of only a single citizen (i.e. $N = 1$). As the sole voter, this citizen has complete control over the voting outcome. Nevertheless, she abstains in equilibrium with 0.33 probability. Before the election, candidates adopt platforms at ± 0.44; if she abstains, they implement the 0 policy instead; if she votes, they respond by implementing ± 0.67.

Proposition 5 *Let $N = 1$ be known and let F be uniform on $[0, 1]$. If candidates are responsive then there is a unique belief threshold strategy σ_{-T^*,T^*} such that $(x_A^*, x_B^*, \sigma_{-T^*,T^*}, y_A^*, y_B^*)$ is a perfect symmetric Bayesian equilibrium. In this equilibrium, $T^* = \frac{1}{3}$, $x_B^* = -x_A^* \approx 0.44$, $y_j^*(0,0) = 0$, and $y_j^*(0,1) = -y_j^*(1,0) \approx 0.67$, and expected turnout is approximately 67%.*

Proof. See Appendix. ■

This exaggerated example elucidates the logic behind the signaling voter's curse: because the winning candidate does not know the citizen's type, he interprets her vote as though her information quality is average. When it is below average, therefore, she anticipates that the candidate will overreact to her vote, implementing a policy more extreme than her information merits. By abstaining, she achieves a more moderate policy outcome.

The results of this section exhibit the same behavioral prediction: whether policy outcomes are exogenous (as in Lemma 4), determined by binding platform commitments (as in Proposition 3), or chosen ex post by the winning candidate (as in Theorem 4) informed citizens vote in equilibrium and uninformed citizens abstain. This prediction is consistent with the empirical evidence, reviewed in Section **??**, that voter turnout is correlated with information variables such as education, and age. As Feddersen and Pesendorfer (1996) point out, it also provides an explanation for voter abstention when voting is costless, such as roll-off. Since equilibrium belief thresholds depend implicitly on the underlying distribution of information, these results are also consistent with evidence in McMurray (2010b), that the empirical importance of information is relative, rather than absolute. Turnout is also highest among those with extreme policy preferences, consistent with evidence from Palfrey and Poole (1987).

5 Welfare

The Condorcet (1785) jury theorem states that, as an electorate grows large, the majority decision identifies the better of two alternatives with probability approaching one. As originally stated, this result assumed sincere voting with no abstention; in this model, voting is instead strategic. Nevertheless, the same result is obtained in Theorem 5, for each of the above specifications of the model.

Theorem 5 (Jury theorem) *Let $\{\mu_k\}_{k=1}^{\infty}$ be a sequence of population size parameters with $\lim_{k\to\infty}\mu_k = \infty$. Then the following are true:*

1. If y_A and y_B are exogenous then

(a) for any k, the symmetric voting strategy $\sigma^*_{\mu_k}$ that maximizes expected utility is a symmetric Bayesian (partial) equilibrium, and

(b) $p\lim_{k\to\infty} Y^*_{\mu_k} = \begin{cases} y_A & \text{if } Z = -1 \\ y_B & \text{if } Z = 1 \end{cases}$.

2. If policy platform commitments are binding and candidates are policy-motivated then

(a) for any k, the voting strategy $\sigma^*_{\mu_k}$ and platform pair $\left(x^*_{A\mu_k}, x^*_{B\mu_k}\right)$ that together maximize expected utility constitute a perfect Bayesian equilibrium, and

(b) $p\lim_{k\to\infty} Y^*_{\mu_k} = Z$.

3. If policy-motivated candidates choose policy responses y_A, y_B to vote totals (N_A, N_B) then

(a) for any k, the voting strategy $\sigma^*_{\mu_k}$ and policy responses $\left(y^*_{A\mu_k}, y^*_{B\mu_k}\right)$ that together maximize expected utility, together with any platform pair (x_A, x_B), constitute a perfect Bayesian equilibrium, and

(b) $p\lim_{k\to\infty} Y^*_{\mu_k} = Z$.

4. Claims 1 through 3 remain true if voter abstention is allowed.

Proof. See Appendix. ■

Theorem 5 has a number of implications for institutional design. For example, the lack of credibility underlying campaign promises is often bemoaned for introducing uncertainty about candidates' future behavior. Part 3 of Theorem 5 implies, however, that responsive candidates will utilize information gleaned from electoral results to implement the socially optimal policy; policies that bind candidates to campaign platform policies will therefore only inhibit welfare.

A related implication of Theorem 5 is that, when campaign platform commitments are binding, welfare is higher when candidates are policy-motivated than when they are office-motivated. Specifically, Lemma 1 implies identical voter behavior regardless of candidate motivation; given this behavior, Part 2 of Theorem 5 implies that the policy platforms adopted by policy-motivated candidates (which differ in equilibrium from those adopted by office-motivated candidates, by Theorems 1 and 2) are socially optimal. This result may have relevance for determining optimal financial rewards for office holders.

The result that large electorates do well at selecting good policies might motivate popular "get out the vote" efforts to encourage voter participation. Some nations have gone as far as to make voting mandatory, levying fines on non-voters. Similar policies have been recommended for the United States (e.g. Lijphart, 1997). In an environment such as this, such policies might seem particularly useful, since every citizen possesses valuable private information; by allowing abstention, a voluntary election fails to utilize this information. On the other hand, it is also sometimes argued that voters who lack information should be prohibited from voting.

An implication of Theorem 5, however, is that equilibrium voter abstention is socially optimal. Specifically, Part 4 of Theorem 5 states that, allowing voter abstention in each version of the model, the optimal combination of voter and candidate behavior constitutes a Bayesian equilibrium. As discussed in Section 4, any such equilibrium involves voter abstention. One way to understand this result is that, as McMurray (2010b) points out, an optimal election mechanism would place greater weight on the votes of citizens with high-quality information than on those of poorly informed citizens; allowing abstention is a crude way of accomplishing this. With responsive candidates, an alternative intuition comes from viewing voters and candidates as senders and receivers in a "cheap talk" game (a la Crawford and Sobel, 1982). Within that framework, allowing abstention amounts to expanding the size of the message space from two messages to three. This interpretation is immediately evident in Proposition 5, where the single citizen divides her type space into three equal segments, voting according to $\sigma_{-\frac{1}{3},\frac{1}{3}}$ in equilibrium.

6 Multiple Candidates

In this section, the set $\{A, B, C, D\}$ of candidates is expanded from two to four. As in Section 3.4, candidates are responsive; as in Section 4, abstention is allowed. Definition 3 redefines the concept of a belief threshold strategy for this setting, using four belief thresholds instead of two. Under such a strategy, citizens with strong private opinions vote for candidates A or D, citizens with moderate opinions vote for candidates B or C, and citizens with only weak opinions abstain.

Definition 3 *$\sigma_{T_{AB},T_{A0},T_{0B},T_{CD}}$ is a belief threshold strategy with abstention if*

$$\sigma_{T_{AB},T_{B0},T_{0C},T_{CD}}(q,s) = \begin{cases} A \text{ if } qs \in (-1, T_{AB}) \\ B \text{ if } qs \in (T_{AB}, T_{B0}) \\ 0 \text{ if } qs \in (T_{B0}, T_{0C}) \\ C \text{ if } qs \in (T_{0C}, T_{CD}) \\ D \text{ if } qs \in (T_{CD}, 1) \end{cases} .$$

If $T_{0B} = -T_{A0} \equiv T_1$ and $T_{CD} = -T_{AB} \equiv T_2$ then $\sigma_{-T_2,-T_1,T_1,T_2}$ is a symmetric belief threshold strategy.

Theorem 6 now states the existence of a perfect Bayesian equilibrium, characterized by belief threshold voting. As prescribed by Lemma 2, the winning candidate implements his expectation of the state, conditional on vote totals; as in Lemma 3, the effect of a single vote is to push the policy outcome in one direction or another. Because more extreme citizen types vote for candidates A and D than B and C, votes for these two candidates have a greater impact on the winning candidate's beliefs. Thus, voting for an extreme candidate pushes policy by more than voting for a moderate candidate.

Theorem 6 *If candidates A, B, C, and D are responsive and abstention is allowed then there exist quality thresholds $0 < T_1^* < T_2^* < 1$ such that $\left(x^*, \sigma_{-T_2^*,-T_1^*,T_1^*,T_2^*}, y^*\right)$ is a perfect Bayesian equilibrium if $y^* = \left(y_j^*\right)_{j\in\{A,B,C,D\}}$ is the vector of policy response functions defined by $y_j^*(a,b,c,d) = \hat{z}_{a,b,c,d} \equiv E\left(Z|N_A = a, N_B = b, N_C = c, N_D = d; \sigma_{-T_2^*,-T_1^*,T_1^*,T_2^*}\right)$ for any voting outcome $(a,b,c,d) \in \mathbb{Z}_+^4$ and $x^* = \left(x_j^*\right)_{j\in\{A,B,C,D\}}$ is any vector of candidate platforms.*

Proof. See Appendix. ■

Similar to Proposition 5 in Section 3.4, Proposition 6 illustrates the equilibrium identified in Theorem 6 for the electorate comprised of only a single citizen. In that equilibrium, all candidates expect a positive vote share, and the citizen also abstains with positive probability (expected turnout is 80%).

Proposition 6 *Let $N = 1$ be known, and let F be uniform on $[0,1]$. If candidates A, B, C, and D are responsive then (x^*, σ^*, y^*) is a perfect Bayesian equilibrium for the symmetric belief threshold voting strategy $\sigma^* = \sigma_{-.6,-.2,.2,.6}$, the vector $y^* = \left(y_j^*\right)_{j\in\{A,B,C,D\}}$ of policy responses defined by $y_j^*(0,0,0,1) = -y_j^*(1,0,0,0) = 0.8$, $y_j^*(0,0,1,0) = -y_j^*(0,1,0,0) = 0.4$, $y_j^*(0,0,0,0) = 0$, and any vector $x^* = \left(x_j^*\right)_{j\in\{A,B,C,D\}}$ of candidate platforms. In this equilibrium, expected voter turnout is 80%.*

Proof. See Appendix. ■

Voting behavior in Proposition 6 is similar to that in Proposition 5, in that the type space is divided in equilibrium into equal segments—this time five instead of three. Applied here, the logic of Theorem 5 suggests that this addition of candidates improves welfare.

One noteworthy comparative static result from this section is that the intensity of a citizen's political preference is positively related to her information quality. That is, citizens with poor information quality tend not to support extreme candidates. This is despite the modeling assumption that the ideal policy is commonly known to lie at one of the extremes of the policy spectrum: if information were perfect, every citizen would be an extremist. In essence, the same "signaling voter's curse" that in Section 4 caused citizens with the poorest information to abstain altogether from voting causes moderately informed citizens to vote for a moderate rather than an extreme candidate.

7 Conclusion

In generalizing the Condorcet (1785) environment, this paper begins to bridge the gap between models of conflicting interests and conflicting information. For example, asymmetric information provides a novel explanation for single-peaked preferences; consistent with empirical evidence from Palfrey and Poole (1987), citizens with the most information favor the most extreme policies, and are the most likely to vote. If candidates commit to implement campaign platform policies, office motivation yields a standard median voter theorem, while policy motivation yields an equilibrium with divergent platforms. If pre-election policy commitments are nonbinding, the candidate who wins office can utilize information reflected in vote totals when implementing policy. This gives rise to the popular notion of electoral mandates—by which large vote margins prompt more extreme policy movements. Similarly, narrow victories lead candidates to moderate their policy stances. Votes for extreme candidates impact the eventual policy outcome more strongly than votes for moderate candidates, providing a possible rationale for candidates who are unlikely to win office but nevertheless campaign, and do receive some votes. The possibility of extreme policy outcomes is desirable here, underscoring the importance of reevaluating standard models, in which extreme policies represent a failure to compromise between competing interests.[14]

When platform commitments are binding, the swing voter's curse dissuades uninformed citizens from voting, even though voting is costless—an unsurprising result, given the similarity between this model and the models of McMurray (2010b) and Feddersen and Pesendorfer

[14] Even relaxing the assumption that the ideal policy lies at one of the two extremes, movements away from the center would likely reflect welfare improvements.

(1996). The logic of the swing voter's curse, however, stems from the observation that a citizen's vote only influences her payoff when it changes the election outcome, so that a citizen should condition her behavior on this rare but informative event. With responsive candidates, a citizen's vote always influences the policy outcome; in fact, the identity of the election winner is irrelevant to her decision. The result that uninformed citizens continue to abstain, therefore, may be surprising. Abstention is commonly viewed as a threat to democracy, but improves welfare here—contrary to the intuition that nonvoters fail to contribute socially valuable information—by enriching voters' message space. The comparative static result that a citizen's information quality (relative to others in her electorate) makes her more likely to vote is also important for explaining phenomena such roll-off, as well as the empirical correlations between information and voting discussed by McMurray (2010a). As noted above, one justification for the common-value model is its ability to explain results such as these, which are difficult to accommodate in a standard framework.

A natural extension of this model would be to endogenize entry. In that context, a multidimensional policy space would be particularly interesting: just as the vote shares of two candidates identify the optimal policy in one dimension, the vote shares of $k+1$ candidates (with linearly independent platforms) would likely identify the optimal policy in k dimensions. If so, candidates may continue to enter a costly campaign, in efforts to draw attention to neglected policy dimensions and enrich the menu of messages that voters can send. An important component for such an analysis might be some sort of cost for deviating from policy platforms: when platforms are completely non-credible, candidates are perfectly responsive, implying that the identity of the winning candidate actually doesn't matter, and entry always has benefit; if platform commitments instead limited a candidate's responsiveness, then the identity of the winning candidate would matter, and an entrant would risk splitting the vote share of the most attractive candidate, upsetting the election in favor of an inferior opponent.

It is worth noting that nothing about this signaling mechanism of elections need limit its interpretation to the candidates of a given election; electoral results could just as easily send a "message" to other elected officials. For example, a recent victory by Republican Scott Brown in a 2010 special Massachusetts election to fill the senate seat vacated by the passing of Democratic senator Ted Kennedy, who had held the seat for decades, was popularly interpreted as a call for moderation in health care reform and other policies under consideration by a Democratic congress and Democratic president Barack Obama.[15] For

[15] Cooper, Michael (2010, January 10). G.O.P. Victory Stuns Democrats. *The New York Times*, http://www.nytimes.com/2010/01/21/us/politics/21elect.html?scp=9&sq=Scott%20Brown&st=cse (accessed 6 February 2010).

that matter, similar motivations may influence other political activities, such as writing letters to public officials and participating in protests, or even public opinion surveys. This model could also be reinterpreted an informational model of proportional rule elections, in which the vote share that one party receives determines its fraction of seats in parliament.

A Appendix: Proofs

Lemma 1 *If y_A, y_B are fixed exogenously then the best response to any symmetric voting strategy σ is a belief threshold strategy σ_T. Furthermore, there exists a belief threshold T^* such that σ_{T^*} is a symmetric Bayesian (partial) equilibrium. σ_0 is such an equilibrium if and only if policy outcomes $y_B = -y_A$ are symmetric around zero.*

Proof. For a citizen of information quality $q \in [0,1]$ and signal $s \in \{-1,1\}$, the difference $\Delta_{AB}(q,s;\sigma)$ in expected utility from voting for candidate B instead of candidate A (given opponent voting strategy σ) is given by the following:

$$\begin{aligned}
\Delta_{AB}(q,s;\sigma) &= \sum_{z=1,-1} [u(y_B,z) - u(y_A,z)] [\Pr(piv_B|z) - \Pr(piv_A|z)] \frac{1}{2}(1+zqs) \\
&= 2(y_B - y_A)(1-\bar{y})\left(P_1 - \tilde{P}_1\right)\frac{1}{2}(1+qs) \\
&\quad +2(y_B - y_A)(-1-\bar{y})\left(\tilde{P}_{-1} - P_{-1}\right)\frac{1}{2}(1-qs) \\
&= (y_B - y_A)\left\{\begin{array}{c} \left[\begin{array}{c} qs\bar{y}\left(\tilde{P}_1 + \tilde{P}_{-1} - P_1 - P_{-1}\right) \\ +\left(\tilde{P}_{-1} + P_1 - \tilde{P}_1 - P_{-1}\right) \end{array}\right] \\ -\left[\begin{array}{c} \bar{y}\left(\tilde{P}_{-1} + P_1 - \tilde{P}_1 - P_{-1}\right) \\ +\left(\tilde{P}_{-1} + P_{-1} - P_1 - \tilde{P}_1\right) \end{array}\right] \end{array}\right\}.
\end{aligned}$$

When $y_A \neq y_B$, this benefit is positive if and only if qs exceeds a belief threshold $T_{AB}(\sigma)$, defined by (9) below.

$$T_{AB}(\sigma) = \frac{\bar{y}\left(\tilde{P}_1 + P_1 + \tilde{P}_{-1} + P_{-1}\right) + \left(\tilde{P}_1 + P_1 - \tilde{P}_{-1} - P_{-1}\right)}{\bar{y}\left(\tilde{P}_1 + P_1 - \tilde{P}_{-1} - P_{-1}\right) + \left(\tilde{P}_1 + P_1 + \tilde{P}_{-1} + P_{-1}\right)}. \tag{9}$$

Thus, the best response to any voting strategy σ is a belief threshold strategy $\sigma_{T_{AB}(\sigma)}$. In particular, the best response to a belief threshold strategy σ_T is another belief threshold strategy $\sigma_{T_{AB}(\sigma)}$. The best-response threshold function $T_{AB}(\sigma_T)$ is thus a continuous function from the compact set $[-1,1]$ into itself, so a fixed point $T^* = T_{AB}(\sigma_{T^*})$ exists by Brouwer's theorem, and the corresponding belief threshold strategy σ_{T^*} is a symmetric Bayesian (partial) equilibrium. Citizens have no incentive to deviate from σ_{T^*} when $y_A = y_B$ because in that case voting has no influence on policy.

A threshold at $T = 0$ is special because voting probabilities are symmetric in that case: $p(A|1;\sigma_0) = p(B|-1;\sigma_0)$, $p(A|-1;\sigma_0) = p(B|1;\sigma_0)$ and $\psi(a,b|z) = \psi(b,a|-z)$. This implies symmetric pivot probabilities $\pi^0_{\sigma_0}(z) = \pi^0_{\sigma_0}(-z)$, $\pi^A_{\sigma_0}(z) = \pi^B_{\sigma_0}(-z)$, $P_1 = P_{-1}$, and $\tilde{P}_1 = \tilde{P}_{-1}$, in turn reducing (9) to $T_{AB}(\sigma_0) = \bar{y}$. Thus $T^* = 0$ is a fixed point of (9) if and only if $\bar{y} = 0$ or, equivalently, if policy outcomes $y_B = -y_A$ are symmetric around zero. ■

Theorem 1 (Median Voter Theorem) *If policy platform commitments are binding and candidates are office-motivated, and if there is a unique equilibrium threshold $T^*(x_A, x_B)$ in the stage-game associated with every pair $x_A \neq x_B$ of candidate platforms, then (σ^*, x_A^*, x_B^*) is a perfect symmetric Bayesian equilibrium if and only if σ^* is the belief threshold strategy σ_{T^*} and $x_A^* = x_B^* = 0$.*

Proof. The voting stage-game equilibrium condition that σ^* be a belief threshold strategy is stated in Lemma 1. Rewriting the best response threshold (9) in terms of the probabilities $A_1 = \tilde{P}_1 + P_1$ and $A_2 = \tilde{P}_{-1} + P_{-1}$ of being pivotal in states 1 and -1 yields the following:

$$T_{AB}(\sigma) = \frac{\bar{y}(A_1 + A_2) + (A_1 - A_2)}{\bar{y}(A_1 - A_2) + (A_1 + A_2)}.$$

This best response threshold is increasing in $\bar{y} = \frac{y_A + y_B}{2} = \frac{x_A + x_B}{2}$, and therefore in x_A and in x_B:

$$\begin{aligned} & \frac{d}{d\bar{y}} T_{AB}(\sigma) \\ = & \frac{(A_1 + A_2)[\bar{y}(A_1 - A_2) + (A_1 + A_2)] - [\bar{y}(A_1 + A_2) + (A_1 - A_2)](A_1 - A_2)}{[\bar{y}(A_1 - A_2) + (A_1 + A_2)]^2} \\ = & \frac{4A_1A_2}{[\bar{y}(A_1 - A_2) + (A_1 + A_2)]^2} > 0. \end{aligned}$$

If $T^*(x_A, x_B)$ is unique for every pair $x_A \neq x_B$ of candidate platforms then this implies that T^* also increases in x_A and x_B.[16]

Citizens are more likely to vote candidate B than for candidate A if and only if $T^* < 0$. The two candidates expect equal vote shares if and only if $T^* = 0$. As Lemma 1 states, this occurs in equilibrium only if $\frac{y_A + y_B}{2} = \frac{x_A + x_B}{2} = 0$ or, in other words, if $x_B = -x_A$. The

[16] If there are mutliple partial equilibrium thresholds $T^*(x_A, x_B)$ in response to candidate platforms $x_A \neq x_B$ then the result that $T(\sigma)$ increases with $\bar{y} = \frac{y_A + y_B}{2} = \frac{x_A + x_B}{2}$ implies that both the minimum and the maximum partial equilibrium thresholds increase with x_A and x_B.

result that T^* increases with $\bar{x}$ therefore implies that $T^* > 0$ whenever $x_B > -x_A$ and that $T^* < 0$ whenever $x_B < -x_A$. In other words, the candidate whose platform is closer to zero receives a larger (expected) share of votes. Thus no pair (x_A, x_B) of candidate platforms other than $(0, 0)$ can be sustained in equilibrium. ■

Theorem 2 (Policy Divergence) *If policy platform commitments are binding and candidates are policy-motivated, and if there is a unique equilibrium threshold* $T^*(x_A, x_B)$ *in the stage-game for every pair* $x_A \neq x_B$ *of candidate platforms, then* (σ^*, x_A^*, x_B^*) *is a perfect symmetric Bayesian equilibrium only if* σ^* *is the belief threshold strategy* $\sigma_{T^*(x_A^*, x_B^*)}$ *and* $x_A^* = \hat{z}_A(\sigma^*, x_A^*, x_B^*) < \hat{z}_B(\sigma^*, x_A^*, x_B^*) = x_B^*$*, where* $\hat{z}_j(\sigma, x_A, x_B) \equiv E(Z|W = j; \sigma, x_A, x_B)$*. Furthermore, such an equilibrium exists, with* $T^* = 0$*.*

Proof. McLennan (1998) shows, for common interest games, that the strategy that maximizes the common objective function is an equilibrium—since no player can profit by deviating—and that the symmetric strategy that maximizes the common objective function is a symmetric equilibrium. In the context of this model, the latter of these two results implies that the optimal voting response $\sigma^*(x_A, x_B)$ to candidate platforms x_A and x_B is an equilibrium in the voting stage-game, and therefore (by Lemma 1) can be characterized by a belief threshold T^*. If there is a unique equilibrium strategy in each voting stage-game, therefore, then that strategy is an optimal response to x_A and x_B (i.e. maximizes $Eu(Z, Y)$).

Policy-motivated candidates, like voters, seek to maximize $Eu(Z, Y)$. Taking optimal voter responses as given, McLennan's (1998) first result therefore implies that the platform pair (x_A^*, x_B^*) that maximizes $Eu(Z, Y)$ is an equilibrium in a reduced game between the two candidates. The strategy combination (σ^*, x_A^*, x_B^*) that maximizes $Eu(Z, Y)$ generally, then, is a perfect Bayesian equilibrium in the original game. Such an optimal strategy combination exists by the Weierstrass theorem, since $Eu(Z, Y; \sigma, x_A, x_B)$ is continuous in (σ, x_A, x_B) on the compact set $[-1, 1]^2 \times \Sigma$.

For any set Ω of information, expected utility

$$Eu(x, Z|\Omega) = \sum_{z=1,-1} -(x - z)^2 \Pr(z|\Omega)$$

is strictly concave in x, and is uniquely maximized at $x^* = E(Z|\Omega)$. If voting behavior were fixed as $\sigma(x_A, x_B)$ for an arbitrary pair (x_A, x_B) of candidate platforms, therefore, then candidate j could increase $Eu(Z, Y|W = j)$ by deviating to $x_j = \hat{z}_j(\sigma, x_A, x_B)$. Allowing voters next to replace their previous voting strategy with the stage-game equilibrium response to the new platform pair would only increase $Eu(Z, Y)$ further, by the above logic of optimal voting. By these two steps, therefore, candidates can improve upon any pair (x_A^*, x_B^*) of candidate platforms unless, for the stage-game equilibrium voting strategy $\sigma^* = \sigma_{T^*\left(x_A^*, x_B^*\right)}$, it is the case that $x_j^* = \hat{z}_j\left(\sigma^*, x_A^*, x_B^*\right)$. That $\hat{z}_A < \hat{z}_B$ follows because σ^* is a belief threshold strategy, so $W = A$ is more likely when $Z = -1$ and $W = B$ is more likely when $Z = 1$, because a high proportion of citizens receive negative and positive signals, respectively, in the two states. That σ_0 together with $\hat{z}_A\left(\sigma_0, \hat{z}_A, \hat{z}_B\right)$ and $\hat{z}_B\left(\sigma_0, \hat{z}_A, \hat{z}_B\right)$ constitute an equilibrium follows from the last part of Lemma 1, because $\hat{z}_B\left(\sigma_0, \hat{z}_A, \hat{z}_B\right) = -\hat{z}_A\left(\sigma_0, \hat{z}_A, \hat{z}_B\right)$. ■

Lemma 3 *If σ_T is a belief threshold strategy (with $-1 < T < 1$) then $\hat{z}_{a+1,b}\left(\sigma_T\right) < \hat{z}_{a,b}\left(\sigma_T\right) < \hat{z}_{a,b+1}\left(\sigma_T\right)$ for all $a, b \in \mathbb{Z}_+$.*

Proof. If $0 \leq T < 1$ then 4 simplifies as follows,

$$\begin{array}{ll} p_{\sigma_T}\left(A|-1\right) = F\left(T\right) + \int_T^1 \frac{1}{2}\left(1+q\right) dF\left(q\right) & p_{\sigma_T}\left(B|-1\right) = \int_T^1 \frac{1}{2}\left(1-q\right) dF\left(q\right) \\ p_{\sigma_T}\left(A|1\right) = F\left(T\right) + \int_T^1 \frac{1}{2}\left(1-q\right) dF\left(q\right) & p_{\sigma_T}\left(B|-1\right) = \int_T^1 \frac{1}{2}\left(1+q\right) dF\left(q\right), \end{array}$$

so $p_{\sigma_T}\left(A|-1\right) > p_{\sigma_T}\left(A|1\right)$ and $p_{\sigma_T}\left(B|1\right) > p_{\sigma_T}\left(B|-1\right)$. If $-1 < T < 0$ then 4 reduces similarly, yielding the same inequalities. From (5) it can be seen that $\psi_{\sigma_T}\left(a, b+1|z\right) = \frac{\mu p_{\sigma_T}(B|z)}{b+1}\psi_{\sigma_T}\left(a, b|z\right)$ and $\psi_{\sigma_T}\left(a+1, b|z\right) = \frac{\mu p_{\sigma_T}(A|z)}{b+1}\psi_{\sigma_T}\left(a, b|z\right)$. The expectation of Z given $N_A = a$ and $N_B = b+1$ is therefore

$$\begin{aligned} \hat{z}_{a,b+1}\left(\sigma_T\right) &= \frac{\psi_{\sigma_T}\left(a, b+1|Z=1\right) - \psi_{\sigma_T}\left(a, b+1|Z=-1\right)}{\psi_{\sigma_T}\left(a, b+1|Z=1\right) + \psi_{\sigma_T}\left(a, b+1|Z=-1\right)} \\ &= \frac{p_{\sigma_T}\left(B|Z=1\right)\psi_{\sigma_T}\left(a, b|Z=1\right) - p_{\sigma_T}\left(B|Z=-1\right)\psi_{\sigma_T}\left(a, b|Z=-1\right)}{p_{\sigma_T}\left(B|Z=1\right)\psi_{\sigma_T}\left(a, b|Z=1\right) + p_{\sigma_T}\left(B|Z=-1\right)\psi_{\sigma_T}\left(a, b|Z=-1\right)} \\ &> \frac{p_{\sigma_T}\left(B|Z=1\right)\psi_{\sigma_T}\left(a, b|Z=1\right) - p_{\sigma_T}\left(B|Z=1\right)\psi_{\sigma_T}\left(a, b|Z=-1\right)}{p_{\sigma_T}\left(B|Z=1\right)\psi_{\sigma_T}\left(a, b|Z=1\right) + p_{\sigma_T}\left(B|Z=1\right)\psi_{\sigma_T}\left(a, b|Z=-1\right)} \\ &= \hat{z}_{a,b}\left(\sigma_T\right). \end{aligned}$$

That $\hat{z}_{a+1,b}\left(\sigma\right) < \hat{z}_{a,b}\left(\sigma\right)$ follows from similar reasoning. ■

Theorem 3 (Signaling) *If candidates are responsive then the best voting response to a belief threshold voting strategy is another belief threshold strategy. Furthermore, if $T^* = 0$ and $y_j^*\left(a, b; \sigma\right) = \hat{z}_{a,b}\left(\sigma\right)$ is the policy choice described in Lemma 2 for $j = A, B$, then $\left(x_A^*, x_B^*, \sigma_{T^*}, y_A^*, y_B^*\right)$ is a perfect symmetric Bayesian equilibrium for any pair $x_A^*, x_B^* \in [-1, 1]$ of policy platforms.*

Proof. Given that a responsive candidate will implement $y_{a,b}^*\left(\sigma\right)$ in response to vote totals a and b and voting strategy σ, the benefit $\Delta_{AB}\left(\sigma\right)$ to an individual citizen of voting B instead of A is given simply by the following:

$$\begin{aligned}
\Delta_{AB} &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left[-\left(y^*_{a,b+1}-z\right)^2+\left(y^*_{a+1,b}-z\right)^2\right]\psi_\sigma\left(a,b|z\right)\frac{1}{2}\left(1+zqs\right) \\
&= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}[\left(y^*_{a,b+1}-y^*_{a+1,b}\right)\times \\
&\quad \left(z-\frac{y^*_{a+1,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma\left(a,b|z\right)\left(1+zqs\right). \qquad (10)
\end{aligned}$$

If σ is a belief threshold strategy then $y^*_{a,b+1}(\sigma)=\hat{z}_{a,b+1}(\sigma)>\hat{z}_{a+1,b}(\sigma)=y^*_{a+1,b}(\sigma)$, as pointed out by Lemma 3. Since $\hat{z}_{a,b+1}$ and $\hat{z}_{a+1,b}$ are bounded between -1 and 1, Δ_{AB} is strictly increasing in qs:

$$\begin{aligned}
\frac{d\Delta_{AB}}{d\left(qs\right)} &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(y^*_{a,b+1}-y^*_{a+1,b}\right)\left(z-\frac{y^*_{a+1,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma\left(a,b|z\right)z \\
&= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a+1,b}\right)\left(1-\frac{\hat{z}_{a+1,b}+\hat{z}_{a,b+1}}{2}\right)\psi_\sigma\left(a,b|Z=1\right) \\
&\quad -\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a+1,b}\right)\left(-1-\frac{\hat{z}_{a+1,b}+\hat{z}_{a,b+1}}{2}\right)\psi_\sigma\left(a,b|Z=-1\right) \\
&> 0.
\end{aligned}$$

Thus the benefit Δ_{AB} to voting B is positive if and only if qs exceeds the belief threshold $T^*(\sigma_T)$, so the best response to σ_T, given responsive candidates, is another belief threshold strategy $\sigma_{T^*(\sigma_T)}$.

If $T=0$ then voting is symmetric with respect to the two candidates (i.e. $p(A|z)=p(B|-z)$, $\psi(a,b|z)=\psi(b,a|z)$ for $z=-1,1$ and $a,b\in\mathbb{Z}_+$) and candidates' policy responses are likewise symmetric (i.e. $y^*_{a,b}(\sigma)=\hat{z}_{a,b}(\sigma)=-\hat{z}_{b,a}(\sigma)=-y^*_{b,a}(\sigma)$ for $a,b\in\mathbb{Z}_+$), so Δ_{AB} reduces from (10) to the following:

$$\begin{aligned}
&\Delta_{AB} \\
= &\sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=a+1}^{\infty}\left(y^*_{a,b+1}-y^*_{a+1,b}\right)\left(z-\frac{y^*_{a+1,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma\left(a,b|z\right)\left(1+zqs\right) \\
&+\sum_{z=1,-1}\sum_{b=0}^{\infty}\sum_{a=b+1}^{\infty}\left(-y^*_{b+1,a}+y^*_{b,a+1}\right)\left(z-\frac{-y^*_{b,a+1}-y^*_{b+1,a}}{2}\right)\psi_\sigma\left(b,a|-z\right)\left(1+zqs\right) \\
= &\sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=a+1}^{\infty}\left(y^*_{a,b+1}-y^*_{a+1,b}\right)\left(z-\frac{y^*_{a+1,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma\left(a,b|z\right)\left(1+zqs\right) \\
&+\sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=a+1}^{\infty}\left(-y^*_{a+1,b}+y^*_{a,b+1}\right)\left(z-\frac{-y^*_{a,b+1}-y^*_{a+1,b}}{2}\right)\psi_\sigma\left(a,b|-z\right)\left(1+zqs\right) \\
= &\sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=a+1}^{\infty}\left(y^*_{a,b+1}-y^*_{a+1,b}\right)\left(z-\frac{y^*_{a+1,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma\left(a,b|z\right)\left(1+zqs\right) \\
&+\sum_{z=-1,1}\sum_{a=0}^{\infty}\sum_{b=a+1}^{\infty}\left(y^*_{a,b+1}-y^*_{a+1,b}\right)\left(-z+\frac{y^*_{a+1,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma\left(a,b|z\right)\left(1-zqs\right).
\end{aligned}$$

This expression is equal to zero if $qs = 0$, which is the case for a citizen right at the threshold $T = 0$. Thus $\sigma^* = \sigma_0$, together with the policy functions $y^*_{a,b}$ defined by Lemma 2, constitute a perfect Bayesian equilibrium. ■

Lemma 4 (Swing Voter's Curse) *If policy outcomes $y_A < y_B$ are exogenous and abstention is allowed then the best response to any voting strategy σ is a belief threshold strategy σ_{T_1,T_2}. Such a strategy is a symmetric Bayesian (partial) equilibrium only if $T_1 < T_2$. Furthermore, there exist belief thresholds $T_1^* < T_2^*$ such that $\sigma_{T_1^*,T_2^*}$ is a symmetric Bayesian (partial) equilibrium. If $y_B = -y_A$ then there exists a threshold $T^* > 0$ such that σ_{-T^*,T^*} is a symmetric Bayesian (partial) equilibrium.*

Proof. For a citizen of information quality $q \in [0,1]$ and signal $s \in \{-1,1\}$, the difference $\Delta_{0B}(q,s;\sigma)$ in expected utility between voting for candidate B and abstaining in response to policy outcomes y_A, y_B and voting strategy σ is given by the following, where $\bar{y} = \frac{y_A+y_B}{2}$

denotes the midpoint between the two policy outcomes, as before:

$$
\begin{aligned}
\Delta_{0B}(q,s;\sigma) &= \sum_{z=1,-1}[u(y_B,z)-u(y_A,z)]\Pr(piv_B|z)\frac{1}{2}(1+zqs) \\
&= 2(y_B-y_A)(1-\bar{y})P_1\frac{1}{2}(1+qs) \\
&\quad +2(y_B-y_A)(-1-\bar{y})\tilde{P}_{-1}\frac{1}{2}(1-qs) \\
&= (y_B-y_A)\{\left[qs\bar{y}\left(\tilde{P}_{-1}-P_1\right)+\left(\tilde{P}_{-1}+P_1\right)\right] \\
&\quad -\left[\bar{y}\left(\tilde{P}_{-1}+P_1\right)+\left(\tilde{P}_{-1}-P_1\right)\right]\}.
\end{aligned}
$$

A citizen prefers voting B to abstaining if and only if this benefit is positive. When $y_A \neq y_B$, Δ_{0B} is positive if and only if qs exceeds a belief threshold $T_{0B}(\sigma)$, defined by (12) below. Similarly, a citizen prefers voting A to abstaining if and only if qs is below the threshold $T_{A0}(\sigma)$, as defined in (11). Also, just as in Lemma 1, a citizen again prefers voting B to voting A if and only if qs exceeds $T_{AB}(\sigma)$, defined above in (9) and rewritten here as (13).

$$T_{A0}(\sigma) = \frac{\bar{y}\left(\tilde{P}_1+P_{-1}\right)-\left(\tilde{P}_1-P_{-1}\right)}{-\bar{y}\left(\tilde{P}_1-P_{-1}\right)+\left(\tilde{P}_1+P_{-1}\right)} \tag{11}$$

$$T_{0B}(\sigma) = \frac{\bar{y}\left(\tilde{P}_{-1}+P_1\right)+\left(\tilde{P}_{-1}-P_1\right)}{\bar{y}\left(\tilde{P}_{-1}-P_1\right)+\left(\tilde{P}_{-1}+P_1\right)} \tag{12}$$

$$T_{AB}(\sigma) = \frac{\bar{y}\left(\tilde{P}_1+P_1+\tilde{P}_{-1}+P_{-1}\right)+\left(\tilde{P}_1+P_1-\tilde{P}_{-1}-P_{-1}\right)}{\bar{y}\left(\tilde{P}_1+P_1-\tilde{P}_{-1}-P_{-1}\right)+\left(\tilde{P}_1+P_1+\tilde{P}_{-1}+P_{-1}\right)} \tag{13}$$

Setting $T_1 = \min\{T_{A0}(\sigma), T_{BA}(\sigma)\}$ and $T_2 \equiv \max\{T_{0B}(\sigma), T_{BA}(\sigma)\}$ therefore defines a belief threshold strategy σ_{T_1,T_2} that is a best response to σ.

The belief threshold strategy σ_{T_1,T_2} is not an equilibrium if $T_1 = T_2 = T$ because the best response thresholds to $\sigma_{T,T}$ do not coincide: $T_{A0}(\sigma_{T,T}) < T_{0B}(\sigma_{T,T})$. To see this, consider the case in which $T \geq 0$. In that case, a citizen votes for B (i.e. $qs \geq T$) only if she receives a high-quality positive signal; citizens with low-quality or negative signals all vote for A (i.e. $qs < T$). Voting probabilities therefore reduce from (4) to the following,

$$
\begin{aligned}
p_\sigma(A|z) &= F(T_B)+\frac{1}{2}\int_{T_B}^{1}(1-zq)f(q)\,dq \\
p_\sigma(B|z) &= \frac{1}{2}\int_{T_B}^{1}(1+zq)f(q)\,dq,
\end{aligned}
$$

yielding the following inequalities:

$$p_\sigma(A|-1)\,p_\sigma(B|1) > p_\sigma(A|1)\,p_\sigma(B|-1) \tag{14}$$

$$p_\sigma(A|-1) > F(T_B)+p_\sigma(B|-1) \tag{15}$$

$$F(T_B)+p_\sigma(B|1) > p_\sigma(A|1) \tag{16}$$

$$p_\sigma(A|1)\,p_\sigma(B|1) > p_\sigma(A|-1)\,p_\sigma(B|-1). \tag{17}$$

These imply that $\tilde{P}_1\tilde{P}_{-1} > P_1P_{-1}$, which is algebraically equivalent to $T_{A0}(\sigma_{T,T}) < T_{0B}(\sigma_{T,T})$. This can be seen as follows,

$$\begin{aligned}
& \tilde{P}_{-1}\tilde{P}_1 - P_{-1}P_1 \\
= \;& \frac{1}{2}\left[\pi_\sigma^0(-1)+\pi_\sigma^B(-1)\right]\frac{1}{2}\left[\pi_\sigma^0(1)+\pi_\sigma^A(1)\right] \\
& -\frac{1}{2}\left[\pi_\sigma^0(-1)+\pi_\sigma^A(-1)\right]\frac{1}{2}\left[\pi_\sigma^0(1)+\pi_\sigma^B(1)\right] \\
= \;& \frac{1}{4}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\psi_\sigma(j,j|-1)\,\psi_\sigma(k,k|1)\left[1+\frac{\mu p_\sigma(A|-1)}{j+1}\right]\left[1+\frac{\mu p_\sigma(B|1)}{k+1}\right] \\
& -\frac{1}{4}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\psi_\sigma(j,j|-1)\,\psi_\sigma(k,k|1)\left[1+\frac{\mu p_\sigma(B|-1)}{j+1}\right]\left[1+\frac{\mu p_\sigma(A|1)}{k+1}\right] \\
> \;& \frac{1}{4}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\psi_\sigma(j,j|-1)\,\psi_\sigma(k,k|1)\left[\frac{\mu F(T_B)}{j+1}-\frac{\mu F(T_B)}{k+1}\right] \\
= \;& \sum_{j=0}^{\infty}\sum_{k=j+1}^{\infty}\frac{F(T_B)\,e^{-\mu}\mu^{2j+2k+1}}{4(j+1)!\,(k+1)!}p_\sigma^j(A|-1)\,p_\sigma^j(B|-1)\,p_\sigma^k(A|1)\,p_\sigma^k(B|1)\,[k-j] \\
& -\sum_{k=0}^{\infty}\sum_{j=k+1}^{\infty}\frac{F(T_B)\,e^{-\mu}\mu^{2j+2k+1}}{4(j+1)!\,(k+1)!}p_\sigma^j(A|-1)\,p_\sigma^j(B|-1)\,p_\sigma^k(A|1)\,p_\sigma^k(B|1)\,[j-k] \\
= \;& \sum_{j=0}^{\infty}\sum_{k=j+1}^{\infty}\frac{F(T_B)\,e^{-\mu}\mu^{2j+2k+1}}{4(j+1)!\,(k+1)!}p_\sigma^j(A|-1)\,p_\sigma^j(B|-1)\,p_\sigma^j(A|1)\,p_\sigma^j(B|1)\times \\
& \left[p_\sigma^{k-j}(A|1)\,p_\sigma^{k-j}(B|1)-p_\sigma^{k-j}(A|-1)\,p_\sigma^{k-j}(B|-1)\right](k-j) \\
> \;& 0,
\end{aligned}$$

where the first inequality follows from (14) through (16) and the final inequality follows from (17). Similar reasoning applies if $T_{A0}=T_{0B}=T<0$.

Since the best response to a threshold strategy is another threshold strategy, the best response threshold functions $T_{A0}(\sigma_{T_A,T_B}) < T_{0B}(\sigma_{T_A,T_B})$ can together be thought of as a single continuous function from the set $\{(T_1,T_2): 0 \le T_1 \le T_2 \le 1\}$ of voting thresholds into itself. Since this set is compact, a fixed point (T_1^*,T_2^*) exists by Brouwer's theorem; the corresponding threshold strategy $\sigma_{T_1^*,T_2^*}$ is therefore a symmetric Bayesian (partial) equilibrium.

Symmetric voting thresholds $T_2 = -T_1 = T$ produce symmetric vote probabilities $p_{\sigma_{-T,T}}(A|z) = p_{\sigma_{-T,T}}(B|-z)$, symmetric outcome probabilities $\psi_{\sigma_{-T,T}}(a,b|z) = \psi_{\sigma_{-T,T}}(b,a|-z)$, and symmetric pivot probabilities $\pi^A_{\sigma_{-T,T}}(z) = \pi^B_{\sigma_{-T,T}}(-z)$, $\tilde{P}_1 = \tilde{P}_{-1} \equiv \tilde{P}$, and $P_1 = P_{-1} \equiv P$. Symmetric platforms $y_B = -y_A$ imply that $\bar{y} = 0$, so that best response thresholds $T_{0B}(\sigma_{-T,T}) = -T_{A0}(\sigma_{-T,T}) = \frac{\tilde{P}-P}{P+\tilde{P}} > 0$ are symmetric as well. In that case, the best response function $T_{0B}(\sigma_{-T,T})$, is a continuous function from the compact interval $[0,1]$ into itself, again implying by Brouwer's theorem the existence of a fixed point T^* such that $T_{0B}(\sigma_{-T^*,T^*}) = -T_{A0}(\sigma_{-T^*,T^*}) = T^* > 0$; the corresponding threshold strategy σ_{-T^*,T^*} is a symmetric Bayesian (partial) equilibrium. ■

Proposition 3 *If policy platform commitments are binding, candidates are policy-motivated and abstention is allowed, and if there is a unique equilibrium voting response $\sigma^*(x_A,x_B)$ in the stage-game for every pair $x_A \neq x_B$ of candidate platforms, then there exists a threshold T^* such that $(\sigma_{-T^*,T^*}, x_A^*, x_B^*)$ is a perfect Bayesian equilibrium, where $x_B^* = -x_A^* = \hat{z}_B(\sigma_{-T^*,T^*})$.*

Proof. McLennan (1998) shows, for common interest games, that the strategy that maximizes the common objective function is an equilibrium—since no player can profit by deviating—and that the symmetric strategy that maximizes the common objective function is a symmetric equilibrium. In the context of this model, the latter of these two results implies that the optimal voting response $\sigma^*(x_A,x_B)$ to candidate platforms x_A and x_B is an equilibrium in the voting stage-game, and therefore (by Lemma 4) can be characterized as a belief threshold strategy $\sigma_{T_1^*,T_2^*}$, with $T_1^* < T_2^*$. If there is a unique stage-game equilibrium voting strategy, therefore, then that strategy is necessarily an optimal response to x_A and x_B (i.e. maximizes $Eu(Z,Y)$).

Policy-motivated candidates, like voters, seek to maximize $Eu(Z,Y)$. Taking optimal voter responses as given, McLennan's (1998) first result therefore implies that the platform pair (x_A^*, x_B^*) that maximizes $Eu(Z,Y)$ is an equilibrium in a reduced game between the two candidates. The strategy combination (σ^*, x_A^*, x_B^*) that maximizes $Eu(Z,Y)$ generally, then, is a perfect Bayesian equilibrium in the original game. Such an optimal strategy combination exists by the Weierstrass theorem, since $Eu(Z,Y;\sigma,x_A,x_B)$ is continuous in (σ,x_A,x_B) on the compact set $\Sigma \times [-1,1]^2$.

For any information set Ω, the policy that maximizes $E\left[-(x-Z)^2|\Omega\right]$ is simply $x = E(Z|\Omega)$. If voting behavior were fixed as $\sigma(x_A,x_B)$ for an arbitrary pair (x_A,x_B) of candidate platforms, therefore, then candidate j could increase $Eu(Z,Y|W=j)$ by deviating to $x_j = \hat{z}_j(\sigma,x_A,x_B)$. Allowing voters next to replace their previous voting strategy with the stage-game equilibrium response to the new platform pair would only increase $Eu(Z, Y)$ further, by the above logic of optimal voting. By these two steps, therefore, candidates can improve upon any pair (x_A^*, x_B^*) of candidate platforms unless, for the stage-game equilibrium voting strategy σ^*, it is the case that $x_j^* = \hat{z}_j(\sigma^*, x_A^*, x_B^*)$. That $\hat{z}_A(\sigma^*, x_A^*, x_B^*) < \hat{z}_B(\sigma^*, x_A^*, x_B^*)$ follows because σ^* is a belief threshold strategy, so $W = A$ is more likely when $Z = -1$ and $W = B$ is more likely when $Z = 1$, because a high proportion of citizens receive negative and positive signals, respectively, in the two states.

Symmetric voting thresholds $T_2 = -T_1 = T$ produce symmetric vote probabilities $p_{\sigma_{-T,T}}(A|z) = p_{\sigma_{-T,T}}(B|-z)$, symmetric outcome probabilities $\psi_{\sigma_{-T,T}}(a, b|z) = \psi_{\sigma_{-T,T}}(b, a|-z)$, and symmetric pivot probabilities $\pi^A_{\sigma_{-T,T}}(z) = \pi^B_{\sigma_{-T,T}}(-z)$, $\tilde{P}_1 = \tilde{P}_{-1} \equiv \tilde{P}$, and $P_1 = P_{-1} \equiv P$, and lead to symmetric expectations $\hat{z}_B(\sigma_{-T,T}) = -\hat{z}_A(\sigma_{-T,T})$. This implies symmetric platforms $x_B^* = -x_A^*$ in equilibrium, implying that best response thresholds $T_{0B}(\sigma_{-T,T}) = -T_{A0}(\sigma_{-T,T}) = \frac{\tilde{P}-P}{P+\tilde{P}} > 0$ are symmetric as well. In that case, Lemma 4 demonstrates the existence of an equilibrium threshold T^* such that $(\sigma_{-T^*,T^*}, x_A^*, x_B^*)$ is a perfect Bayesian equilibrium for $x_j^* = \hat{z}_j(\sigma_{-T^*,T^*})$. ■

Theorem 4 (Signaling Voter's Curse) *If candidates are policy-motivated and responsive to vote totals and voter abstention is allowed then the best response to any belief threshold strategy σ_{T_1,T_2} is another belief threshold strategy $\sigma_{T_1^*(\sigma_{T_1,T_2}),T_2^*(\sigma_{T_1,T_2})}$, where $T_1^*(\sigma_{T_1,T_2}) < T_2^*(\sigma_{T_1,T_2})$; furthermore, a threshold T^* exists such that $(\sigma_{-T^*,T^*}, x_A, x_B, y_{a,b}^*)$ is a perfect Bayesian equilibrium for any candidate platform pair (x_A, x_B), where $y_{a,b}^*$ is the candidate response function described in Lemma 2.*

Proof. Given that a responsive candidate will implement $y_{a,b}^*(\sigma)$ in response to vote totals a and b and voting strategy σ, as Lemma 2 prescribes, the benefit $\Delta_{0B}(\sigma)$ to an individual citizen of type qs of voting B instead of abstaining is given simply by the following:

$$\begin{aligned}
\Delta_{0B} &= \sum_{z=1,-1} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left[-\left(y_{a,b+1}^* - z\right)^2 + \left(y_{a,b}^* - z\right)^2 \right] \psi_\sigma(a, b|z) \frac{1}{2}(1 + zqs) \\
&= \sum_{z=1,-1} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left(y_{a,b+1}^* - y_{a,b}^*\right) \left(z - \frac{y_{a,b}^* + y_{a,b+1}^*}{2} \right) \psi_\sigma(a, b|z)(1 + zqs).
\end{aligned}$$

For a belief threshold strategy σ_{T_1,T_2}, Lemma 3 points out that $y_{a,b+1}^*(\sigma_{T_1,T_2}) = \hat{z}_{a,b+1}(\sigma_{T_1,T_2}) > \hat{z}_{a,b}(\sigma_{T_1,T_2}) = y_{a,b}^*(\sigma_{T_1,T_2})$. Since $\hat{z}_{a,b+1}$ and $\hat{z}_{a,b}$ are bounded between -1 and 1, Δ_{0B} is strictly increasing in qs,

$$\begin{aligned}
\frac{d\Delta_{0B}}{d(qs)} &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(y^*_{a,b+1}-y^*_{a,b}\right)\left(z-\frac{y^*_{a,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma(a,b|z)\,z \\
&= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(y^*_{a,b+1}-y^*_{a,b}\right)\left(1-\frac{y^*_{a,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma(a,b|Z=1) \\
&\quad -\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(y^*_{a,b+1}-y^*_{a,b}\right)\left(-1-\frac{y^*_{a,b}+y^*_{a,b+1}}{2}\right)\psi_\sigma(a,b|Z=-1) \\
&> 0,
\end{aligned}$$

implying the existence of a threshold $T_{0B}(\sigma_{T_1,T_2})$ such that a citizen prefers voting for B to abstaining if and only if $qs \geq T_{0B}(\sigma_{T_1,T_2})$. The benefit Δ_{AB} of voting B instead of A and the benefit Δ_{A0} of abstaining instead of voting A are likewise increasing in qs, implying the existence of thresholds $T_{AB}(\sigma_{T_1,T_2})$ and $T_{A0}(\sigma_{T_1,T_2})$ above which citizens prefer to vote B and abstain, respectively, and below which citizens prefer to vote A. Setting $T_1^*(\sigma_{T_1,T_2}) = \min\{T_{AB}(\sigma_{T_1,T_2}), T_{A0}(\sigma_{T_1,T_2})\}$ and $T_2^*(\sigma_{T_1,T_2}) = \max\{T_{AB}(\sigma_{T_1,T_2}), T_{0B}(\sigma_{T_1,T_2})\}$ defines a belief threshold strategy $\sigma_{T_1^*(\sigma_{T_1,T_2}),T_2^*(\sigma_{T_1,T_2})}$ that is a best response to σ_{T_1,T_2}.

It remains to show that $T_A(\sigma) < T_B(\sigma)$. To do so I show that perfectly uninformed (i.e. $Q_i = 0$) individuals—and therefore, by continuity, sufficiently uninformed individuals—strictly prefer to abstain from voting. Given candidate behavior as described in Lemma 2, a citizen essentially chooses between three policy functions: by voting A, B, or 0, respectively, she can cause the winning candidate to implement $y(a,b) \in \left\{y^*_{a+1,b}, y^*_{a,b+1}, y^*_{a,b}\right\}$. Which action is optimal therefore depends on which of these policy functions maximizes the expectation of $u(y(a,b),Z)$ with respect to the state Z, vote totals a and b, and her private information (q,s). Since a perfectly uninformed citizen's posterior beliefs about Z are the same as her prior beliefs, however, her expectation reduces to

$$\begin{aligned}
E_Z\left(E_{a,b}\left\{u\left[y(a,b),Z\right]|Z\right\}|q,s\right) &= E_Z\left(E_{a,b}\left\{u\left[y(a,b),Z\right]|Z\right\}\right) \\
&= E_{a,b}\left(E_Z\left\{u\left[y(a,b),Z\right]|a,b\right\}\right).
\end{aligned}$$

The inner component $E_Z\{u[y(a,b),Z]|a,b\}$ of this expression is identical to candidates' objective function; by Lemma 2, it is uniquely maximized at $y^*_{a,b} = \hat{z}_{a,b}$. Therefore $E_Z\{u[y(a,b),Z]|a,b\}$ is greater for $y(a,b) = y^*_{a,b}$ than for $y(a,b) = y^*_{a+1,b}$ or $y(a,b) = y^*_{a,b+1}$. Since this is true for any voting outcome (a,b), it is true for the expectation $E_{a,b}(E_Z\{u[y(a,b),Z]|a,b\})$, as well. Thus, the perfectly uninformed citizen prefers the policy function $y^*_{a,b}$ to either $y^*_{a+1,b}$ or $y^*_{a,b+1}$, and therefore prefers to abstain.

If thresholds $T_2 = -T_1 = T$ are symmetric around zero then voting probabilities are symmetric with respect to the state (i.e. $p(A|z) = p(B|-z)$, $\psi(a,b|z) = \psi(b,a|z)$ for

$z = -1, 1$ and $a, b \in \mathbb{Z}_+$), prompting symmetric policy responses from candidates:

$$\begin{aligned} y^*_{a,b} &= E(Z|a,b;\sigma) = \frac{\psi_\sigma(a,b|Z=1) - \psi_\sigma(a,b|Z=-1)}{\psi_\sigma(a,b|Z=1) + \psi_\sigma(a,b|Z=-1)} \\ &= \frac{-\psi_\sigma(b,a|Z=-1) + \psi_\sigma(b,a|Z=1)}{-\psi_\sigma(b,a|Z=-1) - \psi_\sigma(b,a|Z=1)} \\ &= -E(Z|b,a;\sigma) = -y^*_{b,a}. \end{aligned}$$

As in the proof of Theorem 3, therefore, a perfectly-uninformed individual is indifferent between voting for A and B (i.e. $q = 0$ implies $\Delta_{AB} = 0$). Furthermore, the benefit Δ_{0B} of voting B instead of abstaining and the benefit $-\Delta_{A0}$ of voting A instead of abstaining are the same, for individuals who receive opposite signals:

$$\begin{aligned} \Delta_{0B}(q,-s) &= \sum_{z=1,-1} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left[-\left(y^*_{a,b+1} - z\right)^2 + \left(y^*_{a,b} - z\right)^2 \right] \psi_\sigma(a,b|z) \frac{1}{2}(1 - zsq) \\ &= \sum_{\tilde{z}=-1,1} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left[-\left(-y^*_{b+1,a} + \tilde{z}\right)^2 + \left(-y^*_{b,a} + \tilde{z}\right)^2 \right] \times \\ &\quad \psi_\sigma(b,a|\tilde{z}) \frac{1}{2}(1 + \tilde{z}sq) \\ &= \sum_{\tilde{z}=-1,1} \sum_{b=0}^{\infty} \sum_{a=0}^{\infty} \left[-\left(y^*_{a+1,b} - \tilde{z}\right)^2 + \left(y^*_{a,b} - \tilde{z}\right)^2 \right] \psi_\sigma(a,b|\tilde{z}) \frac{1}{2}(1 + \tilde{z}sq) \\ &= \Delta_{A0}(q,s). \end{aligned}$$

This implies that best response thresholds $T^*_2(\sigma_{-T,T}) = -T^*_1(\sigma_{-T,T})$ are symmetric as well. $T^*_2(\sigma_{-T,T})$ is therefore a continuous function from the compact interval $[0,1]$ into itself; by Brouwer's theorem, a fixed point T^* exists (where $T^* > 0$ by the logic above). The corresponding symmetric belief threshold strategy σ_{-T^*,T^*}, together with the policy response functions $y^*_{a,b}$ identified in Lemma 2 and any pair (x_A, x_B) of policy platforms, constitute a perfect Bayesian equilibrium. ■

Proposition 4 *Let* $N = 2$ *be known and let* F *be uniform on* $[0,1]$*. If candidates are responsive then* $(x^*_A, x^*_B, \sigma_{-T^*,T^*}, y^*_A, y^*_B)$ *is a perfect Bayesian equilibrium, where* $x^*_B = -x^*_A \approx 0.5242$*,* $y^*_B(0,0) = y^*_A(0,0) = 0$*,* $y^*_B(0,1) = -y^*_A(1,0) \approx 0.7907$*, and* $y^*_B(0,2) = -y^*_A(2,0) \approx 0.9730$*, and where* σ_{-T^*,T^*} *is a symmetric belief threshold strategy with* $T^* \approx 0.5814$*. In this equilibrium, expected turnout is approximately* 42%*.*

Proof. Given a symmetric belief threshold $\sigma_{-T,T}$ and conditional on the state, an individual votes with the following probabilities,

$$
\begin{aligned}
p_{\sigma_{-T,T}}(A|-1) &= p_{\sigma_{-T,T}}(B|1) = \int_T^1 \frac{1}{2}(1+q)\,dq \\
&= \frac{1}{2}\left(1-T+\frac{1-T^2}{2}\right) = \frac{1}{4}(1-T)(T+3)
\end{aligned}
$$

and

$$
\begin{aligned}
p_{\sigma_{-T,T}}(A|1) &= p_{\sigma_{-T,T}}(B|-1) = \int_T^1 \frac{1}{2}(1-q)\,dq \\
&= \frac{1}{2}\left(1-T-\frac{1-T^2}{2}\right) = \frac{1}{4}(1-T)^2,
\end{aligned}
$$

and abstains with probability $p_{\sigma_{-T,T}}(0|z) = \int_0^T dq = T$. Given any voting outcome (a,b), these probabilities determine the winning candidate's expectation $\hat{z}_{a,b}$ of Z. As Lemma 2 states, $\hat{z}_{a,b}$ is the winning candidate's optimal policy response to (a,b).

If no one votes, or if the two citizens vote for opposite candidates, then the election winner is determined by a coin toss, and implements the zero policy:

$$
\begin{aligned}
\hat{z}_{00} &= \frac{-p^2_{\sigma_{-T,T}}(0|-1) + p^2_{\sigma_{-T,T}}(0|1)}{p^2_{\sigma_{-T,T}}(0|-1) + p^2_{\sigma_{-T,T}}(0|1)} = 0 \\
\hat{z}_{11} &= \frac{-2p_{\sigma_{-T,T}}(A|-1)\,p_{\sigma_{-T,T}}(B|-1) + 2p_{\sigma_{-T,T}}(A|1)\,p_{\sigma_{-T,T}}(B|1)}{2p_{\sigma_{-T,T}}(A|-1)\,p_{\sigma_{-T,T}}(B|-1) + 2p_{\sigma_{-T,T}}(A|1)\,p_{\sigma_{-T,T}}(B|1)} = 0.
\end{aligned}
$$

If both citizens vote B then candidate B wins the election and implements

$$
\begin{aligned}
\hat{z}_{02} &= \frac{-p^2_{\sigma_{-T,T}}(B|-1) + p^2_{\sigma_{-T,T}}(B|1)}{p^2_{\sigma_{-T,T}}(B|-1) + p^2_{\sigma_{-T,T}}(B|1)} \\
&= \frac{-\frac{1}{16}(1-T)^4 + \frac{1}{16}(1-T)^2(T+3)^2}{\frac{1}{16}(1-T)^4 + \frac{1}{16}(1-T)^2(T+3)^2} = \frac{4T+4}{T^2+2T+5};
\end{aligned}
$$

if one citizen votes B while the other abstains then candidate B wins and implements

$$
\begin{aligned}
\hat{z}_{01} &= \frac{-2p_{\sigma_{-T,T}}(0|-1)\,p_{\sigma_{-T,T}}(B|-1) + 2p_{\sigma_{-T,T}}(0|1)\,p_{\sigma_{-T,T}}(B|1)}{2p_{\sigma_{-T,T}}(0|-1)\,p_{\sigma_{-T,T}}(B|-1) + 2p_{\sigma_{-T,T}}(0|1)\,p_{\sigma_{-T,T}}(B|1)} \\
&= \frac{-p_{\sigma_{-T,T}}(B|-1) + p_{\sigma_{-T,T}}(B|1)}{p_{\sigma_{-T,T}}(B|-1) + p_{\sigma_{-T,T}}(B|1)} = \frac{-(1-T)^2 + (1-T)(T+3)}{(1-T)^2 + (1-T)(T+3)} = \frac{T+1}{2}.
\end{aligned}
$$

Symmetrically, $\hat{z}_{20} = -\hat{z}_{02}$ and $\hat{z}_{10} = -\hat{z}_{01}$.

Given these values, the benefit Δ_{0B} for an individual of type (q,s) is given by the following,

$$\begin{aligned}\Delta_{0B}\left(q;\sigma_{-T,T}\right) &= \left(\hat{z}_{11}-\hat{z}_{10}\right)\left[\begin{array}{c}\left(1-\frac{\hat{z}_{11}+\hat{z}_{10}}{2}\right)\left(1+qs\right)p_{\sigma_{-T,T}}\left(A|1\right)\\ +\left(-1-\frac{\hat{z}_{11}+\hat{z}_{10}}{2}\right)\left(1-qs\right)p_{\sigma_{-T,T}}\left(A|-1\right)\end{array}\right]\\ &\quad+\left(\hat{z}_{01}-\hat{z}_{00}\right)\left[\begin{array}{c}\left(1-\frac{\hat{z}_{01}+\hat{z}_{00}}{2}\right)\left(1+qs\right)p_{\sigma_{-T,T}}\left(0|1\right)\\ +\left(-1-\frac{\hat{z}_{01}+\hat{z}_{00}}{2}\right)\left(1-qs\right)p_{\sigma_{-T,T}}\left(0|-1\right)\end{array}\right]\\ &\quad+\left(\hat{z}_{02}-\hat{z}_{01}\right)\left[\begin{array}{c}\left(1-\frac{\hat{z}_{02}+\hat{z}_{01}}{2}\right)\left(1+qs\right)p_{\sigma_{-T,T}}\left(B|1\right)\\ +\left(-1-\frac{\hat{z}_{02}+\hat{z}_{01}}{2}\right)\left(1-qs\right)p_{\sigma_{-T,T}}\left(B|1\right)\end{array}\right].\end{aligned}$$

Solving $\Delta_{0B}\left(T;\sigma_{-T,T}\right)=0$ numerically yields $T^*\approx 0.58$, implying that $\sigma_{-0.58,0.58}$ (together with candidate platforms and $y_j^*\left(a,b\right)=\hat{z}_{a,b}$) is a perfect Bayesian equilibrium. Evaluating the formulas above for $T^*\approx 0.58$ yields $\hat{z}_{01}=-\hat{z}_{10}\approx 0.7907$, and $\hat{z}_{02}=-\hat{z}_{20}\approx 0.9730$. Expected turnout is $1-T^*\approx 0.42$.

In this equilibrium, candidate B wins with probability

$$\begin{aligned}\Pr\left(W=B|-1\right) &= \left[p^2_{\sigma_{-T^*,T^*}}\left(B|-1\right)+2p_{\sigma_{-T^*,T^*}}\left(0|-1\right)p_{\sigma_{-T^*,T^*}}\left(B|-1\right)\right]\\ &\quad+\frac{1}{2}\left[p^2_{\sigma_{-T^*,T^*}}\left(0|-1\right)+2p_{\sigma_{-T^*,T^*}}\left(A|-1\right)p_{\sigma_{-T^*,T^*}}\left(B|-1\right)\right]\\ &= \frac{1}{16}\left(1-T^*\right)^4+2T^*\frac{1}{4}\left(1-T^*\right)^2\\ &\quad+\frac{1}{2}\left[T^{*2}+2\frac{1}{4}\left(1-T^*\right)\left(T^*+3\right)\frac{1}{4}\left(1-T^*\right)^2\right]\\ &\approx 0.2379\end{aligned}$$

in state -1 and probability

$$\begin{aligned}\Pr\left(W=B|1\right) &= \left[p^2_{\sigma_{-T^*,T^*}}\left(B|1\right)+2p_{\sigma_{-T^*,T^*}}\left(0|1\right)p_{\sigma_{-T^*,T^*}}\left(B|1\right)\right]\\ &\quad+\frac{1}{2}\left[p^2_{\sigma_{-T^*,T^*}}\left(0|1\right)+2p_{\sigma_{-T^*,T^*}}\left(A|1\right)p_{\sigma_{-T^*,T^*}}\left(B|1\right)\right]\\ &= \frac{1}{16}\left(1-T^*\right)^2\left(T^*+3\right)^2+2T^*\frac{1}{4}\left(1-T^*\right)\left(T^*+3\right)\\ &\quad+\frac{1}{2}\left[T^{*2}+2\frac{1}{4}\left(1-T^*\right)^2\frac{1}{4}\left(1-T^*\right)\left(T^*+3\right)\right]\\ &\approx 0.7621\end{aligned}$$

in state 1. Conditional only on winning, therefore, candidate B's expectation of Z is given by

$$\begin{aligned}\hat{z}_B &= \frac{-\Pr\left(W=B|-1\right)+\Pr\left(W=B|1\right)}{\Pr\left(W=B|-1\right)+\Pr\left(W=B|1\right)}\\ &\approx 0.5242\end{aligned}$$

and candidate A's expectation is symmetric $\hat{z}_A = -\hat{z}_B$.

In equilibrium, four out of five policy outcomes (i.e. ± 0.7907 and ± 0.9730, but not 0) are more extreme than the campaign platforms $x_j^* = \hat{z}_j \approx \pm 0.5242$. The probability of a tie occurring in equilibrium is

$$\begin{aligned} & \frac{1}{2}\left[p_{\sigma_{-T,T}}^2(0|-1) + 2p_{\sigma_{-T,T}}(A|-1)\,p_{\sigma_{-T,T}}(B|-1)\right] \\ & +\frac{1}{2}\left[p_{\sigma_{-T,T}}^2(0|1) + 2p_{\sigma_{-T,T}}(A|1)\,p_{\sigma_{-T,T}}(B|1)\right] \\ = & \frac{1}{2}\left[T^2 + 2\frac{1}{4}(1-T)(T+3)\frac{1}{4}(1-T)^2\right] \\ & +\frac{1}{2}\left[T^2 + 2\frac{1}{4}(1-T)^2\frac{1}{4}(1-T)(T+3)\right] \\ = & T^2 + \frac{1}{8}(T+3)(1-T)^3 \approx 0.3696. \end{aligned}$$

■

Proposition 5 *Let $N = 1$ be known and let F be uniform on $[0,1]$. If candidates are responsive then there is a unique belief threshold strategy σ_{-T^*,T^*} such that $(x_A^*, x_B^*, \sigma_{-T^*,T^*}, y_A^*, y_B^*)$ is a perfect symmetric Bayesian equilibrium. In this equilibrium, $T^* = \frac{1}{3}$, $x_B^* = -x_A^* \approx 0.44$, $y_j^*(0,0) = 0$, and $y_j^*(0,1) = -y_j^*(1,0) \approx 0.67$, and expected turnout is approximately 67%.*

Proof. With only a single citizen, there are only three possible voting outcomes: $(1,0)$, $(0,0)$, and $(0,1)$. As in Proposition 4, a symmetric belief threshold voting strategy $\sigma_{-T,T}$ leads the citizen to vote with probabilities $p_{\sigma_{-T,T}}(A|-1) = p_{\sigma_{-T,T}}(B|1) = \frac{1}{4}(1-T)(T+3)$ and $p_{\sigma_{-T,T}}(A|1) = p_{\sigma_{-T,T}}(B|-1) = \frac{1}{4}(1-T)^2$, and to abstain with probability $p_{\sigma_{-T,T}}(0|1) = p_{\sigma_{-T,T}}(0|-1) = F(T) = T$. The winning candidate's expectation $\hat{z}_{a,b}$ of Z is therefore given by the following:

$$\begin{aligned} \hat{z}_{0,0} &= \frac{-T+T}{T+T} = 0 \\ \hat{z}_{0,1} &= -\hat{z}_{1,0} = \frac{-\frac{1}{4}(1-T)^2 + \frac{1}{4}(1-T)(T+3)}{\frac{1}{4}(1-T)^2 + \frac{1}{4}(1-T)(T+3)} = \frac{T+1}{2}. \end{aligned}$$

Conditional only on winning, his expectation $\hat{z}_j$ is given by

$$\begin{aligned} \hat{z}_B &= -\hat{z}_A = \frac{-\left[\frac{1}{4}(1-T)^2 + \frac{1}{2}T\right] + \left[\frac{1}{4}(1-T)(T+3) + \frac{1}{2}T\right]}{\left[\frac{1}{4}(1-T)^2 + \frac{1}{2}T\right] + \left[\frac{1}{4}(1-T)(T+3) + \frac{1}{2}T\right]} \\ &= \frac{-(1-T)^2 + (1-T)(T+3)}{(1-T)^2 + (1-T)(T+3) + 4T} = \frac{1-T^2}{2}. \end{aligned}$$

As Lemma 2 states, the winning candidate prefers to implement his expectation $\hat{z}_{a,b}$ of the state. Anticipating such a policy response, the citizen perceives the benefit $\Delta_{0B}(q,s)$ of voting B rather than abstaining, given her type (q,s):

$$\begin{aligned}\Delta_{0B}(q,s) &= (y_{0,1}-y_{0,0})\left[\begin{array}{c}\left(1-\frac{y_{0,0}+y_{0,1}}{2}\right)(1+qs)\\ +\left(-1-\frac{y_{0,0}+y_{0,1}}{2}\right)(1-qs)\end{array}\right]\\ &= 2y_{0,1}\left(qs-\frac{y_{0,1}}{2}\right),\end{aligned}$$

which is positive if and only if $qs \geq \frac{y_{0,1}}{2}$. Thus, the optimal voting response to $(y_{1,0}, y_{0,0}, y_{0,1})$ is a quality threshold strategy with $T=\frac{y_{0,1}}{2}$. Solving $T=\frac{y_{0,1}}{2}$ and $y_{0,1}=\frac{T+1}{2}$ simultaneously yields a unique solution at $T^*=\frac{1}{3}$, $y^*_{0,1}=\frac{2}{3}\approx 0.67$ (and, by symmetry, $y^*_{1,0}=-\frac{2}{3}$), implying that the citizen votes with probability $1-F(T^*)=1-T^*=\frac{2}{3}\approx 0.67$. If candidate platforms reflect ex ante expectations of the state, as in Lemma 2, then $x^*_B=-x^*_A=\frac{4}{9}\approx 0.44$.

In equilibrium, two out of three policy outcomes (i.e. ± 0.67, but not 0) are more extreme than the campaign platforms (i.e. $x^*_j=\hat{z}_j\approx\pm 0.5242$). The probability of a tie occurring in equilibrium is $\frac{1}{2}p_{\sigma_{-T,T}}(0|1)+\frac{1}{2}p_{\sigma_{-T,T}}(0|-1)=T=\frac{1}{3}$. ■

Theorem 5 (Jury Theorem) *Let $\{\mu_k\}_{k=1}^{\infty}$ be a sequence of population size parameters with $\lim_{k\to\infty}\mu_k=\infty$. Then the following are true:*

1. If y_A and y_B are exogenous then

(a) for any k, the symmetric voting strategy $\sigma^*_{\mu_k}$ that maximizes expected utility is a symmetric Bayesian (partial) equilibrium, and

(b) $p\lim_{k\to\infty} Y^*_{\mu_k}=\begin{cases} y_A & \text{if } Z=-1\\ y_B & \text{if } Z=1\end{cases}$.

2. If policy platform commitments are binding and candidates are policy-motivated then

(a) for any k, the voting strategy $\sigma^*_{\mu_k}$ and platform pair $\left(x^*_{A\mu_k}, x^*_{B\mu_k}\right)$ that together maximize expected utility constitute a perfect Bayesian equilibrium, and

(b) $p\lim_{k\to\infty} Y^*_{\mu_k}=Z$.

3. If policy-motivated candidates choose policy responses y_A, y_B to vote totals (N_A, N_B) then

(a) for any k, the voting strategy $\sigma^*_{\mu_k}$ and policy responses $\left(y^*_{A\mu_k}, y^*_{B\mu_k}\right)$ that together maximize expected utility, together with any platform pair (x_A, x_B), constitute a perfect Bayesian equilibrium, and

(b) $p\lim_{k\to\infty} Y^*_{\mu_k}=Z$.

4. Claims 1 through 3 remain true if voter abstention is allowed.

Proof. Claims 1(a), 2(a), and 3(a) follow from an argument similar to that made by McLennan (1998): consider the strategy combination that maximizes expected utility $Eu(Y,Z)$.

Since this is the objective function maximized by citizens and candidates alike, no citizen or candidate has incentive to deviate when all are playing the social optimum. This is true whether the relevant strategy space is a voting strategy alone, as in 1(a); a voting strategy and platform pair, as in 2(a); or a voting strategy and policy function pair, as in 3(a); and whether abstention is allowed (as in claim 4) or not. Note that the existence of a welfare-maximizing strategy combination is guaranteed in most of these cases by the Weierstrass theorem, since (under the uniform topology) expected utility is continuous and the (products of) strategy spaces Σ, $\Sigma \times [-1,1]^2$, Σ', and $\Sigma' \times [-1,1]^2$ are compact. The set $\left\{y : \mathbb{Z}_+^2 \to [-1,1]\right\}$ of policy response functions is not compact under the uniform topology, but Lemma 2 states that the optimal policy response function given a particular voting strategy σ is $\hat{z}_{a,b}(\sigma)$; given this policy response function, an optimal voting strategy σ^* exists because $Eu\left(\hat{z}_{N_A,N_B}(\sigma), Z; \sigma\right)$ is continuous in σ and Σ (and Σ') is compact. So a social optimum exists in every case.

Let $Y_{\mu_k}^0$ denote the policy outcome if citizens sincerely report their private information (i.e. vote A if $s = -1$ and B if $s = 1$, with no abstention), as directed by the belief threshold strategy σ_0 (i.e. $\sigma_{0,0}$, if abstention is allowed). Under this strategy, the expected share of A votes in state -1 or B votes in state 1 is $p(A|-1) = p(B|1) = EQ$, and the expected share of A votes in state 1 or B votes in state -1 is $p(A|1) = p(B|-1) = E(1-Q)$. The expected difference $E(N_A - N_B)$ between A and B votes, therefore, is positive in state -1 and negative in state 1. By the law of large numbers, therefore, $\lim_{k\to\infty} \Pr\left(\left|Y_{\mu_k}^0 - y_A\right| |Z = -1\right) = \lim_{k\to\infty} \Pr\left(\left|Y_{\mu_k}^0 - y_B\right| |Z = 1\right) = 1$. In general, σ_0 is inconsistent with equilibrium. The result that equilibrium strategy combinations are optimal, however, implies that $Eu\left(Y_{\mu_k}^*, Z\right) > Eu\left(Y_{\mu_k}^0, Z\right)$. In state -1, therefore, $Y_{\mu_k}^0 \to_p y_A$ implies $Y_{\mu_k}^* \to_p y_A$. Similarly, $Y_{\mu_k}^0 \to_p y_B$ implies $Y_{\mu_k}^* \to_p y_B$ in state 1. Together, these establish 1(b).

If platforms commitments are binding, the above logic implies that equilibrium voting elects candidate A in state -1 and candidate B in state 1, with probabilities approaching 1. As Proposition 2 states, policy-motivated candidates commit to platforms based on their expectations of Z, conditional on winning the election. As the expected number μ_k of citizens grows large, these expectations converge to Z (i.e. $E(Z|W = A; \sigma_0, \mu_k) \to -1$ and $E(Z|W = B; \sigma_0, \mu_k) \to 1$), implying that $x_{A\mu_k}^0 \to_p -1$ and $x_{B\mu_k}^0 \to_p 1$. Expected utility therefore approaches zero (i.e. $Eu\left(Y_{\mu_k}^0, Z\right) \to_p 0$), even for non-equilibrium voting σ_0; by the optimality argument above, the same must be true in equilibrium, so $Y_{\mu_k}^* \to_p Z$, establishing claim 2(b). Similarly, a responsive candidate implements his expectation of Z, given vote totals N_A and N_B; the expectation of which is simply the state variable Z. As a population grows large, therefore, the law of large numbers implies again that $Y_{\mu_k}^0 \to_p Z$, and the optimality of equilibrium again implies that $Y^*_{\mu_k} \to_p Z$ as well, establishing 3(b). Claim 4 follows identically, since an equilibrium voting strategy with abstention is likewise superior to $\sigma_{0,0}$. ■

Theorem 6 *If candidates A, B, C, and D are responsive and abstention is allowed then there exist quality thresholds $0 < T_1^* < T_2^* < 1$ such that $\left(x^*, \sigma_{-T_2^*,-T_1^*,T_1^*,T_2^*}, y^*\right)$ is a perfect Bayesian equilibrium if $y^* = \left(y_j^*\right)_{j\in\{A,B,C,D\}}$ is the vector of policy response functions defined by $y_j^*\left(a,b,c,d\right) = \hat{z}_{a,b,c,d} \equiv E\left(Z|N_A = a, N_B = b, N_C = c, N_D = d; \sigma_{-T_2^*,-T_1^*,T_1^*,T_2^*}\right)$ for any voting outcome $(a,b,c,d) \in \mathbb{Z}_+^4$ and $x^* = \left(x_j^*\right)_{j\in\{A,B,C,D\}}$ is any vector of candidate platforms.*

Proof. By logic identical to that used in Lemma 2, the winning candidate's optimal policy choice $y^*_{a,b,c,d} = \hat{z}_{a,b,c,d}$ in response to any voting strategy σ is to implement the conditional expectation $\hat{z}_{a,b,c,d}$ of Z in response to vote totals $(a,b,c,d) \in \mathbb{Z}_+^4$. If voting follows a symmetric belief threshold strategy $\sigma_{-T_2,-T_1,T_1,T_2}$ then, by logic identical to that used in Lemma 3, $\hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$ for any $(a,b,c,d) \in \mathbb{Z}_+^4$. The effect of a single vote, therefore, is to push the ultimate policy outcome either to the left or to the right, with A and D votes pushing policy further than B and C votes. Also, the symmetry of vote probabilities

$$
\begin{aligned}
p_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(A|z\right) &= p_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(D|-z\right) \\
p_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(B|z\right) &= p_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(C|-z\right) \\
p_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(0|z\right) &= p_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(0|-z\right) \\
\psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(a,b,c,d|z\right) &= \psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(d,c,b,a|-z\right)
\end{aligned}
$$

generates symmetric expectations:

$$
\begin{aligned}
\hat{z}_{d,c,b,a} &= \frac{\psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(d,c,b,a|1\right) - \psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(d,c,b,a|-1\right)}{\psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(d,c,b,a|1\right) + \psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(d,c,b,a|-1\right)} \\
&= \frac{\psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(a,b,c,d|-1\right) - \psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(a,b,c,d|1\right)}{\psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(a,b,c,d|-1\right) + \psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(a,b,c,d|1\right)} \\
&= -\hat{z}_{a,b,c,d}.
\end{aligned}
$$

Given this response to $\sigma_{-T_2,-T_1,T_1,T_2}$, let Δ_{j_1,j_2} denote the expected benefit of voting for candidate j_2 instead of candidate j_1, where $j_1, j_2 \in \{A,B,C,D,0\}$. For example, the benefit Δ_{CD} of voting for candidate D instead of candidate C is given by

$$
\begin{aligned}
\Delta_{CD}\left(q,s\right) &= \sum_{z=1,-1}\sum_{(a,b,c,d)\in\mathbb{Z}_+^4} 2\left(\hat{z}_{a,b,c,d+1} - \hat{z}_{a,b,c+1,d}\right)\times \\
&\quad \left(z - \frac{\hat{z}_{a,b,c+1,d} + \hat{z}_{a,b,c,d+1}}{2}\right)\psi_{\sigma_{-T_2,-T_1,T_1,T_2}}\left(a,b,c,d|z\right)\frac{1}{2}\left(1+zqs\right),
\end{aligned}
$$

for a citizen of type (q,s). Since $\hat{z}_{a,b,c,d}$ is bounded between -1 and 1, Δ_{CD} is increasing in qs:

$$\begin{aligned}\frac{d\Delta_{CD}}{d(qs)} &= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4}\left(\hat{z}_{a,b,c,d+1}-\hat{z}_{a,b,c+1,d}\right)\times \\ &\quad [\left(1-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_{\sigma_{-T_2,-T_1,T_1,T_2}}(a,b,c,d|1) \\ &\quad -\left(-1-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_{\sigma_{-T_2,-T_1,T_1,T_2}}(a,b,c,d|-1)] \\ &> 0.\end{aligned}$$

By a similar derivation, Δ_{j_1,j_2} is increasing in qs whenever j_1 precedes j_2 in the ordering $\{A,B,0,C,D\}$. This implies the existence of thresholds $T_{AB}\leq T_{B0}\leq T_{0C}\leq T_{CD}$ such that the belief threshold strategy $\sigma_{T_{AB},T_{A0},T_{0B},T_{CD}}$ is a best response to $\sigma_{-T_2,-T_1,T_1,T_2}$.

The symmetry of $\hat{z}_{a,b,c,d}$ implies that voting benefits are symmetric as well. For example,

$$\begin{aligned}\Delta_{CD}(q,s) &= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4}\left(\hat{z}_{a,b,c,d+1}-\hat{z}_{a,b,c+1,d}\right)\times \\ &\quad [\left(1-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_{\sigma_{T_1,T_2}}(a,b,c,d|1)\frac{1}{2}(1+qs) \\ &\quad +\left(-1-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_{\sigma_{T_1,T_2}}(a,b,c,d|-1)\frac{1}{2}(1-qs)] \\ &= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4}\left(-\hat{z}_{d+1,c,b,a}+\hat{z}_{d,c+1,b,a}\right)\times \\ &\quad [\left(1-\frac{-\hat{z}_{d+1,c,b,a}-\hat{z}_{d,c+1,b,a}}{2}\right)\psi_{\sigma_{T_1,T_2}}(d,c,b,a|-1)\frac{1}{2}(1+qs) \\ &\quad +\left(-1-\frac{-\hat{z}_{d,c+1,b,a}-\hat{z}_{d+1,c,b,a}}{2}\right)\psi_{\sigma_{T_1,T_2}}(d,c,b,a|1)\frac{1}{2}(1-qs)] \\ &= \sum_{(d,c,b,a)\in\mathbb{Z}_+^4}\left(\hat{z}_{a,b+1,c,d}-\hat{z}_{a+1,b,c,d}\right)\times \\ &\quad [\left(1+\frac{\hat{z}_{a+1,b,c,d}+\hat{z}_{a,b+1,c,d}}{2}\right)\psi_{\sigma_{T_1,T_2}}(a,b,c,d|-1)\frac{1}{2}(1+qs) \\ &\quad +\left(-1+\frac{\hat{z}_{a+1,b,c,d}+\hat{z}_{a,b+1,c,d}}{2}\right)\psi_{\sigma_{T_1,T_2}}(a,b,c,d|1)\frac{1}{2}(1-qs)] \\ &= -\Delta_{AB}(q,-s).\end{aligned}$$

Similarly, $\Delta_{0C}(q,s)=-\Delta_{B0}(q,-s)$. This symmetry implies that belief thresholds $T_{0C}=-T_{B0}\equiv T_1$ and $T_{CD}=-T_{AB}\equiv T_2$ are symmetric around zero. Thus, the best response to a symmetric belief threshold strategy is a symmetric belief threshold strategy.

Since best response belief thresholds are a continuous function from the compact set $\{(T_1, T_2) : 0 \leq T_1 \leq T_2 \leq 1\}$ of possible thresholds into itself, Brouwer's theorem guarantees the existence of equilibrium thresholds $T_1^* \leq T_2^*$. Furthermore, it must be that $T_2^* < 1$ in equilibrium because a perfectly-informed citizen prefers to vote for the most extreme candidate available:

$$\begin{aligned} \Delta_{CD}(1,1) &= \sum_{z=1,-1} \sum_{(a,b,c,d)\in\mathbb{Z}_+^4} 2\left(\hat{z}_{a,b,c,d+1} - \hat{z}_{a,b,c+1,d}\right) \times \\ &\quad \left(z - \frac{\hat{z}_{a,b,c+1,d} + \hat{z}_{a,b,c,d+1}}{2}\right) \psi_{\sigma_{T_1,T_2}}(a,b,c,d|z) \frac{1}{2}(1+z) \\ &= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4} \left(\hat{z}_{a,b,c,d+1} - \hat{z}_{a,b,c+1,d}\right) \times \\ &\quad \left(1 - \frac{\hat{z}_{a,b,c+1,d} + \hat{z}_{a,b,c,d+1}}{2}\right) \psi_{\sigma_{T_1,T_2}}(a,b,c,d|1)(1+1) \\ &> 0. \end{aligned}$$

That $T_1^* > 0$ in equilibrium follows by the same logic as in Theorem 4: given the vote totals (a, b, c, d) of her fellow citizens and candidates' optimal responses, a citizen essentially chooses $y(a,b,c,d) \in \{\hat{z}_{a+1,b,c,d}, \hat{z}_{a,b+1,c,d}, \hat{z}_{a,b,c,d}, \hat{z}_{a,b,c+1,d}, \hat{z}_{a,b,c,d+1}\}$ from a menu of five policy functions to maximize the expectation of $u(Y, Z)$. Since a perfectly uninformed citizen's posterior beliefs about Z are the same as her prior beliefs, however, this expectation reduces to

$$\begin{aligned} E_Z\left(E_{a,b,c,d}\left\{u\left[y(a,b,c,d), Z\right]|Z\right\}|q,s\right) &= E_Z\left(E_{a,b,c,d}\left\{u\left[y(a,b,c,d), Z\right]|Z\right\}\right) \\ &= E_{a,b,c,d}\left(E_Z\left\{u\left[y(a,b,c,d), Z\right]|a,b,c,d\right\}\right). \end{aligned}$$

The inner component $E_Z\{u[y_{a,b,c,d}, Z]|a,b,c,d\}$ of this expression is identical to candidates' objective function, and is uniquely maximized at $y^*_{a,b,c,d} = \hat{z}_{a,b,c,d}$. Since this is true for any voting outcome (a, b, c, d), it is true for the expectation $E_{a,b,c,d}(E_Z\{u[y_{a,b,c,d}, Z]|a,b,c,d\})$, as well. The perfectly uninformed citizen—and, by continuity, a sufficiently poorly informed citizen—thus prefers the policy function $y^*_{a,b,c,d} = \hat{z}_{a,b,c,d}$, and therefore prefers to abstain. ■

Proposition 5 *Let $N = 1$ be known, and let F be uniform on $[0,1]$. If candidates A, B, C, and D are responsive then (x^*, σ^*, y^*) is a perfect Bayesian equilibrium for the symmetric belief threshold voting strategy $\sigma^* = \sigma_{-.6,-.2,.2,.6}$, the vector $y^* = (y_j^*)_{j\in\{A,B,C,D\}}$ of policy responses defined by $y_j^*(0,0,0,1) = -y_j^*(1,0,0,0) = 0.8$, $y_j^*(0,0,1,0) = -y_j^*(0,1,0,0) = 0.4$, $y_j^*(0,0,0,0) = 0$, and any vector $x^* = (x_j^*)_{j\in\{A,B,C,D\}}$ of candidate platforms. In this equilibrium, expected voter turnout is 80%.*

Proof. According to Lemma 2, a responsive candidate's optimal response to vote totals (a, b, c, d) is to implement his expectation $\hat{z}_{a,b,c,d}$ of Z. If the citizen votes according to the symmetric belief threshold strategy $\sigma_{-T_2,-T_1,T_1,T_2}$ then the winning candidate's expectations are as follows:

$$\begin{aligned}
\hat{z}_{1,0,0,0} &= E\left(Z|qs \in [-1, -T_2]\right) = -\frac{T_1 + T_2}{2} \\
\hat{z}_{0,1,0,0} &= E\left(Z|qs \in [-T_2, -T_1]\right) = -\frac{T_2 + 1}{2} \\
\hat{z}_{0,0,0,0} &= E\left(Z|qs \in [-T_1, T_1]\right) = 0 \\
\hat{z}_{0,0,1,0} &= E\left(Z|qs \in [T_1, T_2]\right) = \frac{T_1 + T_2}{2} \\
\hat{z}_{0,0,0,1} &= E\left(Z|qs \in [T_2, 1]\right) = \frac{T_2 + 1}{2}.
\end{aligned}$$

The benefit $\Delta_{0C}(q, s)$ to a citizen of type (q, s) of voting for C instead of abstaining is therefore given by

$$\begin{aligned}
\Delta_{0C}(q, s) &= \sum_{z=1,-1} 2\left(\hat{z}_{0,0,1,0} - 0\right)\left(z - \frac{0 + \hat{z}_{0,0,1,0}}{2}\right)\frac{1}{2}(1 + zqs) \\
&= 2\hat{z}_{0,0,1,0}\left(qs - \frac{\hat{z}_{0,0,1,0}}{2}\right),
\end{aligned}$$

which is positive if and only if $qs \geq \frac{\hat{z}_{0,0,1,0}}{2} \equiv T_1\left(\sigma_{-T_2,-T_1,T_1,T_2}\right)$. Similarly, the benefit $\Delta_{CD}(q, s)$ of voting D instead of C is given by

$$\begin{aligned}
\Delta_{CD}(q, s) &= \sum_{z=1,-1} 2\left(\hat{z}_{0,0,0,1} - \hat{z}_{0,0,1,0}\right)\left(z - \frac{\hat{z}_{0,0,1,0} + \hat{z}_{0,0,0,1}}{2}\right)\frac{1}{2}(1 + zqs) \\
&= \left(\hat{z}_{0,0,0,1} - \hat{z}_{0,0,1,0}\right)\left(qs - \frac{\hat{z}_{0,0,1,0} + \hat{z}_{0,0,0,1}}{2}\right),
\end{aligned}$$

which is positive if and only if $q \geq \frac{\hat{z}_{0,0,1,0}+\hat{z}_{0,0,0,1}}{2} \equiv T_2\left(\sigma_{-T_2,-T_1,T_1,T_2}\right)$. Solving $T_1 = \frac{\hat{z}_{0,0,1,0}}{2}$, $T_2 = \frac{\hat{z}_{0,0,1,0}+\hat{z}_{0,0,0,1}}{2}$, $\hat{z}_{0,0,1,0} = \frac{T_1+T_2}{2}$, and $\hat{z}_{0,0,0,1} = \frac{T_2+1}{2}$ simultaneously yields $T_1^* = 0.2$, $T_2^* = 0.6$, $\hat{z}_{0,0,1,0} = 0.4$, $\hat{z}_{0,0,0,1} = 0.8$. Abstention is given by $1 - F\left(T_1^*\right) = T_1^* = 0.2$, so turnout is 80%. ■

References

[1] Arrow, Kenneth J. 1963. *Social Choice and Individual Values*, 2nd ed. New York: Wiley.

[2] Austen-Smith, David and Jeffrey S. Banks. 1996. "Information Aggregation, Rationality, and the Condorcet Jury Theorem." *The American Political Science Review*, 90(1): 34-45.

[3] Bade, Sophie. 2006. "A Simple Model of the Swing Voter's Curse." Unpublished, Penn State University.

[4] Besley, Timothy, and Anne Case. 2003. "Political Institutions and Policy Choices: Evidence from the United States." *Journal of Economic Literature*, 41(1): 7-73.

[5] Besley, Timothy, and Stephen Coate. 1997. "An Economic Model of Representative Democracy", *Quarterly Journal of Economics*, 112(1): 85-114.

[6] Black, Duncan. 1948. "On the Rationale of Group Decision-Making." *The Journal of Political Economy*, 56(1): 23-34.

[7] Condorcet, Marquis de. 1785. *Essai sur l'application de l'analyse a la probabilite' des decisions rendues a la pluralite' des voix.* Paris: De l'imprimerie royale. Trans. Iain McLean and Fiona Hewitt. 1994.

[8] Crawford, Vincent P. and Joel Sobel. 1982. "Strategic Information Transmission." *Econometrica*, 50(6): 1431-1451.

[9] Downs, Anthony. 1957. *An Economic Theory of Democracy.* New York: Harper and Row.

[10] Duggan, John. 2005. "A Survey of Equilibrium Analysis in Spatial Models of Elections." Unpublished.

[11] Duverger, M. 1954. *Political Parties*, New York: Wiley.

[12] Feddersen, Timothy J. and Wolfgang Pesendorfer. 1996. "The Swing Voter's Curse." *The American Economic Review*, 86(3): 408-424.

[13] Lijphart, Arend. 1997. "Unequal Participation: Democracy's Unresolved Dilemma." *The American Political Science Review*, 91(1): 1-14.

[14] McLennan, Andrew. 1998. "Consequences of the Condorcet Jury Theorem for Beneficial Information Aggregation by Rational Agents." *American Political Science Review*, 92(2): 413-418.

[15] McMurray, Joseph C. 2010a. "Empirical Evidence of Strategic Voter Abstention." Unpublished.

[16] McMurray, Joseph C. 2010b. "Information and Voting: the Wisdom of the Experts versus the Wisdom of the Masses." Unpublished

[17] Meirowitz, Adam, and Kenneth W. Shotts. 2007. "Pivots Versus Signals in Elections." Paper presented at the 14th Annual Conference on Political Economy of the Wallis Institute of Political Economy, Rochester, NY.

[18] Myerson, Roger. 1998. "Population Uncertainty and Poisson Games." *International Journal of Game Theory*, 27: 375-392.

[19] Myerson, Roger. 2000. "Large Poisson Games." *Journal of Economic Theory*, 94: 7-45.

[20] Osborne, Martin J., and Al Slavinski. 1996. "A Model of Political Competition with Citizen-Candidates." *Quarterly Journal of Economics*, 111(1): 65-96.

[21] Palfrey, Thomas R. and Keith T. Poole. 1987. "The Relationship between Information, Ideology, and Voting Behavior." *American Journal of Political Science*, 31(3): 511-530.

[22] Piketty, Thomas. 2000. "Voting as Communicating." *The Review of Economic Studies*, 67: 169-191.

[23] Razin, Ronny. 2003. "Signaling and Election Motivations in a Voting Model with Common Values and Responsive Candidates." *Econometrica*, 71(4): 1083-1119.

[24] Riker, William H. and Peter C. Ordeshook. 1968. "A Theory of The Calculus of Voting." *American Political Science Review*, 62: 25-42.

[25] Shapley, Lloyd and Bernard Grofman. 1984. "Optimizing Group Judgmental Accuracy in the Presence of Interdependencies." *Public Choice*, 43: 329-343.

[26] Shotts, Kenneth W. 2006. "A Signaling Model of Repeated Elections." *Social Choice and Welfare*, 27: 251-261.

[27] Wittman, Donald. 1977. "Candidates with Policy Preferences: A Dynamic Model." Journal of Economic Theory, 14: 180-89.

[28] Wittman, Donald. 1983. "Candidate Motivation: A Synthesis of Alternative Theories." *American Political Science Review*, 77(1): 142-157.

[29] Wolfinger, Raymond E. and Rosenstone, Stephan J. 1980. *Who Votes?* New Haven: Yale University Press.