# Information Aggregation in Elections with Endogenous Platforms 

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## PRELIMINARY - WORK IN PROGRESS


#### Abstract

I study a model of representative democracy with asymmetric information. Two solely office-motivated candidates know the state of nature, while each voter receives an independent imperfect signal. I show that a minimal amount of private information for the voters ensures the existence of robust, sincere voting rules, i.e. rules which lead to full information equivalence in all sequential equilibria. Moreover, if there is a strictly positive probability for the candidates to always propose the ex-post optimal platform, then there is only one possible electoral outcome, that is, one in which even office-motivated candidates always propose the ex-post optimal platform. This result is robust to some plausible extensions of the model, including fully strategic voting, policy-motivated candidates, sequential political campaigns and different majority rules.


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[^0]The secret of the demagogue is to appear as dumb as his audience so that these people can believe themselves as smart as he. Kraus 1990, p. 113)

## 1 Introduction

It is a common adage that the human mind is easily persuaded of what it already thinks. In politics, this principle stands at the basis of what is perhaps the most common and ancient criticism of representative democracy. Electors are often not well informed of the dimension or even the nature of the problem in question, while politicians have access to better sources of information. Hence, voters would prefer that candidate whose platform most concurs with the information available to them, and an office-motivated candidate would have an advantage over a candidate whose platform always reflects the real ex-post preferences of the electorate. In conclusion, there are situations in which asymmetric information condemns democracy to fail to choose the right policy.

This criticism of representative democracy demands for a deeper understanding of the mechanisms of information aggregation when the set of alternatives among which voters are called to express their preferences are endogenously determined by the electoral competition among multiple candidates.

The Condorcet jury literature gives us as a rationale for voting: the group has better information than single individuals, and information can be aggregated through the voting process. This literature assumes that voters vote for an alternative (which can be interpreted as a candidate) and the uncertainty resides on which of these alternatives is best.

On the other side, we often think of candidates as agents selecting their platforms from a larger set of possible alternatives, i.e. from a set of feasible policies. Therefore, it is natural to distinguish the two concepts of candidates and policies: candidates are strategic agents while policies are collective actions.

Since candidates are strategic in choosing their platform, we might expect an officemotivated candidate to propose that platform that is ex-ante perceived as best by the majority of the voters. Hence, when this policy is not ex-post optimal, the set of alternatives voters might end up voting for might not contain at all the optimal alternative.

In this paper, I study an election with two informed strategic candidates, each proposing a policy to a continuum of less informed voters. I find the argument that information asymmetries condemn democracy to failure partly right but mostly wrong: a minimal amount of private information for the voters ensures the existence of robust voting rules, i.e. rules which lead to full information equivalence in all sequential equilibria. Moreover, if there is a strictly positive probability for each candidate to always propose the ex-post optimal
platform, then there is only one possible electoral outcome, that is, one in which even officemotivated candidates always propose the ex-post optimal platform. Hence, my argument is that electoral competition among candidates is sufficient to select the best of all the feasible alternatives even when voters' ex-ante preferences are in favor of a different alternative. The reason why this is the case is precisely that the argument that an office-motivated candidate as an advantage over a truthful candidate is not true. In any equilibrium, for any strategy the office-motivated candidate might play, her expected payoff is less than or equal to that of the truthful candidate.

It is not difficult to imagine situations in which candidates are better informed than voters about the state of nature. After all, parliamentarians and members of the government spend most of their time and resources trying to gain information about the results of different policies. For example, were a polity to decide whether to enter a war, politicians would have access to informative of the secret services not available to the general public. Or were the debate about whether to invest public resources in a major infrastructure, a candidate would receive detailed information on the costs and benefits of such a project from her own party and other institutional sources, information that would be available to the general public only through more noisy media reports.

It is worth noticing at this point that all the results in this paper are reached assuming voters act naively, that is to say they vote for the candidate whose platform gives them the highest expected payoff. Hence, the mechanism through which information is aggregated is not related to strategic voting (in the sense of Myerson and Weber (1993)). On the opposite, it is the strategic choice of the candidates that delivers in equilibrium the optimal policy in each state of nature. In section 6, I show that, if voters are fully strategic, full information equivalence is satisfied by the unique equilibrium

A large literature has contributed to our understanding of the mechanisms of information aggregation in elections. A strain of this literature has made use of the metaphor of the jury: voters choose the among pre-determined alternatives and the uncertainty relies on which alternative is best. The arguments in this paper are somewhat related to recent works on Condorcet jury theorem and voting equilibria (for example, Myerson and Weber, 1993; Feddersen and Pesendorfer, 1997; Bhattacharya, 2008). While in the jury metaphor policy alternatives are assumed to be exogenous, in a representative democracy the alternatives among which the electorate is called to express its preferences are not exogenous, but determined in equilibrium by the strategic choice of candidates and parties. I show that the introduction of this level of strategic behavior provides a different source of information aggregation. To the best of my knowledge, this is the first work to address the question of how electoral competition combines with voting equilibria: Bond and Eraslan (2010) consider a
signaling game with one agent and many principal-voters.
On the other hand, since Downs (1957), another strain of literature has investigated the role of politicians and parties in selecting the alternatives (platforms) voters should choose from. The arguments in this paper are related in some respect to those of Schultz 1995, 1996) who studied a model of political competition in representative democracy with mainly policy motivated candidates. Schultz (1996) shows that, when the political parties have sufficiently non-polarized preferences, the informational asymmetry problem is solved by electoral competition and policy adjusts to the median voters' ex-post preferences. Callander and Wilkie (2007) develop a model allowing policy-motivated candidates in the campaign stage to misrepresent their policy intentions if elected in office. Harrington (1993) considers the incentives of an incumbent politician to target voters' prior beliefs: the extent to which the incumbent chooses policy according to the voters' prior is larger if the voters' prior is strong and the incumbent is mostly office concerned. Frisell (2004) and Binswanger and Prufer (2009) suggest that the more informed politicians are about the opinion of the electorate, and the more bounded is the use of Bayesian updating on the side of the voters, the more likely it is that politicians will provide voters with an independent viewpoint. Finally, Austen-Smith and Banks (1988) consider a similar problem in the contest of a three party proportional system. In this second strain of literature, the uncertainty still relies on the set of alternatives the voters can choose from, though here the alternatives are the candidates themselves, rather than policies.

To the best of my knowledge, the question of how the electoral competition can aggregate information when politicians select their platforms from the set of alternatives and the uncertainty relies on the underlying set of alternatives, and not on candidates' preferences, has received little attention. One example can be the work of Ghosh and Tripathi (2009), considering voters' behavior in an election between an ideologue committed to a fixed policy and an idealist candidate who implements the ex-post socially optimal policy.

The arguments of this paper also relate to the literature on cheap talk (e.g. Crawford and Sobel (1982)) and in particular to the strain of this literature that has studied the case when there is more than one sender (examples include Austen-Smith (1993); Battaglini (2002)).

The remainder of the paper is structured as follows. Section 2 introduces the model. I provide results regarding pooling equilibria in the case of two purely office-motivated candidates in section 3. Section 4 presents a characterization of the set of separating (and partially separating) equilibria. In section 5, I show that the equilibrium is essentially unique and satisfies full information equivalence if there is a strictly positive probability that one of the two candidates proposes the ex-post optimal platform in all states. I then turn my attention to some plausible extensions of the model and section 9 concludes.

## 2 The model

Consider a polity facing a finite set of different policy alternatives, $e \in \mathcal{E}:=\{1,2, \ldots, \mathcal{E}\}$. There is a continuum ${ }^{1}$ of voters whose utility depends on the policy chosen and on an uncertain state of nature $\theta \in \Theta:=\{1,2, \ldots, \Theta\}$. Denote by $u^{i}(e, \theta):=u(e, \theta)$ the Von Neumann-Morgenstern utility function of voter $i$ and denote by

$$
e^{*}(\theta):=\arg \max _{e \in \mathcal{E}} u(e, \theta)
$$

the optimal policy in state $\theta$.
Each voter is uncertain about the realization of the state and the common prior over the set of states $\Theta$ is given by $\boldsymbol{q}:=\left(q_{1}, q_{2}, \ldots, q_{\Theta}\right)$, where $q_{j}:=\operatorname{Pr}(\theta=j)$. Each voter receives a signal $s^{i}$, conditionally independent and identically distributed over a finite set $\mathcal{S}:=\{1,2, \ldots, S\}$. Denote by $p(s, \theta):=\operatorname{Pr}(s \mid \theta)>0$ the conditional probability of a voter receiving signal $s$ if state $\theta$ is realized. I assume that each signal is informative about all states so that $p(s, \theta) \neq p\left(s, \theta^{\prime}\right)$ for each $\theta^{\prime} \neq \theta$. I will denote by $\nu(\theta, s)$ the probability that state $\theta$ is realized, conditional on receiving signal $s$ :

$$
\nu(\theta, s):=\frac{p(s, \theta) q_{\theta}}{\sum_{\hat{\theta} \in \Theta} p(s, \hat{\theta}) q_{\hat{\theta}}} .
$$

Two candidates, $A$ and $B$, simultaneously determine platforms $x:=\left(x_{T}, x_{D}\right) \in \mathcal{E}^{2}$. Each candidate $c \in\{A, B\}$ can be of two types $\tau^{c} \in\{s, t\}$, with $\operatorname{Pr}\left(\tau^{c}=s\right)=1-\delta$. Type $s$ is a strategic, purely office-motivated agent, and receives a rent $R>0$ if elected. If candidate $c$ is of type $t$, then she is truthful and always proposes $x_{c}(\theta)=e^{*}(\theta)$. The analysis of the following sections makes an explicit differentiation between two cases, namely when both candidates are surely strategic $(\delta=0)$ and when there is a strictly positive probability that a candidate is of the truthful type $(\delta>0)$. The type of each candidate is private information of the candidate and voters ignore if any of the candidates is of the truthful type and, if any, which one ${ }^{2}$.

A candidate is elected with probability 1 if she receives the largest share of votes and with probability $\frac{1}{2}$ in case of a tie. The platform of the winning candidate is implemented with probability 1 . Voters are assumed to cast their vote sincerely in the sense that voter $i$

[^1]casts his vote in favor of candidate $c \in\{A, B\}$ if $E_{\Theta}\left[u\left(x_{c}, \theta\right)\right] \geq E_{\Theta}\left[u\left(x_{-c}, \theta\right)\right]$, where $x_{-c}$ is the platform presented by candidate $c$ 's opponent. Notice that a voter can mix between the two candidates if the two platforms give the same expected payoff.

The timing of the electoral game is as follows: at stage 1 , nature chooses a state $\theta \in \Theta$ and candidates' types; at stage 2, candidates $A$ and $B$ observe the state $\theta$ and propose platforms $x_{A}$ and $x_{B}$, respectively; at stage 3 , each voter $i$ observes signal $s^{i} \in \mathcal{S}$ and candidates' platform profile $x$, and casts a vote in favor of either candidate $A$ or $B$. The candidate with the largest share of votes receives a rent $R$ and her platform is implemented.

I make the following assumptions:
Assumption 1. For all $\theta \in \Theta$, there is only one optimal policy, i.e. $\exists!e \in \mathcal{E}: e \in e^{*}(\theta)$ and there exists at least two states $\theta \neq \theta^{\prime}$ for which $e^{*}(\theta) \neq e^{*}\left(\theta^{\prime}\right)$.

If the platforms are identical $\left(x_{A}=x_{B}\right)$, all voters mix equally among the two candidates. $u(e, \theta)$ is bounded.

Assumption 1 says that a pooling equilibrium never satisfies full information equivalence. This is merely for expositional purposes, since when an equilibrium satisfying full information equivalence exists, then this exists even if it is a pooling equilibrium.

Assumption 2 greatly simplifies the analysis by eliminating equilibria in which a candidate is indifferent among different policies only because all voters would prefer the other candidate when indifferent. I justify this assumption by claiming that since when both candidates chose the same platforms, the two are perfectly indistinguishable and the policy outcome is predetermined, a voter has no interest in choosing any particular mixed strategy over another and there is no reason why a particular mixed strategy should be chosen by all voters.

Assumption 3 is a mere technicality.
In the remainder of the paper, I will characterize the set of electoral equilibria of this model, where this would correspond to the set of sequential equilibria (Kreps and Wilson, 1982). I will refer to the (behavioral) strategy of candidate $c$ as $x_{c}(\theta)$, with $x(\theta):=$ $\left(x_{A}(\theta), x_{B}(\theta)\right)^{3}$. I impose no further refinement on the equilibrium concept, though not trembling hand perfection, intuitive criterion or properness would bite here.

Notice that in an election in which the state is known, voters would not have any difficulty in approving the optimal policy $e^{*}(\theta)$ in any state $\theta \in \Theta$. It is therefore a natural question whether representative democracy is capable of aggregating information such that the political outcome of the elections is as if the state was known. Following Feddersen and Pesendorfer (1997), I label this condition full information equivalence ${ }^{4}$ :

[^2]Definition 1. Call $\lambda_{e}(\theta)$ the probability that the elections will be won by a candidate with $x_{c}=e$ in state $\theta$. An electoral equilibrium satisfies full information equivalence if, for all $\theta \in \Theta, \lambda_{e}(\theta)>0 \Longleftrightarrow e=e^{*}(\theta)$.

### 2.1 A $2 \times 2$ example

In order to expose the intuition that drives my analysis, in the remainder of the paper I will often refer to the following example. There are two states, $\Theta=\left\{\theta_{1}, \theta_{2}\right\}$, with $q_{\theta_{1}}<\frac{1}{2}$ (for the numerical example: $q_{\theta_{1}}=.2$ ), and two policies, $\mathcal{E}=\left\{e_{1}, e_{2}\right\}$. I will refer to $\theta_{1}$ as the rare state, since it verifies with lower probability than $\theta_{2}$.

A continuum of voters indexed by $i$ has preferences given by the utility function $u^{i}(e, \theta)=$ $u(e, \theta)$ such that $u\left(e_{k}, \theta_{k}\right)=1>u\left(e_{j}, \theta_{k}\right)=0, k \neq j$, all $k, j=1,2$. Hence, policy $e_{1}$ is optimal in the rare state $\theta_{1}$, while policy $e_{2}$ would be preferred otherwise. Voters do not know the state of nature, but each voter $i$ receives signal $s^{i} \in\left\{s_{1}, s_{2}\right\}$, with $\frac{1}{2}>p\left(s_{1}, \theta_{1}\right)>$ $p\left(s_{1}, \theta_{2}\right)$, where $p(s, \theta):=\operatorname{Pr}(s \mid \theta)$ (for the numerical example I will use $p\left(s_{1}, \theta_{1}\right)=.2>$ $\left.p\left(s_{1}, \theta_{2}\right)=.1\right)$. Therefore, signal $s_{1}$ is relatively more likely in the rare state $\theta_{1}$. Nevertheless, a large majority of voters observes signal $s_{2}$ in both states. Furthermore, notice that in the numerical example the expected payoff of policy $e_{2}$ is greater than the expected payoff of policy $e_{1}$, independently of the signal received: $E_{\Theta}\left[u\left(e_{1}, \theta\right) \mid s_{1}\right]=\frac{1}{3}<\frac{2}{3}=E_{\Theta}\left[u\left(e_{2}, \theta\right) \mid s_{1}\right]$. If the voters were called to express their preference over one of the two policies, they would prefer policy $e_{2}$ independently of the signal received. Hence, we can refer to policy $e_{2}$ as the ex-ante Condorcet winner. Nevertheless, if both candidates were to propose the same policy, say $e_{1}$, voters would remain with no choice and policy $e_{1}$ would be implemented.

## 3 Pooling equilibria with only strategic candidates ( $\delta=$ $0)$

In this section I characterize the set of pooling equilibria in the case of two strategic candidates $(\delta=0)$. Indeed, proposition 2, in section 4, shows that no such an equilibrium can exist otherwise. The main result in this section informs us that while it is possible for both candidates to converge to the same policy in all states, this is not true for all policies. In particular, consider a policy $e$ for which there exists $e^{\prime} \in \mathcal{E}$ such that $u(e, \theta)<u\left(e^{\prime}, \theta\right)$ in all states. I will refer to such a policy as a dominated policy:
stringent since it requires that an optimal policy is chosen with probability 1. Aside from technicalities, the two definitions are conceptually equivalent.

Definition 2. A policy $e \in \mathcal{E}$ is said to be dominated if there exists a policy $e^{\prime}$ such that $u(e, \theta)<u\left(e^{\prime}, \theta\right)$ for all $\theta \in \Theta$.

Consider a dominated policy $e$ and suppose that candidate $A$ is choosing strategy $x_{A}(\theta)=$ $e$ for some state $\theta$. If candidate $B$ imitates $A$, then she would receive a share of votes equal to $\frac{1}{2}$ and expect her payoff to be $\frac{R}{2}$. On the other hand, were $B$ to deviate to $x_{B}(\theta)=e^{\prime}$, voters would grant her their preferences. This is because the expected payoff of $e^{\prime}$ is greater than the expected payoff of $e$ for all voters, independently of one's beliefs about the realized state. Therefore, we can conclude that policy $e$ is never the platform of any candidate.

Proposition 1 says $(i)$ that any non dominated policy $e$ can be chosen in some equilibrium by both candidates in some states and (ii) that there exist pooling equilibria characterized by $x_{A}(\theta)=x_{B}(\theta)=e$, for all $\theta$.

Proposition 1. If $\delta=0$, a pooling electoral equilibrium with $x(\theta)=(e, e)$, all $\theta$, $e \in \mathcal{E}$, exists if and only if e is a non dominated policy.

To give an intuition, recall that if a policy $e$ is non dominated, for any other policy $e^{\prime}$, there is at least one state $\theta$ in which $e$ is better than $e^{\prime}$. If voters are convinced that state $\theta$ is realized every time they observe $x=\left(e, e^{\prime}\right)$, then no candidate has incentives to deviate from platforms $x_{c}=e$ to $x_{c}=e^{\prime}$. Notice that this assessment is consistent since it is the limit of mixed strategies converging to $x_{c}=e$ with probability 1 in a slower fashion in state $\theta$ than in any other state. The proof of proposition 1 generalizes this argument.

Proof. I first proof that if $e \in \mathcal{E}$ is not dominated, then $x(\theta)=(e, e)$ for all $\theta \in \Theta$ is a sequential equilibrium. Consider therefore a sequence of assessments in which both candidates always converge to the same non dominated platform $e$. Also, recall that if a voter is indifferent between the two candidates, he votes for each candidate with probability $\frac{1}{2}$.

If $e$ is not dominated, then for all $e^{\prime} \neq e$, there exists a non-empty set $\mathcal{O}\left(e, e^{\prime}\right) \subseteq \Theta$ such that $u(e, \theta) \geq u\left(e^{\prime}, \theta\right)$ for all $\theta \in \mathcal{O}\left(e, e^{\prime}\right)$. Consider then a sequence $n \rightarrow \infty$ of assessments such that $x_{c}^{n}(\theta)=e$ with probability $1-\alpha^{n}, \lim _{n \rightarrow \infty} \alpha^{n}=0$, for all $\theta \in \Theta$ and all $c \in\{A, B\}$. Furthermore,

$$
\operatorname{Pr}\left(x_{c}(\theta)=e^{\prime}\right)= \begin{cases}\epsilon^{n} & \text { if } \theta \in \mathcal{O}\left(e, e^{\prime}\right) \\ \eta^{n} & \text { otherwise }\end{cases}
$$

with $\epsilon^{n}, \eta^{n} \in \mathbb{R}_{++}$such that $\epsilon^{n}|\mathcal{O}|+\eta^{n}|\Theta \backslash \mathcal{O}|=\alpha^{n}$ and $\lim _{n \rightarrow \infty} \frac{\eta^{n}}{\epsilon^{n}}=0$. Then, as $n \rightarrow \infty$, if $x=(e, e), \operatorname{Pr}(\theta \mid x, s) \rightarrow \nu(\theta, s)$ while if $x \in\left\{\left(e, e^{\prime}\right),\left(e^{\prime}, e\right)\right\}$,

$$
\operatorname{Pr}(\theta \mid x, s) \rightarrow \begin{cases}1 & \text { if } \theta \in \mathcal{O}\left(e, e^{\prime}\right) \\ 0 & \text { otherwise }\end{cases}
$$

It follow that any deviation from $x_{c}(\theta)=e$ for any candidate $c \in\{A, B\}$ cannot constitute an individual gain. We can therefore conclude that $x(\theta)=(e, e)$, all $\theta$, is part of a sequential equilibrium.

I turn now to demonstrate that if $e$ is a dominated policy, then there cannot be a sequential equilibrium such that $x(\theta)=(e, e)$, all $\theta$. Suppose in fact that such an equilibrium exists and consider a deviation of a candidate, say $A$, to $x_{A}=e^{\prime}$, where $u(e, \theta)<u\left(e^{\prime}, \theta\right)$ for all $\theta$. Then, for any beliefs about the state $\theta$, each voter would cast his vote in favor of candidate $A$. We can conclude that if $e$ is dominated, $x(\theta)=(e, e)$ cannot be a part of any equilibrium for any $\theta \in \Theta$.

### 3.1 Pooling equilibria in the $2 \times 2$ example

The $2 \times 2$ example of section 2.1 has two pooling sequential equilibria: one in which both candidates converge to $x_{c}=e_{2}$ in both states, and another in which both candidates converge to $x_{c}=e_{1}$ in both states. In order to construct the beliefs necessary to sustain the second equilibrium, it is sufficient to think of a sequence of completely mixed strategies for the candidates in which both candidates chose with increasing probability $e_{1}$ in both states, but the probability of deviating from this pure strategy is of an order of magnitude larger in state $\theta_{1}$ than in state $\theta_{2}$. If this is the case, a voter observing a candidate deviating from $x_{c}=e_{1}$, will think that it must be in state $\theta_{1}$, in which case he will prefer to cast his vote for the candidate non deviating from the equilibrium path. It is worth noticing that the electoral equilibrium with outcome $e_{1}$ in all states has a lower expected payoff for the voters than choosing the ex-ante preferred policy $e_{2}$.

Clearly this is a very simple example, with only two states and two policies. One might expect that if there was a third alternative giving utility close enough to the optimal policy in each state, an unilateral deviation for candidate $D$ to such a policy would be a winning strategy. Furthermore, with a large number of states and policy, suppose that policy $e$ is optimal and slightly better than policy $e^{\prime}$ only in state $\theta$, but it is dramatically worse than a policy $e^{\prime}$ in all other states. Then we could expect to be very difficult for the polity to chose policy $e$ if and only if the state $\theta$ is verified. However, I show that the same intuition described in this simple example holds for generic finite sets of states, policies and signals and is robust to some plausible extensions of the model.

## 4 Separating equilibria and full information equivalence

In this section, I characterize the set of separating (and partially separating) equilibria of the electoral model described in section 2. First, I prove (lemma 1) that in all electoral equilibria candidates must converge to the same pure strategy with probability 1. This means that only equilibria in pure strategies can exist and that the pure strategies of the two candidates must be identical. To give an intuition, consider the simple $2 \times 2$ example of section 2.1 and imagine that in equilibrium candidate $A$ would mix between the two platforms in state $\theta_{1}$. This implies that she would not gain by deviating and choosing platform $e_{1}$ (or $e_{2}$ ) for sure. Notice that this means that voters have to form beliefs using Bayes' rule if the node $\left(e_{1}, e_{2}\right)$ is reached. There are three possibilities: $(i)$ the majority of the voters strictly prefers $e_{1}$, in which case it would be fool for $A$ not to deviate to $e_{1},(i i)$ the majority of the voters strictly prefer $e_{2}$, in which case it would be fool not to deviate to $e_{2}$, and (iii) some, but not all, voters are indifferent. If the majority of the voters is indifferent, which implies they are those receiving signal $s_{2}$, then it is possible that they might mix between the two candidates in such a way that, in state $\theta_{1}$, $A$ 's chances of winning the elections are equal if she plays $x_{A}\left(\theta_{1}\right)=e_{1}$ or $x_{A}\left(\theta_{1}\right)=e_{2}$. But this cannot be true if the realized state is $\theta_{2}$, since those voters that were mixing between the two policies are more in state $\theta_{2}$ than in state $\theta_{1}$. Hence, if $\theta_{2}$ is realized, both candidates would chose $x_{c}=e_{2}$ with probability 1 . We can now turn to ask whether it is possible for some voter to be indifferent between the two different policies in state $\theta_{1}$ when candidates present different platforms. The answer is clearly negative, since this event verifies only in state $\theta_{1}$, in which case all the voters strictly prefer policy $e_{1}$. A very similar argument guarantees convergence among the two candidates.

Lemma 1. In any electoral equilibrium, in each state $\theta \in \Theta$, both candidates converge to the same platform with probability 1, i.e. the strategy profile for the candidate is $x(\theta)=$ $\left(x_{A}(\theta), x_{B}(\theta)\right)$ with $x_{A}(\theta)=x_{B}(\theta)$, all $\theta$.

Proof. Suppose, by contradiction, that an equilibrium in mixed strategies exists. Then it must be that $\operatorname{Pr}\left(\right.$ cwins $\left.\mid x_{c}=e, \theta\right)=\operatorname{Pr}\left(c\right.$ wins $\left.\mid x_{c}=e^{\prime}, \theta\right)$ for some $e, e^{\prime} \in \mathcal{E}$ and some opponent's strategy $x_{-c}(\theta)$. Since this is an equilibrium in mixed strategies, beliefs on the nodes reached after the moves $x \in\left\{\left(e, e^{\prime}\right),\left(e^{\prime}, e\right)\right\}$ are to be computed through Bayesian updating. Since the signal is always correlated with the states, then it must be that for any $s \neq s^{\prime}, \operatorname{Pr}(\theta \mid x, s) \neq \operatorname{Pr}\left(\theta \mid x, s^{\prime}\right)$. This implies that for at most one signal, $s_{\sim}$, $E_{\Theta}\left[u(e, \theta) \mid x, s_{\sim}\right]=E_{\Theta}\left[u\left(e^{\prime}, \theta\right) \mid x, s_{\sim}\right]$. For any mixed strategy of the voters receiving signal $s_{\sim}$, there is a majority of voters who would vote for one of the two candidates in all but
at most one state $\theta_{\sim}$. In all other states, both candidates will converge with probability 1 to one single platform. It follows that if a voter observes $x \in\left\{\left(e, e^{\prime}\right),\left(e^{\prime}, e\right)\right\}$, then it must be that the state is $\theta_{\sim}$, implying that a deviation to $x_{c}\left(\theta_{\sim}\right)=e^{*}\left(\theta_{\sim}\right)$ constitutes a gain for candidate $c$, contradicting the hypothesis that in state $\theta_{\sim}$ the candidates would have randomized among different platforms.

A similar argument applies to show that, in any sequential equilibrium, candidates must always converge to identical platforms. Suppose in fact that in state $\theta_{\sim}$ the candidates chose different platforms, i.e. $x_{A}\left(\theta_{\sim}\right) \neq x_{B}\left(\theta_{\sim}\right)$. For this to be true, it must be that each of the candidates has a probability equal to $\frac{1}{2}$ to win the elections. It follows that voters must be uncertain about the state $\theta$ when they observe $x=\left(x_{A}\left(\theta_{\sim}\right), x_{B}\left(\theta_{\sim}\right)\right)$. For any mixed strategy of the voters receiving a single signal $s_{\sim}$, there is a majority of voters who would vote for one of the two candidates in at least all states but state $\theta_{\sim}$. Therefore, in all other states, candidates' action profile would be different. It follows that if a voter observes $x=\left(x_{A}\left(\theta_{\sim}\right), x_{B}\left(\theta_{\sim}\right)\right)$, then it must be that the state is $\theta_{\sim}$, implying that a deviation to $x_{c}\left(\theta_{\sim}\right)=e^{*}\left(\theta_{\sim}\right)$ constitutes a gain for any candidate $c$, contradicting the hypothesis that in state $\theta_{\sim}$ the candidates would have chosen different platforms.

I turn now prove the existence of a separating equilibrium satisfying full information equivalence. The fundamental intuition relies on the arguments made in the introduction about the existence of fully revealing equilibria in the $2 \times 2$ example. What is important to notice is that, outside the equilibrium path, it is possible to have some voters preferring the optimal policy and others preferring an alternative policy (in particular one that is optimal in some other state). But in an equilibrium satisfying full information equivalence, such a platform profile is expected to be possible almost exclusively in states in which at least one of the two policies is optimal. Since the signals are completely correlated on the states, the share of voters preferring policy $e^{*}(\theta)$ over $e^{\prime} \neq e^{*}(\theta)$ is larger in state $\theta$ than in any state $\theta^{\prime}$ for which $e^{\prime}=e^{*}\left(\theta^{\prime}\right)$. Therefore it is possible to have some (possibly mixed) strategy for the voters, conditional on the signal received, such that the majority of them prefers $e^{*}(\theta)$ to $e^{\prime}$ in state $\theta$, and viceversa in state $\theta^{\prime}$. Proposition 2 formalizes this argument.

Proposition 2. There exists an electoral equilibrium satisfying full information equivalence.
Proof. Consider a sequence of completely mixed assessments ordered by $n$ such that $x_{c}^{n}(\theta)=$ $e^{*}(\theta)$ with probability $1-\alpha_{\theta}^{n}, \lim _{n \rightarrow \infty} \alpha_{\theta}^{n}=0$, for all $\theta \in \Theta$ and all candidates $c \in\{A, B\}$, where $\alpha_{\theta}^{n} \in \mathbb{R}_{++}$, all $\theta$. Also, for all $e \neq e^{*}(\theta), x_{c}^{n}(\theta)=e$ with probability $\epsilon_{e}^{n}(\theta)$ such that $\sum_{e \neq e^{*}(\theta)} \epsilon_{e}^{n}(\theta)=\alpha_{\theta}^{n}$. I want to show now that it is possible to construct $\left\{\epsilon_{e}(\theta)\right\}_{e \in \mathcal{E} \backslash e^{*}(\theta)}$, all $\theta$, such that $\lim _{n \rightarrow \infty} \operatorname{Pr}(\theta \mid x, s), x \in\left\{\left(e^{*}(\theta), e\right),\left(e, e^{*}(\theta)\right)\right\}$ is finite if $e=e^{*}\left(\theta^{\prime}\right)$ for some state $\theta^{\prime} \neq \theta$, and 1 if $e \neq e^{*}(\theta)$ for all $\theta \in \Theta$. Call $P^{\sigma}(x \mid \theta)$ the probability that the action
profile $x$ is chosen in state $\theta$ under the mixed strategies $\sigma$ of each assessment. Then it must be that (omitting the superscript $n$ )

$$
\begin{aligned}
& \operatorname{Pr}\left(\hat{\theta} \mid\left(e^{*}(\hat{\theta}), e^{*}(\hat{\hat{\theta}})\right), s\right)=\frac{P^{\sigma}(x \mid \hat{\theta}) p(s, \hat{\theta}) \nu(\hat{\theta}, s)}{\sum_{\Theta} P^{\sigma}(x \mid \theta) p(s, \theta) \nu(\theta, s)}= \\
&= \overbrace{\left(1-\alpha_{\hat{\theta}}\right) p(s, \hat{\theta}) \nu(\hat{\theta}, s)}^{\sum_{D_{\Theta \backslash\{\hat{\theta}, \hat{\theta}\}} \epsilon_{e}(\theta)^{2} p(s, \theta)}^{\epsilon_{e^{*}(\hat{\hat{\theta}})}(\hat{\theta})}}+\underbrace{N}_{D_{1}} \\
& \underbrace{\left(1-\alpha_{\hat{\theta}}\right) p(s, \hat{\theta}) \nu(\hat{\theta}, s)}_{D_{2}} \underbrace{\frac{\epsilon_{e^{*}(\hat{\theta})}(\hat{\hat{\theta}})}{\epsilon_{e^{*}(\hat{\hat{\theta}})}(\hat{\theta})}\left(1-\alpha_{\hat{\theta}}\right) p(s, \hat{\hat{\theta}}) \nu(\hat{\hat{\theta}}, s)}_{D_{3}}
\end{aligned}
$$

When $\alpha_{\theta} \rightarrow 0(n \rightarrow \infty)$, for all $\theta \in \Theta, D_{1} \rightarrow 0$. This implies that, in the closure of this sequence of assessments, if an action profile $x=\left(e, e^{\prime}\right)$ is observed by voters, the only possible states are those for which either $e$ or $e^{\prime}$ are optimal.

Furthermore, it is possible to construct the series of $\left\{\epsilon_{e}(\theta)\right\}_{e \in \mathcal{E} \backslash e^{*}(\theta)}$, all $\theta$, such that $\operatorname{Pr}\left(\hat{\theta} \mid\left(e^{*}(\hat{\theta}), e^{*}(\hat{\hat{\theta}})\right), s\right) \rightarrow k \in(0,1)$, where $k$ is always finite. Hence, for any pair of states, there exists at most one signal, $s_{\sim}$, such that a voter receiving signal $s_{\sim}$ is indifferent among the two policies optimal in each state if both are presented by one candidate, i.e.

$$
E_{\Theta}\left[u\left(e^{*}(\hat{\theta}), \theta\right) \mid\left(e^{*}(\hat{\theta}), e^{*}(\hat{\hat{\theta}})\right), s_{\sim}\right]=E_{\Theta}\left[u\left(e^{*}(\hat{\hat{\theta}}), \theta\right) \mid\left(e^{*}(\hat{\theta}), e^{*}(\hat{\hat{\theta}})\right), s_{\sim}\right] .
$$

Therefore, any such a sequence of assessments partitions the set of signals $\mathcal{S}$ in three possibly empty subsets:

$$
\begin{aligned}
& \mathcal{S}_{e^{*}(\hat{\theta})}:=\left\{\begin{array}{l}
s \in \mathcal{S}: E_{\Theta}\left[u\left(e^{*}(\hat{\theta}), \theta\right) \mid\left(e^{*}(\hat{\theta}), e^{*}(\hat{\hat{\theta}})\right), s_{\sim}\right]> \\
>E_{\Theta}\left[u\left(e^{*}(\hat{\hat{\theta}}), \theta\right) \mid\left(e^{*}(\hat{\theta}), e^{*}(\hat{\hat{\theta}})\right), s_{\sim}\right]
\end{array}\right\} \\
& \mathcal{S}_{e^{*}(\hat{\hat{\theta}})}:=\left\{\begin{array}{l}
s \in \mathcal{S}: E_{\Theta}\left[u\left(e^{*}(\hat{\theta}), \theta\right) \mid\left(e^{*}(\hat{\theta}), e^{*}(\hat{\hat{\theta}})\right), s_{\sim}\right]< \\
<E_{\Theta}\left[u\left(e^{*}(\hat{\hat{\theta}}), \theta\right) \mid\left(e^{*}(\hat{\theta}), e^{*}(\hat{\hat{\theta}})\right), s_{\sim}\right]
\end{array}\right\}
\end{aligned}
$$

and $s_{\sim}$.
Consider now a mixed strategy for those voters receiving signal $s_{\sim}$ such that, with probability $a \in[0,1]$, they vote for the candidate proposing platform $e^{*}(\hat{\theta})$ and, with probability $1-a$, they vote for the candidate proposing platform $e^{*}(\hat{\theta})$, if they observe an action profile
containing both platforms. Notice that

$$
\begin{equation*}
\frac{\sum_{\mathcal{S}_{e^{*}(\hat{\hat{\theta}}}} p(s, \hat{\theta})+(1-a) p\left(s_{\sim}, \hat{\theta}\right)}{\sum_{\mathcal{S}_{e^{*}(\hat{\hat{\theta}})}} p(s, \hat{\hat{\theta}})+(1-a) p\left(s_{\sim}, \hat{\hat{\theta}}\right)}<\frac{\sum_{\mathcal{S}_{e^{*}(\hat{\theta})}} p(s, \hat{\theta})+a p\left(s_{\sim}, \hat{\theta}\right)}{\sum_{\mathcal{S}_{e^{*}(\hat{\theta})}} p(s, \hat{\hat{\theta}})+a p\left(s_{\sim}, \hat{\hat{\theta}}\right)} \tag{1}
\end{equation*}
$$

and the sum of the numerators (denominators) on the two sides of (1) is equal to 1 . This implies that the share of voters voting in favor of the candidate presenting platform $e^{*}(\hat{\theta})$ $\left(e^{*}(\hat{\hat{\theta}})\right)$ is larger (smaller) in state $\hat{\theta}$ than in state $\hat{\hat{\theta}}$. This in turn means that there always exists a partition $\mathcal{S}_{e^{*}(\hat{\theta})}, \mathcal{S}_{e^{*}(\hat{\hat{\theta}})}, s_{\sim}$ and a voting rule $a$ for those voters receiving signal $s_{\sim}$ such that the majority of the voters is voting in favor of the candidate presenting the optimal platform in each state.

Given the arguments above, we can conclude that there exist a series $\left\{\epsilon_{e}(\theta)\right\}_{e \in \mathcal{E} \backslash e^{*}(\theta)}$ for which, when $\alpha_{\theta} \rightarrow 0$, if only one candidate presents the ex-post optimal platform, then she wins for sure. Thus, there exists an electoral equilibrium satisfying full information equivalence.

An interesting feature of the proof of proposition 2 is that it is based on constructing an inference rule and a strategy (a voting rule) for the voters such that, given this strategy, full information equivalence is achieved. Lipman and Seppi (1995) call this a robust inference rule. In this sense, even in the presence of a multiplicity of equilibria, some of which are far from satisfying full information equivalence, representative democracy could ensure ex-post efficiency were the electorate capable of committing to the robust voting rule. As mentioned further on (see section 8) the result of proposition 2 holds even if instead of a continuum of voters there is only one principal (or perhaps a small number of principals). In this case, the robust voting rule would be focal in the sense of constituting an optimal mechanism design for the principal, who could announce his scheme in advance, before candidates move.

Remark 1. There exists an inferential voting rule for the voters inducing full information equivalence in all electoral equilibria.

Lemma 1 is a very partial characterization of the set of separating electoral equilibria. Unfortunately, while in the $2 \times 2$ example of section 2.1 it is possible to reduce this set to a large degree, in general this is not possible. It is nevertheless possible to state two general characteristics: (i) a dominated policy is never chosen by any candidate in any state, and (ii) among non dominated policies, there is no sequential equilibrium characterized by candidates converging on the worst possible policy in all states. The proof of this last result is indeed a corollary of the proof of the existence of an equilibrium satisfying full information
equivalence.

### 4.1 Separating equilibria in the $2 \times 2$ example

Proposition 2 tells us that a sequential equilibrium satisfying full information equivalence must exist. In this case this is equivalent to say that $x_{c}\left(\theta_{k}\right)=e_{k}$ for all $k=1,2$ and all $c \in\{A, B\}$. In order to sustain this equilibrium, consider a sequence of completely mixed strategies such that, in the limit, the voters receiving signal $s_{2}$ are indifferent between the two policies (so that those receiving signal $s_{1}$ will strictly prefer $e_{1}$ to $e_{2}$ ). Suppose that voters receiving signal $s_{2}$ vote for a candidate presenting platform $x_{c}=e_{2}$ with probability $a$ if the two candidates have presented different platforms. In order to sustain full information equivalence it is sufficient to impose

$$
.1+(1-a) .9<\frac{1}{2}<.2+(1-a) .8
$$

or $\frac{5}{9}<a<\frac{5}{8}$.
In this example there exists no other separating electoral equilibrium. This is true for a general $p\left(s_{1}, \theta_{1}\right)>p\left(s_{1}, \theta_{2}\right)$ and $\operatorname{Pr}\left(\theta_{1}\right) \in(0,1)$. Indeed, consider an equilibrium with $x_{c}\left(\theta_{k}\right)=e_{j}, j \neq k$, all $k, j=1,2$, all $c \in\{A, B\}$. In order to sustain this equilibrium, there should exist a sequence of completely mixed strategies such that, in the limit, the voters receiving signal $s_{2}$ would be indifferent between the two policies and by mixing between the two candidates the majority would always vote for the candidate proposing the ex-post sub-optimal policy. Suppose that the voters receiving signal $s_{2}$ will vote for a candidate presenting platform $x_{c}=e_{1}$ with probability $b$ if the two candidates have presented different platforms. In order to sustain this equilibrium, it should be

$$
\begin{gathered}
p\left(s_{1}, \theta_{2}\right)+b\left[1-p\left(s_{1}, \theta_{2}\right)\right]>\frac{1}{2}>p\left(s_{1}, \theta_{1}\right)+b\left[1-p\left(s_{1}, \theta_{1}\right)\right] \\
\Rightarrow \frac{\frac{1}{2}-p\left(s_{1}, \theta_{2}\right)}{1-p\left(s_{1}, \theta_{2}\right)}<b<\frac{\frac{1}{2}-p\left(s_{1}, \theta_{1}\right)}{1-p\left(s_{1}, \theta_{1}\right)} \\
\Rightarrow p\left(s_{1}, \theta_{2}\right)>p\left(s_{2}, \theta_{1}\right)
\end{gathered}
$$

contradicting the hypothesis that $s_{1}$ was the signal relatively more likely in state $\theta_{1}$.

## 5 Truthfuls vs strategists ( $\delta>0$ )

In this section I derive the fundamental result for the case of $\delta \in(0,1]$, i.e. when there is a strictly positive probability for a candidate of being truthful and always choose a policy $x_{c}(\theta)=e^{*}(\theta)$. It is important to notice that voters do not know whether there is a candidate who is of the truthful type, or, if there is only one, which of the candidates is. Nevertheless, they know that in an equilibrium in which a strategist would chose a policy $e \neq e^{*}(\theta)$ in state $\theta$, still there would be a non zero probability of observing $x \in\left\{\left(e, e^{*}(\theta)\right),\left(e^{*}(\theta), e\right)\right\}$ if the realized state is $\theta$. Hence, if a voter observes such a platform profile, he would infer that the realized state must actually be $\theta$ unless he expects $e^{*}(\theta)$ to be chosen in some other state $\theta^{\prime}$ in which also $e$ is chosen often enough. But this can be true only if $e=e^{*}\left(\theta^{\prime}\right)$, and this turns out to contradict the hypothesis of such an equilibrium to exist if the signal is completely correlated with states. Indeed, given any voting strategy, if there exists a majority preferring $e^{*}(\theta)$ to $e$ in state $\theta^{\prime}$, then there is a larger majority preferring $e^{*}(\theta)$ to $e$ in state $\theta$. This implies that the only possible equilibrium outcome is one satisfying full information equivalence. On the other hand, the proof of proposition 2 still holds, since in an equilibrium satisfying full information equivalence, the probability $\delta$ does not play any role in shaping out-of-equilibrium beliefs.

The following proposition formalizes this intuition.
Proposition 3. If $\delta>0$, the unique electoral equilibrium outcome satisfies full information equivalence. A strategic candidate's optimal strategy is to act as if she was of the truthful type.

An implication of proposition 3 is that a strategic candidate's optimal strategy is to imitate a truthful candidate, whether a candidate of the truthful type exists or not. Hence, in equilibrium, the strategist has no advantage over a truthful candidate. Suppose that a strategic candidate, say $A$, knows for sure that her opponent, $B$, is truthful. She also knows that the electorate ignores which of the two candidates is of the truthful type. Imagine that, as in the $2 \times 2$ example, a large majority of the voters receives signal $s_{2}$ and that, independently of the signal received, policy $e_{1}$ is expected by all voters to be strictly better than policy $e_{2}$. One could expect $A$ to target voters' prior, i.e. propose platform $e_{2}$ in all states, and defeat her truthful opponent. After all, voters know that by mistrusting a candidate proposing $e_{1}$ in favor of one proposing $e_{2}$, they give incentives to the candidates to propose always $e_{2}$. Hence, this way voters would commit a mistake only in the rare state ( 1 percent of the times). Were they to mistrust the candidate proposing $e_{2}$, they would commit a mistake in the abundant state $\theta_{2}$ ( 80 percent of the times). Proposition 3 says that this would be a suboptimal strategy for $A$ and that, despite her advantage over candidate $B$
(after all, candidate $A$ can choose which platform to propose, while candidate $B$ cannot), in the unique electoral equilibrium, the winning chances of candidate $A$ are equal to those of candidate $B$.

Proof. The proof of the existence of an equilibrium satisfying full information equivalence follows the same arguments of the proof of proposition 2. It remains to prove that there exists no other equilibrium. Suppose, by contradiction, that there exists $\hat{\theta} \in \Theta$ such that $x_{c}(\hat{\theta}) \neq e^{*}(\hat{\theta})$. Then, there is a probability $2(1-\delta) \delta>0$ of observing $x \in$ $\left\{\left(e^{*}(\hat{\theta}), x_{c}(\hat{\theta})\right),\left(x_{c}(\hat{\theta}), e^{*}(\hat{\theta})\right)\right\}$ if state $\hat{\theta}$ is verified. Therefore, for $\operatorname{Pr}(\hat{\theta} \mid x, s) \nrightarrow 1$, it must be that there exists another state $\theta^{\prime} \neq \hat{\theta}$ such that $x_{c}\left(\theta^{\prime}\right)=e^{*}(\theta)$ and $e^{*}\left(\theta^{\prime}\right)=x_{c}(\hat{\theta})$. Recall that for any signal $s \in \mathcal{S}, p\left(s, \theta^{\prime}\right) \neq p(s, \hat{\theta})$. This means that we can partition $\mathcal{S}$ in three possibly empty subsets $\mathcal{S}_{e^{*}(\hat{\theta})}, \mathcal{S}_{e^{*}\left(\theta^{\prime}\right)}, s_{\sim}$ as in the proof of proposition 2 . Then it must be that, for some $a \in[0,1]$

$$
\left\{\begin{array}{l}
\sum_{\mathcal{S}_{e^{*}(\hat{\theta})}} p\left(s, \theta^{\prime}\right)+a p\left(s_{\sim}, \theta^{\prime}\right)>\frac{1}{2} \\
\sum_{\mathcal{S}_{e^{*}(\hat{\theta})}} p(s, \hat{\theta})+a p\left(s_{\sim}, \hat{\theta}\right)>\sum_{\mathcal{S}_{e^{*}(\hat{\theta})}} p\left(s, \theta^{\prime}\right)+a p\left(s_{\sim}, \theta^{\prime}\right)
\end{array}\right.
$$

which implies that a candidate presenting platform $e^{*}(\hat{\theta})$ would win against a candidate presenting $e^{*}\left(\theta^{\prime}\right)$ in state $\hat{\theta}$, reaching a contradiction with the hypothesis $x_{c}(\hat{\theta}) \neq e^{*}(\hat{\theta})$.

## 6 Fully strategic voting

In the previous sections, I have assumed that voters express their preferences towards the candidate whose platform maximizes their expected payoff. In this sense, though voters are not naive, they still express their preferences sincerely. The literature on voting equilibria has highlighted the importance for voters to consider how likely their vote is to change the result of the elections, that is their pivotal probability. In this section I show that if voters are fully strategic, there is only one electoral equilibrium and it satisfies full information equivalence.

There are $n$ voters, where $n$ is a random variable distributed according to a Poisson distribution $\mathcal{P}: \mathbb{N} \rightarrow[0,1]$. The remainder of the model is essentially identical to the model of section 2.1, but for the voters casting their vote considering the expected payoff of their actions, i.e. taking into account the probability of their vote being pivotal between policies. In particular, notice that assumption 2 still holds.

I will now present the analysis of the set of sequential equilibria for $n$ large. I will show that, for fully strategic voting, equivalent statements to propositions 2 and 3 exist, and that for the simple $2 \times 2$ model, full information equivalence is the unique electoral outcome even for $\delta=0$.

Consider a voter observing signal $s_{2}$ and platform profile $x \in\left\{\left(e_{1}, e_{2}\right),\left(e_{2}, e_{1}\right)\right\}$. Denote by $\operatorname{Pr}\left(p i v_{e_{j} e_{k}} \mid \theta\right)$ the probability of a vote being pivotal for a candidate proposing platform $e_{j}$ against one proposing platform $e_{k}$ in state $\theta$. The expected gain from taking the action of voting for the candidate proposing $e_{1}$ is given by $G\left(e_{1}, s_{2}, x\right)$ :

$$
G\left(e_{1}, s_{2}, x\right):=\operatorname{Pr}\left(p i v_{e_{1} e_{2}} \mid \theta_{1}\right) \operatorname{Pr}\left(\theta_{1} \mid s_{2}, x\right)-\operatorname{Pr}\left(p i v_{e_{1} e_{2}} \mid \theta_{2}\right) \operatorname{Pr}\left(\theta_{2} \mid s_{2}, x\right)
$$

symmetrically the expected payoff of voting for the candidate proposing $e_{2}$ is given by $G\left(e_{2}, s_{2}\right)$ :

$$
G\left(e_{2}, s_{2}, x\right):=-\operatorname{Pr}\left(p i v_{e_{2} e_{1}} \mid \theta_{1}\right) \operatorname{Pr}\left(\theta_{1} \mid s_{2}, x\right)+\operatorname{Pr}\left(p i v_{e_{2} e_{1}} \mid \theta_{2}\right) \operatorname{Pr}\left(\theta_{2} \mid s_{2}, x\right)
$$

From section 2 we know that a necessary condition for a separating equilibrium satisfying full information equivalence is that voters receiving signal $s_{2}$ vote for the candidate proposing $e_{2}$ with probability $a \in[0,1]$ satisfying

$$
\begin{equation*}
p\left(s_{1}, \theta_{1}\right)+(1-a)\left[1-p\left(s_{1}, \theta_{1}\right)\right]>\frac{1}{2}>p\left(s_{1}, \theta_{2}\right)+(1-a)\left[1-p\left(s_{1}, \theta_{2}\right)\right] . \tag{2}
\end{equation*}
$$

Hence, voters receiving signal $s_{2}$ must be indifferent between voting for one or the other candidate when observing $x \in\left\{\left(e_{1}, e_{2}\right),\left(e_{2}, e_{1}\right)\right\}$, meaning $G\left(e_{1}, s_{2}, x\right)=G\left(e_{2}, s_{2}, x\right)$ :

$$
\begin{equation*}
\frac{\operatorname{Pr}\left(\operatorname{piv}_{e_{1} e_{2}} \mid \theta_{1}\right)+\operatorname{Pr}\left(\operatorname{piv}_{e_{2} e_{1}} \mid \theta_{1}\right)}{\operatorname{Pr}\left(\operatorname{piv}_{e_{1} e_{2}} \mid \theta_{2}\right)+\operatorname{Pr}\left(\operatorname{piv}_{e_{2} e_{1}} \mid \theta_{2}\right)}=\frac{\operatorname{Pr}\left(\theta_{2} \mid s_{2}, x\right)}{\operatorname{Pr}\left(\theta_{1} \mid s_{2}, x\right)} \tag{3}
\end{equation*}
$$

with the $L H S$ of this equation converging to a finite, non-zero real $\kappa>0$, as $n \rightarrow \infty$. By the magnitude theorem (Theorem 1, Myerson (2000)), this implies

$$
\begin{gathered}
\operatorname{mag}\left(\operatorname{piv}_{e_{1} e_{2}} \mid \theta_{1}\right)=\operatorname{mag}\left(\operatorname{piv}_{e_{1} e_{2}} \mid \theta_{2}\right) \\
\Rightarrow-\left(\sqrt{\tau_{e_{2}, \theta_{2}}}-\sqrt{\tau_{e_{1}, \theta_{2}}}\right)^{2}=-\left(\sqrt{\tau_{e_{2}, \theta_{1}}}-\sqrt{\tau_{e_{1}, \theta_{1}}}\right)^{2}
\end{gathered}
$$

where $\tau_{e, \theta}$ is the expected vote share for the candidate proposing policy $e$ in state $\theta$. Notice that $\tau_{e_{1}, \theta_{i}}=1-\tau_{e_{2}, \theta_{i}}$, all $i=1,2$ and it must be, from equation (2), $\tau_{e_{1}, \theta_{1}}, \tau_{e_{2}, \theta_{2}}>\frac{1}{2}$. Hence,
$\tau_{e_{1}, \theta_{1}}=\tau_{e_{2}, \theta_{2}}$. This in turn implies

$$
\begin{equation*}
a=\frac{1}{2-p\left(s_{1}, \theta_{1}\right)-p\left(s_{1}, \theta_{2}\right)} \in\left(\frac{1}{2}, 1\right) . \tag{4}
\end{equation*}
$$

Notice that equations (2) and (4) coexist for all $p\left(s_{1}, \theta_{1}\right)>p\left(s_{1}, \theta_{2}\right)$, which was assumed. Hence, to construct an electoral equilibrium satisfying full information equivalence, it is sufficient to construct a sequence of assessments converging to $e_{c}(\theta)=e^{*}(\theta)$ such that the $R H S$ of equation (5) converges to $\kappa>0$. Therefore, we can state the following proposition.

Proposition 4. There exists a unique electoral equilibrium with fully strategic voting satisfying full information equivalence.

Suppose now that there exists a pooling equilibrium of the form $x_{c}\left(\theta_{i}\right)=e_{2}$, for all $i=1,2$. For this strategy to be sequentially rational, it must be that a candidate proposing $e_{1}$ would lose vis-à-vis a candidate proposing $e_{2}$ in all states. Suppose voters observing signal $s_{2}$ are not indifferent among the two policies when observing candidates presenting different platforms. In particular, since they would prefer policy $e_{2}$, it must be $G\left(e_{1}, s_{2}, x\right)<$ $G\left(e_{2}, s_{2}, x\right)$ :

$$
\begin{equation*}
\frac{\operatorname{Pr}\left(p^{2} v_{e_{1} e_{2}} \mid \theta_{1}\right)+\operatorname{Pr}\left(\text { piv }_{e_{2} e_{1}} \mid \theta_{1}\right)}{\operatorname{Pr}\left(\operatorname{piv}_{e_{1} e_{2}} \mid \theta_{2}\right)+\operatorname{Pr}\left(\operatorname{piv}_{e_{2} e_{1}} \mid \theta_{2}\right)}<\frac{\operatorname{Pr}\left(\theta_{2} \mid s_{2}, x\right)}{\operatorname{Pr}\left(\theta_{1} \mid s_{2}, x\right)}=\frac{\nu\left(\theta_{2}, s_{2}\right)}{\nu\left(\theta_{1}, s_{1}\right)} \tag{5}
\end{equation*}
$$

Since the RHS is finite by assumption, then it must be

$$
\operatorname{mag}\left(\operatorname{piv}_{e_{1} e_{2}} \mid \theta_{1}\right) \leq \operatorname{mag}\left(\operatorname{piv}_{e_{1} e_{2}} \mid \theta_{2}\right) \Rightarrow-\left(\sqrt{\tau_{e_{2}, \theta_{2}}}-\sqrt{\tau_{e_{1}, \theta_{2}}}\right)^{2} \geq-\left(\sqrt{\tau_{e_{2}, \theta_{1}}}-\sqrt{\tau_{e_{1}, \theta_{1}}}\right)^{2}
$$

Hence, the election should be expected to at least as close in state $\theta_{2}$ than in state $\theta_{1}$. This implies $\tau_{e_{2}, \theta_{2}} \leq \tau_{e_{2}, \theta_{1}}$, which is not possible since for any strategy of the voters observing $s_{1}$ and any pure strategy of the voters observing $s_{2}$, then $\tau_{e_{2}, \theta_{2}}>\tau_{e_{1}, \theta_{1}}$ since the $p\left(s_{2}, \theta_{2}\right)>p\left(s_{1}, \theta_{1}\right)$. Therefore, it must be that if a pooling equilibrium exists, then voters receiving signal $s_{2}$ should be indifferent when candidates propose different platforms. From the arguments above, we know that this implies $\tau_{e_{1}, \theta_{1}}=\tau_{e_{2}, \theta_{2}}$, which is coherent only with a mixed strategy for voters receiving signal $s_{2}$ such that equation $(2)$ holds, and hence it is coherent only with a separating equilibrium satisfying full information equivalence. A symmetric argument applies to the case of a pooling equilibrium with $x_{c}\left(\theta_{i}\right)=e_{1}$, all $i=1,2$. The following proposition formalizes this argument.

Proposition 5. The unique electoral equilibrium outcome with fully strategic voting satisfies full information equivalence.

I turn now to to the case of $\delta>0$ and show that full information equivalence is the unique electoral outcome.

From the proof of proposition 3 we know that, if $\delta>0$, it is impossible to construct consistent beliefs for the voters such that voting for a candidate proposing $e_{2}$ vis-à-vis another candidate proposing $e_{1}$ is not a dominated strategy in state $\theta_{1}$ (with sincere voting). Hence, either $i) \operatorname{Pr}\left(\theta_{1} \mid s_{i}, x\right) \rightarrow 1$, all $i=1,2$, in which case for any distribution of pivotal probabilities, voting for candidate $e_{2}$ is a dominated strategy, or $\left.i i\right) \operatorname{Pr}\left(\theta_{1} \mid s_{1}, x\right) \rightarrow 1$ and $\operatorname{Pr}\left(\theta_{1} \mid s_{2}, x\right) \rightarrow \xi \in(0,1)$, in which case the result of proposition 4 holds and full information equivalence is achieved. We can then conclude that the following proposition holds

Proposition 6. If $\delta>0$, the unique electoral equilibrium outcome with fully strategic voting satisfies full information equivalence.

## 7 Policy-motivated candidates

In the developing of the previous sections, I have assumed that strategic candidates are solely office-motivated. While this might be considered as a useful benchmark case, candidates might be policy-motivated as well as office motivated. Moreover, a candidate's preferences might be misaligned with voters' preferences and biased towards one specific policy. In this section, I show that, in equilibrium, such a biased strategic candidate would always propose the same policy of a truthful candidate, i.e. the policy ex-post preferred by voters, despite she would prefer a different policy to be implemented at least in some state of nature.

Consider the following extension of the $2 \times 2$ model of section 2.1 with $\delta>0$ : there are two types of candidates, one being of the truthful type and one being a strategic candidate motivated by the utility function $\pi(w, o)=R w+\psi o$, where $w=1$ if the candidate wins the elections and 0 otherwise, and $o=1$ if policy $e_{1}$ is implemented and 0 otherwise. $\psi>0$ is a parameter describing the relative intensity of the candidate's preferences towards $e_{1}$. I assume $\psi<\frac{R}{2}$. Hence, a strategic candidate desires to win the elections and that policy $e_{1}$ is implemented. The type of each candidate is private information (that is to say that voters cannot observe which candidate, if any, is truthful).

Suppose that in equilibrium strategic candidates propose $e_{1}$ in both states. Along the equilibrium path, when observing the two candidates presenting different platforms, all voters would believe to be in state $\theta_{2}$ (i.e. that one of the candidates is truthful) and cast their vote in favor of the candidate proposing $e_{2}$, that is to say the one they believe to be truthful, and the strategist's payoff is 0 . Were a strategic candidate to deviate to $e_{2}$, then she would win the elections with probability 1 if her opponent is strategic, and with probability $\frac{1}{2}$ if her opponent is truthful. Hence, her expected payoff is $\delta \frac{R}{2}+(1-\delta) R$ if she proposes $e_{2}$, and
$(1-\delta)\left[\frac{R}{2}+\psi\right]$ if she proposes $e_{1}$. Since $\psi<\frac{R}{2}$, deviating to $x_{c}\left(\theta_{2}\right)=e_{2}$ would increase candidate $c$ 's payoff. This contradicts the hypothesis of the existence of an equilibrium in which strategic candidates always proposes $e_{1}$. Symmetrically, there is no electoral equilibrium in which strategic candidates always plays $e_{2}$.

Consider then the possibility that strategic candidates play devil's advocate: propose $e_{1}$ in state $\theta_{2}$ and $e_{2}$ in state $\theta_{1}$. In this case, along the equilibrium path, voters cannot infer the state of nature from the strategy profile of the candidates. It follows that they would vote for the ex-ante optimal policy, say $e_{2}$, all the time. Hence, in state $\theta_{2}$, a strategic candidate would do better by imitating the truthful type and propose $e_{2}$.

It remains to verify that there exists an electoral equilibrium in which strategic candidates always propose the policy ex-post preferred by voters. Along the equilibrium path, both candidates win with probability $\frac{1}{2}$, but a strategy candidate's expected payoff is $\frac{1}{2}+1$ if the realized state is $\theta_{1}$. Consider a sequence of assessments as in the proof of proposition 2; in each assessment the demagogue deviates from the equilibrium strategy more often in state $\theta_{k}, k=1,2$, such that in the limit, if candidates diverge, voters receiving the more abundant signal $s_{j}, j \neq k$, are indifferent among the two policies. As shown in the introduction, there exists a mixed strategy for those voters observing signal $s_{j}$ such that the majority of the voters always express its preference in favor of the candidate proposing the ex-post optimal policy. Hence, were a strategic candidate to deviate from the proposed equilibrium strategy, she would never win and hence her expected payoff would be 0 instead of $\frac{1}{2}$ in state $\theta_{2}$, and $\frac{1}{2}$ instead of $\frac{1}{2}+1$ in state $\theta_{1}$ ). Hence, there exists an electoral equilibrium satisfying full information equivalence.

The following proposition formalizes this argument.
Proposition 7. If $\psi<\frac{R}{2}$, the unique equilibrium outcome with a biased demagogue satisfies full information equivalence. A biased demagogue's optimal strategy is to act as if she was of the truthful type.

As an extreme example, consider the situation in which one candidate is strategist and the other is truthful. Voters know that one of the two candidates is truthful, but ignore which one. We might expect the strategic candidate to have some advantage over the truthful one. On the contrary, the equilibrium strategy for the strategic candidate is to imitate the truthful, for all $\psi, R>0$.

## 8 Extensions

In this section I present some results regarding a few plausible extensions of the model introduced in section 2. I show that the result of proposition 3 s not sensitive to the assumption that candidates act simultaneously or that the number of voters is infinite. Also, I extend my model to encompass different majoritarian voting rules. In order to illustrate these results, I consider again the $2 \times 2$ model of section 2.1 .

Finite number of voters In sections $2 \sqrt{5}$, I have assumed that there is a continuum of voters. This assumption ensures that a law of large numbers holds such that the share of voters receiving a signal $s$ is known with certainty. One might think that this assumption is driving the results above, since they rely on candidates' presumptions about the distribution of voters' beliefs. It turns out that this assumption is on the opposite irrelevant as long as candidates are risk neutral and maximize expected payoff. Consider indeed a variation of the model in section 2 such that the electorate is composed by a finite number $N \geq 1$ of voters. Notice that this includes the case of a single principal. When a candidate chooses her own platform, she does not know how many of the voters have received each of the signals, nevertheless, she expects each voter to receive signal $s_{1}$ with probability equal to $\operatorname{Pr}\left(s_{1} \mid \theta\right)$, where $\theta$ is the verified state (known to the candidate). Hence, her decision must be taken on the basis of the expected distribution of the signal, that is as if a law of large numbers was holding. The unique electoral equilibrium outcome of proposition 3 is such that both candidates chose to present the same (optimal) policy. This means that, along the equilibrium path, voters do not have to chose among the candidates (indeed I have assumed that they would vote for each candidate with probability $\frac{1}{2}$ ). Hence, the exact number of voters receiving each signal is irrelevant for the results of the elections along the equilibrium path.

As briefly mentioned above, the case of a single principal-voter, i.e. $N=1$, suggests another consideration. Even when $\delta=0$, that is to say when both candidates are necessarily of the demagogue type, proposition 4 guarantees the existence of an inference rule and a strategy (a voting rule) for the voters such that, given this strategy, full information equivalence is achieved. Lipman and Seppi (1995) call this a robust inference rule. In this sense, there exists a revealing mechanism to which a single voter could commit such that both candidates' optimal strategy is to propose the ex-post optimal (for the voter) policy in all states.

Different majority rules and unanimity In the analysis of the previous sections, I have considered elections with simple majority rule. Although simple majority is the most widely
used voting rule in democratic societies and committees, other voting rules have been studied in the literature and used in practice. Proposition 2 shows that under simple majority rule, an electoral equilibrium satisfying full information equivalence always exists. The argument of the proof relies on the fact that it is always possible to construct a sequence of assessments such that in its limit, the voters receiving one and only one particular signal are indifferent among the two candidates if the two candidates propose different policies. This in turn implies that there exists a mixed strategy for these voters such that the majority of the electorate always cast its vote for the candidate proposing the ex-post optimal policy. I show here that the same logic applies for different majority rules, but it does not hold for unanimity rule.

Consider now the simple $2 \times 2$ model of the introduction and extend it to allow for different majority rules. In particular, assume that a candidate proposing the policy optimal in the rare state, that is $e_{1}$, is chosen only if voted by a share $\tau \geq \frac{1}{2}$ of the voters. Also, the probability of an individual voter $i$ to observe signal $s^{i}=s_{1}$ in state $\theta \in \Theta$ is $p\left(s_{1}, \theta\right)<\frac{1}{2}$. If $\tau<p\left(s_{1}, \theta_{1}\right)$, then it suffices for the voters receiving signal $s_{2}$ to be indifferent among the two candidates when $x_{T} \neq x_{D}$. Indeed, as long as these voters vote for the candidate whose platform is $e_{2}$ with probability $a$ such that $p\left(s_{1}, \theta_{1}\right)+(1-a) p\left(s_{1}, \theta_{1}\right)<\tau<p\left(s_{1}, \theta_{2}\right)+$ $(1-a) p\left(s_{1}, \theta_{2}\right)$, then the majority of the voters would always favor the candidate proposing the ex-post optimal policy.

Suppose, on the other hand, that $1>\tau>p\left(s_{1}, \theta_{1}\right)$. Then in this case we would need the voters observing signal $s_{1}$ to be indifferent among the two candidates. Nevertheless, the same logic would apply and an electoral equilibrium satisfying full information equivalence would hold.

On the contrary, if $\tau=1$, i.e. unanimity is required to approve $e_{1}$, in no case it is possible to achieve full information equivalence. Suppose in fact that $p\left(s_{1}, \theta_{2}\right) \geq 0$ (that is, allow for the extreme case of nobody receiving signal $s_{1}$ in state $\theta_{2}$, then for a candidate proposing policy $e_{1}$ to win the elections we would need all voters to vote for him. But if this is true in state $\theta_{1}$, then it must be true as well in state $\theta_{2}$, contradicting the hypothesis of full information equivalence.

Heterogeneous preference intensities In the previous sections, voters were assumed to have identical preferences, that is $u^{i}(e, \theta)=u(e, \theta)$. In this paragraph, I relax this assumption by assuming that there is a finite set of voter types $v \in \mathcal{V}:=\{1,2, \ldots, \mathcal{V}\}$ with identical preference order, i.e. for all $e \in \mathcal{E}, u^{v}(e, \theta) \geq u^{v}\left(e, \theta^{\prime}\right) \Longleftrightarrow u^{v^{\prime}}(e, \theta) \geq u^{v^{\prime}}\left(e, \theta^{\prime}\right)$, all $v, v^{\prime} \in \mathcal{V}$ and all $\theta, \theta^{\prime} \in \mathcal{E}$. I assume that types are independently and identically distributed among the voters' population.

Consider the simple $2 \times 2$ model of section 2.1. We can partition the set $\mathcal{V}$ as follows:

$$
\begin{gathered}
\mathcal{V}_{1}:=\left\{v \in \mathcal{V}: u^{v}\left(e_{1}, \theta\right)>u^{v}\left(e_{2}, \theta\right), \text { all } \theta \in \Theta\right\} \\
\mathcal{V}_{2}:=\left\{v \in \mathcal{V}: u^{v}\left(e_{2}, \theta\right)>u^{v}\left(e_{1}, \theta\right), \text { all } \theta \in \Theta\right\} \\
\mathcal{V}_{0}:=\mathcal{V} \backslash\left(\mathcal{V}_{1} \cup \mathcal{V}_{2}\right)
\end{gathered}
$$

All voters of type $v \in \mathcal{V}_{1}$ would vote for a candidate proposing $e_{1}$ for any belief over the realized state. Symmetrically, voters of types in $\mathcal{V}_{2}$ would always prefer a candidate proposing $e_{2}$. The remaining voters, that is to say those with a type in $\mathcal{V}_{0}$, would behave exactly as the voters in the previous sections.

Call $v_{i}$ the type of voter $i$. Suppose that $\operatorname{Pr}\left(v_{i} \in \mathcal{V}_{1}\right)<\frac{1}{2}$ and $\operatorname{Pr}\left(v_{i} \in \mathcal{V}_{2}\right)<\frac{1}{2}$, that neither group of voters with a dominant strategy constitutes a majority. Then the result of the elections would depend on the strategy of those voters with a type in $\mathcal{V}_{0}$. In particular, if the two candidates propose different platforms, then the candidate proposing policy $e_{1}$ would win the elections if and only if a share $\hat{\tau}$ of voters with types in $\mathcal{V}_{0}$ vote for her, with

$$
\begin{gathered}
\operatorname{Pr}\left(v_{i} \in \mathcal{V}_{1}\right)+\hat{\tau} \operatorname{Pr}\left(v_{i} \in \mathcal{V}_{0}\right)=\frac{1}{2} \\
\Longleftrightarrow \hat{\tau}=\frac{\frac{1}{2}-\operatorname{Pr}\left(v_{i} \in \mathcal{V}_{1}\right)}{\operatorname{Pr}\left(v_{i} \in \mathcal{V}_{0}\right)} .
\end{gathered}
$$

From the analysis of the previous paragraph we know that, if $\delta>0$, then the unique electoral equilibrium outcome satisfies full information equivalence for any majority rule $\tau$. Hence, we can conclude that, if $\delta>0$ and $\operatorname{Pr}\left(v_{i} \in \mathcal{V}_{1}\right)<\frac{1}{2}$ and $\operatorname{Pr}\left(v_{i} \in \mathcal{V}_{2}\right)<\frac{1}{2}$, then the unique electoral equilibrium outcome with heterogeneous preference intensities satisfies full information equivalence.

Sequential political campaigns I here consider the possibility that the electoral campaigns of the two candidates are run at different stages such that candidate $A$ moves first and $B$ follows. The analysis assumes $\delta>0$. Notice that when $B$ moves, she knows what $A$ has proposed. Hence, I will denote a strategy for candidate $B$ as a function $x_{B}\left(\theta, x_{A}\right): \Theta \times \mathcal{E} \rightarrow \mathcal{E}$.

The timing of the electoral game is modified as follows: at stage 1 , nature chooses a state $\theta \in \Theta$ and candidates' types; at stage 2, candidate $A$ observes the state $\theta$ and propose platforms $x_{A}$; at stage 3 , candidate $B$ observes the state $\theta$ and $A$ 's platform $x_{A}$ and chooses her platform $x_{B}$; at stage 4 , each voter observes signal $s^{i} \in \mathcal{S}$ and casts a vote in favor of either candidate $A$ or $B$. The candidate with the largest share of votes receives a rent $R$ and
her platform is implemented.
As for the case of simultaneous moves, an electoral equilibrium satisfying full information equivalence exists. Consider a strategy profile for which both candidates always propose the ex-post optimal policy in both states, i.e. $x_{A}\left(\theta_{i}\right)=x_{B}\left(\theta_{i}, x_{A}\right)=e_{i}$, all $i=1,2$. The only beliefs consistent with this strategy profile are such that a voter observing $x=\left(e_{1}, e_{1}\right)$ would believe to be in state $\theta_{1}$ with probability 1 , and viceversa, when observing $x=\left(e_{2}, e_{2}\right)$, he would be sure to be in state $\theta_{2}$. When $x \in\left\{\left(e_{1}, e_{2}\right),\left(e_{2}, e_{1}\right)\right\}$, consider beliefs such that voters observing the rare signal $s_{1}$ strictly prefer $e_{1}$ to $e_{2}$, while instead those voters that have received signal $s_{2}$ are indifferent. Suppose that the voters receiving signal $s_{2}$ will vote for a candidate presenting platform $x_{c}=e_{2}$ with probability $a$ if the two candidates have presented different platforms. Then the strategy profile of the candidates is sequentially rational in

$$
p\left(s_{1}, \theta_{1}\right)+(1-a) p\left(s_{1}, \theta_{1}\right)<\frac{1}{2}<p\left(s_{1}, \theta_{2}\right)+(1-a) p\left(s_{1}, \theta_{2}\right)
$$

Indeed, proposition 8 says that full information equivalence is sustained in a sequential equilibrium.

Proposition 8. There exists an electoral equilibrium with sequential political campaigns satisfying full information equivalence.

Proof. Omitted.
I turn now to the analysis of the entire set of sequential equilibria. Lemma 2 says that there is no sequential equilibrium in which candidate $T$ would propose the same policy in both states.

Lemma 2. There exists no electoral equilibrium with sequential political campaigns with $x_{A}\left(\theta_{1}\right)=x_{A}\left(\theta_{2}\right)$.

To give an intuition, suppose that $A$ always plays $e_{1}$. Then $B$ might limit herself to play $e_{2}$ and win the elections for sure, since candidates' strategy profile is constant across states and therefore voters' beliefs are simply equal to the prior. Suppose instead that $A$ always proposes $e_{2}$. Then $B$ might imitate him and the probability of each candidate to win the elections is equal to $\frac{1}{2}$. This is trivially not an equilibrium, because if any of the candidates deviate from these strategies, then all voters would think that the deviating candidate must be of the truthful type, and therefore prefer to vote for her.

[^3]Proof. I first prove that there exists no equilibrium with $x_{A}\left(\theta_{1}\right)=x_{A}\left(\theta_{2}\right)=e_{1}$. If $B$ plays $e_{1}$ in both states as well, then notice that any sequence of assessments converging to this strategy profile has voters believing that if they observe one of the two candidates proposing $e_{2}$, then it must be that this candidate is of the truthful type, since this probability must converge to a real number bounded away from zero. But this implies that this strategy profile is not sequentially rational since $B$ has an incentive to play $e_{2}$. If, on the opposite, $B$ plays $e_{2}$, then platforms are identical across states. This implies that voters' beliefs must be equal to the prior along the equilibrium path, in turn implying that $B$ would win the elections with probability 1. Again, the strategy of $A$ is not sequentially rational. It is possible to imagine that $B$ could play a strategy of the kind $x_{B}\left(\theta_{1}, x_{A}\right)=x_{A}$. In this case both candidates would win the elections with probability $\frac{1}{2}$. Nonetheless in any such an equilibrium, if voters observe $x_{A} \neq x_{B}$, then the only possibility is that $B$ is of the truthful kind. It follows that $B$ has an incentive to deviate from the proposed strategy. The same is true if $B$ 's strategy is to imitate $A$ only when $A$ 's platform is ex-post optimal: voters' beliefs must be such that if $x_{A} \neq x_{B}$, then $x_{B}$ must be the ex-post optimal policy, giving $B$ an incentive not to imitate $A$ even if $A$ 's platform is ex-post optimal.

I now prove that there exists no equilibrium with $x_{A}\left(\theta_{1}\right)=x_{A}\left(\theta_{2}\right)=e_{2}$. Notice that $x_{B}\left(\theta_{k}, x_{A}\left(\theta_{k}\right)\right)=e_{1}$, all $k=1,2$, is never a best response in any consistent assessment (along the equilibrium path, beliefs must be equal to the prior). Hence, it must be that $B$ proposes $e_{2}$ at least for some combination $\left(\theta, x_{A}\right)$. If $B$ always proposes $x_{B}=e_{2}$, then $A$ has an incentive to move to $x_{A}=e_{1}$, since voters would interpret this platform profile as a sign that $A$ is truthful. Suppose then that $B$ always imitates $A$, i.e. $x_{B}\left(\theta_{1}, x_{A}\right)=x_{A}$. Then, if $B$ is of the demagogue type, $A$ is indifferent between proposing any of the two policies. Nevertheless, there is a probability $\delta>0$ that $B$ is of the truthful type and hence will propose the optimal policy in each state. This implies that in state $\theta_{1}, A$ would win the elections with probability $\frac{1}{2}(1-\delta)$ by playing $x_{A}=e_{2}$, and with probability $\frac{1}{2}>\frac{1}{2}(1-\delta)$ by playing $x_{A}=e_{1}$. Hence $A$ has an incentive to play $e_{1}$ in state $\theta_{1}$.

The next lemma says $i$ ) that there is no sequential equilibrium in which candidate $A$ would never (in none of the states) propose the ex-post optimal policy, and $i$ ) that if candidate $A$ always propose the ex-post optimal policy, then $B$ would do the same in any sequential equilibrium. The results in proposition 8 and in lemmata 2 and 3 prove the result in proposition 9 .

Proposition 9. The unique equilibrium outcome with sequential political campaigns satisfies full information equivalence. A demagogue's optimal strategy is to act as if she was of the truthful type.

Lemma 3. i) there is no electoral equilibrium with sequential political campaigns in which $x_{A}\left(\theta_{k}\right)=e_{j}, k \neq j$, all $k, j=1,2 ;$ ii) if in a consistent assessment, $x_{A}\left(\theta_{k}\right)=e_{k}$, all $k=1,2$, then the assessment is sequentially rational only if $x_{B}\left(\theta_{k}, x_{A}\right)=e_{k}$, all $k=1,2$.

Proof. I begin by proving the first statement in lemma 3. Suppose that there exists an equilibrium with sequential political campaigns in which $x_{A}\left(\theta_{k}\right)=e_{j}, k \neq j$, all $k, j=1,2$. For A's strategy to be sequentially rational it must be that voters' beliefs are different in the two states along the equilibrium path, in turn implying that the two strategy profiles are different in the two states. Said otherwise, it must be a completely revealing equilibrium. Hence, for $A$ to have any chance of winning it must be that, in equilibrium, $B$ imitates $A$, playing strategy $x_{B}\left(\theta_{k}, x_{A}\left(\theta_{k}\right)\right)=x_{A}\left(\theta_{k}\right)$, all $k=1,2$. We can distinguish two cases: $a$ ) $x_{B}\left(\theta_{k}, x_{A}\left(\theta_{k}\right)\right)=x_{A}$, and b) $x_{B}\left(\theta_{k}, x_{A}\left(\theta_{i}\right)\right)=e_{j}, k \neq j$, all $k, j=1,2$.

In case $a) B$ is supposed to always match $A$ 's platform. If they differ, then it must be that $B$ is truthful. Hence $A$ would win with probability $\frac{1}{2}(1-\delta)$ in each state. By deviating and playing $x_{A}\left(\theta_{k}\right)=e_{k}, A$ would win the elections with probability $\frac{1}{2}>\frac{1}{2}(1-\delta)$.

In case $b$ ) both candidates are supposed to propose the sub-optimal platform. Suppose that one of the two candidates deviates and voters observe $x \in\left\{\left(e_{1}, e_{2}\right),\left(e_{2}, e_{1}\right)\right\}$. For the supposed strategies to be part of an equilibrium, then it must be that in state $\theta_{2}$ the majority of the voters would prefer policy $e_{1}$ to $e_{2}$, i.e. all voters receiving signal $s_{1}$ would strictly prefer $e_{1}$ and voters receiving signal $s_{2}$ would at least weakly prefer $e_{1}$. Since $s_{1}$ is more likely in state $\theta_{1}$, then the majority of the voters would prefer policy $e_{1}$ in state $\theta_{1}$. It follows that there is an incentive for both candidates to deviate to $x_{c}\left(\theta_{1}\right)=e_{1}$, contradicting the hypothesis that $x_{A}\left(\theta_{k}\right)=e_{j}, k \neq j$, all $k, j=1,2$ can be part of an equilibrium.

It remains to prove the second statement of lemma 3. Suppose that $A$ 's strategy is $x_{A}\left(\theta_{k}\right)=e_{k}$, all $k=1,2$. Then $A$ 's platform is always supposed to be ex-post optimal, no matter what $B$ proposes. In fact, suppose that $B$ follows any strategy different than $x_{B}\left(\theta_{k}, x_{A}\right)=e_{k}$, all $k=1,2$, or $x_{B}\left(\theta_{k}, x_{A}\right)=x_{A}$. In particular, suppose that in some state $\hat{\theta} \in \Theta x_{B}\left(\hat{\theta}, x_{A}\right) \neq x_{A}(\hat{\theta})$. Then if voters observe $x=\left(x_{A}(\hat{\theta}), x_{B}\left(\hat{\theta}, x_{A}\right)\right)$, then their belief must be such that $\operatorname{Pr}(\hat{\theta} \mid x)=1$ and will all vote for candidate $A$. Hence, $B$ has an incentive to imitate $A$ and win the elections with probability $\frac{1}{2}$.

## 9 Conclusions

In the preceding sections, I have shown that a representative democracy can effectively aggregate information even if the candidates are purely office-motivated and voters' prior is extremely skewed towards a suboptimal (at least in some states) policy alternative. Fur-
thermore, if there is a strictly positive probability, even if extremely small, that candidates always propose the ex-post optimal policy alternative, then there exists a unique equilibrium outcome, one that satisfies full information equivalence. Thus, the optimal strategy for a solely office-motivated candidate is to propose the ex-post optimal platform in all states, even though this implies that her chances of winning the elections are not larger than those of a truthful candidate.

These results are derived assuming voters express their preferences sincerely, casting their vote in favor of that candidate whose platform gives voters the highest expected payoff. Indeed, it is the strategic choice of candidates (and not voters) driving the results. Despite this, fully strategic voting guarantees full information equivalence even when both candidates are surely only office-motivated.

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[^1]:    ${ }^{1}$ The assumption of there being a continuum of voters instead of a finite number is mostly for computational purposes and does not affect results as long as voters' information is private. See section 8 for a more detailed discussion.
    ${ }^{2}$ As it shall be clear, it is not relevant whether one's type is known by one's opponent, as long as this is not known by the voters.

[^2]:    ${ }^{3}$ I do not explicitly label candidates' mixed strategies. When required, from time to time, I shall describe mixed strategies by specifying a probability distribution over the set of policies in each state.
    ${ }^{4}$ The definition in Feddersen and Pesendorfer (1997) is different from mine and indeed mine is more

[^3]:    ${ }^{5}$ Notice that an $a \in[0,1]$ satisfying this condition always exists as long as $p\left(s_{1}, \theta_{1}\right)>p\left(s_{1}, \theta_{2}\right)$.

