Knowledge-Belief Space Approach to Robust Implementation

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In this paper, we talk about *robust implementation*, which was first studied by Bergemann-Morris (2005). After the work by Palfrey-Srivastava (1989) and Jackson (1991), *Bayesian implementation* has been a widely acknowledged as a method to study the mechanism design problem with incomplete information. However, in order to Bayesian implement a social choice function, the designer must know the information structure of the agents. Usually it is impossible in applications. To overcome this difficulty, Bergemann-Morris invented a concept of belief-free implementation, that is, robust implementation.

That a mechanism *robustly implements* a social choice function means that it Bayesian implements the function in every possible Harsanyi type space over the same ex-post payoff parameter space. Thus, the designer does not have to know the agents' information structure in order to implement the function once he has such a mechanism. Bergemann-Morris gave a characterization to robust implementation of social choice functions in their series of papers (2003, *etc*). However there are some problems not solved. First, it is hard to extend their characterization to social choice correspondences. They did not deal with Bayesian implementation directly. They first established the equivalence between the set of Bayesian equilibria and the set of rationalizable actions as shown in Brandenburger-Dekel (1988), and characterized the implementation done with rationalizable actions. However it is not easy to apply this method to social choice correspondences.

Second, they only dealt with the private signal model where each agent only knows their ex-post payoff type and has no information about the other agents' payoff types. However, in applications, it is common that private signals convey some noisy information about other agents' types. In order to deal with such cases, implementation on Harsanyi type spaces is not enough. We have to think about implementation on more general *knowledge-belief spaces* as Palfrey-Srivastava did. However, it is also not easy to extend Bergemann-Morris's way using rationalizability to robust implementation on knowledge-belief spaces.

To solve these problems, we adopted a totally different way without using rationalizability. Our approach is to consider Bayesian implementation on the *universal type space*. The universal type space, composed of the sequential beliefs over the ex-post payoff parameter space and shown by Mertens-Zamir (1985) and Brandenburger-Dekel (1993), is a Harsanyi type space where we can embed any Harsanyi type space over the same ex-post payoff parameter space. Intuitively, it seems robustly implementable if a social choice function is Bayesian implementable on the universal type space. But, there are two serious problems. The first one is the existence of *redundant types*. Some different types in a Harsanyi type space fall onto the same sequential belief. With such redundant types, we cannot consider the Harsanyi type space to be a sub type space of the universal type space. Second, the set of Bayesian equilibrium strategies on the universal type space does not necessarily coincide with that on its sub type space. It is possible that an equilibrium on a sub type space cannot be supported by any equilibrium on the universal type space. It implies that Bayesian implementability on the universal type space does not imply Bayesian implementability on sub type spaces.

To deal with the first problem, we adopted an extended universal type space, which was developed by Yokotani (2009), by introducing a payoff irrelevant parameter space. There we can embed any Harsanyi type space even if it has redundant types. For the second problem, we introduced a concept of *Harsanyi type space with context* (hereafter HTC). It is an extreme case of knowledge-belief space where the partition consists of the entire set. Then we can distinguish two Harsanyi type spaces which are homeomorphic to each other but one is a subspace of a larger HTC and the other is HTC itself. We showed that there exists a HTC large enough to embed any HTC. Since this "universal" space can be decomposed to sub type spaces due to the knowledge of context, the equivalence of Bayesian equilibria between the entire type space and sub type space is recovered.

Since we eliminated the two obstacles for Bayesian implementation, Bayesian implementation on the "universal" HTC space implies robust implementation. By applying the argument by Jackson, we got a sufficient condition of robust implementation not only of functions but of correspondences using Bayesian monotonicity. Also, we constructed a "universal" space for general knowledge belief spaces, and got a characterization result of robust implementation in more general information structures which allows noisy information about other agents.

References

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