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Fund-Raising Games Played on a Network

By

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Abstract

It is well known among fund-raisers that many people contribute to charities or organizations only when asked and that large donations are more likely to occur as a fund-raiser increases the time spent soliciting and/or researching a potential donor. As fund-raisers can only spend time with or research donors that they are aware of, the relationship (or links) between fund-raisers and donors is quite important. We model a fund-raising game where fund-raisers can only solicit donors whom they are tied to and analyze how this network influences donation requests. We show that if this network is incomplete and if donors experience extreme donor fatigue, then fund-raisers will spend more time soliciting donors that other fund-raisers are also tied to and less time soliciting donors that they are the only fund-raiser tied to. If instead donors experience mild donor fatigue, then fund-raisers prefer donors that they are the only fund-raiser tied to over donors that are shared with other fund-raisers. If donors are potential givers with no donor fatigue, then multiple equilibria may exist. Stochastic stability is used to refine the number of equilibria in this case and conditions are given under which the unique stochastically stable equilibrium is efficient.

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1 Introduction

In 2005, seventy percent of all U.S. households gave to charity with donations totalling over $200 billion; see Andreoni (2008). This high charitable contribution rate is due in part to the "power of the ask" as most people contribute to charities or organizations only when asked; see Yörük (2009), Andreoni (2006), Andreoni and Payne (2003), and Keegan (1994). Large donations are more likely to occur as a fund-raiser increases the time spent with the potential donor and/or increases the time spent researching the potential donor.[1] However, a fund-raiser can only research or solicit a donor that he is aware of or that he has some sort of tie to; for instance a fund-raiser may research or solicit a donor who has responded in the past to a mail out campaign. Thus, the relationship or ties between fund-raisers and donors is quite important. However, the network of ties between fund-raisers and donors and this network's influence on donations has not been explored in the fund-raising literature. We analyze such a network and examine how this network influences "the ask" or donation requests.

In particular, we show that if the fund-raiser donor network is incomplete, then whether or not a fund-raiser shares a donor tie with other fund-raisers becomes important and will influence donation requests. For instance, if donors experience extreme "donor fatigue", fund-raisers prefer shared donor ties while if donors experience mild "donor fatigue" then fund-raisers prefer un-shared donor ties. Additionally, if donors are identified as potential givers with no "donor fatigue", then multiple equilibria may exist. We use stochastic stability to refine the number of equilibria and give conditions under which the unique stochastically stable state is efficient.

The basic model and results are as follows. Fund-raisers are tied to donors in a

[1] For instance, Della Vigna, List, and Malmendier (2009) compare donation rates from a door to door fund-raising drive to donations received via the mail or the Internet. They find that the mail and Internet donation rate is around 0.0001 percent while the face to face donation rate is over 6 percent. Thus, fund-raisers can greatly increase donations by using the more time consuming face to face solicitation technique.

network and fund-raisers may only ask donors that they are tied to for donations. If multiple fund-raisers are tied to the same donor, then they compete with each other for donations in an average donation sharing game. In such a game, a donor's total donation increases as the total time all fund-raisers spend with the donor increases and donations are split among fund-raisers in proportion to the time each fund-raiser spends soliciting or researching the donor. If there is a monopoly fund-raiser who is tied to all donors, then this fund-raiser splits his solicitation efforts evenly among all donors if donors experience "donor fatigue" or if donations increase at a decreasing rate with solicitation time and thus donors become less and less responsive to solicitation appeals. The fund-raiser picks a single donor and spends all his time soliciting this donor if donors are thought of as potential givers with no donor fatigue whose donations increase at an increasing rate with solicitation time. These results are echoed if multiple fund-raisers compete on the complete network where every fund-raiser is tied to every donor. Here fund-raisers split their solicitation efforts evenly among donors if donor fatigue is present of if the returns to solicitation time are increasing at a decreasing rate. While all fund-raisers spend all their time soliciting the same donor if the returns to solicitation time are increasing at an increasing rate.

The results change if the fund-raiser donor network is incomplete or if not all donors are tied to all fund-raisers. For instance, alumni donors are tied to colleges and/or universities that they attended and not to all institutions, religious donors are tied to religious organizations of which they are members, and charities in general have lists of donors who have responded to past mail out campaigns where again not all donors will respond to all campaigns.

First, we assume that donors experience donor fatigue or that donations are increasing at a decreasing rate with time spent soliciting donations. Here, fund-raisers will no longer split their time evenly among donors. In an incomplete network, some fund-raisers may have access to donors that others do not while other donors may be tied to multiple fund-raisers. If donors experience extreme donor fatigue or if the donation-solicitation function is fairly curved, then as a fund-raiser spends extra time with a donor although the total donation does not increase much the fund-raiser's share of the donation increases quite a bit. Thus, the fund-raiser spends more time soliciting donors that other fund-raisers are also tied to and less time soliciting donors that he is the only fund-raiser tied to. If donors experience mild donor fatigue of it the donation-solicitation function is not as curved, then the fund-raiser prefers un-shared donor ties and the fund-raiser will spend more time soliciting donors that other fund-raisers are not tied to and less time soliciting donors that other fund-raisers are also tied to.

Next, we consider donors who experience no donor fatigue and we allow donations to increase in solicitation time at an increasing rate. Here all fund-raisers would prefer to solicit the same donor. If the network is incomplete, then this may not be possible. Even if there exists a donor who all fund-raisers are tied to, not all fund-raisers will have access to every other donor and multiple equilibria will exist. At most equilibria, multiple donors will be solicited since once a fund-raiser solicits a potential donor, the incompleteness of the network may not allow other fund-raisers also to solicit this same donor; there can exist a large number of these inefficient equilibria. As the number of equilibria may be quite large, stochastic stability is used in order to refine the number of equilibria. Trembles or mistakes are allowed to occur where after a fund-raiser has decided how much time to spend with a donor, there is a positive probability that a mistake occurs and that his solicitation decisions are reallocated. If the network is such that there is a donor whom all fund-raisers are tied to, then at the unique stochastically stable state all fund-raisers will solicit this donor which coincides with the efficient Nash equilibrium.

Our paper contributes to several literatures. First, it contributes to the literature on the strategic analysis of fund-raising. Andreoni (2006) offers an excellent literature review on this topic. This literature answers questions such as how fund-raisers react to government grants (see Andreoni and Payne (2003)) as well as how competition among charities can result in too many or too few solicitations (see Mungan and Yörük (2009)). Our paper adds to this literature by allowing fund-raisers to behave strategically while competing for donations on a network. Other fund-raising literatures analyze the strategic behavior of donors in a public goods context; see Marx and Matthews (2000), Bagnoli and Lipman (1989), and Andreoni (2006) for a literature review. Recent empirical studies show that fund-raising behavior is crucial to donations; see Yörük (2009) and Della Vigna, List, and Malmendier (2009).

Second, our paper contributes to the literature on economic and social networks; a literature review is given by Jackson (2009) and Jackson and Wolinsky (1996) is a founding paper of this literature. Specifically, our model adds to the literature on applications of social and economic networks. In our model donations or money can only flow along social links. Previous papers have looked at the flow of information regarding coauthors or job prospects along links (see Jackson and Wolinsky (1996) and Calvo-Armengol and Jackson (2004)) as well as the flow of financial help along social links in developing countries (see Bramoullé and Kranton (2007) and Fafchamps and Lund (2003)); for further examples of network applications see Jackson (2009). Additionally, we employ stochastic stability in a network context as does Jackson and Watts (2002a and 2002b).

Lastly, we assume donations are divided among fund-raisers in proportion to the time spent by the fund-raiser with the donor. Thus, our paper is related to the literature on average surplus and average cost sharing games of Moulin (2008), Moulin and Watts (1997), and Watts (1996). The current paper differs from these previous papers in that the average surplus sharing game is played on a network and each fund-raiser may solicit multiple donors.

The paper proceeds as follows. The basic model and monopoly fund-raiser results are presented in sections 2 and 3. In section 4, fund-raisers compete on the complete network. While the incomplete network results are presented in section 5 and concluding remarks are presented in section 6.

2 Basic Model

There are $N = \{1, 2, ..., n\}$ potential donors. Let $M = \{1, 2, ..., m\}$ represent the set of fund-raisers. Each $i \in M$ has total time $T = 1$ to spend asking for donations. Fund-raiser i can divide T among the set N where t_j^i is the time i spends with donor $j \in N$ to solicit his donation and $\sum_{j=1}^n t_j^i \leq 1$. Let $t^i \equiv \{t_1^i, t_2^i, ..., t_n^i\}$, $t_j \equiv \{t_j^1, t_j^2, ..., t_j^m\}$, and $t \equiv \{t^1; t^2; ...; t^m\}$. Let $t^{-i} \equiv \{t^1; t^2; ..., t^{i-1}, t^{i+1}, ..., t^m\}$ and $t_j^{-i} \equiv \{t_j^1, t_j^2, ...t_j^{i-1}, t_j^{i+1}, ..., t_j^m\}$.

The profit $i \in M$ receives equals his donations net fund-raising costs, where we assume a constant marginal cost of fund-raising equal to c. The donation i receives from $j \in N$ depends on the time all fund-raisers spend with j and is represented by $\pi_j^i(t_j) = y(t_j^i, t_j^{-i}) - ct_j^i$. We assume $\frac{\partial y}{\partial t_j^i} > 0$ and $y(0, t_j^{-i}) = 0$. Additionally, we assume that if $\widetilde{t_j^{-i}}$ is any permutation of t_j^{-i} then $y(t_j^i; \widetilde{t_j^{-i}}) = y(t_j^i; t_j^{-i})$. Let the total profits that i receives be represented by $\pi^i(t) = \sum_{j=1}^n \pi_j^i(t_j)$. For brevity, we will often abbreviate $\pi_j^i(t_j)$ by π_j^i and $\pi^i(t)$ by π^i.

We assume that c is small enough so that in all of the following results each fund-raiser will select t^i such that $\sum_{j=1}^n t_j^i = 1$. Thus, for simplicity we will ignore fund-raising costs in what follows and will set $\pi_j^i(t_j) = y(t_j^i, t_j^{-i})$.

3 Monopoly Fund-raiser

First we consider the case where there is no competition for donors by fund-raisers, or where $m = 1$. Here agent $i \in M$ wants to choose t^i to maximize his total expected profits $\pi^i = \sum_{j=1}^n y(t_j^i)$.

Proposition 1 *Let $m = 1$ and assume $y'' > 0$. Then $i \in M$ maximizes total expected profits by setting $t_j^i = 1$ for some $j \in N$ and $t_k^i = 0$ for all $k \neq j$, $k \in N$.*

Proof. Agent i wants to maximize π^i or to $\max_{t_1^i,...,t_n^i} \sum_{j=1}^n y(t_j^i)$ such that $\sum_{j=1}^n t_j^i \leq 1$. It is easy to check that given $y' > 0$ and $y'' > 0$, the solution to the maximization problem is a corner solution where $t_j^i = 1$ for some $j \in N$. ■

Proposition 2 *Let $m = 1$ and assume $y'' < 0$. Then $i \in M$ maximizes total expected profits by setting $t_j^i = \frac{1}{n}$ for all $j \in N$.*

Proof. As $y' > 0$, agent i will always choose to use all of his time or to set $\sum_{j=1}^n t_j^i = 1$. Agent i wants to maximize π^i or $\max_{t_1^i,...,t_n^i} \sum_{j=1}^n y(t_j^i)$ subject to $\sum_{j=1}^n t_j^i = 1$. First order conditions yield $y'(t_j^i) = y'(1 - t_1^i - t_2^i - ... - t_n^i)$ for all $j \in \{1, 2, ..., n-1\}$. Given $y' > 0$ and $y'' < 0$, these first order conditions are only met if $t_1^i = t_2^i = ...t_n^i$ or if $t_j^i = \frac{1}{n}$ for all $j \in N$. Since $y'' < 0$, the corresponding bordered Hessian has leading principal minors which alternate in sign. Thus π^i is negative definite on $\sum_{j=1}^n t_j^i = 1$ and our solution is a maximum. ■

4 Fund-raisers Competing on the Complete Network

Next we consider the case where the number of fund-raisers $m > 1$. In this section, we assume that each fund-raiser has access to every donor. If we consider a fund-raiser's access to a donor as a link between a fund-raiser and a donor, then every fund-raiser is linked to every donor and the fund-raiser donor network is complete. The case of an incomplete network is considered in section 5.

4.1 Average Donation Sharing Model on the Complete Network

Fund-raisers compete against each other in an average donation sharing game for each agent $j \in N$'s donation. Fund-raisers simultaneously choose time spent with donors and if multiple fund-raisers solicit the same donor, then the donation is split among fund-raisers in proportion to the time spent with the donor. Formally, the donation $i \in M$ expects from $j \in N$ equals

$$\pi_j^i = y(t_j^i; t_j^{-i}) = \frac{t_j^i}{\sum_{k=1}^m t_j^k} f(\sum_{k=1}^m t_j^k)$$

for some function f such that $f(0) = 0$ and $f' > 0$. Here $f(\sum_{k=1}^m t_j^k)$ represents the total donations that donor j will make given the total time all fund-raisers spend soliciting j's donation. For simplicity, we assume this function is the same for all donors. Additionally, note that we use the convention that if $t_j^k = 0$ for all $k \in M$, then $\pi_j^i = 0$.

4.2 Average Donation Sharing Results on the Complete Network

Proposition 3 *Let $m > 1$, $f'' < 0$, and assume fund-raisers compete in an average donation sharing game. At the symmetric Nash equilibrium of the average donation sharing game, $t_j^i = \frac{1}{n}$ for all $j \in N$ and $i \in M$.*

Note that we focus on only the symmetric Nash equilibrium in this proposition and in the first part of section 5 where the donation function is concave. The reason is threefold. First, we want to compare the Nash equilibria of the game played on the complete network to the game played on the incomplete network. Concentrating on the symmetric Nash in each situation facilitates this comparison. Second, focusing on the symmetric Nash simplifies our calculations. The first order conditions of our maximization problem are quite complex making it extremely difficult to compute other possible equilibria. Lastly, we hypothesize that in many cases the symmetric Nash is the unique Nash, although proving this is quite difficult and is beyond the scope of the current paper.

Proof. We find $i \in M$'s best response to $t_j^k = \frac{1}{n}$ for all $k \neq i$, $k \in M$, and $j \in N$. Since $\pi_j^i = (\frac{t_j^i}{\sum_{k=1}^m t_j^k}) f(\sum_{k=1}^m t_j^k)$ we know that $\pi^i = \sum_{j=1}^n((\frac{t_j^i}{\sum_{k=1}^m t_j^k}) f(\sum_{k=1}^m t_j^k))$. Thus i's maximization problem is

$$\max_{t_i^i, t_2^i, ,\ldots, t_n^i} \pi^i \text{ such that } \sum_{j=1}^{n} t_j^i = 1$$

with first order conditions

$$\frac{\partial \pi^i}{\partial t_j^i} = (\frac{\sum_{k \neq i} t_j^k}{(t_j^i + \sum_{k\neq i} t_j^k)^2}) f(t_j^i + \sum_{k \neq i} t_j^k) + (\frac{t_j^i}{(t_j^i + \sum_{k \neq i} t_j^k)}) f'(t_j^i + \sum_{k \neq i} t_j^k) = 0 \qquad \text{for all } j \in N$$

$$\text{and } \sum_{j=1}^{n} t_j^i = 1.$$

If $t_j^k = \frac{1}{n}$ for all $k \neq i$, $k \in M$ and $j \in N$, then $\sum_{k \neq i} t_j^k = \frac{m-1}{n}$ for all $j \in N$ and the solution to the above first order conditions is $t_j^i = \frac{1}{n}$ for all $j \in N$. To check the second order conditions, let $O_j^i(\frac{m-1}{n}) = p(\frac{t_j^i}{(t_j^i + \frac{m-1}{n})}) f(t_j^i + \frac{m-1}{n})$ represent i's opportunity set for donations from j given $t_j^k = \frac{1}{n}$ for all $k \neq i$, $k \in M$. Thus, $\pi^i(t^i, \frac{m-1}{n}) = \sum_{j=1}^{n} O_j^i(\frac{m-1}{n})$. It is easy to check that $\frac{\partial O_j^i}{\partial t_j^i} > 0$ and $\frac{\partial^2 O_j^i}{\partial (t_j^i)^2} < 0$ for all $0 < t_j^i \leq 1$. Thus, $\pi^i(t^i, \frac{m-1}{n})$ is negative definite on $\sum_{j=1}^{n} t_j^i = 1$. So $t_j^i = \frac{1}{n}$ for all $j \in N$ is i's best response to $t_j^k = \frac{1}{n}$ for all $k \neq i$, $k \in M$, and $j \in N$, and our solution is a Nash equilibrium. ■

Proposition 4 *Let $m > 1$, $f'' > 0$, and assume fund-raisers compete in an average donation sharing game. At all Nash equilibria there exists $k \in N$ such that $t_k^i = 1$ for all $i \in M$.*

Proof. Assume there exists $k \in N$ such that $t_k^\ell = 1$ for all $\ell \neq i$, $\ell \in M$. We find $i \in M$'s best response. Here, $\pi^i = \sum_{j \in N, j \neq k} (\frac{t_j^i}{(t_j^i)}) f(t_j^i) + (\frac{t_k^i}{(t_k^i + (m-1))}) f(t_k^i + (m-1))$ with $\sum_{j=1}^{n} t_j^i = 1$. Since $f' > 0$ and $f'' > 0$, i does best by choosing $t_k^i = 1$ and $t_j^i = 0$ for all $j \in N$, $j \neq k$. Next we show that all Nash equilibria are of this type. Consider i's best response to a fixed t^{-i}. Here i's best response is to set $t_k^i = 1$ for some k such that $k \in \arg\max_j \sum_{\ell \in M, \ell \neq i} t_j^\ell$. Given that this is true for all $i \in M$, all Nash equilibria must be as described. ■

5 Competition on an Incomplete Network

Let g represent a network of connections between fund-raisers and donors. If $i \in M$ and $j \in N$ then $ij \in g$ if i and j are linked. Note that for all links, ij, we will use the convention that the fund-raiser is always listed first.

Let $B(j,g) = \{i \mid ij \in g,\, i \in M\}$; thus $B(j,g)$ represents the set of fund-raisers that $j \in N$ is directly linked with in graph g. Let $\mid S \mid$ represent the cardinality of set $S \subset \mathbb{Z}$. Let $A_z(i,g) = \{j \mid ij \in g,\, j \in N \text{ and} \mid B(j,g) \mid = z\}$; thus $A_z(i,g)$ represents the set of donors that i is linked to in g such that each donor in the set has exactly z direct links.

Let fund-raisers compete in an average donation sharing game and assume g is fixed. Here

$$\pi^i = \sum_{j \in A_1(i,g)} f(t_j^i) + \sum_{j \in A_2(i,g)} \frac{t_j^i}{t_j^i + t_j^{k(j)}} f(t_j^i + t_j^{k(j)}) + ... + \sum_{j \in A_n(i,g)} \frac{t_j^i}{\sum_{\ell=1}^n t_j^\ell} f(\sum_{i=1}^n t_j^\ell)$$

where $k(j) \in M$ such that $kj \in g$.

5.1 Competition on an Incomplete Network with a Concave Donation Function

The following example illustrates how time spent soliciting donations depends on both the network configuration and the curvature of $f(t)$.

Example 1 *Let $M = \{1,2\}$ and $N = \{1,2,3\}$. Let $f(t) = t^{\frac{1}{2}}$. Let $g_1 = \{11,12,21,22\}$ and $g_2 = \{11,12,22,23\}$. Notice that in network g_1 each fund-raiser is tied to the same two donors. In network g_2 each fund-raiser again has two ties, but this time only one donor is common to both fund-raisers while each fund-raiser also has one tie to a donor that the other has no ties to. By Proposition 3, the symmetric Nash equilibrium of the average donation sharing game with network g_1 is $t_j^i = \frac{1}{2}$ for all $i \in M$, $j \in N$. Next we consider the symmetric Nash equilibrium of the average donation sharing game with network g_2. Here*

$\pi^1 = (t_1^1)^{\frac{1}{2}} + \frac{t_2^1}{t_2^1+t_2^2}(t_2^1 + t_2^2)^{\frac{1}{2}}$ and π^2 is similar. First order conditions for fund-raiser 1 yield $\frac{1}{2}(t_1^1)^{-\frac{1}{2}} = \left(\frac{t_2^2}{(t_2^1+t_2^2)^2}\right) (t_2^1 + t_2^2)^{\frac{1}{2}} + \frac{\frac{1}{2}t_2^1}{t_2^1+t_2^2}(t_2^1 + t_2^2)^{-\frac{1}{2}}$ and $t_1^1 + t_2^1 = 1$, with fund-raiser 2 having similar first order conditions. At the symmetric Nash equilibrium $t_1^1 = t_3^2 = .47$ and $t_2^1 = t_2^2 = .53$. Thus, fund-raisers spend more time with the shared donor than they do in network g_1 and less time with the unshared donor. However, if we change $f(t)$ so that $f(t) = t^{.7}$, then at the symmetric Nash of the game with g_2, $t_1^1 = t_3^2 = .502$ and $t_2^1 = t_2^2 = .498$. Thus, fund-raisers now spend less time with the shared donor.

Next we increase the number of donors and compare three networks: one where all fund-raisers are linked to the same set of donors, one where each fund-raiser has only one shared tie and many unshared ties, and one where each fund-raiser has only one unshared tie and many shared ties.

Example 2 *Let $M = \{1,2\}$ and $N = \{1,2,3,...,2a-1\}$ for $a \geq 2$ and $a \in \mathbb{Z}$. Let $f(t) = t^{\frac{1}{2}}$. Let $g_1 = \{11,12,...,1a,21,22,...,2a\}$, $g_2 = \{11,12,...,1a,2a,2(a+1),...,2(2a-1)\}$, and $g_3 = \{11,12,...,1a,22,23,...,2(a+1)\}$. By Proposition 3, the symmetric Nash equilibrium of the average donation sharing game with network g_1 is $t_j^i = \frac{1}{a}$ for all $i \in M$, $j \in N$. For g_2 at the symmetric Nash $t_k^1 = t_j^2 = \frac{1}{a+.125}$ for $k \in \{1,...,a-1\}$, $j \in \{a+1,...,2a-1\}$ and $t_a^1 = t_a^2 = \frac{1.125}{a+.125}$. While at the symmetric Nash for g_3, $t_1^1 = t_{a+1}^2 = \frac{1}{1.125a-.125}$ and $t_k^1 = t_j^2 = \frac{1.125}{1.125a-.125}$ for k, $j \in \{2,...,a\}$. Again, the fund-raisers spend more time with the shared donors and less with time with the unshared donors. Notice that fund-raisers spend more time with unshared links in g_2 than in g_3, but the ratio of time spent with unshared to shared links $(\frac{t_1^1}{t_a^1})$ is the same for these two networks.*

Next we consider the general case of any concave donation function, f, and compare two symmetric networks: one were all fund-raisers are linked to the same set of donors, and one where each fund-raiser has the same number of unshared ties and has the same set of shared ties.

Proposition 5 *Let* $M = \{1, 2\}$ *and* $N = \{1, 2, ..., 2a+b\}$ *for* $a, b \in \mathbb{Z}$, $a \geq 1$ *and* $b \geq 1$. *Let* $g_1 = \{11, 12, ..., 1(a+b), 21, 22, ..., 2(a+b)\}$ *and let* $g_2 = \{11, 12, ..., 1(a+b), 2(a+1), 2(a+2), ..., 2(2a+b)\}$. *At the symmetric Nash equilibrium of the average donation sharing game with* g_1, $t^i_j = \frac{1}{a+b}$ *for all* $j \in N$, $i \in M$. *At the symmetric Nash equilibrium of the average donation sharing game with* g_2,

(i) if $f'(\frac{t}{2}) < \frac{f(t)}{2t} + \frac{f'(t)}{2}$ *for all* $0 \leq t \leq 1$, *then* $t^i_j < \frac{1}{a+b}$ *for all* $i \in M$ *and* $j \in \{1, 2, ..., a, a+b+1, a+b+2, ..., 2a+b\}$ *and* $t^i_j > \frac{1}{a+b}$ *for all* $i \in M$ *and* $j \in \{a+1, a+2, ..., a+b\}$, *and*

(ii) if $f'(\frac{t}{2}) > \frac{f(t)}{2t} + \frac{f'(t)}{2}$ *for all* $0 \leq t \leq 1$, *then* $t^i_j > \frac{1}{a+b}$ *for all* $i \in M$ *and* $j \in \{1, 2, ..., a, a+b+1, a+b+2, ..., 2a+b\}$ *and* $t^i_j < \frac{1}{a+b}$ *for all* $i \in M$ *and* $j \in \{a+1, a+2, ..., a+b\}$.

Notice that our condition $f'(\frac{t}{2}) < \frac{f(t)}{2t} + \frac{f'(t)}{2}$ for all $0 \leq t \leq 1$ is met by $f(t) = t^\alpha$ for $0 < \alpha \leq .63$ while the condition $f'(\frac{t}{2}) > \frac{f(t)}{2t} + \frac{f'(t)}{2}$ for all $0 < t \leq 1$ is met by $f(t) = t^\alpha$ for $.64 \leq \alpha < 1$.

Proposition 5 says that if the donation function, $f(t)$, is fairly curved, then fund-raisers spend more time with shared ties and less time with unshared ties. While if the donation function is straighter, then fund-raisers spend more time with unshared donors and less time with shared donors. Thus, if the donation function is more curved, then the loss from decreasing time with unshared donors is small while the gain from spending more time with shared donors is large, because even though shared donations do not increase that much the fund-raiser's proportion of these donations increases quite a bit. If however the donation function is less curved, then there is a substantial gain from increasing time spent with unshared donors.

Proof. The statement regarding the symmetric Nash equilibrium for the game with g_1 is proven true by Proposition 3. Next we find the symmetric Nash equilibrium for the average donation sharing game with g_2. Here $\pi^1 = \sum_{j=1}^{a} f(t^1_j) + \sum_{k=a+1}^{a+b} \frac{t^1_k}{t^1_k + t^2_k} f(t^1_k + t^2_k)$

while π^2 is similar. The first order conditions for the Nash equilibrium require that $f'(t_j^1) = \frac{t_k^2}{(t_k^1+t_k^2)^2} f(t_k^1 + t_k^2) + \frac{t_k^1}{t_k^1+t_k^2} f'(t_k^1 + t_k^2)$ for all $j \in \{1, 2, ..., a\}$ and $k \in \{a+1, a+2, ..., a+b\}$. At the symmetric Nash equilibrium, $t_1 \equiv t_j^1 = t_\ell^2$ for all $j \in \{1, 2, ..., a\}$ and $\ell \in \{a+b+1, a+b+2, ..., 2a+b\}$ and $t_{a+1} \equiv t_k^1 = t_k^2$ for all $k \in \{a+1, a+2, ..., a+b\}$. In addition we know that $\sum_{j=1}^{a+b} t_j^1 = 1$. Thus, we can rewrite the first order conditions as

$$f'(t_1) = \frac{t_{a+1}}{(2t_{a+1})^2} f(2t_{a+1}) + \frac{t_{a+1}}{2t_{a+1}} f'(2t_{a+1}) \text{ where } at_1 + bt_{a+b} = 1. \tag{1}$$

If to the contrary we let $t_1 = t_{a+1} = \frac{1}{a+b}$ then the first order conditions simplify to $f'(\frac{1}{a+b}) = \frac{1}{2}\frac{f(\frac{2}{a+b})}{\frac{2}{a+b}} + \frac{1}{2}f'(\frac{2}{a+b})$. If $f'(\frac{t}{2}) < \frac{f(t)}{2t} + \frac{f'(t)}{2}$ for all $0 \leq t \leq 1$, then our first order condition will not be met. Fund-raiser 1 must increase the left hand side of equation (1) and decrease the right hand side. Since $f' > 0$ and $f'' < 0$, we know that t_1 must be decreased and thus t_{a+b} increased. Thus, at the Nash equilibrium $t_1 < \frac{1}{a+b}$ and $t_{a+1} > \frac{1}{a+b}$. If instead $f'(\frac{t}{2}) > \frac{f(t)}{2t} + \frac{f'(t)}{2}$ for all $0 \leq t \leq 1$, then in order to meet equation 1 we must increase t_1 and so at the Nash $t_1 > \frac{1}{a+b}$ and $t_{a+1} < \frac{1}{a+b}$. ■

Example 3 *Let $M = \{1, 2, 3\}$ and $N = \{1, 2, 3, 4, 5, 6, 7\}$. Let*

$$g_1 = \{11, 12, 13, 14, 21, 22, 23, 24, 31, 32, 33, 34\}$$

and let

$$g_2 = \{11, 12, 13, 14, 22, 24, 25, 26, 33, 34, 35, 37\}.$$

Thus, in network g_2 each fund-raiser has one tie to an unshared donor, two ties to donors shared with one other fund-raiser, and one tie to donor 4, while in network g_1 all fund-raisers are tied to the same set of four donors. By Proposition 3, for any increasing and concave $f(t)$ the symmetric Nash equilibrium of the average donation sharing game with network g_1 is $t_j^i = \frac{1}{4}$ for all $i \in M$, $j \in N$. Next we consider the symmetric Nash equilibrium of the game with network g_2. Here $\pi^1 = f(t_1^1) + \sum_{j\in\{2,3\}} \frac{t_j^1}{t_j^1+t_j^j} f(t_j^1 + t_j^j) + \frac{t_4^1}{t_4^1+t_4^2+t_4^3} f(t_4^1 + t_4^2 + t_4^3)$

and π^2 *and* π^3 *are similar. First order conditions for fund-raiser 1 are* $f'(t_1^1) = \frac{t_j^j}{(t_j^1+t_j^j)^2} f(t_j^1 + t_j^j) + \frac{t_j^1}{t_j^1+t_j^j} f'(t_j^1 + t_j^j) = \frac{t_4^2+t_4^3}{(t_4^1+t_4^2+t_4^3)^2} f(t_4^1 + t_4^2 + t_4^3) + \frac{t_4^1}{t_4^1+t_4^2+t_4^3} f'(t_4^1 + t_4^2 + t_4^3)$ *for all* $j \in \{2,3\}$ *and* $\sum_{j=1}^4 t_j^1 = 1$*; first order conditions for fund-raisers* 2 *and* 3 *are similar. If* $f(t) = t^{.5}$*, then at the symmetric Nash equilibrium* $t_1^1 = t_6^2 = t_7^3 = .24$*,* $t_j^i = .27$*, and* $t_4^i = .22$ *for* $i \in M$*,* $j \in \{2,3,5\}$*. Thus, the tie to donor* 4 *who is tied to all fund-raisers is weighted least and the ties to donors shared among two fund-raisers are weighted most. If* $f(t) = t^{.9}$*, then at the symmetric Nash equilibrium* $t_1^1 = t_6^2 = t_7^3 = .29$*,* $t_j^i = .25$*, and* $t_4^i = .20$ *for* $i \in M$*,* $j \in \{2,3,5\}$*. Here again the tie to donor* 4 *who is tied to all fund-raisers is weighted least, but the tie to the donor shared with no one is weighted most. If* $f(t) = t^{.1}$*, then at the symmetric Nash equilibrium* $t_1^1 = t_6^2 = t_7^3 = .09$*,* $t_j^i = .32$*, and* $t_4^i = .27$ *for* $i \in M$*,* $j \in \{2,3,5\}$*. Here the tie to the donor shared with no one is weighted least and the ties to donors shared among two fund-raisers are weighted most.*

Let there be m fund-raisers and n donors. Let g^{sym} be a symmetric network such that each fund-raiser has a_1 ties with to donors shared with no one, a_2 ties to donors shared with one other fund-raiser, a_3 ties to donors shared with two other fund-raisers,..., and a_n ties to donors shared with all other fund-raisers, where $a_j \in \mathbb{Z}_+$ for all $j \in \{1,2,...,n\}$. Let g^1 be a network where each fund-raiser is tied to each $j \in \{1,2,...,\sum_{i=1}^n a_i\}$ and $j \in N$.

Proposition 6 *If* $f'(t) > \frac{1}{2}\frac{f(2t)}{2t} + \frac{1}{2}f'(2t)$*, then at the symmetric Nash equilibrium of the average donation sharing game with any* g^{sym}*,* $t_{\alpha_1} > t_{\alpha_2} > ... > t_{\alpha_n}$ *where* t_{α_j} *is the time* $i \in M$ *spends with each of his* a_j *ties,* $j \in \{1,2,...,n\}$ *and* $ij \in g^{sym}$*. At the symmetric Nash equilibrium of the game with* g^1*,* $t_j^i = \frac{1}{\sum_{j=1}^n a_j}$ *for all* $i \in M$*,* $j \in N$*.*

Proof. Since $f' > 0$ and $f'' < 0$ and by assumption $f'(t) > \frac{1}{2}\frac{f(2t)}{2t} + \frac{1}{2}f'(2t)$, it follows that $\frac{k-1}{k}\frac{f(kt)}{kt} + \frac{1}{k}f'(kt) > \frac{k}{k+1}\frac{f((k+1)t)}{(k+1)t} + \frac{1}{k+1}f'((k+1)t)$ for all $k \in \{1,2,...,n-1\}$. Consider any fixed network g^{sym}. Let $A_j \subseteq N$ represent the set of donors each having j ties in g^{sym},

$j \in \{1, 2, ..., n\}$. Let $\alpha_j \in A_j$ represent an element of this set. Profits for fund-raiser 1 are represented as

$$\pi^1 = \sum_{\alpha_1 \in A_1; 1\alpha_1 \in g^{sym}} f(t^1_{\alpha_1}) + \sum_{\alpha_2 \in A_2; 1\alpha_2 \in g^{sym}, \ell\alpha_2 \in g^{sym}} \frac{t^1_{\alpha_2}}{t^1_{\alpha_2} + t^\ell_{\alpha_2}} f(t^1_{\alpha_2} + t^\ell_{\alpha_2}) + ... \quad (2)$$

$$+ \sum_{\alpha_n \in A_n; 1\alpha_n \in g^{sym}} \frac{t^1_{\alpha_n}}{\sum_{i=1}^n t^i_{\alpha_n}} f(\sum_{i=1}^n t^i_{\alpha_n})$$

with $\ell \in M$ and $\ell \neq 1$. First order conditions require

$$f'(t^1_{\alpha_1}) = \frac{t^\ell_{\alpha_2}}{(t^1_{\alpha_2} + t^\ell_{\alpha_2})^2} f(t^1_{\alpha_2} + t^\ell_{\alpha_2}) + \frac{t^1_{\alpha_2}}{t^1_{\alpha_2} + t^\ell_{\alpha_2}} f'(t^1_{\alpha_2} + t^\ell_{\alpha_2}) = ... = \quad (3)$$

$$= \frac{t^2_{\alpha_n} + t^3_{\alpha_n} + ... + t^n_{\alpha_n}}{(t^1_{\alpha_n} + t^2_{\alpha_n} + ... + t^n_{\alpha_n})^2} f(t^1_{\alpha_n} + t^2_{\alpha_n} + ... + t^n_{\alpha_n}) +$$

$$\frac{t^1_{\alpha_n}}{t^1_{\alpha_n} + t^2_{\alpha_n} + ... + t^n_{\alpha_n}} f'(t^1_{\alpha_n} + t^2_{\alpha_n} + ... + t^n_{\alpha_n})$$

and $\sum_{j=1}^n a_j t^1_{\alpha_j} = 1$, where $\alpha_i \in A_i$, $1\alpha_i \in g^{sym}$, $\ell\alpha_i \in g^{sym}$ for $i \in \{1, 2, ..., n\}$, $\ell \neq 1$, $\ell \in M$. The first order conditions for $i \neq 1$, $i \in M$ are similar.

At the symmetric Nash equilibrium $t_{\alpha_j} \equiv t^i_{\alpha_j} = t^\ell_{\alpha_j}$ for all $i \neq \ell$, $i, \ell \in M$, $\alpha_j \in A_j$, $i\alpha_j \in g^{sym}$, $\ell\alpha_j \in g^{sym}$ and $j \in \{1, 2, ..., n\}$. We can rewrite the first order conditions as

$$f'(t_{\alpha_1}) = \frac{1}{4t_{\alpha_2}} f(2t_{\alpha_2}) + \frac{1}{2} f'(2t_{\alpha_2}) = \frac{2}{9t_{\alpha_3}} f(3\alpha_3) + \frac{1}{3} f'(3\alpha_3) = ... = \frac{(n-1)}{n^2 t\alpha_n} f(nt_{\alpha_n}) + \frac{1}{n} f'(nt_{\alpha_n})$$

where $\sum_{j=1}^n a_j t_{\alpha_j} = 1$.

Assume to the contrary that $t_{\alpha_k} = \frac{1}{\sum_{j=1}^n a_j} \equiv \bar{t}$ for all $k \in \{1, 2, ..., n\}$. Then, by assumption

$$f'(\bar{t}) > \frac{1}{2} \frac{f(2\bar{t})}{2\bar{t}} + \frac{1}{2} f'(2\bar{t}) > \frac{2}{3} \frac{f(3\bar{t})}{3\bar{t}} + \frac{1}{3} f'(3\bar{t}) > ... > \frac{(n-1)}{n} \frac{f(n\bar{t})}{n\bar{t}} + \frac{1}{n} f'(n\bar{t})$$

and the first order conditions are not met. In order to meet the first order conditions we must decrease $f'(t_{\alpha_1})$ in comparison to $\frac{1}{4t_{\alpha_2}}f(2t_{\alpha_2})+\frac{1}{2}f'(2t_{\alpha_2})$ and decrease $\frac{1}{4t_{\alpha_2}}f(2t_{\alpha_2})+\frac{1}{2}f'(2t_{\alpha_2})$ in comparison to $\frac{2}{9t_{\alpha_3}}f(3\alpha_3)+\frac{1}{3}f'(3\alpha_3)$, etc. Since $f'>0$, $f''<0$, we know that t_{α_i} must be increased in comparison to $t_{\alpha_{i+1}}$ for all $i \in \{1,2,...,n-1\}$. Thus, in equilibrium $t_{\alpha_1} > t_{\alpha_2} > ... > t_{\alpha_n}$. ■

Proposition 7 *If $f'(t) < \frac{1}{2}\frac{f(2t)}{2t}+\frac{1}{2}f'(2t)$, then at the symmetric Nash equilibrium of the average donation sharing game with any g^{sym}, $t_{\alpha_2} > t_{\alpha_1}$.*

Proof. Profits for fund-raiser 1 are as given by equation 2 and first order conditions are as given by equation 3. Thus, in equilibrium we must have $f'(t^1_{\alpha_1}) = \frac{t^j_{\alpha_2}}{(t^1_{\alpha_2}+t^j_{\alpha_2})^2}f(t^1_{\alpha_2}+t^j_{\alpha_2}) + \frac{t^1_{\alpha_2}}{t^1_{\alpha_2}+t^j_{\alpha_2}}f'(t^1_{\alpha_2}+t^j_{\alpha_2})$ for $\alpha_1 \in A_1$, $\alpha_2 \in A_2$, $1\alpha_1 \in g^{sym}$, $j\alpha_2 \in g^{sym}$, $j \neq 1$ and $j \in M$. Assume to the contrary that $t_{\alpha_1} = t_{\alpha_2} = \bar{t}$. Then, by assumption $f'(\bar{t}) < \frac{1}{2}\frac{f(2\bar{t})}{2\bar{t}}+\frac{1}{2}f'(2\bar{t})$. In order to meet our first order conditions we must increase the left hand side of this inequality and decrease the right hand side. Since $f'>0$ and $f''<0$ this requires that we decrease t_{α_1} and increase t_{α_2} and so in equilibrium $t_{\alpha_1} < t_{\alpha_2}$. ■

Remark 1 *We have assumed that the fund-raiser's time constraint is binding and that the fund-raiser's cost constraint is not. If instead the cost constraint is binding but not the time constraint, then all results regarding the $f''<0$ case would change quantitatively but not qualitatively. If the cost constraint is binding, then all first order conditions would remain the same but would now also all equal c. Thus, every t^i_j would decrease from the case where only the time constraint is binding, but the relationships between every t^i_k and t^i_j, $j,k \in N$, would remain the same as those given in the results above. Thus, our results would not qualitatively change. The same logic applies to the complete network case and to the monopoly fund-raiser case.*

In the next section we assume $f''>0$. Here it does not make sense to consider the case where the cost constraint is binding. If $f''>0$, and if cost is small enough so that it *is worthwhile for the fund-raiser to solicit donors, then the fund-raiser always does best by choosing* $\sum_{j=1}^{n} t_j^i = 1$. *Again the same logic applies to the complete network case and to the monopoly fund-raiser case.*

5.2 Competition on an Incomplete Network with a Convex Donation Function

When the donation function is convex, all of the time spent with a donor significantly increases donations. Thus agents want to spend time with donors that others are soliciting. However, the network may not allow all fund-raisers to solicit a popular donor; this can create multiple equilibria where several donors are solicited at the same time as can be seen in the following example.

Example 4 *Let* $M = \{1,2,3\}$ *and* $N = \{1,2,3,4,5,6,7\}$. *Let* $f' > 0$ *and* $f'' > 0$. *Let* $g = \{11,12,13,14,22,24,25,26,33,34,35,37\}$. *Thus, each fund-raiser has one tie to a donor untied to other fund-raisers, two ties to donors shared with one other fund-raiser, and one tie to a donor (donor* 4*) shared with all fund-raisers. As* $f' > 0$ *and* $f'' > 0$*, each fund-raiser wants to solicit only one donor and prefers to solicit a donor who is already being solicited by others. In this example, there are 14 pure Nash equilibrium: 1)* $t_1^1 = t_6^2 = t_7^3 = 1$*, all other* $t_j^i = 0$*; 2)* $t_4^1 = t_4^2 = t_4^3 = 1$*, all other* $t_j^i = 0$*; 3)* $t_2^1 = t_2^2 = t_3^3 = 1$*, all other* $t_j^i = 0$*; 4)* $t_2^1 = t_2^2 = t_4^3 = 1$*, all other* $t_j^i = 0$*; 5)* $t_2^1 = t_2^2 = t_7^3 = 1$*, all other* $t_j^i = 0$*. There are 9 other Nash equilibria similar to those described in 3), 4), and 5) above where two fund-raisers put all their weight on a shared tie and the third fund-raiser is left to put all his weight on a donor shared with no other fund-raisers. There are also several mixed Nash. For instance there is a mixed Nash where* $t_2^1 = t_2^2 = t_7^3 = 1$ *each with probability* $\frac{1}{2}$ *and* $t_3^1 = t_6^2 = t_5^3 = 1$ *each with probability* $\frac{1}{2}$ *and all other* $t_j^i = 0$*. There are also a number of mixed Nash similar to the following one where* $t_2^1 = t_2^2 = 1$ *and fund-raiser* 3 *plays* $t_3^3 = 1, t_4^3 = 1, t_7^3 = 1$ *all with positive probability. Note that pure Nash* 2) *above where all fund-raisers solicit donor* 4 *generates* *the highest payoff for the fund-raisers. However, at all of the other Nash equilibria, donor* 4 *is either unsolicited or solicited by only one fund-raiser.*

As the above example shows the number of Nash equilibria can be quite large for the case where $f'' > 0$. Next, we try to refine the number of Nash equilibria using stochastic stability.

Dynamics. Let time be discrete and represented as $\{0, 1, 2, ..., \tau, ...\}$. Let $t(\tau - 1)$ represent the time $t(\tau - 1) = \{t^1(\tau - 1), t^2(\tau - 1), ..., t^m(\tau - 1)\}$ the fund-raisers spend soliciting donations in period $(\tau - 1)$. At each period τ, one fund-raiser i is chosen at random to update his strategy, t^i. Fund-raiser i will choose a myopic best response to $t^{-i}(\tau - 1)$. After this choice is made, there is a small probability $1 > \varepsilon > 0$ that another strategy is chosen instead. Thus, with probability ε there is a mistake and the strategy $t^i(\tau)$ is chosen at random with each possible t^i receiving positive probability.

Stochastic Stability. This process determines a finite state, irreducible, aperiodic Markov chain and has a unique invariant probability distribution μ^ε over strategy configurations. A strategy t is *stochastically stable* if it is in the support of $\mu = \lim_{\varepsilon \to 0} \mu^\varepsilon$.[2]

Let g^1 represent a network such that there is exactly one donor, say donor 1, that all fund-raisers are tied to; all other donors have strictly less than m ties.

Proposition 8 *Assume $f' > 0$ and $f'' > 0$. At the unique stochastically stable equilibrium of the average donation sharing game with any g^1, $t_1^i = 1$ for all $i \in M$, $m \geq 2$.*

Proof. First we show that setting $t_1^i = 1$ for all $i \in M$ is a Nash equilibrium. Given all $k \neq i$, $k \in M$ set $t_1^k = 1$, we show that i's best response is to set $t_1^i = 1$. Assume to the contrary that $t_1^i = a$ such that $0 \leq a < 1$ and that $t_j^i = 1 - a$ for some $j \in N$. Then $\pi^i = f(1-a) + \frac{a}{n-1+a} f(n-1+a)$. Since $f' > 0$ and $f'' > 0$, $f(1-a) + \frac{a}{n-1+a} f(n-1+a) <$

[2] See Freidlin and Wentzell (1984) or Young (1998) for further discussion of stochastic stability.

$\frac{1}{n}f(n)$, which is i's payoff if $t_1^i = 1$. Similarly, for any other $t^i \neq (1, 0, ..., 0)$, i's payoff will be smaller than $\frac{1}{n}f(n)$. Thus, i sets $t_1^i = 1$ and our proposed strategy is a Nash equilibrium. (Note that at all Nash equilibrium each fund-raiser i sets $t_j^i = 1$ for some $j \in N$ since i's payoff, $\frac{t}{t+a}f(t+a)$, from soliciting a donor who others are soliciting with total time $a \geq 0$, is increasing in both t and a; thus i would like to spend all his time soliciting the donor with the largest such a.)

Let t^* represent the equilibrium where $t_1^i = 1$ for all $i \in M$. If t^* is the unique Nash equilibrium of the game, then it is trivially stochastically stable. Next, we assume that there is at least one other Nash equilibrium say $\widetilde{t} = (\widetilde{t}^1, \widetilde{t}^2, ..., \widetilde{t}^m)$. Let $m \geq 3$, we will consider the case of $m = 2$ below. To leave state t^* and move to state $\widetilde{t}$ takes at least $\lceil \frac{m}{2} \rceil$ trembles (where $\lceil a \rceil$, $a \in \mathbb{R}$, represents the integer $z \in \mathbb{Z}$ closest to a such that $z \geq a$). To see this note that if $\lceil \frac{m}{2} - 1 \rceil$ trembles occur, then the largest payoff a trembling agent could have is if all trembling fund-raisers solicit the same new donor resulting in a payoff for each trembling agent of $\frac{1}{\lceil \frac{m}{2} - 1 \rceil} f(\lceil \frac{m}{2} - 1 \rceil)$; this is because $\frac{1}{t}f(t)$ is strictly increasing in t whenever $f' > 0$ and $f'' > 0$. If the trembling agent goes back to his t^* strategy his payoff would be $\frac{1}{m+1-\lceil \frac{m}{2} - 1 \rceil} f(m + 1 - \lceil \frac{m}{2} - 1 \rceil)$ which is larger than $\frac{1}{\lceil \frac{m}{2} - 1 \rceil} f(\lceil \frac{m}{2} - 1 \rceil)$. Thus, $\lceil \frac{m}{2} - 1 \rceil$ trembles is not enough to leave state t^*.

However, to go from state $\widetilde{t}$ to state t^* takes at most $\lceil \frac{m}{2} - 1 \rceil$ trembles for $m \geq 3$. To see this notice that at the $\widetilde{t}$ equilibrium not all fund-raisers are soliciting the same donor, since in g^1 there is only one donor who is linked to all fund-raisers and this donor is solicited by all fund-raisers at the t^* equilibrium. Thus, at the $\widetilde{t}$ equilibrium there exists a donor, say 2, such that the number of fund-raisers soliciting 2 is less than or equal to $\frac{m}{2}$. First, assume that the number of fund-raisers soliciting 2 is less than $\frac{m}{2}$. Let $\lceil \frac{m}{2} - 1 \rceil$ fund-raisers not soliciting 2 tremble to the t^* equilibrium. Then any fund-raiser soliciting 2 has incentive to move to the t^* equilibrium as his payoff will increase from at most $\frac{1}{\lceil \frac{m}{2} - 1 \rceil} f(\lceil \frac{m}{2} - 1 \rceil)$ to $\frac{1}{\lceil \frac{m}{2} - 1 \rceil + 1} f(\lceil \frac{m}{2} - 1 \rceil + 1)$. Similarly, all remaining fund-raisers will move to the t^* equilibrium. Second, let the number of fund-raisers soliciting 2 equal $\frac{m}{2}$. Again let $\lceil \frac{m}{2} - 1 \rceil$

other fund-raisers tremble to the t^* equilibrium. Next, the remaining fund-raiser who is also not soliciting donor 2 will move to the t^* equilibrium as his payoff will increase from $f(1)$ to $\frac{1}{\lceil \frac{m}{2}-1\rceil+1} f(\lceil \frac{m}{2}-1\rceil+1)$. Similarly, all remaining fund-raisers will move to t^*. Thus, $\lceil \frac{m}{2}-1\rceil$ trembles is enough to move from state $\widetilde{t}$ to state t^*.

Lastly, consider the case where $m=2$ and where an alternative Nash, $\widetilde{t}$, exists. To move from t^* to $\widetilde{t}$ takes two trembles, since if either fund-raiser is soliciting donor 1, then the other fund-raiser has incentive to also solicit donor 1. However, to leave $\widetilde{t}$ and move to t^* takes just one tremble. Here if one fund-raiser trembles to his t^* strategy, then the other fund-raiser will also switch to solicit donor 1. By Young (1993) only states with the minimum resistance are stochastically stable; so t^* is stochastically stable while $\widetilde{t}$ is not. ■

Notice here that the unique stochastically stable state coincides with the efficient Nash equilibrium, since all players are better off soliciting the same donor when the donation function is increasing and convex.

Remark 2 *If the network has multiple donors each of whom are linked to all fund-raisers, then the above proof can be modified to show that there will be multiple stochastically stable states. At each stochastically stable state, each fund-raiser spends all his time soliciting the same donor.*

6 Conclusion

We showed that donation requests are influenced by the fund-raiser donor network and in particular by whether or not a fund-raiser shares a donor tie with other fund-raisers. For instance, in an incomplete network with extreme donor fatigue fund-raisers spend more time soliciting donors shared with other fund-raisers as compared with unshared donor ties. If donors experience instead mild donor fatigue, then fund-raisers prefer to solicit donors unshared with other fund-raisers. Additionally, we examined the case of no donor fatigue and showed that multiple equilibria may exist; stochastic stability was used to refine the number of equilibria.

The model may be extended in several ways. For instance, we could allow heterogeneous donors to exist on the same network where some donors are more willing to donate money than others. It would be interesting to see how the placement of such a donor onto the network would influence donation requests. As the current model is already quite complicated, solving this issue is beyond the scope of the current paper and is left for future research.

Additionally, we have assumed for simplicity that the network is exogenous, however it would be interesting to allow the network to be endogenous. For instance, perhaps every period fund-raisers could learn of new donors and have a chance to add a link to such a new donor or not. As fund-raisers do not have time to solicit all donors, it would be interesting to explore both the circumstances under which a fund-raiser would add and would refuse a new donor link.

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