Price Discrimination for Bayesian Buyers

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Abstract

The paper studies *2.5-degree price discrimination* to buyers whose prior valuations are initially observable to a seller but receive private information about a product or service. The buyers interpret new information via Bayes rule. In this environment, we show that prices are not monotonic in buyers' *ex ante* expected valuation. Surprisingly, a seller may offer a higher price to a low-valuation buyer than to a high-valuation buyer. This result is sharply contrasting to the standard result of price discrimination. The *reverse price discrimination* is caused by slightly different reasons in monopoly and duopoly markets. (JEL: D4, D8)

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1 Introduction

When you open the door to a car dealership, a salesperson will welcome you and ask how you are. At this moment, the salesperson tries to figure out your willingness to pay by looking at your appearance such as your gender, race, age, and asking your occupation, residence and etc. Based on this observation, he or she may provide different information and offer different prices. This looks a typical third-degree price discrimination. Salespeople may be able to observe each consumer's willingness to pay and try to sell to high valuation consumers at a higher price.

However, a crucial feature differing from a standard (third-degree) price discrimination is that consumers are uncertain about their valuations for products or services and their valuations can be changed depending on information provided by sellers. For example, car buyers can do test drives and home buyers can visit houses. Professionals provide their customers with a brief presentation or summary brochure. As a result of a seller's information provision, buyers have partial private information. In this situation, although consumers' prior willingness to pay might be observable to a seller, their posterior valuations are private information.[1] A key feature is that buyers update their valuations in a Bayesian way. That is, buyers' posterior valuations are systemically dependent on their prior beliefs through Bayes rule.[2]

A surprising result is that prices are not monotonic in buyers' *prior* valuation. The seller offers a higher price to low-valuation buyers than to high-valuation buyers. Needless to say, this result is sharply contrasting to the standard result of price discrimination which is a high price to high valuation buyers and a low price to low valuation buyers.

[1] Moscarini and Ottaviani (2001) and Damiano and Li (2007) share the same feature with our model such that a seller and buyers share a prior valuation, but buyers observe a private signal about products.

[2] Nelson (1970) classifies products into two categories: search goods and experience goods. He defines a search good as one whose qualities can be easily evaluated by the consumer before purchase. Similarly, he defines an experience good as one whose qualities are difficult to observe in advance before purchase. In fact, there are many goods and services which have both features together or which fall somewhere between the two types of goods. In addition, a seller's information provision can determine the characteristic of a good between search and experience goods. In this sense, our setup can be thought of as a general framework.

Here is the intuition underlying this result.

The seller faces a traditional trade-off in pricing decision-making between getting a higher margin and getting a higher market share. This trade-off depends crucially on the price elasticity of demand, which is the distribution of posterior valuation. The elasticity is determined by the interaction between a buyer's prior and the precision of information provided by the seller. When information precision is not extreme, neither too precise nor too imprecise, we find that the demand is elastic when the buyer's prior valuation is high while the demand is inelastic when the buyer's prior is low. In other words, if the buyer's prior is high, the seller chooses to serve all buyers at a low price because buyers' updated valuation does not decline significantly even if they receive a bad signal. On the other hand, it is more profitable for seller to charge a high price and target only buyers who receive a good signal if their prior valuation is low. This leads to the result that a higher price is offered to consumers with lower valuations and vice versa.

This result may provide another explanation to discrimination in car sales observed by Ayres (1995) and Ayres and Siegelman (1995).[3] They experiment how new-car dealerships quoted differently across customers' race and gender. They find that dealers offered significantly lower prices to white males than to non-white or female buyers even though non-white people are believed to have a lower willingness to pay than white people.[4] This is why price discrimination appears to be racial or gender discrimination. However, our theory suggests that salespeople offer a higher price to the minority group who are more likely to have a lower willingness to pay because they prefer to target those who receive a good signal.

Our result can also explain rewards to frequent customers. When a seller can identify

[3] See Yinger (1998) for a good survey and summary of the discrimination literature in consumer markets.

[4] Ayres and Siegelman (1995) discuss three hypotheses in terms of consumer information, bargaining costs and search costs. The first two hypotheses are related to negotiations. More people in minority groups are not aware of the fact that the sticker price is negotiable or averse to conducting negotiations. The last explanation is directly related to price discrimination based on the difference in consumers' willingness to pay. That is, black American might have higher willingness to pay in terms of search costs. On the contrary, our hypothesis is that salespeople may charge a higher price to black American because they are more likely to have lower willingness to pay.

repeat customers, she regards them as consumers with high willingness to pay. The recent literature on behavior-based price discrimination has been studying this issue extensively.[5] A common finding in this literature is the so called "ratchet effect", which describes price discrimination against loyal customers by charging a higher price to repeat customers. However, there are many opposite examples. Airline companies provide discounts and free upgrades to frequent customers. Automobile companies offer loyalty rebates to customers who currently own the same brand car. In other words, they price discriminate against non-loyal customers. Now, in our model, a buyer's prior can be thought of as brand loyalty. Our theory suggests that the seller may offer discount price to loyal customers when they have private information about the quality of a product.

We next consider a duopoly market. Competing sellers share buyers' prior valuations and buyers draw independent signals from both sellers. Again, we show that the reverse price discrimination can arise. However, the reason is slightly different from the monopoly case. In the duopoly, new information plays a role of differentiating two products. When a buyer draws the same signal from both sellers, there is no change in the buyer's *ex post* relative preferences for the two sellers. On the contrary, when the buyer draws different signals, the buyer perceives that two products are more differentiated and price competition can be mitigated. Interestingly, we find that the degree of differentiation is greater when the buyer has an intermediate prior valuation and so are the equilibrium prices. This is because the Bayesian buyer is more likely to draw the same signal when he is too decisive by having either very optimistic or pessimistic expectation, while the likelihood that the buyer draws different signals is greater when he is in-between. As a result, the equilibrium prices do not change monotonically with the buyer's prior valuation.

At its heart, our theory begins by departing from the standard classification of price discrimination. The third-degree price discrimination is based on the individual customer's observable identity. However, when the demand of individual consumers or groups is not observable, a seller may offer a menu of bundles which induce consumers to reveal their

[5] See Armstrong (2006) and Fudenberg and Villas-Boas (2005) for the literature review.

willingness to pay. This is the second-degree price discrimination. On the other hand, we consider the intermediate case between two types of price discrimination. Although a buyers' prior valuation can be observable to the seller and so she price discriminate based on this, the buyer receives new information and updates his valuation in the process of purchase. As a consequence, the buyer's posterior valuation is private information. In this sense, we will refer to this as *2.5-degree price discrimination*.[6] In addition, our main result also departs from the standard result of price discrimination in a significant way in that the seller charges a higher price to buyers with lower willingness to pay as a profitable strategy. Now, to emphasize the contrast to the standard result, we refer to this outcome as *reverse price discrimination*.

Price discrimination with incomplete information has been studied in the environment of screening or self-selection mechanism followed by Mussa and Rosan (1978) and Maskin and Riley (1984). In particular, Courty and Li (2000) is the closest paper to ours because they study the environment where consumers are initially uncertain of their valuations. While consumers learn their actual valuations after contracts are signed in their model, the seller provides consumers with product information before purchase in our model. In addition, their paper studies the second-degree price discrimination through refund policy in which firms are assumed not to be able to observe consumers' expected valuations.[7] On the other hand, in our paper, the seller can observe it and price discriminate based on it.

The rest of this article proceeds as follows. Section 2 describes the structure and assumptions of the model. In Section 3, we study the monopolist's price discrimination when a buyer has a unit demand and receives a binary signal. In Section 4, we show that the main result carries over to the duopoly. In Section 5, we elaborate on extensions

[6] This type of price discrimination is prevalent in consumer markets, in particular, when a seller can choose different prices to different individuals. Examples abound. Providers of professional services such as lawyers, car mechanics, medical doctors, etc. are what we have in mind.

[7] In this paper, we confine our attention to the case that the seller makes a take-it-or-leave-it offer after the buyer receives an informative signal. On the other hand, the seller may allow the buyer to purchase the product before the signal is received by the buyer. This can be thought of as the second-degree price discrimination by providing different level of information. This is the issue that our companion paper, Bang and Kim (2010), studies.

of the model. The seller is now allowed to engage in not only price discrimination but information discrimination. We show that the seller provides different levels of information to different groups of consumers. Next, we incorporate a general information structure and non-unit demand function sequentially in the model and show the robustness of our main result with respect to information and demand structure. Section 6 concludes.

2 Basic Model

Players. There is a continuum of buyers with total mass of one. The buyers have a unit demand for the good.[8] The true value of the good is $w \in \{H, L\}$ and those are mutually exclusive. If $w = H$, the good is a good match with a buyer and if $w = L$, it is a bad match. A buyer's prior belief for $w = H$ is denoted as $\theta \in [0, 1]$. We normalize the buyer's valuation for the good to be 1 for $w = H$ and 0 for $w = L$. Thus, θ can be thought of as the buyer's *ex ante* expected valuation for the good. We consider two types of buyers and type-i buyer's valuation is $\theta_i \in \{\theta_L, \theta_H\}$, where $\theta_H > 1/2 > \theta_L > 0$. We define that the buyer is a type-H if $\theta_i = \theta_H$ and type-L if $\theta_i = \theta_L$. The buyer's type is public information and observable to the seller. For simplicity, the seller's reservation value for the good is assumed to be 0.

Information. The seller provides a buyer with information about the good. Although the buyers are offered the identical information, they may draw different signals as follows. The buyer observes a binary signal $s \in \{s_H, s_L\}$ on his match value.[9] Whether the buyer observes $s = s_H$ or $s = s_L$ is private information. If $s = s_H$, the buyer infers that the good is a good match with him. If $s = s_L$, it denotes the opposite case. The signal $s \in \{s_H, s_L\}$ partially reveals the true match value of the good in the sense of Blackwell.

[8] We will consider the non-unit demand case later.

[9] Later we extend the model to incorporate a general information structure.

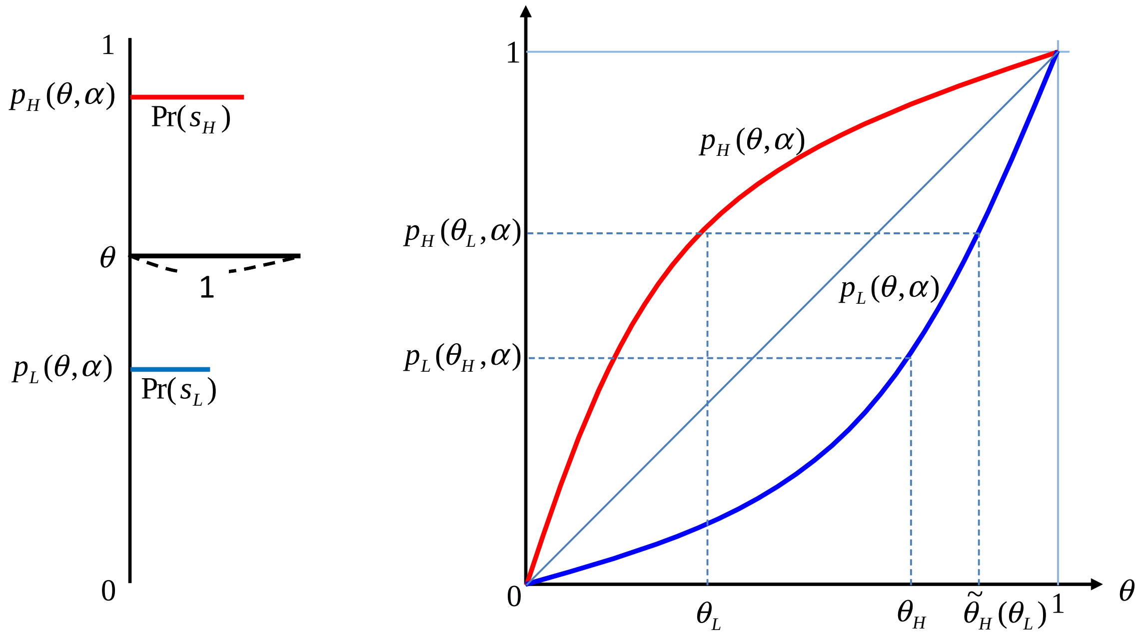

Figure 1: Demand

$$\begin{aligned}\Pr(s_H|w &= H) = \Pr(s_L|w = L) = \alpha \\ \Pr(s_L|w &= H) = \Pr(s_H|w = L) = 1 - \alpha\end{aligned}$$

where $\alpha \in \left(\frac{1}{2}, 1\right)$. Here, α is the precision of signal s and therefore can be interpreted as the quality of information.

Let us refer to $p_H(\theta_i, \alpha)$ and $p_L(\theta_i, \alpha)$ as the buyer's posterior valuation when type i buyer observes a signal s_H and s_L, respectively. Then, Bayes' rule leads to

$$p_H(\theta_i, \alpha) = \frac{\alpha\theta_i}{\alpha\theta_i + (1-\alpha)(1-\theta_i)} \text{ and} \tag{1}$$

$$p_L(\theta_i, \alpha) = \frac{(1-\alpha)\theta_i}{\alpha(1-\theta_i) + (1-\alpha)\theta_i} \tag{2}$$

because we normalize the buyer's valuation for the good to be 1 for $w = H$ and 0 for $w = L$.

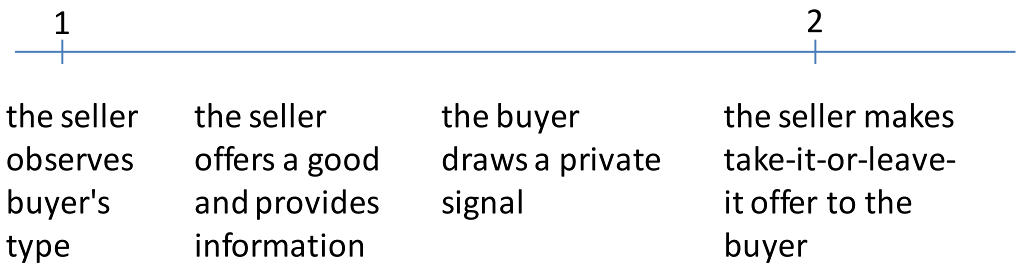

Figure 2: Timing

The buyer's posterior valuation is a mean-preserving spread of his prior belief. A signal disperses the prior belief to two-point distribution: $p_H(\theta_i, \alpha)$ with the probability $\Pr(s_H)$ and $p_L(\theta_i, \alpha)$ with the probability $\Pr(s_L)$. The probabilities that the buyer receive a good signal and bad signal are given by $\Pr(s_H) = \sum_{w \in \{L,H\}} \Pr(s_H|w)\Pr(w)$ and $\Pr(s_L) = \sum_{w \in \{L,H\}} \Pr(s_L|w)\Pr(w)$, respectively. In this information structure, the following properties are well-known. First, the posterior valuations $p_H(\theta_i, \alpha)$ and $p_L(\theta_i, \alpha)$ are increasing in θ at a decreasing rate and at an increasing rate respectively:

$$\frac{\partial p_H(\theta_i, \alpha)}{\partial \theta} > 0, \ \frac{\partial p_L(\theta_i, \alpha)}{\partial \theta} > 0, \ \frac{\partial^2 p_H(\theta_i, \alpha)}{\partial \theta} < 0, \text{ and } \frac{\partial^2 p_L(\theta_i, \alpha)}{\partial \theta} > 0 \tag{3}$$

as represented in Figure 1. Second, $p_H(\theta_i, \alpha)$ is an increasing function and $p_L(\theta_i, \alpha)$ is a decreasing function in α:

$$\frac{\partial p_H(\theta_i, \alpha)}{\partial \alpha} > 0 \text{ and } \frac{\partial p_L(\theta_i, \alpha)}{\partial \alpha} < 0 \tag{4}$$

Timing. The timing of the game is summarized in Figure 2. In the first stage, the seller observes the buyer's type. She knows whether the buyer's prior valuation is θ_H or θ_L. The seller provides information about the good. Then, the buyer draws a private signal about the match value of the good. In the second stage, the seller quotes price, $P \in R_+$ to the buyer based on observable θ_i and makes take-it-or-leave-it offer. We assume that the buyer prefers to purchase the good when he is indifferent between buying and not buying the good.

3 Monopoly: 2.5-degree Price Discrimination

Given the information structure, we find the optimal prices that the seller offers to each type of buyer. Note that when buyer i receives a good signal, he will purchase the good if and only if $p_H(\theta_i, \alpha) - P \geq 0$. Likewise, when he receives a bad signal, he will purchase if and only if $p_L(\theta_i, \alpha) - P \geq 0$. Hence, the expected demand is the equilibrium probability that buyer i accepts an offer for a given price P, which is given by

$$D(P) = \begin{cases} 1, & \text{if } P \leq p_L(\theta_i, \alpha), \\ \Pr(s_H) = \alpha\theta_i + (1-\alpha)(1-\theta_i), & \text{if } p_L(\theta_i, \alpha) < P \leq p_H(\theta_i, \alpha) \\ 0, & \text{if } P > p_H(\theta_i, \alpha). \end{cases}$$

Figure 2 illustrates the demand function depending on the realization of private information.

For each type buyer, the seller has to choose one between two alternative prices: (i) a high price at which only the buyer who receives a good signal $s = s_H$ can afford to buy, i.e., $P = p_H(\theta_i, \alpha)$ and (ii) a low price at which everyone including the buyer who receives a bad signal $s = s_L$ can afford to buy, i.e., $P = p_L(\theta_i, \alpha)$. When a seller charges $p_H(\theta_i, \alpha)$ to type-i buyer, she can sell the good with probability $\Pr(s_H)$ which is the probability that the buyer receive a signal $s = s_H$. On the other hand, she can sell the good with probability 1 if she charges $p_L(\theta_i, \alpha)$. Keep in mind that the seller can observe the buyer's type (θ_i, *ex ante* valuation) and therefore can offer different prices to two different types of buyer.[10]

Let $\pi_H(\theta_i, \alpha)$ and $\pi_L(\theta_i, \alpha)$ be the seller's expected profit when she charges $p_H(\theta_i, \alpha)$

[10] As with any model of price discrimination, we rule out a resale possibility.

and $p_L(\theta_i, \alpha)$ to a type-i buyer. Then, we obtain from (1) and (2)

$$\begin{aligned} \pi_H(\theta_i, \alpha) &= p_H(\theta_i, \alpha) \Pr(s_H) = \alpha\theta_i \text{ and} & (5) \\ \pi_L(\theta_i, \alpha) &= p_L(\theta_i, \alpha) \cdot 1 = \frac{(1-\alpha)\theta_i}{\alpha(1-\theta_i) + (1-\alpha)\theta_i}. & (6) \end{aligned}$$

Comparing $\pi_H(\theta_i, \alpha)$ and $\pi_L(\theta_i, \alpha)$ determines the price a seller charges to type-i buyer. In fact, the comparison tells us whether the expected demand is inelastic, unit-elastic, or elastic. Since the expected demand is a two-point distribution, we use the midpoint method for calculating the price elasticity of demand.

$$\varepsilon_p = -\frac{\Pr(s_H) - 1}{p_H(\theta_i, \alpha) - p_L(\theta_i, \alpha)} \cdot \frac{\frac{p_H(\theta_i,\alpha)+p_L(\theta_i,\alpha)}{2}}{\frac{1+\Pr(s_H)}{2}}.$$

Lemma 1 $\pi_H(\theta_i, \alpha) \gtreqless \pi_L(\theta_i, \alpha)$ *as* $\varepsilon_p \lesseqgtr 1$. *The seller charges a higher price when the demand is inelastic and a lower price when the demand is elastic.*

The tension in determining the price is the trade-off between getting a higher margin by charging $p_H(\theta_i, \alpha)$ and getting a greater market share by charging $p_L(\theta_i, \alpha)$. The seller's pricing decision is determined by the price elasticity of demand. Now, let us show that the elasticity systemically depends on the interaction between a buyer's prior valuation and the precision of information. There are three possible cases. The following proposition summarizes the result.

Proposition 2 *Let* $\alpha_2 = \alpha^*(\theta_i = \theta_H)$ *and* $\alpha_1 = \alpha^*(\theta_i = \theta_L)$ *where*

$$\alpha^*(\theta_i) = \frac{\left((\theta_i + 1) - \sqrt{\theta_i^2 - 6\theta_i + 5}\right)}{2(2\theta_i - 1)}.$$

Then there exist $\alpha_1, \alpha_2 \in \left(\frac{1}{2}, 1\right)$ *such that the following results hold.*

(i) If $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, a seller charges $p_L(\theta_i, \alpha)$ for each type i.

(ii) If $\alpha \in [\alpha_1, \alpha_2]$, a seller charges $p_H(\theta_L, \alpha)$ for L buyer and $p_L(\theta_H, \alpha)$ for H buyer.

(iii) If $\alpha \in (\alpha_2, 1)$, a seller charges $p_H(\theta_i, \alpha)$ for each type i.

If information is sufficiently precise, the demand becomes inelastic. The seller prefers to get a higher margin because she has to lower the price too much in order to sell to the buyer with a bad signal. On the other hand, the price $p_H(\theta_i, \alpha)$ is high enough to compensate the loss in profit from losing the buyer with a bad signal. Hence the seller charges $p_H(\theta_i, \alpha)$ to each type of buyers. By contrast, if information is sufficiently imprecise, the demand is now elastic. In this case, the seller prefers to have the large market share and serve all buyers by charging $p_L(\theta_i, \alpha)$. In these two cases, the elasticity is solely determined by the quality of information.

The most interesting case is the one where information quality is intermediate and the buyer's prior valuation determines the elasticity. When a buyer is type-H, it is more important to increase a market share because type-H buyer's posterior valuation even with $s = s_L$ is relatively high enough to make the seller offer $p_L(\theta_L, \alpha)$ and serve all H buyers. On the other hand, when a buyer is type-L, the seller has to lower the price significantly if she wants to serve all the buyers with $s = s_L$. Hence increasing a market share is not attractive and the seller charges the higher price $p_H(\theta_L, \alpha)$ to L buyers.[11] Now we compare the prices offered to each type of buyers.

Proposition 3 *If* $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$ *or* $\alpha \in (\alpha_2, 1)$*, the price offered to type-H buyer is higher than that offered to type-L buyer.*

If information quality is sufficiently precise, i.e., $\alpha \in (\alpha_2, 1)$, the seller charges $p_H(\theta_i, \alpha)$ for each type i. Since $p_H(\theta_i, \alpha)$ is an increasing function in θ_i from (1), we obtain

[11] Even if perfect price discrimination is allowed, it does not obtain full efficiency when the quality of goods and services is uncertain and buyers are partially informed.

$p_H(\theta_H,\alpha) > p_H(\theta_L,\alpha)$. On the other hand, if information quality is sufficiently imprecise, i.e., $\alpha \in \left(\frac{1}{2},\alpha_1\right)$, the seller charges $p_L(\theta_i,\alpha)$ for each type i. Again, we obtain $p_L(\theta_H,\alpha) > p_L(\theta_L,\alpha)$ because $p_L(\theta_i,\alpha)$ is also an increasing function in θ_i from (2). Hence, if information quality is either sufficiently precise or imprecise, the price offered to H buyer should be higher than that for L buyer. In fact, this is the standard result of price discrimination.

Now, the striking result is that we find the possibility of *reverse price discrimination* for the intermediate case where $\alpha \in [\alpha_1,\alpha_2]$. The price offered to L buyer can be higher than the price offered to H buyer. As shown in Figure 1, since $p_H(\theta_i,\alpha)$ is concave in θ_i and $p_L(\theta_i,\alpha)$ is convex in θ_i, we can easily denote the case of $p_H(\theta_L,\alpha) > p_L(\theta_H,\alpha)$ by looking at a certain θ_L and θ_H. We summarize this result more formally in the following proposition.

Proposition 4 *Suppose that $\alpha \in [\alpha_1,\alpha_2]$. Let us define $\widetilde{\theta}_H(\theta_L)$ such that $p_L(\widetilde{\theta}_H(\theta_L),\alpha) = p_H(\theta_L,\alpha)$ for given θ_L and $\widetilde{\theta}_L(\theta_H)$ such that $p_H(\widetilde{\theta}_L(\theta_H),\alpha) = p_L(\theta_H,\alpha)$ for given θ_H. We obtain the reverse price discrimination,*

$$p_H(\theta_L,\alpha) > p_L(\theta_H,\alpha),$$

if $\theta_H < \widetilde{\theta}_H(\theta_L) = \frac{\alpha^2\theta_L}{\alpha^2\theta_L+(1-\alpha)^2(1-\theta_L)}$, or equivalently $\widetilde{\theta}_L(\theta_H) = \frac{(\alpha-1)^2\theta_H}{(\theta_H-2\alpha\theta_H+\alpha^2)} < \theta_L$.

For the reverse price discrimination to arise, the two prior valuations θ_L and θ_H should not differ too much. For given θ_L, we need to have $\theta_L < \theta_H < \widetilde{\theta}_H(\theta_L)$. For given θ_H, we need to have $\widetilde{\theta}_L(\theta_H) < \theta_L < \theta_H$. In other words, the difference between θ_L and θ_H should be bounded above. In addition, note that $\frac{\partial\left(\widetilde{\theta}_H(\theta_L)\right)}{\partial\alpha} > 0$ and $\frac{\partial\left(\widetilde{\theta}_L(\theta_H)\right)}{\partial\alpha} < 0$.[12] That is, for given θ_L, as α increases, the value of $\widetilde{\theta}_H(\theta_L)$ below which $p_H(\theta_L,\alpha) > p_L(\theta_H,\alpha)$ increases.

[12] $\frac{\partial\left(\widetilde{\theta}_H(\theta_L)\right)}{\partial\alpha} = \frac{2(\theta_L-1)(\alpha-1)\alpha\theta_L}{\left(2\alpha\theta_L-\theta_L-2\alpha+\alpha^2+1\right)^2} > 0$ and $\frac{\partial\left(\widetilde{\theta}_L(\theta_H)\right)}{\partial\alpha} = \frac{2(1-\theta_H)(\alpha-1)\alpha\theta_H}{\left(2\alpha\theta_H-\theta_H-\alpha^2\right)^2} < 0$.

Also for given θ_H, as α increases, the value of $\widetilde{\theta}_L(\theta_H)$ above which $p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha)$ decreases.

Corollary 3.1 *For given $\alpha \in [\alpha_1, \alpha_2]$, as information quality α increases (decreases), the parameter set of θ_L and θ_H for which the reverse price discrimination is derived increases (decreases).*

As information quality increases, L buyer's posterior evaluation after observing a good signal becomes relatively high enough to compensate the initial low prior belief. Also although H buyer's initial prior belief is high, his posterior belief after observing a bad signal becomes relatively low. Hence even though the prior valuation differs much, as information quality increases, the reverse price discrimination is more likely to be derived.

We believe that our result is a new explanation for Ayres and Siegelman (1995). According to their findings, the car dealers offered the \$1,061 higher price to non-white buyers, although the non-white buyers are believed to have a lower willingness to pay than white buyers. Our model suggests that their findings can be due to the dealer's strategy that she targets only the buyers having a higher posterior valuation for the case of non-white buyers. Provided that their initial willingness to pay, from a low prior valuation, is relatively low, the dealer has to lower the price too significantly if she wants to serve everyone including even the ones with low posterior valuation. Thus, it may yield a larger profit for a seller to target only the ones with high posterior valuation. On the other hand, even though the white buyers are believed to have relatively high willingness to pay, the seller may want to offer a relatively low price in order to entice all the buyers regardless of the signals they receive, since even the ones getting a low signal still have sufficiently high valuation.

4 Duopoly: Hotelling Model

In this section, we consider a duopoly market where two sellers are located at the two end points on the Hotelling line of unit length. Two sellers, A and B, supply products A and B respectively. As in a standard Hotelling model, each buyer is indexed $x \in [0, 1]$ which denotes a buyer's location or brand preference and is uniformly distributed. Buyers purchase either one unit of a good from only one firm or nothing. On average, type $i-$buyer's value is $v + \theta_i - tx$ for good A and $v + \theta_i - t(1 - x)$ for good B. We add to the model the common value v which is sufficiently large so that the market is fully covered. The parameter t reflects the degree of product differentiation. To keep consistency with Monopoly case in the previous section, the buyers' prior θ_i is known to both the buyers and the sellers. For analytical simplicity, we assume that the symmetric information structure is exogenously given.

Now the buyers independently receive a private signal from each seller.[13] There are four different cases by the four possible combinations of the signal realizations: (group HH) good signals from both sellers with probability $\Pr(s_H)^2$, (group LL) bad signals from both sellers with probability $\Pr(s_L)^2$, (group HL) a good signal from seller A and a bad signal from seller B with probability $\Pr(s_H)\Pr(s_L)$, and (group LH) a bad signal from seller A and a good signal from seller B with probability $\Pr(s_H)\Pr(s_L)$.

Figure 3 illustrates the demand structure as a result of the realization of private signals. When the buyers receive the same signals from both sellers, there is no difference from the standard Hotelling model because their relative preference between the two goods does not change. This is the case for groups HH and LL and the location of marginal consumers who are indifferent between the two goods is denoted by

$$x_{HH} = x_{LL} = \frac{1}{2} + \frac{P_B - P_A}{2t},$$

[13] The signals do not need to be drawn independently unless they are perfectly correlated. This feature can be generalized to allow the draws to be imperfectly correlated.

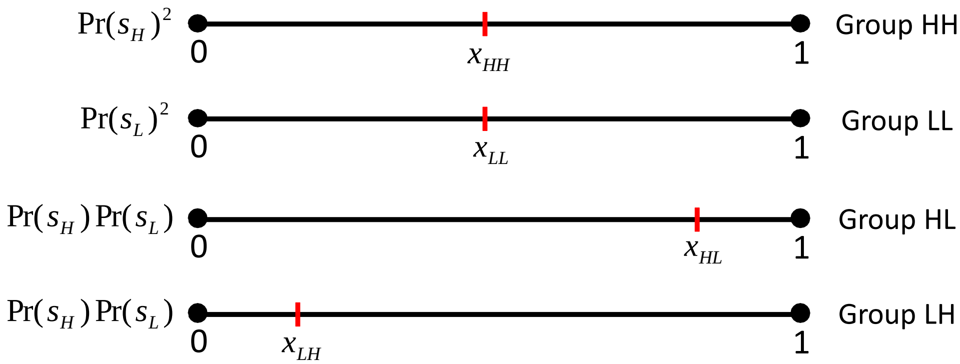

Figure 3: Demand in the Hotelling Model

where P_A and P_B are the prices offered by firms A and B, respectively. On the other hand, when they draw different signals, the buyers become biased toward one good over the other by $p_H(\theta) - p_L(\theta)$.[14] Let us denote this bias by $\Delta(\theta) \equiv p_H(\theta) - p_L(\theta)$. Note that $0 < \Delta(\theta) < 1$, for $\frac{1}{2} < \alpha < 1$. The marginal consumers for group HL and group LH are respectively

$$x_{HL} = \frac{1}{2} + \frac{P_B - P_A + \Delta(\theta)}{2t} \text{ and } x_{LH} = \frac{1}{2} + \frac{P_B - P_A - \Delta(\theta)}{2t}.$$

In fact, Figure 3 represents the case where the bias is not so large compared to the price difference that $x_{HL} < 1$ and $x_{LH} > 0$. Thus, for example, firm A's demand function is written by

$$D_A = (\Pr(s_H)^2 + \Pr(s_L)^2)x_{HH} + \Pr(s_H)\Pr(s_L)(x_{HL} + x_{LH}), \text{ if } x_{HL} < 1 \text{ and } x_{LH} > 0.$$

By contrast, when the bias is large enough relative to the price difference, all group HL buyers purchase good A, while all group LH buyers purchase good B. The two sellers can avoid competition for these two groups of buyers, if both firms find the bias too large to

[14] When $\alpha = 1/2$, the model comes down to a standard Hotelling model because $p_H(\theta) = p_L(\theta)$.

be overcome by lowering the price. In this case, seller A's demand function is written by

$$D_A = (\Pr(s_H)^2 + \Pr(s_L)^2)x_{HH} + \Pr(s_H)\Pr(s_L), \text{ if } x_{HL} > 1 \text{ and } x_{LH} < 0.$$

Similarly, we can write down seller B's demand function.[15] Each firm's demand is summarized by

$$D_A = \begin{cases} \frac{1}{2} + \frac{P_B - P_A}{2t}, & \text{if } (P_B - P_A) \in [\Delta(\theta) - t, t - \Delta(\theta)] \text{ and } \Delta(\theta) \leq t, \\ \left(\Pr(s_H)^2 + \Pr(s_L)^2\right)\left(\frac{1}{2} + \frac{P_B - P_A}{2t}\right) + \Pr(s_H)\Pr(s_L), & \\ & \text{if } (P_B - P_A) \in [t - \Delta(\theta), \Delta(\theta) - t] \text{ and } \Delta(\theta) > t, \end{cases}$$

$$D_B = \begin{cases} \frac{1}{2} + \frac{P_A - P_B}{2t}, & \text{if } (P_B - P_A) \in [\Delta(\theta) - t, t - \Delta(\theta)] \text{ and } \Delta(\theta) \leq t, \\ \left(\Pr(s_H)^2 + \Pr(s_L)^2\right)\left(\frac{1}{2} + \frac{P_A - P_B}{2t}\right) + \Pr(s_H)\Pr(s_L), & \\ & \text{if } (P_B - P_A) \in [t - \Delta(\theta), \Delta(\theta) - t] \text{ and } \Delta(\theta) > t. \end{cases}$$

In turn, we solve the two sellers' maximization problems, the symmetric equilibirum prices can be readily shown as follows.

$$P^*(\theta) = P_A^* = P_B^* = \begin{cases} t, & \text{if } \Delta(\theta) \leq t, \\ t\left(1 + 2\frac{\Pr(s_H)\Pr(s_L)}{\Pr(s_H)^2 + \Pr(s_L)^2}\right), & \text{if } \Delta(\theta) > t. \end{cases}$$

When the bias is less than the product differentiation parameter t, the equilibrium price is t as in the standard Hotelling model. However, when the bias is greater than t, the equilibrium price is also greater than t. In addition, the equilibrium price does not behave monotonically with respect to θ. It is increasing in $\theta < 1/2$ and decreasing in $\theta > 1/2$ as shown in Figure 4, since the probability that buyers receive different signals from each seller thereby being biased toward one good over the other is maximized at $\theta = 1/2$.

Now, let us study how $\Delta(\theta)$ behaves with θ. $\Delta(\theta)$ is strictly concave and has a unique

[15] Of course, there are two asymmetric cases such as $x_{HL} < 1$ and $x_{LH} < 0$ and $x_{HL} > 1$ and $x_{LH} > 0$. However, we exclude those cases from the demand functions, since we focus on the symmetric equilibrium in this section.

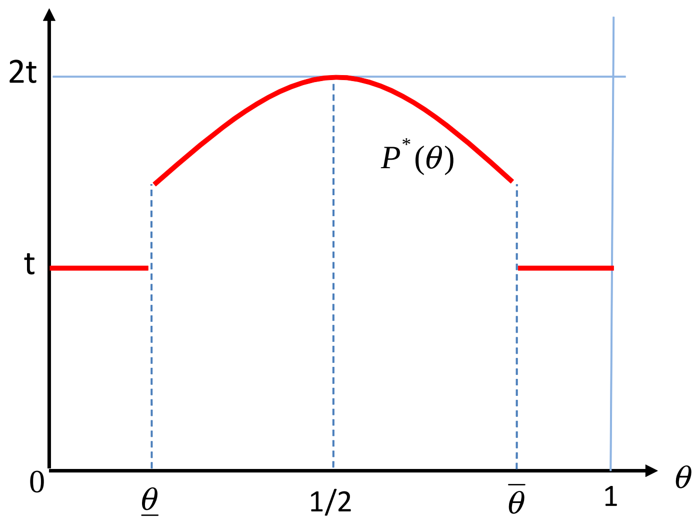

Figure 4: Equilibrium prices in Duopoly

maximum. It can be easily shown that $\partial\Delta(\theta)/\partial\theta = 0$ at $\theta = 1/2$ and the maximum of $\Delta(\theta)$ is $2\alpha - 1$.

Lemma 5 *When $\alpha > \frac{t+1}{2}$, there exist two cutoff values $\overline{\theta} \in (\frac{1}{2}, 1)$ and $\underline{\theta} \in (0, \frac{1}{2})$ such that*

$$\begin{aligned} \Delta(\theta) &> t, \text{ if } \theta \in (\underline{\theta}, \overline{\theta}), \text{ and} \\ \Delta(\theta) &\leq t, \text{ if } \theta \in [0, \underline{\theta}] \text{ or } [\overline{\theta}, 1]. \end{aligned}$$

Interestingly, in the duopoly market, there is also a possibility that both firms offer a higher price to a buyer with lower willingness to pay than to the one with higher willingness to pay. However, the driving force is different from the monopoly case. Here, one sufficient condition for reverse price discrimination is that $\theta_L \in (\underline{\theta}, \overline{\theta})$ and $\theta_H \in [\overline{\theta}, 1]$, given $\alpha > \frac{t+1}{2}$. If the low type buyer's prior belief falls into the intermediate level, then after receiving private signals he would view two products more differentiated than the high type buyer would do, and therefore two sellers compete less intensely for the low type than the high type buyer. The intuition is as follows. The consumers having θ close to $\frac{1}{2}$ can be regarded as "less confident" about their valuation of the product – i.e., the probability that their match value turns out to be either $w = H$ or $w = L$ is similar. Thus, they are more likely to become biased toward one of the products when observing different signals from each seller, which in turn mitigates the competition between the firms and leads to a higher equilibrium price. Whereas the consumers having θ close to either 0 or 1 are so decisive and stubborn that they hardly incline to one product over the other even if they receive different signals. To put it simply, the easier consumers are swayed by the signals, the more they are exploited by the firms. The conditions for the reverse price discrimination are summarized in the following proposition.

Proposition 6 *Suppose that $\alpha \in (\frac{t+1}{2}, 1)$. We obtain the reverse price discrimination in a symmetric equilibrium,*

$$P^*(\theta_L) > P^*(\theta_H),$$

if (i) $\theta_L \in (\underline{\theta}, \overline{\theta})$ and $\theta_H \in [\overline{\theta}, 1]$ or (ii) $\theta_H \in (\frac{1}{2}, \overline{\theta})$ and $\theta_L \in (1 - \theta_H, \theta_H)$.

Our paper may provide complementary but sharply contrary results to those shown by Armstrong (2006). He shows that it has no effect on the firms' prices and profits in the Hotelling model that the firms can observe a consumer's valuation and target a personalized price to the consumers. In our setting with private information, however, the sellers may offer different personalized prices based on buyers' prior valuations.[16]

[16] Damiano and Li (2007) study a very similar issue such as price competition for privately informed buyers. A crucial difference is that they focus on the case in which the prior is fixed as $\theta = 1/2$. In other words, price discrimination is not an issue in their paper. On the other hand, our focus is to find what prices two sellers offer to buyers based on θ. In addition, it is worthwhile to explain the difference of the modeling strategy between the two papers. In their model, the two goods are ex ante identical. Thus, there is no pure-strategy Nash equilibrium and it is hard to characterize and compare the equilibrium prices with our general prior θ. This is the reason why we model the duopoly market by using the Hotelling model of product differentiation.

5 Extensions

5.1 Information Provision

In the previous section, the information structure α has been given exogenously. We relax this assumption so that the seller can choose the level of information. In other words, the seller chooses the optimal level of information. To focus attention on the strategic effect of information, we abstract from any costs the seller might incur in providing information. We allow the seller to provide different level of information to different types of buyer. For example, the salesperson may provide different level of information to white and non-white buyers.

It can be easily understood that the seller has opposite incentives in providing information to different people. The seller prefers low-valuation buyers to be more informed about the good, while she prefers H buyer to be less informed. Since the seller's optimal strategy is to serve a portion of low-valuation buyers at a high price, she wants to make these buyers update their match value more precisely and have a higher ex post value. On the contrary, for H buyers, since the seller serves all buyers at a low price, she wants to make buyers update less.

Proposition 7 *When the seller can information discriminate, the seller provides as more information as possible for L buyer, whereas she provides as less information as possible for H buyer.*

Ayres and Siegelman (1995) test whether salespeople might discriminate white and non-white buyers simply because of their animus or bigotry. However, they find that salespeople actually spent longer time with non-white consumers.[17] Our result about

[17] The page 316 states "[I]n addition, we would expect that bigoted salespeople would want to spend less time with non-white-male testers than with white males. In fact, however, salespeople spent nearly 13-percent longer negotiating with the "minority" testers than with the white males, which cast doubt on salesperson animus as the source of price differences."

information provision may be able to explain their observation in a different way. Since the seller targets the low-valuation buyer only with receiving a good signal, salespeople try to inform non-white consumers as much as possible. On the other hand, in our model, the seller does not have incentives to provide any information to H buyer because the seller prefers the high-valuation buyer to receive a bad signal as little as possible.

5.2 General Information Structure

In this section, we incorporate a general information structure. The buyer observes a signal $\alpha \in [0,1]$, distributed according to density $f_H(\alpha)$ if a good is H and $f_L(\alpha)$ if it is L. Both densities are bounded away from zero and we assume the monotone likelihood ratio property such that the likelihood function $f_L(\alpha)/f_H(\alpha)$ is decreasing in α. Then, the buyer's posterior valuation, given prior θ, can be written by

$$p(\theta,\alpha) = \frac{\theta f_H(\alpha)}{\theta f_H(\alpha) + (1-\theta) f_L(\alpha)} = 1 \Bigg/ \left[1 + \frac{(1-\theta)}{\theta}\frac{f_L(\alpha)}{f_H(\alpha)}\right].$$

Given his prior belief and a signal about the quality, the buyer will make a decision of whether to buy or not. The buyer decides to purchase the good if he receives a signal better than his standard. Since a buyer's expected net payoff from buying the good is $p(\theta,\alpha) - P$, he purchases if and only if $p(\theta,\alpha) \geq P$, i.e., $\frac{f_L(\alpha)}{f_H(\alpha)} \leq \frac{\theta}{(1-\theta)}\frac{1-P}{P}$. The cutoff signal is defined as

$$\widehat{\alpha}(\theta,P) \equiv \min\left\{\alpha \in [0,1] \,\middle|\, \frac{f_L(\alpha)}{f_H(\alpha)} \leq \frac{\theta}{(1-\theta)}\frac{1-P}{P}\right\}. \tag{7}$$

If the buyer receives a signal greater (smaller) than the standard $\widehat{\alpha}(\theta,P)$, he decides to buy (not buy) the good or service. $\widehat{\alpha}(\theta,P)$ is downward sloping in θ. The group with a more optimistic belief sets a lower standard because $\widehat{\alpha}(\theta,P)$ is decreasing in θ. In contrast, buyers set higher standards for higher prices obviously because $\widehat{\alpha}(\theta,P)$ is increasing in P.

We turn to the seller's problem. The seller chooses the optimal price P. Since the buyer purchases the good when he receives a signal $\alpha \geq \widehat{\alpha}$, the expected demand can be written by

$$D(\widehat{\alpha}) = \theta[1 - F_H(\widehat{\alpha})] + (1-\theta)[1 - F_L(\widehat{\alpha})].$$

Thus, the seller's profit function is $\Pi = PD(P)$. This can be rewritten in terms of $\widehat{\alpha}$ as $\Pi = p(\theta, \widehat{\alpha})D(\widehat{\alpha})$. The first-order condition is $\frac{\partial p(\theta,\widehat{\alpha})}{\partial \widehat{\alpha}} D(\widehat{\alpha}) + p(\theta, \widehat{\alpha})\frac{\partial D(\widehat{\alpha})}{\partial \widehat{\alpha}} \leq 0$. This gives us the optimal price $P^*(\theta)$. However, it is hard to find the sign of $\frac{\partial P^*(\theta)}{\partial \theta}$ in this general apporach.

Let us look at the simplest case of this model in which a good signal follows the uniform distribution on $[0, \underline{\alpha}]$ and a bad signal follows the uniform distribution on $[0, \overline{\alpha}]$, where $\overline{\alpha} < \underline{\alpha}$. When $\alpha > \underline{\alpha}$, buyers can be sure that this is certainly good H, while when $\alpha < \overline{\alpha}$, this is certainly good L. When $\alpha \in [\overline{\alpha}, \underline{\alpha}]$, the quality of the good is unclear. Since the likelihood ratio $\frac{1-\overline{\alpha}}{\underline{\alpha}}$ is constant, buyers purchase the good as long as $\frac{\underline{\alpha}\theta}{\underline{\alpha}\theta + (1-\overline{\alpha})(1-\theta)} \geq P$.

Proposition 8 *There exists a cutoff value $\widehat{\theta}$ so that*

$$P^* = \begin{cases} 1 & \text{if } \theta \leq \widehat{\theta} \\ \frac{\underline{\alpha}\theta}{\underline{\alpha}\theta + (1-\overline{\alpha})(1-\theta)} & \text{if } \theta > \widehat{\theta}. \end{cases}$$

The result is very similar to our basic model. When the buyer has a relatively low valuation, the seller charges the maximum price 1 and serves only the buyer who can be sure of the high quality. By contrast, when the buyer's valuation is greater than the threshold $\widehat{\theta}$, the seller offers a lower price to serve the buyer who is uncertain of the quality. The reverse price discrimination, $P(\theta_L) > P(\theta_H)$, arises if $\theta_L \in [0, \widehat{\theta}]$ and $\theta_H \in (\widehat{\theta}, 1]$.

5.3 Non-unit Demand: Linear Demand Case

We now relax the buyer's unit demand to a linear demand function. A buyer demand function is given by $Q(P) = \theta - P$. After the buyer draws a good or bad signal from

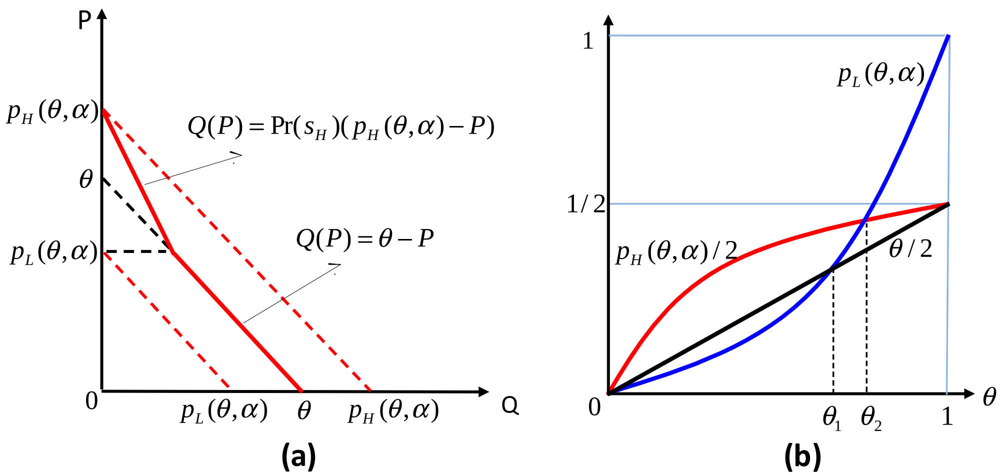

Figure 5: (a) Linear demand function (b) The optimal price

the seller, his demand function is either $Q(P) = p_H(\theta, \alpha) - P$ or $Q(P) = p_L(\theta, \alpha) - P$ respectively. The seller's expected demand function for the Bayesian buyer should be an inwardly kinked demand function as shown in Figure.

$$Q(P) = \begin{cases} \theta - P & \text{if } P \leq p_L(\theta, \alpha), \\ \Pr(s_H)\,(p_H(\theta, \alpha) - P) & \text{if } P \in (p_L(\theta, \alpha), p_H(\theta, \alpha)], \\ 0 & \text{if } P > p_H(\theta, \alpha). \end{cases}$$

When $P \leq p_L(\theta, \alpha)$, the expected demand is simply $Q(P) = \Pr(s_H)\,(p_H(\theta, \alpha) - P) + \Pr(s_L)\,(p_L(\theta, \alpha) - P)$, which turns out to be $Q(P) = \theta - P$. On the contrary, when $P > p_L(\theta, \alpha)$, notice that the buyer has a demand only when he draws a good signal with probability $\Pr(s_H)$. As a result, the demand curve must be inwardly kinked in Figure 4 (a) and there may be two local maximum.

It turns out that the profit maximizing price is either $p_H(\theta, \alpha)/2$ or $\theta/2$. Without a production cost, the monopoly price is simply the half of the price intercept of the linear demand curve. We find a threshold at which the seller switches from the high price to the low price. The price is not monotonic in θ, again.

Proposition 9 *If $\alpha > \frac{\sqrt{5}-1}{2}$, there exist cutoff values θ_1, θ_2, and $\widetilde{\theta}$ so that*

$$P^* = \begin{cases} p_H(\theta,\alpha)/2 & \text{if } \theta \leq Min\{Max\{\theta_1, \widetilde{\theta}\}, \theta_2\} \\ \theta/2 & \text{otherwise.} \end{cases}.$$

On the other hand, if $\alpha \leq \frac{\sqrt{5}-1}{2}$, $Min\{Max\{\theta_1, \widetilde{\theta}\}, \theta_2\} = \theta_2$.

Essentially, the case with non-unit demand function does not differ from the unit-demand. Again, there is a downward jump in the price at a certain threshold, $\theta \in \{\theta_1, \theta_2, \widetilde{\theta}\}$. As a result, we can obtain the reverse price discrimination such as $P(\theta_L) > P(\theta_H)$.

6 Concluding Remarks

This paper characterizes price discrimination under partially incomplete information in the sense that a buyer's expected valuation can be observed by the seller but the buyer draws a private signal from information provided by the seller. In this case, it is possible that the buyer with a higher willingness to pay is offered a lower price and vice versa. We have also shown that this reverse price discrimination can arise in both monopoly and duopoly markets. In the monopoly market, the seller targets low type buyers who draw a good signal, whereas she wants to serve all of high type buyers at a low price. On the other hand, in the duopoly market, buyers perceive that two competing products are more differentiated by new information when they have intermediate prior valuations.

Our result might be considered restrictive because the information quality should be intermediate, i.e., $\alpha \in [\alpha_1, \alpha_2]$ for the reverse price discrimination to arise. In other words, the signals buyers observe should be neither too precise nor too vague. However, we believe this is more realistic in many cases, especially for experience goods. Car dealership can set a good example since consumers are usually given some opportunities to inspect and test drive a car before the purchase is made, but it is still hard for them to completely figure out how well the car fits their taste. In addition, if we consider that the seller has a positive reservation price in the model, the reverse price discrimination can arise even when the information quality is relatively low, i.e., $\alpha < \alpha_1$. Suppose that the seller's reservation price is r, which is greater $p_L(\theta_L, \alpha)$ but smaller than $p_L(\theta_H, \alpha)$. In this case, the seller charges $p_L(\theta_H, \alpha)$ to H type and $p_H(\theta_L, \alpha)$ to L type buyer. Again, there is a possibility to get $p_L(\theta_H, \alpha) < p_H(\theta_L, \alpha)$.

Another perspective to the intermediate level of information quality is that the type of goods and services we are interested in are somewhere between search goods and experience goods. While $\alpha = 1$ is comparable to the case where buyers are able to observe the quality of a good completely before purchase, $\alpha = 1/2$ represents the other case where they are not able to observe in advance. In fact, these are the definitions of search goods and experience goods respectively. One can think that the standard result holds for almost search and experience goods. (Proposition 3) However, the result is reversed for many goods and services that are neither search goods nor experience goods perfectly. (Proposition 4)

Appendix

Proof of Proposition 1

$\pi_H(\theta_i, \alpha) \gtreqless \pi_L(\theta_i, \alpha) \Longrightarrow \alpha\theta_i \gtreqless \frac{(1-\alpha)\theta_i}{\alpha(1-\theta_i)+(1-\alpha)\theta_i} \Longrightarrow \theta_i\alpha^2(1-2\theta_i) + \alpha\left(\theta_i + \theta_i^2\right) - \theta_i \gtreqless 0$. Let $f(\alpha) = \theta_i\alpha^2(1-2\theta_i) + \alpha\left(\theta_i + \theta_i^2\right) - \theta_i$.

Case 1) Let $\theta = \theta_H > \frac{1}{2}$

Then, (a) $f(\alpha)$ is a concave function, (b) $f(\alpha)$ attains max value at $\alpha = \frac{\theta_H+1}{2(2\theta_H-1)} > 1$ and $f\left(\alpha = \frac{\theta_H+1}{2(2\theta_H-1)}\right) = \frac{(\theta_H-5)(\theta_H-1)\theta_H}{4(2\theta_H-1)} > 0$, (c) $f\left(\alpha = \frac{1}{2}\right) = \left(-\frac{1}{4}\right)\theta_H < 0$, (d) $f(\alpha = 1) = -\theta_H(\theta_H - 1) > 0$. So $\exists\alpha_2$ such that if $\alpha \in \left(\frac{1}{2}, \alpha_2\right]$, $f(\alpha) \leq 0$ and if $\alpha \in (\alpha_2, 1)$, $f(\alpha) > 0$. This implies that if $\alpha \in \left(\frac{1}{2}, \alpha_2\right]$, $\pi_H(\theta_H, \alpha) \leq \pi_L(\theta_H, \alpha)$ and if $\alpha \in (\alpha_2, 1)$, $\pi_H(\theta_H, \alpha) > \pi_L(\theta_H, \alpha)$.

Case 2) Let $\theta = \theta_L < \frac{1}{2}$

Then, (a) $f(\alpha)$ is a convex function, (b) $f(\alpha)$ attains min value at $\alpha = \frac{\theta_L+1}{2(2\theta_L-1)} < \frac{1}{2}$ and $f\left(\alpha = \frac{\theta_L+1}{2(2\theta_L-1)}\right) = \frac{(\theta_L-5)(\theta_L-1)\theta_L}{4(2\theta_L-1)} < 0$, (c) $f\left(\alpha = \frac{1}{2}\right) = \left(-\frac{1}{4}\right)\theta_L < 0$, (d) $f(\alpha = 1) = -\theta_L(\theta_L - 1) > 0$. So $\exists \alpha_1$ such that if $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $f(\alpha) < 0$ and if $\alpha \in [\alpha_1, 1)$, $f(\alpha) \geq 0$. This implies that if $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $\pi_H(\theta_L, \alpha) < \pi_L(\theta_L, \alpha)$ and if $\alpha \in [\alpha_1, 1)$, $\pi_H(\theta_L, \alpha) \geq \pi_L(\theta_L, \alpha)$.

Now let us define $\alpha^*(\theta_i) = \frac{1}{2\theta_i-1}\left(\frac{1}{2}\theta_i - \frac{1}{2}\sqrt{-6\theta_i + \theta_i^2 + 5} + \frac{1}{2}\right)$. The computation yields that $\alpha_2 = \alpha^*(\theta_i = \theta_H)$ and $\alpha_1 = \alpha^*(\theta_i = \theta_L)$. Also

$$\frac{\partial(\alpha^*(\theta_i))}{\partial\theta_i} = -\frac{\left(5\theta_i - 7 + 3\sqrt{\theta_i^2 - 6\theta_i + 5}\right)}{2(2\theta_i - 1)^2\left(\sqrt{\theta_i^2 - 6\theta_i + 5}\right)} > 0$$

Here, $\left(3\sqrt{\theta^2 - 6\theta + 5}\right)^2 - (7 - 5\theta)^2 = (-4)(2\theta - 1)^2 < 0$, which implies that $3\sqrt{\theta^2 - 6\theta + 5} < 7 - 5\theta$ because $3\sqrt{\theta^2 - 6\theta + 5} > 0$ and $7 - 5\theta > 0$ for $\theta \in [0, 1]$. As the numerator is negative, $\frac{\partial(\alpha^*(\theta_i))}{\partial\theta_i} > 0$. Then, this implies that $\alpha_2 > \alpha_1$ because $\theta_H > \frac{1}{2} > \theta_L$. Then, (i) If $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $\pi_H(\theta_i, \alpha) < \pi_L(\theta_i, \alpha)$. (ii) If $\alpha \in [\alpha_1, \alpha_2]$, $\pi_H(\theta_L, \alpha) > \pi_L(\theta_L, \alpha)$ and $\pi_H(\theta_H, \alpha) < \pi_L(\theta_H, \alpha)$. (iii) If $\alpha \in (\alpha_2, 1)$, $\pi_H(\theta_i, \alpha) > \pi_L(\theta_i, \alpha)$. Note that $\pi_H(\theta_i, \alpha)$ ($\pi_L(\theta_i, \alpha)$) is the profit when a seller charges $p_H(\theta_i, \alpha)$ ($p_L(\theta_i, \alpha)$) for type i. Then this proves Proposition 1. ■

Proof of Proposition 3

Let us define $\widetilde{\theta}_H(\theta_L)$ such that $p_L(\widetilde{\theta}_H(\theta_L), \alpha) = p_H(\theta_L, \alpha)$ for given θ_L. Then, $p_L(\theta_H, \alpha) < p_H(\theta_L, \alpha)$ for $\theta_L < \theta_H < \widetilde{\theta}_H(\theta_L)$. By solving $\frac{\alpha\theta_L}{\alpha\theta_L+(1-\alpha)(1-\theta_L)} = \frac{(1-\alpha)\widetilde{\theta}_H(\theta_L)}{\alpha(1-\widetilde{\theta}_H(\theta_L))+(1-\alpha)\widetilde{\theta}_H(\theta_L)}$ in terms of $\widetilde{\theta}_H(\theta_L)$, we obtain $\widetilde{\theta}_H(\theta_L) = \frac{\alpha^2\theta_L}{\alpha^2\theta_L+(1-\alpha)^2(1-\theta_L)}$. Or if we define $\widetilde{\theta}_L(\theta_H)$ such that $p_H(\widetilde{\theta}_L(\theta_H), \alpha) = p_L(\theta_H, \alpha)$ for given θ_H, $p_L(\theta_H, \alpha) < p_H(\theta_L, \alpha)$ for $\widetilde{\theta}_L(\theta_H) < \theta_L < \theta_H$. By solving $\frac{(1-\alpha)\theta_H}{\alpha(1-\theta_H)+(1-\alpha)\theta_H} = \frac{\alpha\theta_L}{\alpha\theta_L+(1-\alpha)(1-\theta_L)}$, we obtain $\widetilde{\theta}_L(\theta_H) = \frac{(\alpha-1)^2\theta_H}{(\theta_H-2\alpha\theta_H+\alpha^2)}$. ■

Proof of Proposition 6

The seller's pricing decision is either 1 or $\frac{\underline{\alpha}\theta}{\underline{\alpha}\theta+(1-\overline{\alpha})(1-\theta)}$. When $P = 1$, the fraction of buyers $\frac{1-\underline{\alpha}}{1-\overline{\alpha}}\theta$ purchase the good. This is the probability that buyers draw a signal from $(\underline{\alpha}, 1]$. On the other hand, when $P = \frac{\underline{\alpha}\theta}{\underline{\alpha}\theta+(1-\overline{\alpha})(1-\theta)}$, the fraction of buyers $\theta + (1-\theta)\frac{\underline{\alpha}-\overline{\alpha}}{\underline{\alpha}}$. This is the probability that buyers draw a signal from $[\overline{\alpha}, 1]$. We can easily compute the seller's profit at each price as follows.

$$\begin{aligned}
\pi_H &= \frac{1-\underline{\alpha}}{1-\overline{\alpha}}\theta \qquad\qquad \text{at } P = 1, \text{ and} \\
\pi_L &= \frac{\underline{\alpha}\theta + (\underline{\alpha}-\overline{\alpha})(1-\theta)}{\underline{\alpha}\theta + (1-\overline{\alpha})(1-\theta)}\theta \text{ at } P = \frac{\underline{\alpha}\theta}{\underline{\alpha}\theta + (1-\overline{\alpha})(1-\theta)}.
\end{aligned}$$

Note that π_H/π_L is monotonically decreasing in θ and $\pi_H/\pi_L \in [\frac{1-\underline{\alpha}}{\underline{\alpha}-\overline{\alpha}}, \frac{1-\underline{\alpha}}{1-\overline{\alpha}}]$, where $\frac{1-\underline{\alpha}}{\underline{\alpha}-\overline{\alpha}} > 1 > \frac{1-\underline{\alpha}}{1-\overline{\alpha}}$. Thus, we can find a uniqe $\widehat{\theta}$ such that $\pi_H \gtreqless \pi_L$ as $\theta \lesseqgtr \widehat{\theta}$, where $\pi_H(\widehat{\theta}) = \pi_L(\widehat{\theta})$. ■

Proof of Proposition 7

We have to consider two prices, $P > p_L(\theta,\alpha)$ and $P \leq p_L(\theta,\alpha)$. Suppose $P > p_L(\theta,\alpha)$ for the high demand, $\Pr(s_H)\,(p_H(\theta,\alpha) - P)$, to be binding. The seller's profit is $\pi = \Pr(s_H)\,(p_H(\theta,\alpha) - P)\,P$ and the first-order condition is $\Pr(s_H)\,(p_H(\theta,\alpha) - 2P) \leq 0$. On the other hand, when $P \leq p_L(\theta,\alpha)$, the low demand is binding. In this case, the profit is $\pi = (\theta - P)P$ and the first-order condition is $\theta - 2P \leq 0$. Thus, the prices are can be written as

$$P = \begin{cases} p_H(\theta,\alpha)/2 \\ p_L(\theta,\alpha) \end{cases} \text{ and } \pi = \begin{cases} \Pr(s_H)\frac{p_H(\theta,\alpha)^2}{4} & \text{if } p_L(\theta,\alpha) < p_H(\theta,\alpha)/2 \\ p_L(\theta,\alpha)(\theta - p_L(\theta,\alpha)) & \text{if } p_L(\theta,\alpha) \geq p_H(\theta,\alpha)/2 \end{cases} \text{ and}$$

$$P = \begin{cases} \theta/2 \\ p_L(\theta,\alpha) \end{cases} \text{ and } \pi = \begin{cases} \frac{\theta^2}{4} & \text{if } p_L(\theta,\alpha) > \theta/2 \\ p_L(\theta,\alpha)(\theta - p_L(\theta,\alpha)) & \text{if } p_L(\theta,\alpha) \leq \theta/2 \end{cases}.$$

Note that $P = p_L(\theta,\alpha)$ is always dominated by either $p_H(\theta,\alpha)/2$ or $\theta/2$. If $p_L(\theta,\alpha) \geq p_H(\theta,\alpha)/2$, it is dominated by $\theta/2$. Similarly, if $p_L(\theta,\alpha) \leq \theta/2$, it is dominated by $p_H(\theta,\alpha)/2$. Let us define θ_1 such that $p_L(\theta_1,\alpha) = \theta_1/2$ and θ_2 such that $p_L(\theta_2,\alpha) =$

$p_H(\theta_2, \alpha)/2$. We obtain $\theta_1 < \theta_2$ as in Figure. Thus, we find the unique optimal price: $p_H(\theta, \alpha)/2$ for $\theta < \theta_1$ and $\theta/2$ for $\theta > \theta_2$. On the other hand, when $\theta_1 \leq \theta \leq \theta_2$, there are the two local maximum. Thus, we have to compare the two profit functions, $\Pr(s_H)\frac{p_H(\theta,\alpha)^2}{4}$ and $\frac{\theta^2}{4}$. We find the following two cases.

First, when $\alpha > \frac{\sqrt{5}-1}{2}$, there exists a unique $\widetilde{\theta} \in (0, 1]$ such that $\Pr(s_H)\frac{p_H(\widetilde{\theta},\alpha)^2}{4} = \frac{\widetilde{\theta}^2}{4}$ because

$$\frac{\Pr(s_H)\frac{p_H(\theta,\alpha)^2}{4}}{\theta^2/4} = \frac{\alpha^2}{\alpha\theta + (1-\alpha)(1-\theta)} \in [\frac{\alpha^2}{1-\alpha}, \alpha]$$

is decreasing in θ. In this case, we obtain $\Pr(s_H)\frac{p_H(\theta,\alpha)^2}{4} \gtreqless \frac{\theta^2}{4}$ as $\theta \lesseqgtr \widetilde{\theta}$. Depending on α, $\widetilde{\theta}$ can be greater or smaller than either θ_1 or θ_2. Second, it is immediate that $\Pr(s_H)\frac{p_H(\theta,\alpha)^2}{4} < \frac{\theta^2}{4}$ when $\alpha \leq \frac{\sqrt{5}-1}{2}$. That is, $\widetilde{\theta} > \theta_2$. ■

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