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Bid or Wait ? Theory and Evidence of Auctions for Foreclosed Properties in Taipei

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Abstract

The paper explores theoretically and empirically the determinants of the outcomes of a multiple-stage first-price sealed-bid juridical auction for distressed properties. In the event the first auction fails, the court shall call the second, third and fourth auctions with a reduction in reserve prices for each additional auction. We consider first a simple two-stage first price private value auction for a foreclosed property. It is shown that there exists a cutoff value in equilibrium such that a potential bidder chooses not to bid in the first auction if his valuation is below the cutoff value even though it is above the reserve price in the first auction. He submits a bid if only his valuation is above the cutoff value but the bid is less than what he would submit without the second auction. Furthermore, for a property, an increase of the number of potential bidders, a reduction of its reserve price in the first auction, or a reduction of the auction risk costs, raises its expected number of actual bidders, the probability of being sold and its bidding premiums in the first auction. This, however, is not the case in the second auction. To examine the theoretical conjectures, we use the following empirical tests using data of juridical auctions in Taipei City from the first quarter of 2006 through 2009. A multinomial logit regression is used for exploring the determinants of the probability for a property being sold in earlier auctions. A zero-inflated binomial negative regression is employed to examine factors influencing the number of actual bidders in earlier auctions and in later auctions. Finally, a two-stage estimation is used to decompose the direct impacts of characteristic variables on the bidding premiums and their indirect impact on premiums via influencing the number of actual bidders. Empirical results support our theoretical conjectures.

Keywords: Auctions, Real Estate, Regression for Count Data, Multinomial Logit Choice Model.

1 Introduction

The standard auction theory implicitly assumes that the auctioneer can credibly commit to not sell the auctioned object if it cannot be sold at or above the reserve price. Without such a commitment, buyers may anticipate that the object, if durable, will be offered for sale again in a later auction and perhaps with a lower reserve price. This expectation may affect their bidding behavior in the first auction. Typical examples of repeated auctions are juridical auctions for distressed real estate properties in some countries like Taiwan and Japan.[1] In contrast to standard auctions, distressed properties in Taiwan is a multi-stages first price auction where in the event the first auction fails, the court shall call the second, the third and the fourth auctions. The reserve price for each additional auction is reduced by no more than 20%. For a multiple-round first price sealed-bid court auctions for distressed properties, this paper aims to answer the following theoretical and empirical questions

- how do potential bidders make their choices as whether to bid in earlier auctions or to wait and bid in later auctions ?
- what factors influencing the number of actual bidders and
- what are the determinants of auction prices.

We consider first a simple two-stage first-price private value auction where a property is for sale with a reserve price and if the reserve is not met, the property will be re-auctioned with a lower reserve price. It is shown that there exists a cutoff valuation ratio in equilibrium such that a bidder chooses to bid in the first auction if his value ratio is not less than the cutoff point and chooses to wait and then bid in the second auction if his value ratio is less than this cutoff ratio and not less than the corresponding value ratio of the reserve price of the second auction. Furthermore, we show that an increase of the number of potential bidders raises the expected number of actual bidders in the first auction and the probability that

[1] Before the amendment of foreclosed laws in Japan in 1998, the reserved price did not change even if properties failed to sell. The new procedure requires the reserved prices to be determined by considering the state of the real estate market and the particular circumstances of the property. If properties fail to sell, the court can lower the reserved price mechanically, by 30%, for example.

the property is sold in the first auction. A rise of the number of potential bidders not only increases the likelihood of higher value bidders in attendance but also decreases the cutoff ratio of bidding in the first auction and thus raises the probability of successful bids in the first auction. Moreover, it is shown that a reduction of a property's reserve price in the first auction, or a reduction of its auction risk costs, raises its expected number of actual bidders, the probability of being sold and its bidding premiums in the first auction. These, however, are not the cases for the properties sold in the second auction.

To examine the theoretical conjectures, we proceed the following empirical tests using data of court auctions in Taipei City from the first quarter of 2006 through the end of 2009. By employing a multinomial logit regression, we explore the determinants of the relative probabilities of being sold in earlier auctions, being sold in later auctions or being unsold eventually. We then use two zero-inflated negative binomial (hereafter, ZINB) count regressions to examine the determinants of the number of bidders for properties auctioned in the first two auctions and the last two auctions whether they were sold or unsold. Finally we examine the determinants of auction prices by using a two-stage least estimation to solve the endogeneity of number of actual bidders on auction prices. Four types of characteristic variables are included: house and building characteristic variables, auction characteristic variables, neighborhood characteristic variables and time dummy variables.

We preform a multinomial logit estimation by concerning only the final outcomes of properties auctioned within the sample period, and categorizing the observations into three groups : sold in the first auction (treated as the earlier auction), sold in the second auction (treated as the later auction) and sold or unsold after the second auction (teated as unsold eventually). By selecting the properties sold in the first auction to be the reference group, the estimation results indicate that most of those variables having significantly impacts on the tendency of being sold after the second auction or unsold eventually, have no significant effect on the tendency of being sold in the second auction. This empirical result is consistent with our first theoretical conjecture, claiming that the impact of the number of potential bidders, the reserve prices and the risk costs on the probability of being sold in the later auction are indeterminate while they have oppositive impacts between on the probability of being sold in the earlier auction and on the probability of being unsold eventually.

The estimation results of two zero-inflated negative binomial regressions indicate that those variables having significant impacts on the number of actual bidders in the first two auctions, have less significant or smaller impacts on the number of actual bidders in the last two auctions. This result supports our second theoretical conjecture, saying that a fewer potential bidders, or a higher reserve or a higher risk cost, attracts fewer actual bidders in earlier auctions while its impact is indeterminate in later auctions.

To outstand the effects of explanatory variables on the winning prices, more than the reserve price, we use the bidding premium which reflects the deviation of selling price from the reserve price, as the dependent variable. Some omitted variables (due to limited information or difficult to express numerically) affecting auctioning prices, also influence the number of bidders and thus the OLS method is inappropriate for the estimation of an equation in a system of simultaneous equations. Instead, we employ a two-stage estimation where in the second stage the number of bidders is replaced by the predicted number of actual bidders which is estimated in the first stage by running a zero-truncated negative binomial regression. The estimation results indicate most of explanatory variables in the first two auctions have no significant direct impact on the bidding premiums. Increasing one more predicated number of bidders raises 1.24411 of bidding premium in the first two auctions. In contrast, the predicted number of bidders in the last two auctions has no significant impact on the bidding premiums since multicollinearity is serious in a two-stage estimations. By deleting one such variable in the premium regression, the problems of multicollinearity is mitigated and the result suggests that increasing one more predicated number of bidders raises 1.06777 of bidding premium in the last two auctions.

The existing literature on real estate auctions has focused on comparing performances of auctions and private negotiations (Quan, 1992, Dotzour, Moorhead and Winker,1998, Mayer 1998 and Quan, 2002). Less attention has been paid to the question of the determinants of auction success. As the first one of them, DeBoer, Conran and McNamara (1992) analyze sales of tax delinquent peoperties offered at a single property tax auction and find that higher minimum bids decrease the probability of sale. In addition, Ong, Lusht and Mak(2005) provide evidence on the outcomes of auctions of residential real estate in Singapore. In contrast to Singapore where the dominant auction format for real estate properties is English auction, the auction format of Taiwanese judicial auctions is multi-round first price auctions. Our theoretical and empirical models are closer to Ooi, Sirmans, and Turnbull (2006)(hereafter Ooi et al.,2006) and Idee, Iwata and Tabuchi (2009)(hereafter Idee et al.,2009). The paper of Ooi et al.(2006) examines the price formation process under small numbers competition using data from Singapore land auctions in which first-price sealed bid auctions are used. Their theoretical and empirical results show that the expected sale price increases with the number of bidders. The paper of Idee et al.(2009) examines the effect of costly occupants on the auction prices of foreclosed properties. Using data from Osaka District Court, their estimation results suggest that the existence of occupants in properties reduces the auction price through two channels: affecting the reserve price and this changes the auction price and second, the number of bidders changes in response to changes in the reserve price and the number of bidders changes the bidding price. In contrast to the two articles, we propose a two-stage auction where a property is auctioned with a reserve price and if the reserve is not met, the property will be re-auctioned with a lower reserve price. The paper theoretically and empirically explore not only on the determinants of the tendency for a property being sold in the earlier auctions, in the later auctions or being unsold eventually, but also on the factors influencing the number of actual bidders and bidding premiums in the earlier auctions and in the later auctions. The paper may contribute to the literature of real estate auctions both on analyzing bidders' strategies and equilibrium outcomes in a multi-round auctions and on providing empirical evidence for theoretical conjectures.

The paper is organized as follows. The next section describes the process of judical auction in Taiwan. The theoretical model of a two-stage auction is presented in Section 3. The empirical analysis is reported in Section 4. Section 5 provides a short conclusion. Most of the proofs and tables of empirical results appear in the Appendix.

2 The Judicial Foreclosure Process in Taiwan

A judicial auction is a legal process in which a property is auctioned under the jurisdiction of a district court for creditors listing their foreclosure properties to recover their loans. In Taiwan, most of real estate auctions under court jurisdiction were used to dispose of collateral held by financial institutions against non-performing loans. The procedure for disposing of collateralized property in an auction market is as follows. A creditor takes a property to a court in order to foreclose on the mortgage. The district court examines the property, asks the appraiser to evaluate the property, sets reserve prices for each part of a property including lands and buildings, announces an impending auction, and determines the auction time, at which bidders send their bids. The court let potential bidders to examine documents containing details of the property, including floor space (FS), public facility area (Ps), land space ($Land$), the reserve price of land with the property (Lr), the reserve prices of building with the property (Hr), the floor level ($Floor$), total floor level ($Tfloor$), the location, the time of building construction finishing , whether or not issued with eviction order (Eo) and status of tenure of properties auctioned. Whether a property is issued eviction or not depends on these status of tenure. The tenure of status is classified into six categories: vacant, debtor-residing, no-person residing, leased, third-party occupancy and being-controlled by creditor. In addition, the document also indicates whether the property is assigned to the co-owner of the property the preemptive option of buying the property at the winning price (Pr).

In contrast to the Japan judicial auction where bidders can submit their bids within a predetermined period, normally longer than a week and shorter than a month, there is one half hour only, in the Taiwanese judicial auction, for potential bidders of all the auctions at the same preannounced day to submit their bids. No potential bidder knows the exact number of participating bidders for his preferred property until the tender results are revealed. An auctioned property is awarded to the highest sum of bids in each part of the property provided each bid exceeding its reserver price. If a property fails to be sold in the first auction, there may be the second, the third auction. A property failing to be sold in third auction will be put for sale on a first-come -first-served basis at the reserve price of the third auction. If this is still not successful, the property will be entered into the fourth auction. The reserve price for each additional auction is reduced by no more than 20%. A bidder must pay a deposit of equivalent to 20% of the reserve price and deposits are returned in full to unsuccessful bidders. The winner of each auction pays the balance in seven days and registers the estate as his or her property, and the property is delivered to the new owner.

The court, however, provides no guarantee that auction winners can obtain their won houses smoothly. [2] Properties may be occupied and the original residents may refuse to leave auctioned properties. Similar as in Japan and contrast to HUD real estate auctions in Florida (Allen and Swisher 2000), the new owner must evict occupants, if any, at his or her own cost even if they are not protected by the laws. In contrast to Japan, an eviction order in Taiwan can be issued to reduce the risk cost of the winner of the auction.[3] Although, not all properties are issued with eviction orders and even with court eviction orders, there might be risk costs for the property's winner. In addition, when the part of the auctioned property is co-owned by others, the co-owners have preemptive rights of getting the auctioned property at the winning bid.

3 Theoretical Model

The purpose of this section is to link the observable characteristics of auctioned properties to the outcomes of auctions in our empirical study. Similar to Ooi et

[2]There are two main risks of buying foreclosed properties. The principles governing the auction do not hold sellers responsible for defects in properties nor are perspective buyers allowed to enter a nd look at properties prior to bidding.

[3]At the issuing of an eviction order the magistrate will tell the occupiers the date on which they have to vacate the property. If occupiers have not left the property by this date a court appointed bailiff will visit the property and, if necessary, forcibly remove them.

al.(2006) and Idee et al.(2009), we use a model of a first-price sealed-bid auction with symmetric independent values. But in contrast to the two articles, we propose a two-stage mechanism where a property is auctioned with a reserve price and if the reserve is not met, the property will be re-auctioned with a lower reserve price corresponding to the Taiwanese judicial auction.

3.1 Basic Setting

A foreclosed property is put up for sale in a two-stage first-price sealed bid auction. There are n potential bidders interested in the property.[4] Let r be the reserve price of the property in the first auction.[5] If the first auction fails, the property will be re-auctioned with a lower reserve price δr ($0 \leq \delta < 1$).[6] Denote $u = Land * Av + Fs * Abc$ by the basic objective value of a property where Av is the assessed value of lands with the property and Abc is the basic objective cost of building depreciation-adjusted with the property. After examining the property each potential bidder learns his private known ratio of the objective value, θ, and his private value of the property becomes $v(\theta) = \theta * u - c$ where c is the risk cost of buying the foreclosed property. Assume that θ is ex ante independently and identically distributed according to the distribution function, $F(\cdot)$, with a strictly positive density $f(\cdot)$ on $[\underline{\theta}, \bar{\theta}]$ where $\bar{\theta}$ is sufficiently large.[7]

3.2 Strategic Choice of Potential Bidders

Let θ_1^k represent the first statistic of k realizations of the random variable described by F. Denote $G(\cdot) = F(\cdot)^{n-1}$ by the distribution function of θ_1^{n-1}. Let $\theta(v) = \frac{v+c}{u}$

[4] A court releases the public information of properties auctioned fifteen days before the first auctions. Anyone interested in bidding for these foreclosed properties examines the public information of the properties auctioned before the first auction. Because spending time on filed trips to the property interested incurs a cost, each will target only on his interested properties subject to his limited time and budget constraint and becomes a potential bidder of his interested properties.

[5] As mentioned in the above section, auctions for foreclosed properties in Taiwan include a set of reserve prices for each part of properties. For simplicity of analysis, we propose only total reserves r in the theoretical model.

[6] Burguet and Sakovics(1996) (hereafter BS, 1996) consider a two-stage auctions where in the first stage there is certain reserve price r and if no bid is submitted, h more bidders are invited to submit bids, without any reserve price. The work of BS(1996) is a special case of our model when $\delta = 0$ while the paper is a special case of BS(1996) when $h = 0$.

[7] We do not consider the optimal strategy of a seller. In contrast, McAfee, Quan and Vincent (2002) show the optimal reserve price using the common-value model, and then empirically test the real estate auction data.

be the ratio of private value adjusted by risk cost divided by the objective value of the property.

Consider a subgame perfect symmetric equilibrium such that any bidder with $\theta \geq \tilde{\theta}$ will submit a serious bid $\beta_1(\theta)$ in the first auction and any bidder with $\theta(\delta r) \leq \theta < \tilde{\theta}$ will submit a bid $\beta_2(\theta)$, where $\beta_2(\theta(\delta r)) = \delta r$, in the second auction. Namely, any bidder with $\theta(r) \leq \theta < \tilde{\theta}$ chooses to submit no bid in the first auction and bid in the second auction. Denote $\pi^b_{\theta\geq\tilde{\theta}}(b)$ and $\pi^w_{\theta\geq\tilde{\theta}}(b)$ by the payoff of a bidder with θ, $\theta \geq \tilde{\theta}$, by bidding b in the first auction and by bidding b in the second auction, respectively. Denote also $\pi^b_{\theta(\delta r)\leq\theta<\tilde{\theta}}(b)$ and $\pi^w_{\theta(\delta r)\leq\theta<\tilde{\theta}}(b)$ by the payoff of a bidder with valuation θ, $\theta(\delta r) \leq \theta < \tilde{\theta}$, by bidding b in the first auction and by bidding b in the second auction, respectively.

Definition 1 *A strategy profile $\sigma^* = (\tilde{\theta}, \beta_1(\theta), \beta_2(\theta))$ is a symmetric equilibrium given (c, r, δ, n) if*
(i) $\pi^b_{\theta\geq\tilde{\theta}}(\beta_1(\theta)) \geq \pi^w_{\theta\geq\tilde{\theta}}$ for all $\theta \in [\tilde{\theta}, \bar{\theta}]$
(ii) $\pi^w_{\theta(\delta r)\leq\theta<\tilde{\theta}}(\beta_2(\theta)) \geq \pi^b_{\theta(\delta r)\leq\theta<\tilde{\theta}}$ for all $\theta \in [\theta(\delta r), \tilde{\theta})$
where

$$\pi^w_{\theta\geq\tilde{\theta}} = \max_{\delta r\leq b\leq v(\theta)} \pi^w_{\theta\geq\tilde{\theta}}(b), \tag{1}$$

and

$$\pi^b_{\theta(\delta r)\leq\theta<\tilde{\theta}} = \max_{r\leq b\leq v(\theta)} \pi^b_{\theta(\delta)\leq\theta<\tilde{\theta}}(b). \tag{2}$$

That is, given that other bidders play σ^*, it is the optimality of a bidder to submit a serious bid $\beta_1(\theta)$ in the first auction if his private value ratio $\theta \geq \tilde{\theta}$ and wait and bid $\beta_2(\theta)$ in the second auction if $\theta < \tilde{\theta}$.[8] Clearly, $\tilde{\theta}$, the cutoff value in equilibrium, is determined on c,r and δ and n.

3.3 Equilibrium

Similar to BS(1996), the following lemma partially characterize the potential symmetric, monotone equilibria of this game.

Lemma 1 *Given $n, \delta \in (0, 1)$ and $r \in (0, E(\theta_1^{n-1}) * u - c)$, in every monotone, symmetric equilibrium there exists a cutoff pint, $\tilde{\theta} \in (\theta(r), \bar{\theta})$, such that a bidder*

[8]Obviously, not bidding in both auction is the optimality of a bidder if his private value ratio $\theta < \theta(\delta r)$. If he submits a bid $b > \delta r$ and he wins, he obtains $(\theta * u - c - b) < \theta(\delta r)u - c = \delta r - b < 0$.

with θ submits a bid in the first auction if and only if $\theta \geq \tilde{\theta}$.[9] **Moreover, if $\theta = \tilde{\theta}$, he bids exactly r.**

Proof. Suppose to the contrary that no type of bidder bids in the first stage in equilibrium. Then we will have a standard auction in the second period and therefore a bidder with valuation ratio $\bar{\theta}$ would bid $E(\theta_1^{n-1} \mid \theta_1^{n-1} < \bar{\theta}) * u - c$, which is equal to $E(\theta_1^{n-1}) * u - c$, in the second period and obtains the property with probability 1. However, he is better off bidding r in the first period and obtains the good with probability one but pays r, lower than $E(\theta_1^{n-1}) * u - c$ by assumption. Therefore, in any monotone, symmetric equilibrium there is a cutoff point $\tilde{\theta}$ in $(\theta(r), \bar{\theta})$ such that a bidder submits a bid if and only if $\theta \geq \tilde{\theta}$. Second, we show that the bidder with valuation ratio $\tilde{\theta}$ will bid r in equilibrium. Assume that all bidders who bid in the first period, offer strictly more than r. Then a bidder with $\tilde{\theta}$ would benefit from deviating and bidding exactly r since the probability of winning is the same, and the price is strictly lower. □.

Theorem 1 *Consider a two-stage first-price sealed bid auction.*
(i) If $\delta r + c > \bar{\theta} * u$, no type of bidder submits a bid in either the first auction or the second auction.
(ii) If $\delta r + c \leq max\{\bar{\theta} * u, r + c\}$ and $r > E(\theta_1^{n-1})u - c$, there exists a bidder with $\theta \geq \theta(\delta r)$ submits a bid $\beta_2(\theta)$ in the second auction where

$$\begin{aligned} \beta_2(\theta) &= v(\theta) - \frac{\int_{\theta(\delta r)}^{\theta} G(y)udy}{G(\theta)} \\ &= \frac{\delta r G(\theta(\delta r)) + \int_{\theta(\delta r)}^{\theta} v(y)g(y)dy}{G(\theta)} = E[\max\{v(\theta_1^{n-1}), \delta r\} \mid \theta_1^{n-1} < \theta]. \end{aligned} \tag{3}$$

(iii) If $r + c < E(\theta_1^{n-1}) * u$, there is an unique monotone symmetric equilibrium determined by δr and $\tilde{\theta}$, $\theta(\delta r) < \theta(r) < \tilde{\theta} \in (\theta(r), \bar{\theta})$, such that any bidder with valuation ratio θ has the following strategy:

- *Submits no bid in both the first auction and the second auction if $\theta < \theta(\delta r)$.*
- *Submits no bid in the first auction and submits a bid $\beta_2(\theta)$ given by (3) in the second auction when the first auction fails and if $\theta(\delta r) \leq \theta < \tilde{\theta}$ where $\tilde{\theta}$ is*

[9] The inequality $r < E(\theta_1^{n-1}) * u - c$ is merely the sufficient condition of existence of a bid in the first stage.

determined by

$$v(\tilde{\theta}) = r + \frac{\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)udy}{G(\tilde{\theta})}. \tag{4}$$

- *Submits a bid $\beta_1(\theta)$ in the first auction if $\theta \geq \tilde{\theta}$ where*

$$\begin{aligned}\beta_1(\theta) &= \frac{rG(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} v(y)g(y)dy}{G(\theta)} \\ &= v(\theta) - \frac{G(\tilde{\theta})(v(\tilde{\theta}) - r)}{G(\theta)} - \frac{\int_{\tilde{\theta}}^{\theta} G(y)udy}{G(\theta)} < E[\max\{v(\theta_1^{n-1}), r\} \mid \theta_1^{n-1} < \theta].\end{aligned} \tag{5}$$

Proof. Proofs are provided in the Appendix.

Theorem 1 says first that if the reserve price of a property in the first auction is higher than the subject value of the second highest-valued bidder adjusted by the risk cost, all types of bidder do not bid in the first auction. Consider the case that $r + c = E(\theta_1^{n-1}) * u$. If a bidder with $\theta = \bar{\theta}$ bids $\beta_2(\theta) = E[\max\{v(\theta_1^{n-1}), \delta r\} \mid \theta_1^{n-1} < \theta]$, he wins the property with probability 1. He and any type of bidder has no incentive to submit any qualified bid $b \geq r$ in the first auction by assumption. If $r + c < E(\theta_1^{n-1}) * u$, there is a cutoff ratio $\tilde{\theta}$ such that a bidder with $\theta \geq \tilde{\theta}$ bids $\beta_1(\theta)$ in the first auction. The value at the cutoff ratio, $v(\tilde{\theta})$, is greater than the reserve price of the first auction for $\delta < 1$. By (4) and that $\theta(r) = \frac{r+c}{u}$, $\tilde{\theta}$ is a function of r, c, δ and n. Since $\beta_1(\theta) = v(\theta) - \frac{G(\tilde{\theta})(v(\tilde{\theta})-r)}{G(\theta)} - \frac{\int_{\tilde{\theta}}^{\theta} G(y)udy}{G(\theta)} < v(\theta) - \frac{\int_{\theta(r)}^{\theta} G(y)udy}{G(\theta)}$, the bid of any bidder with $\theta > \tilde{\theta}$ in the first auction is less than the bid he would submit without the second auction.[10] A bidder with a ratio lower than $\tilde{\theta}$ but higher than $\theta(\delta r)$ will submit a bid $\beta_2(\theta)$, which is similar in a standard auction.

3.4 Comparative Statics

Equation (4) defines $\tilde{\theta}$ as a function of n, r, δ and c. We have the following corollaries concerning the change of $\tilde{\theta}$ as n, δ, r, c change.

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[10]In equilibrium, each bidder with valuation $\theta \geq \tilde{\theta}$, will bid $\frac{rG(\tilde{\theta})+\int_{\tilde{\theta}}^{\theta} v(y)g(y)dy}{G(\theta)}$, which is the expected value of second-highest value conditional upon his θ is the highest by treating as r in case the second highest $\theta_1^{n-1} < \theta(r)$. By (4), we have $v(\tilde{\theta}) = \tilde{\theta} * u - c > r$ and thus $\tilde{\theta} > \theta(r) = \frac{r+c}{u}$. As a result, $\frac{rG(\tilde{\theta})+\int_{\tilde{\theta}}^{\theta} v(y)g(y)dy}{G(\theta)} < \frac{rG(\theta(r))+\int_{\theta(r)}^{\theta} v(y)g(y)dy}{G(\theta)}$, the equilibrium bid without the second auction.

corollary 1 *Suppose that* $r + c \leq E(\theta_1^{n-1}) * u$. *The cutoff ratio* $\tilde{\theta}$ *is a function of* n, δ, r, c. *We have*

$$\frac{\partial \tilde{\theta}}{\partial n} < 0, \tag{6}$$

$$\frac{\partial \tilde{\theta}}{\partial r} > 0, \tag{7}$$

$$\frac{\partial \tilde{\theta}}{\partial c} > 0, \tag{8}$$

and

$$\frac{\partial \tilde{\theta}}{\partial \delta} < 0. \tag{9}$$

The proof appears provided in the Appendix.

Corollary 1 says that the cutoff ratio $\tilde{\theta}$ is strictly decreasing in n and δ and is strictly increasing in r and c. More potential bidders raise the likelihood of high value rivals and thus it becomes the optimality to bid in the first auction for a bidder with valuation ratio less than the cutoff ratio corresponding to the original n. The less discount of reserve price after the failure of the first auction (larger δ) creates less attraction for the bidder to bid in the second auction. A higher reserve price of the first auction induces a larger gain for bidder to wait in the first auction and bid in the second auction, and thus corresponds a higher cutoff ratio $\tilde{\theta}$.

Denote P_1, P_2 and P_3 by the probabilities that the property is sold in the first auction, sold in the second auction and unsold eventually, respectively. We have

$$P_1 = Prob(\theta_1^n \geq \tilde{\theta}) = 1 - F(\tilde{\theta})^n, \tag{10}$$

$$P_2 = Prob(\theta(\delta r) \leq \theta_1^n < \theta) = F(\tilde{\theta})^n - F(\theta(\delta r))^n, \tag{11}$$

and

$$P_3 = Prob(\theta_1^n < \theta(\delta r)) = F(\theta(\delta r))^n. \tag{12}$$

Equation (10)-(12) define $P_t, t = 1, 2, 3$, as the functions of n, r, δ and c. We have the following corollary concerning the change of $P_t, t = 1, 2, 3$, as n, δ, r, c change.

corollary 2 *Suppose that* $n \geq 2$ $r \leq E(\theta_1^{n-1}) * u - c$. *Then*

(i) $\frac{\partial P_1}{\partial n} > 0$, $\frac{\partial P_3}{\partial n} < 0$ *and the sign of* $\frac{\partial P_2}{\partial n}$ *is indeterminate,*

(ii) $\frac{\partial P_1}{\partial r} < 0$, $\frac{\partial P_3}{\partial r} > 0$ *and the sign of* $\frac{\partial P_2}{\partial r}$ *is indeterminate,*

(iii) $\frac{\partial P_1}{\partial c} < 0$, $\frac{\partial P_3}{\partial c} > 0$ *and the sign of* $\frac{\partial P_2}{\partial c}$ *is indeterminate,*

(iv) $\frac{\partial P_1}{\partial \delta} > 0$, $\frac{\partial P_3}{\partial \delta} > 0$, $\frac{\partial P_2}{\partial \delta} < 0$.

The proof appears in the Appendix.

Corollary 2 says that the probability for a property being sold in the first auction is strictly increasing in n and δ and is strictly decreasing in r and c. The intuition behind this result is simple. In the private value case, an increase in the number of potential bidders not only increases the likelihood of higher value bidders but also decreases the cutoff ratio of participating in the first round auction. Consequently, more potential bidders corresponds a larger probability that auctioned property is sold in the first auction. This is not the case for the probability that the property is sold in the second auction. The effect of the increase of potential bidders on the probability that property is sold in the second auction is indeterminate since it corresponds a lower cutoff ratio and thus induces more bidders to bid in the first auction but it also reduces the probability that the property is unsold eventually.

Note that what can be observed after judicial auctions is the number of actual bidders rather than the number of potential bidders. In equilibrium, the expected numbers of serious bidders in the first auction and in the second auction for a property with n potential bidders are

$$E(n_1) = \sum_{k=0}^{n} k * [1 - F(\tilde{\theta})]^k F(\tilde{\theta})^{n-k} \begin{pmatrix} n \\ k \end{pmatrix}, \tag{13}$$

and

$$E(n_2) = \sum_{k=0}^{n} k * F(\theta(\delta r))^{n-k} [F(\tilde{\theta}) - F(\theta(\delta R))]^k \begin{pmatrix} n \\ k \end{pmatrix}, \tag{14}$$

respectively. Then we have

corollary 3 *Suppose that* $r + c \leq E(\theta_1^{n-1}) * u$. *Then*

(i)

$$E(n_1) = n[1 - F(\tilde{\theta})], \tag{15}$$

(ii)

$$E(n_2) = n[F(\tilde{\theta}) - F(\theta(\delta r))]F(\tilde{\theta})^{n-1}. \tag{16}$$

(iii) $\frac{\partial E(n_1)}{\partial n} > 0$, $\frac{\partial E(n_1)}{\partial r} < 0$, $\frac{\partial E(n_1)}{\partial c} < 0$, $\frac{\partial E(n_1)}{\partial \delta} > 0$ *and* $\frac{\partial E(n_2)}{\partial \delta} < 0$.

(iv) The signs of $\frac{\partial E(n_2)}{\partial n}$, $\frac{\partial E(n_2)}{\partial r}$, *and* $\frac{\partial E(n_2)}{\partial c}$ *are all indeterminate.*

The proof appears in the Appendix.

We show (15) and (16) in Appendix. By (15), the expected number of actual bidders in the first auction is the product of the number of potential bidders and the probability that any potential bidder participates in the first auction. Similarly, by (16), the expected number of actual bidders in the second auction is the product of the number of potential bidders and the probability that a potential bidder participates in the second auction while other potential bidders do not participate in the first auction.[11] It is then clear that $\frac{\partial E(n_1)}{\partial n} = [1 - F(\tilde{\theta})] - nf(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial n} > 0$ since $\frac{\partial \tilde{\theta}}{\partial n} < 0$ by corollary 1. Similarly, $\frac{\partial E(n_1)}{\partial r} = -nf(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial r} < 0$, $\frac{\partial E(n_1)}{\partial c} = -nf(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c} < 0$, and $\frac{\partial E(n_1)}{\partial \delta} = -nf(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial \delta} > 0$ since $\frac{\partial \tilde{\theta}}{\partial r} > 0$, $\frac{\partial \tilde{\theta}}{\partial c} > 0$ and $\frac{\partial \tilde{\theta}}{\partial \delta} < 0$ respectively. An increase in the number of potential bidders not only increases the number of high value bidders but also decreases the cutoff ratio of bidding in the first auction and consequently raises the expected number of actual bidders in the first auction. A higher initial reserve price or a higher risk cost corresponds to a higher cutoff ratio of bidding in the first auction and thus attracts less actual bidders. Finally, a less discount of initial reserve price corresponds a lower ratio of bidding in the first auction and raises the probability that a potential bidder does not participate in either auction. Consequently, a less discount of initial reserve price attracts more actual bidders in the first auction and less bidders in the second auction. The effect of the increase of potential bidders on expected number of bidders in the second auction is indeterminate because it induces more bidders to change to bidding in the first auction while it also reduces the probability that the property is unsold eventually. Similarly, the effects of changing r and c on the expected number of actual bidders are also indeterminate.

We next find the expected auction prices conditional being sold in the first auction and in the second auction, respectively. By Theorem 1, in equilibrium, the

[11] Virtually, $E(n_1)$ of (15) is the mean of n times of binomial experiments where the probability is $1 - F(\tilde{\theta})$. Similarly, $E(n_2)$ of (16) is the product of the probability $F(\tilde{\theta})^{n-1}$ and the mean of n times of binomial experiments where the probability is $F(\tilde{\theta}) - F(\theta(\delta r))$.

expected auction price for a property (given r, δ, c) sold in the first auction is

$$
\begin{aligned}
&E(\beta_1(\theta_1^n)) \\
&= \int_{\tilde{\theta}}^{\bar{\theta}} \beta_1(\theta) u dF^n(\theta) \\
&= \int_{\tilde{\theta}}^{\bar{\theta}} [\frac{rG(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} v(y)g(y)dy}{G(\theta)}] nF^{n-1}(\theta)f(\theta)d\theta \\
&= n * \{rG(\tilde{\theta})[1 - F(\tilde{\theta})] + \int_{\tilde{\theta}}^{\bar{\theta}} [1 - F(\theta)]v(\theta)g(\theta)d\theta\}
\end{aligned}
\tag{17}
$$

while the expected winning prices in the second auction is

$$
\begin{aligned}
&E(\beta_2(\theta_1^n)) \\
&= \int_{\theta(\delta r)}^{\tilde{\theta}} \beta_2(\theta) u dF^n(\theta) \\
&= \int_{\theta(\delta r)}^{\tilde{\theta}} [\frac{\delta r G(\theta(\delta r)) + \int_{\theta(\delta r)}^{\tilde{\theta}} v(y)g(y)dy}{G(\theta)}] nF^{n-1}(\theta)f(\theta)d\theta \\
&= n\{\delta r G(\theta(\delta r))[F(\tilde{\theta}) - F(\theta(\delta r))] + \int_{\theta(\delta r)}^{\tilde{\theta}} [F(\tilde{\theta}) - F(\theta)]v(\theta)g(\theta)d\theta\}.
\end{aligned}
\tag{18}
$$

Then we have the comparative static results as follow.

Proposition 1 *Suppose that* $n \geq 2$ $r \leq E(\theta_1^{n-1}) * u - c$. *Then*
(i) $\frac{\partial E(\beta_1(\theta_1^n))}{\partial c} < 0$,
(ii) the sign of $\frac{\partial E(\beta_2(\theta_1^n))}{\partial c}$ *is indeterminate.*

The proof of Proposition 1 appears in Appendix.

I explain the intuition of Proposition 1 as follows. Note that the auction price of the bidder with the highest valuation ratio θ_1^n in the first auction is his weighted expected value of the second highest ratio θ_1^{n-1} where the weight is $G(\tilde{\theta})$ when $\theta_1^{n-1} \leq \tilde{\theta}$ and $g(\theta_1^{n-1})$ when $\theta_1^{n-1} > \tilde{\theta}$. As the risk cost c increases, the cutoff ratio $\tilde{\theta}$ increases and thus $\beta_1(\theta_1^n)$ puts a larger weight of probability on r, which is less than $v(\theta)$ for $\theta > \tilde{\theta} > \theta(r)$. Note that $v(\theta) = \theta * u - c$ and $v(\theta)$ decreases with c. Thus, $\beta_1(\theta_1^n)$ decreases with c. Furthermore, since $\frac{\partial \tilde{\theta}}{\partial c} > 0$, the probability of $\theta_1^n > \tilde{\theta}$ decreases with c. As a result, $E(\beta_1(\theta_1^n))$ decreases with the risk cost.[12]. Let us now

[12]This is not the case for the winning bid in the second auction. The impact of changing the risk cost on the expected winning bid in the second auction is more complicated and its sign is indeterminate. An increases of risk cost reduces the bids of bidders in the second auction, while it induces a higher cutoff point and thus corresponds to a greater likelihood of higher value bidders in attendance

explain the meaning of the auction price in the second auction. The auction price of the bidder with the highest valuation ratio θ_1^n in the second auction is his weighted expected value of the second highest ratio θ_1^{n-1} where the weight is $G(\theta(\delta r))$ when $\theta_1^{n-1} \leq \theta(\delta r)$ and $g(\theta_1^{n-1})$ when $\theta_1^{n-1} \in (\theta(\delta r), \tilde{\theta})$. As the risk cost c raises, the cutoff ratio $\tilde{\theta}$ increases and thus $\beta_2(\theta_1^n)$ puts a smaller weight of probability on δr, which is less than $v(\theta)$ for $\theta \in (\theta(\delta r), \tilde{\theta})$. There are two oppositive effects on $E(\beta_2(\theta_1^n))$. In one hand, $v(\theta)$ decreases with c and a bidder submits a smaller bid.[13] On the other hand, the probability of the event of $\theta_1^n \in (\theta(\delta r), \tilde{\theta})$ increases with c. As a result, the sign of $\frac{\partial E(\beta_2(\theta_1^n))}{\partial c}$ depends on two conflicting effects.

4 Empirical Analysis

There are some testable implications from our theoretical results. The first is corollary 2, saying that those factors (related to n, r, δ and c) influencing the probability of a property being sold in earlier auctions, have no determinate impact on the probability of being sold in later auctions. By employing a multinomial logit regression, we explore the determinants of the relative probabilities of being sold in earlier auctions, being sold in later auctions or being unsold eventually. The second testable theoretical result is corollary 3, saying that those factors influencing the the expected numbers of actual bidders for a property in earlier auctions, might have no impact on the expected number of bidders in later auctions. We use two zero-inflated negative binomial count regressions (hereafter, ZINB) to examine the determinants of the number of bidders for properties auctioned in the first two auctions and the last two auctions no matter they were sold or unsold. Finally we examine the determinants of auction prices by using a two-stage least estimation to mitigate the endogeneity problem of the number of actual bidders on auction prices.

[13]Note that $\frac{\partial \beta_2(\theta)}{\partial c} = -\int_{\theta(\delta r)}^{\tilde{\theta}} g(\theta) dy < 0$.

4.1 Data and Variables

The auction of property takes place in each area under the jurisdiction of a district court in Taiwan. We use data of auctions under the jurisdiction of the Taipei District Court and Shilin District Court from the first quarter of 2006 through 2009. The database for the Taipei metropolitan area is constructed from the magazine "Tomin Real Estate Journal", which makes the information available in an electronic database. In total, 33,222 auctions were held for foreclosed properties in Taipei city between 2006 and 2009. The sample we used in the analysis is as follows. First, we used only the sample of residential condominiums, excluding detached houses, nonresidential housing, offices, shops, and warehouses. Second, we dropped the sample of properties sold by bulk sale, because we cannot allocate a price to each condominium. Last, we dropped all observations for which all of the necessary information is not available. Screening the data in this manner, we obtained a sample of 5,258 observations of condominiums that sold in auctions.

The sample contains four types of characteristic variables: house and building characteristic variables, auction characteristic variables, neighborhood characteristic variables and time dummy variables. House and building characteristic variables include floor area (Fs), land area ($Land$), public facility space Ps, all measured in square meters, assessed value of the land per square meter (Av), floor level ($Floor$), levels of total floors ($tfloor$), age of building in years (Age), whether building's main frames are made of steel-reinforced concrete (Src).[14] Auction characteristic variables include the number of actual bidders N, the total reserve price r, the reserve price of land Lr and the reserve price of building Hr, whether the properties are issued with eviction order Eo, whether being with pre-emption rights Pre and variables with regard to the status of tenure including $Vacant$, $Debtor$, Nop, $lease$, Occ and $Mort$. To be comparable with the existing literature, the types of occupancy are classified into three different categories: lease-hold, third-party occupancy, and vacant (reference category). Vacant includes a property that is either no-person, occupied by a debtor or controlled by a mortgagee. Both short-

[14] The assessed land value is revalued per year and used for levying the land value increment tax and compensating for the land acquisition in Taiwan.

term leasehold and long-term leasehold have previous rights and obligations still attached to the property. Short-term leasehold is formed by contract after the mortgage is registered, while the long-term leasehold is formed before. The eviction order can be issued to reduce the risk for the properties winners. Whether a property is issued eviction or not depends on these status of tenure. Generally, a property that is either short-term leasehold or illegally occupied can be issued an eviction order while those with leasehold formed before the mortgage is registered or legally occupied is not issued with an eviction order. Other control variables that capture features of the Taiwan judicial auctions are Delinquency. Delinquency cost includes the administration costs of a mortgagor, such as common area maintenance fees, water expenses, etc. A winning bidder must take care of this delinquency instead of the mortgagor. The expected sign of Delinquency on the winning price is negative.

Key factors driving more participants in the auction for a property and thus raising the probability of sale are the ratios of the reserve prices of the property to its market value. The market value of the property, however, can not be estimated since the assessed value of lands are available only for the foreclosed properties but not for the properties sold in search markets. Instead, we use the two following ratios as proxies. The ratio of land reserve to value is defined as $Alr1v = \frac{A1r1}{Av}$ where $Alr1$ is the reserve price of land in the first auction per square meter and Av is the assessed value of land per square meter. Similarly, the ratio of building reserve to cost is defined as $Ahr1c = \frac{Ahr1}{Ahc}$ where $Ahr1$ is the reserve price of building in the first auction per square meter and Ahc is the estimated replacement cost of building per square meter adjusted by its depreciation.[15] A higher $Alr1v$ or a higher $Ahr1c$ represents a higher reserve price, induces a higher $\tilde{\theta}$ and makes the property less likely being sold in earlier auction. We do not include δ as an explanatory variable because bidders do not know δ before the beginning of the first auction and also since most of properties unsold have 20 percentage of reduction of the reserve price in the subsequent auction. Although, when analyzing the number

[15]The replacement cost $Ahc = hc * \frac{durable \quad year-age}{durable \quad year}$ where hc is the assessed construction cost per square meter, depending on its construction structure and total level of floor the property is located in.

of actual bidders, we use δ to distinguish the first auction and the second auction in the earlier auctions and the third auction and the fourth auction in the later auctions.

Other control variables that capture features of the foreclosed properties in Taipei are district dummy variables and time dummy variables. We calculate average selling prices per square meter, shown as Table 1, for each of twelve districts in Taipei metropolitan The reference district is WanHua, which has the lowest average sale price. Finally, we use thirteen time dummy variables (the first quarter of 2008 is a reference group) to control the time trend of fourteen quarters.

The definitions of variables are listed in Table 2. Table 3 shows the summary statistics.

4.2 When to bid - A Multinomial Logit Model

The first question each potential bidder concerns is "at which round a property with characteristic variable vector $\mathbf{x_k}$ will be sold or be eventually unsold ? " Focusing on only the final outcomes of properties auctioned within the sample period, we have 1951 observations. Of the properties, 437, 795, 547, 118, properties were sold in the first auction, in the second auction, in the third auctions, and in the fourth auction, respectively while there are 53 properties failed to be sold in the fourth auction (hereafter, unsold eventually). We employ a multinomial logit choice model to explore the factors influencing the relative probability that properties are sold in earlier auctions, sold in later auctions or unsold eventually.[16] The outcomes of an auctioned property can be categorized into three groups: (i) sold in the first auction, (ii) sold in the second auction, or (iii) sold or unsold after the second auction.[17] We select the properties sold in the first auction to be the reference group ($y = 1$) and those sold in the second auction to be the second group ($y = 2$) while those sold after the second auction and those eventually unsold is assigned to be the third group ($y = 3$). By the multinomial logit model, the log odds of the outcome of $y = j$, $j = 1, 2, 3$ for a property k with characteristic vector $\mathbf{x_k}$,

[16] We tried to use a order logit regression and the assumption of parallel regressions is rejected.

[17] For other categories, the assumptions of independence of irrelevant alternatives (IIA) are rejected in Hausman-McFadden tests. The IIA assumption means adding or deleting alternatives does not affect the odds among the remaining alternatives.

compared with the reference group $y = 1$, can be expressed as:

$$\ln \frac{Prob(y = j \mid \mathbf{x_k})}{Prob(y = 1 \mid \mathbf{x_k})} = \mathbf{x_k}\beta^{\mathbf{j|1}} \tag{19}$$

while the log odd of the third outcome ($y = 3$) to the second outcome ($y = 2$) can be shown to be

$$\ln \frac{Prob(y = 3 \mid \mathbf{x_k})}{Prob(y = 2 \mid \mathbf{x_k})} = \ln \frac{\frac{Prob(y=3|\mathbf{x_k})}{Prob(y=1|\mathbf{x_k})}}{\frac{Prob(y=2|\mathbf{x_k})}{Prob(y=1|\mathbf{x_k})}} = \mathbf{x_k}(\beta^{\mathbf{3|1}} - \beta^{\mathbf{2|1}}). \tag{20}$$

Table 4 presents the results of Hausman tests and Small-Hsiao tests. None of these tests reject the assumption of the independent of irrelevant alternatives and thus the multinomial logit model is appropriate. Regression results for the multinomial logit choice model are described in Table 5. The estimation results suggest that many of those variables having significantly impacts on the log odd $ln\frac{P_3}{P_1}$, have no significant effect on the log odd $ln\frac{P_2}{P_1}$. Look first at the effect of floor area. The significant negative and positive coefficients of floor area and floor area squares respectively in the second regression suggest that properties with the middle-sized floor areas have a larger tendency of being sold in earlier auctions than the property with a too small or too large floor area. The reason is that small-sized properties are more difficult to obtain mortgage loans while large-size properties are not affordable for most potential bidders and thus both types of properties attract less potential bidders. In contrast, floor area and floor area squares are insignificant in the first regression. Both estimation results are consistent with (i) of corollary 2, claiming $\frac{\partial P_1}{\partial n} > 0$, $\frac{\partial P_3}{\partial n} < 0$ and the sign of $\frac{\partial P_2}{\partial n}$ is indeterminate, respectively. The positive coefficients of both ratios of reserver-to-value $Alr1v$ and $Ahr1c$ in both regressions suggest that a higher reserve of the first auction decreases the tendency of the property being sold in earlier auctions. The positive effect is stronger and more significant in the second regression than in the first regression, reflecting (ii) of corollary 2, saying that a higher reserve increases P_3 rather than P_2 while it decreasing P_1. Similar to Idee(2009) finding that leaseholds and third-party occupancy have negative impacts on the reserve prices, leaseholds, third occupancy and preemptive right are significantly positive on the $\frac{p_3}{p_1}$. In contrast to $\frac{p_3}{p_1}$, Third-party occupancy and Preemptive have not a statistically significant impact on $\frac{p_2}{p_1}$. This difference of the results in both regression is again consistent to

(iii) of corollary 2, saying that a higher risk cost for foreclosed properties increases P_3 rather than P_2 while it decreases P_1. This result does not change a lot even when eviction order is issued.[18] Properties with preemptive options attract less potential bidders and thus have larger odd ratios of being sold later or eventually unsold to being sold in the first auction. This effect, however, is not significant in the first regression. Proximity to the most close subway station is positively associated with both odd ratios $\frac{p_2}{p_1}$ and $\frac{p_3}{p_1}$. This result indicates that properties more close the subway stations attract more potential bidders and thus increases the tendency of being sold in earlier auctions. As expected, compared with located in WanHua, properties in most districts have larger tendencies of being sold in earlier auctions. Districts with higher sale price, as described in Table 1, have larger and more significant tendencies of being sold in earlier auctions than districts with lower sale prices. To outstand the effects of time trend and special events, we use the first quarter of 2008 as a comparing group to examine the impacts of the election of Taiwan's present held in March of 2008 and the global financial crisis influencing Taiwan in the third quarter of 2008. Nonsignificant coefficients of $D82$ and $D83$ and positive significant coefficients of $D91$ and $D92$ suggest global financial crisis had a negative impact on the tendency of being sold in earlier auctions while the election of Taiwan president had no significant increase on the probability for foreclosed properties to be sold in earlier auctions.

4.3 Determinants of Number of Actual Bidders

How much a bidder should bid for a property k depends on his valuation and his estimate of the number of bidders, as shown in equations (3) and (5). We examine next the determinants of the number of actual bidders for a property with $\mathbf{x_k}$. To test corollary 3, different from the multinomial logit regression, the data used here contains all the records within sample period for properties auctioned. [19] For

[18]Being issued with an eviction order has positive significant impact on $ln\frac{P_3}{P_1}$ and $ln\frac{P_2}{P_1}$. This is contrast to the intuition that eviction order can reduce the risk of buying foreclosed properties and attract more potential bidders and thus reducing the probability for properties being sold in later auctions. One reason for this contrast result is that introducing dummy variables for the tenure status makes eviction order.

[19]If the final outcome of a property occurs in the t-th auction, we trace its records in the t'-th auction, $1 \leq t' < t$. Thus a property corresponds to t observations.

example, for a property k sold in the second auction with N_{k2} participants, we have two distinct observations: $(0, \mathbf{x_{k1}})$ and $(N_{k_2}, \mathbf{x_{k2}})$. These two observations $(0, \mathbf{x_{k1}})$ and $(N_{k2}, \mathbf{x_{k2}})$ differ at least on the reserve prices and the numbers of actual bidders.

4.4 Statistical methods

Let us divide the whole sample into two sub samples: properties auctioned in the first auctions or in the second auctions (earlier auctions) and properties auctioned in the third auctions or in the fourth auctions (later auctions). The phenomenon of excess zeros is definitely a concern in this study because there are 68.1 $\%(=\frac{2543}{3731})$ and 32.1 $\%(=\frac{315}{980})$ of properties auctioned in the earlier auctions and in the later auctions were not sold, respectively. Attracting no bid after all comes from two possible processes. Properties with $\bar{\theta} * u - c < \delta r$, will certainly attract no actual bidder in both earlier and later auctions. These observations would always attract zero actual bidder, independent of the data generation process. Whereas there are other properties for which $\bar{\theta} * u - c > \delta r$ and the numbers of actual bidders conceivably follow a Poisson process (or negative binomial process), but again are zeros during the sample period since the realized θ_1^n is so low that $\theta_1^n * u - c < \delta r$ (in part due to the data generating process). Similarly, attracting no bid in earlier auctions also comes from two possible processes. By (iii) of Theorem 1, the necessary condition that properties attract no actual bidder in earlier auctions is $E(\theta_1^{n-1}) * u - c \leq r$, by which these observations would always attract zero actual bidder, independent of the data generation process. Whereas there are other properties for which $E(\theta_1^{n-1}) * u - c > r$ and the numbers of actual bidders conceivably follow a Poisson process (or negative binomial process), but again are zeros during the sample period since the realized θ_1^n is so low that $\theta_1^n < \tilde{\theta}$ (in part due to the data generating process). The issue of excess zeros can be dealt with through the application of zero-inflated binomial negative regression model.[20]

[20]The ordinary least squares results have the advantage of easy interpretation. However, since the number of bidders take only integer values, the normality assumption does not hold. Consequently, we estimate a negative binomial count maximum likelihood model which maximizes the likelihood that the number of bidders is equal to the number actually observed assuming a negative binomial distribution. Bajari and Hortacsu(2003) investigate the determinants of entry by regressing the number of

A ZINB model is a modified Poisson regression model designed to deal with two common issues that occur with the application of the Poisson model to count data. These include overdispersion and excess zeros. The ZINB regression assumes that the observed count for observation k is drawn first from

$$y_k = \begin{cases} 0 & \text{with probabality } q_k \\ NB(\lambda_k, \alpha) & \text{with probability } 1 - q_k \end{cases} \tag{21}$$

where

$$q_k = \frac{e^{Z_k' \lambda}}{1 + e^{Z_k' \lambda}}. \tag{22}$$

This model puts extra weight on the probability of observing a zero through a mixing specification. Conceptually, it divides properties auctioned into properties being never sold, with probability q_k, and potential sold property, with probability $1 - q_k$. The unobservable probability q_k is generated as a logistic function of the observable covariates to ensure nonnegativity. An observed zero for $N^{t,k}$ The density of negative binomial negative distribution with $(\lambda_k, \mathbf{x_k})$, is

$$f(y^*_{t,k} \mid \mathbf{x_k}) = Prob(Y = y^*_{t,k} \mid \lambda_k) = \frac{e^{-\lambda_k} \lambda_k^{y^*_{t,k}}}{y^*_{t,k}!}, \quad y^*_{t,k} = 0, ..., \tag{23}$$

$$\lambda_k = e^{\beta' \mathbf{x_k} + \epsilon_k} \tag{24}$$

and e^{ϵ_k} is assumed to have a delta distribution with mean 1 and variance α so that the conditional mean of y^*_k is still λ_k but the conditional variance of y^*_k becomes $\lambda_k(1 + \alpha\lambda_k)$.[21] Therefore

$$\begin{aligned} Prob(y_k = 0 \mid \mathbf{x_k}) &= q_k + (1 - q_k) f(0 \mid \mathbf{x_k}) \\ Prob(y_k = k \mid \mathbf{x_k}) &= (1 - q_k) f(h = k \mid \mathbf{x_k}). \end{aligned} \tag{25}$$

Hence, the ZINB model in which the binary process is estimated by the logit model has the variance

$$Var(y_k) = \lambda_k (1 - q_k)[1 + \lambda(q_k + \alpha)] \tag{26}$$

bidders in an auction on various covariaties and after a specification search, they decide to use a Poisson specification in the regression.

[21] If α approaches zero, y^*_k becomes a Poisson distribution. As α becomes larger, the distribution will be more dispersed. The phenomenon of excess zeros is definitely a concern in this study since the sample contains an abnormal number of zero bids. The issue of excess zeros can be dealt with through two modified count models : hurdle regressions and the application of zero-inflated negative binomial (ZINB) regression model introduced by Green(1994).

and the ratio of the variance to its mean

$$\frac{Var(y_k)}{E(y_k)} = 1 + [\frac{q_k + \alpha}{1 - q_k}]E(y_k). \tag{27}$$

Note that NB and ZINB are not nested, the Vuong non-nested test can be used to decide which model has a better fit as follows. Let $m_k = ln[\frac{\hat{P}_1(y_k|\mathbf{x_k})}{\hat{P}_2(y_k|\mathbf{x_k})}]$ where $\hat{P}_1(y_k \mid \mathbf{x_k})$ and $\hat{P}_2(y_k \mid \mathbf{x_k})$ are the predicted probabilities of the two competing models. Let also $\bar{m}$ to be the mean and let s_m be the standard deviation of m_k. The Vuong statistic, $V = \frac{\sqrt{L}\bar{m}}{s_m}$, is an asymmetric normal distribution where L is the number of observations. Thus, at a 5% level of significance, if the $V > 1.96$, the first model is favored; if $V < -1.96$, the second model is favored. As noted by Greene(1994) and Grootendorst(1995), we can choose the best model among the ZINB, ZIP(zero-inflated Poisson), NB, and Poisson models by the following steps. If Vuong test shows that the NB model is rejected in favor of the ZINB model, we will test if the parameter α in the ZINB model is significant. If the estimate of α is also significant, both the splitting mechanism and individual heterogeneity account for dispersion. To test Corollary 3, we divide the full sample into two subsamples: the first two auctions and the last two auctions. Note that in our theory, there are only two rounds of auctions while our empirical data includes four rounds of auctions. We treat the first two auctions as "earlier auctions" and the last two auctions as "later auctions" and examine the effect of rasing reserve prices of the first auction on the expected number of actual bidders in the earlier auctions and later auctions. To control for the difference between the first auction and the second auction and the difference between the third auction and the fourth auction, we add δ as an explainable variable.

The regression results of ZINB for the two subsamples are shown in Table 6. The Vuong statistics of ZINB and NB model for our samples are $V = 9.4$ and 5.5 in the first two auctions regression and in the last two auctions regression, respectively. Therefore, we choose the ZINB rather than NB model. Next let us examine the hypotheses of no overdispersion for samples in earlier auctions and in later auctions. The likelihood ratio test is developed to examine the null hypothesis of no overdispersion, $H_o : \alpha = 0.$ and the likelihood ratio follows the Chi-squared distribution with one degree of freedom, $LR = 2 * (\ln L_{ZINB} -$

$\ln L_{ZIP}$). The chi-square statistics of no overdispersion for our two sub samples are $\mathbf{2} * ((\mathbf{-5412.756}) - (\mathbf{-7727.148}-)) = \mathbf{2}$ and $2*[-(-2506.252)-(-3474.204)]$ and thus the null hypotheses are rejected, indicating that the ZINB can improve goodness-of-fit over the ZIP.[22] Table 6 presents the results of the ZINB regression model, which include a logit and a NB regression where the logit model reflects the probability of being sold in the first two and in the last two auctions.[23]

While the ratio of land reserve to the assessed value has a significantly negative impact on the expected number of actual bidders in the first two auctions, its coefficient is not statistically significant in the last two auctions. This result supports the theoretical conjecture in part (iii) and (iv) of Corollary 3, saying that a higher reserve, certeris paribus, attracts fewer actual bidders in earlier auctions while its impact is indeterminate in later auctions. One of policy implications for the seller from this result is that reducing reserve prices has a larger effect of increasing expected number of actual bidders and thus rasing the auction revenue for properties more likely sold in earlier auctions since it not only attracts more potential bidders but also reduces the cutoff ratio and induces more potential bidders to become actual bidders. The implication for potential bidders is that the differences of reserve prices between two properties is more meaningful when the two properties are more likely sold in the earlier auctions than when they are more likely sold in later auctions. Namely, for a potential bidder interested in two properties which are more likely sold in earlier auctions, the comparisons of reserve prices, risk costs, districts dummies and time dummies are more important than when they are more likely sold in later auctions.

4.5 Determinants of Auction Prices

We explore next the determinants of winning bids. There are two problems when empirically examining the determinants of winning price. The first one is whether to incorporate the reserve price as an explanatory variable. The second one is that the auction prices may be influenced by those unobservable or omitted variables,

[22]A ZINB model is better than ZIP model also because the estimated results indicate the existence of the overdispersion (i.e. the estimated $Ln\alpha = 0.49$ and the z-statistics is 6.69) for earlier auctions after the excess zero issue is addressed.

[23]The variables included in the regressions of Tables 6, are selected according to (10)-(12), (15) and (16), respectively.

which are also associated with the number of participating bidders. In an open ascending price auctions, the reserve price is the bid-starting price or secret and thus the auction price is not closely correlated with the reserve price. In contrast, in a first-price sealed auction, the auction price of an auctioned object is a function of its reserve price and thus it might explain the most parts of the auction price. To upright the effects of other explanatory variables on the winning prices, we use the bidding premium $Premium$, as the dependent variable, which is defined as

$$Premium = 100 * \frac{Auction \quad Price - reserve}{reserve}. \tag{28}$$

The explained variable $Premium$ reflects the deviation of selling price from the reserve price. Consider a simple structural regression

$$Premium_k = \beta_0 + \beta_N N_k + \sum_{j=1}^{m} \beta_j x_{jk} + u_k \tag{29}$$

where N_k is the number of actual bidders for property k. By (17) and (18), the expected auction price is the function of n, the number of potential bidders. By (15) and (16), the expected number of actual bidders is also the function of the number of potential bidders. Some omitted variables (due to limited information or difficult to express numerically) affecting auctioning prices, also influence the number of actual bidders and thus $cov(N_k, u_k) \neq 0$. The OLS method is inappropriate for the estimation of an equation in a system of simultaneous equations. Instead, we employ a two-stage estimation as follow.[24] Consider the following structural regression

$$Premium_k = \alpha_0 + \alpha_{\hat{N}} \hat{N}_k + \sum_{j=1}^{m} \alpha_j x_{jk} + \tau_k \tag{30}$$

where $\hat{N}_k$ is the predicted number of bidders by running the following zero-truncated negative binomial regression (hereafter ZTNB) :

$$E(N_k \mid \mathbf{x_k}) = \lambda_k = e^{\mathbf{x_k}\beta + \epsilon_k} \tag{31}$$

where N_k is the number of actual bidders for property k and e^{ϵ_k} is assumed to have a delta distribution with mean 1. The probability that N_k equals h conditional on

[24]By contrast, Idee et al.(2009) employ GMM to overcomes the correlation among error terms present.

$\mathbf{x_k}$ and $h > 0$, is

$$Pr(N_k = h \mid h > 0, \mathbf{x_k}) = \frac{e^{-\lambda_k}\lambda_k^h}{h!}\frac{1}{1-e^{-\lambda_k}}, \quad h = 1, ... \tag{32}$$

Table 7 and Table 8 provide the estimation results of number of bidders and the bidding premiums for properties sold in the first two auctions and in the last two auctions, respectively. Each explanatory variable has two possible effects on the bidding premium: the direct effect and indirect effect. The direct impacts of explanatory variables are shown in the second-stage regression of the Two-stage method. The indirect impact of each explanatory variable on the bidding premium is the product of its effect on the number of bidders and the change of the bidding premium by increasing one more predicted number. Namely, the net effect of an explanatory variable x_j on the bidding premium of a property is

$$\begin{aligned} \frac{dPremium}{dx_j}` &= \frac{\triangle Premium}{\triangle x_j} + \frac{\triangle \hat{N}OB}{\triangle x_j} * \frac{\triangle Premium}{\triangle \hat{N}OB} \\ &= \hat{\alpha}_j + \hat{\alpha}_{\hat{N}} * (e^{\hat{\beta}_j \triangle x_j} - 1). \end{aligned} \tag{33}$$

The estimation results of the bidding premium regression excluding the number of bidders (net effect regression) and the OLS result are also presented in Table 7 and Table 8. As shown in Table 7, most of explanatory variables in the first two auctions, except for $Alr1v$, $Ahr1c$, $A4$ (Zhongzheng district dummy) and $D973$, have no significant direct impact on the bidding premiums. Estimation results of Table 7 indicate also that increasing one more predicated number of bidders raises up 1.24411 percentage of bidding premium in the first two auctions. Let us decompose these two effects of increasing $Alr1v$ and $Ahric$ on the bidding premiums. As $Alr1v$ increases by one from his mean $Al\bar{r}1v = 2.003577$, the number of bidders decreases by 31.814%$(= 100 * e^{-0.38293} - 1)$. Notice the average number of bidders of properties with $Al\bar{r}1v$ is 6.68.[25] Thus, increasing $Alr1v$ by one

[25] If we define $E(y \mid \mathbf{x}, x_j)$ as the expected number of for a given $\mathbf{x}$, where we explicitly note the value of x_j, and define $E(y \mid \mathbf{x}, x_j + \delta)$ as be the expected number after increasing x_j by δ units. Then percentage change in the expected number of bidders for a δ unit change in x_j, holding other variables constant, can ne computed as

$$100 * \frac{E(y \mid \mathbf{x}, x_j + \delta) - E(y \mid \mathbf{x}, x_j)}{E(y \mid \mathbf{x}, x_j)} = 100 * (e^{\beta_j \delta} - 1).$$

Thus the change in the expected nunber of bidders for a one unit ($\delta = 1$) change in x_j, holding other variabls cosntant, can ne computed as

$$E(y \mid \mathbf{x}, x_j + \delta) - E(y \mid \mathbf{x}, x_j) = (e^{\beta_j \delta} - 1)E(y \mid \mathbf{x}, x_j).$$

reduces the number of actual bidders by $2.125171006 (= 6.68 * 0.31814)$. As a result, the indirect effect of increasing one of $Alr1v$ on the bidding premium is $-2.63751329 (= -2.12 * 1.24411)$. In total, increasing one of $Alr1v$ changes the bidding premium by $-4.85855329 (= -2.63751329 - 2.22104)\%$, closer to, -3.92, the coefficient of $Alr1v$ on the net effect regression. Similarly, increasing $Alrhc$ by one from his mean $A\bar{hr}1c = 2.31$ reduces the number of bidders by a factor $-0.09252567 (= e^{-0.09709} - 1)$. Notice the average number of bidders of properties with $A\bar{hr}1v$ is 6.37. Thus, the effect of increasing $Ahr1c$ on the number of bidders is $6.37 * (-0.09252567) = -0.589388519$. In total, increasing $Alh1c$ by one reduces $-1.444054151 (= -0.589388519 * 1.24411 - 0.71079 = -0.733264151 - 0.71079)$ of bidding premium, close to, -1.20966%, the coefficient of $Ahr1c$ on the net effect regression.

In contrast to the first two auctions, results of the two-stage estimation described in Table 8 indicate that the predicted number of bidders in the last two auctions has no significant impact on the bidding premiums while most of variables describing risk costs have significantly direct impacts on bidding premiums. One possible reason for this result is that multicollinearity is serious when there are variables having direct impacts on the bidding premiums, also influence the numbers of bidders, such as Eo, $Rent$, Occ, Pre and $D974$. After deleting the variable Eo in the premium regression (second stage), the problems of multicollinearity is mitigated and the result excluding EO variable suggests that increasing one more predicated number of bidders raises up 1.06777 percentage of bidding premium in the last two auctions.[26]

5 Conclusions

What will you suggest for a potential bidder who has a value ratio θ of an interested foreclosed property k with the characteristic $\mathbf{x_k}$? Which round of auction should he submit a serious bid at? How much should he bid ? These questions can be answered by the paper's theoretical conjectures and the estimation results of

[26]By eliminating other variables $Rent$, Occ, Pre and $D974$ have similar results.

empirical models. Nevertheless, some factors influencing participation decision of potential bidders and their bidding strategies are not included in this paper. Common value and affiliated value models should also be considered. These potential issues are on our research schedule.

6 Appendix

Proof of Theorem 1. By Lemma 1, if $r + c < E(\theta_1^{n-1}) * u$ there exists a cutoff ratio, $\tilde{\theta} \in (\theta(r), \bar{\theta})$, such that a bidder with valuation ratio θ submits a bid in the first auction if and only $\theta \geq \tilde{\theta}$.

We now find $\tilde{\theta}$ in equilibrium. Consider the decision problem of a bidder, say bidder 1, whose type is θ, $\theta(\delta r) \leq \theta < \tilde{\theta}$. Suppose all other bidders play according to the proposed strategy. If bidder 1 does not bid in the first auction and submits a bid b in the second auction, he will win the property if the first auction fails and he wins the second auction, namely, $\theta_1^{n-1} \leq min\{\beta_2^{-1}(b), \tilde{\theta}\}$. **Since** $\beta_2(\theta) \leq v(\theta)$, any bid $b \geq \beta_2(\tilde{\theta})$ is not the optimality of the bidder. Thus, $min\{\beta_2^{-1}(b), \tilde{\theta}\} = \beta_2^{-1}(b)$. His payoff by waiting in the first auction and bidding b in the second auction is

$$\pi^w_{\theta(\delta r) \leq \theta < \tilde{\theta}}(b) = (v(\theta) - b)G(\beta_2^{-1}(b)).$$

Maximizing this with respect to b yields the first-order condition:

$$\frac{g(\beta_2^{-1}(b))}{\beta_2'(\beta_2^{-1}(b))}(v - b) - G(\beta_2^{-1}(b)) = 0. \tag{34}$$

where $g = G'$ is the density of θ_1^{n-1}. At a symmetric equilibrium, $b = \beta_2(\theta)$, then (34) yields the differential equation

$$G(\theta)\beta_2'(\theta) + g(\theta)\beta_2(\theta) = v(\theta)g(\theta)$$

or equivalently,

$$\frac{d(G(\theta)\beta_2(\theta))}{d\theta} = v(\theta)g(\theta) \tag{35}$$

and since $\beta_2(\theta(\delta r)) = \delta r$ we have

$$\beta_2(\theta) = \frac{\delta r G(\theta(\delta r))}{G(\theta)} + \frac{\int_{\theta(\delta r)}^{\theta} v(y)g(y)dy}{G(\theta)} = E[max\{v(\theta_1^{n-1}), \delta r\} \mid \theta_1^{n-1} < \theta].$$

By integration by parts, we have [27]

$$\beta_2(\theta) = v(\theta) - \frac{\int_{\theta(\delta r)}^{\theta} G(y)udy}{G(\theta)}.$$

Thus, the payoff for a bidder with type θ ($\theta(\delta r) \leq \theta < \tilde{\theta}$) to not to bid in the first auction and submit $\beta_2(\theta)$ when the first auction fails, is

$$\pi^{w}_{\theta(\delta r)\leq\theta<\tilde{\theta}}(\beta_2(\theta)) = \int_{\theta(\delta r)}^{\theta} G(y)udy.$$

Note that participating in the first auction is not the optimality of a bidder with θ, $\theta < \theta$Consider now the payoff for a bidder with θ, $\theta(r) \leq \theta < \tilde{\theta}$, and suppose all his rivals use the proposed strategies. If he submits a bid $b \geq r$ in the first auction, he will win the first auction if $\theta_1^{n-1} \leq max\{\beta_1^{-1}(b), \tilde{\theta}\}$. **Since** $\beta_1(\tilde{\theta}) = r$ **and** $b \geq r$, $max\{\tilde{\theta}, \beta_1^{-1}(b)\} = \beta_1^{-1}(b)$. Thus his maximal payoff of bidding in the first auction is

$$\pi^{b}_{\theta(r)\leq\theta<\tilde{\theta}} = \max_{r\leq b\leq v(\theta)} (v(\theta) - b)G(max\{\tilde{\theta}, \beta_1^{-1}(b)\}) = \max_{r\leq b\leq v(\theta)} (v(\theta) - b)G(\beta_1^{-1}(b)).$$

Consider next the decision problem of a bidder with type $\theta \geq \tilde{\theta}$. Suppose all other bidders play according to the proposed strategy. If a bidder $\theta \geq \tilde{\theta}$ submits a bid b in the first auction, he wins the first auction if $b \geq r$ and $\theta_1^{n-1} < \max\{\tilde{\theta}, \beta_1^{-1}(b)\}$. **Since** $\beta_1(\tilde{\theta}) = r$, $\max\{\tilde{\theta}, \beta_1^{-1}(b)\} = \beta_1^{-1}(b)$ and his payoff is

$$\pi^{b}_{\theta\geq\tilde{\theta}}(b) = (v(\theta) - b)G(\beta_1^{-1}(b)).$$

Maximizing this with respect to b yields the first-order condition:

$$\frac{g(\beta_1^{-1}(b))}{\beta_1'(\beta_1^{-1}(b))}(v(\theta) - b) - G(\beta_1^{-1}(b)) = 0. \tag{36}$$

At a symmetric equilibrium, $b = \beta_1(\theta)$, then (36) yields the differential equation

$$G(v(\theta))\beta_1'(\theta) + g(\theta)\beta_1(\theta) = v(\theta)g(\theta)$$

[27]let $t(y) = v(y), s(y) = G(y)$

$$\begin{aligned}
&\int_{\theta(\delta r)}^{\theta} v(y)g(y)dy = \int_{\theta(\delta r)}^{\theta} t(y)ds(y) \\
&= t(y)s(y)\,|_{\theta(\delta r)}^{\theta} - \int_{\theta(\delta r)}^{\theta} s(y)dt(y) \\
&= [v(\theta)G(\theta) - v(\theta(\delta r))G(\theta(\delta r))] - \int_{\theta(\delta r)}^{\theta} G(y)v'(y)dy \\
&== [v(\theta)G(\theta) - \delta r G(\theta(\delta r)] - \int_{\theta(\delta r)}^{\theta} G(y)udy.
\end{aligned}$$

or equivalently,

$$\frac{d(G(v)\beta_1(\theta))}{d\theta} = v(\theta)g(\theta)$$

and since $\beta_1(\tilde{\theta}) = r$, we have

$$\beta_1(\theta) = \frac{rG(\tilde{\theta})}{G(\theta)} + \frac{\int_{\tilde{\theta}}^{\theta} yg(y)dy}{G(\theta)}. \tag{37}$$

By the integration by parts, we have

$$\begin{aligned}\beta_1(\theta) &= \frac{rG(\tilde{\theta}) + G(\theta)v(\theta) - G(\tilde{\theta})v(\tilde{\theta}) - \int_{\tilde{\theta}}^{\theta} G(y)udy}{G(\theta)} \\ &= v(\theta) - \frac{G(\tilde{\theta})(v(\tilde{\theta}) - r)}{G(\theta)} - \frac{\int_{\tilde{\theta}}^{\theta} G(y)udy}{G(\theta)}.\end{aligned}$$

As a result, the payoff for a bidder with $\theta \geq \tilde{\theta}$ to bid in the first auction is

$$\pi^b_{\theta\geq\tilde{\theta}}(\beta_1(\theta)) = G(\tilde{\theta})(v(\tilde{\theta}) - r) + \int_{\tilde{\theta}}^{\theta} G(y)udy.$$

On the other hand, if a bidder with $\theta \geq \tilde{\theta}$ submits no bid in the first auction and bids b in the second auction when the first fails, he wins the second auction if $b \geq \delta r$ and $\theta_1^{n-1} < min\{\tilde{\theta}, \beta_2^{-1}(b)\}$. **Since** $\beta_2^{-1}(b) < \tilde{\theta}$, his maximal payoff of bidding in the second auction is

$$\pi^w_{\theta\geq\tilde{\theta}} = \max_{\delta r\leq b\leq v(\theta)} (v(\theta) - b)G(min\{\tilde{\theta}, \beta_2^{-1}(b)\}) = \max_{\delta r\leq b\leq v(\theta)} (v(\theta) - b)G(\beta_2^{-1}(b)).$$

The cutoff ratio $\tilde{\theta}$, $\beta_1(\theta)$ and $\beta_2(\theta)$ in equilibrium must satisfy

$$\pi^b_{\theta\geq\tilde{\theta}}(\beta_1(\theta)) \geq \pi^w_{\theta\geq\tilde{\theta}} = \max_{\delta r\leq b\leq v(\theta)} (v(\theta) - b)G(\beta_2^{-1}(b)), \quad \forall\theta \geq \tilde{\theta}$$

and

$$\pi^w_{\theta(r)\leq\theta<\tilde{\theta}}(\beta_2(\theta)) > \pi^b_{\theta(r)\leq\theta<\tilde{\theta}} = \max_{r\leq b\leq v(\theta)} (v(\theta) - b)G(\beta_1^{-1}(b)), \quad \forall\theta(\delta r) \leq \theta < \tilde{\theta}.$$

By the continuity of the payoff functions, for the conjectured strategies to characterize an equilibrium it is necessary for a bidder with $\theta = \tilde{\theta}$ to have the same expected payoff from bidding $\beta_1(\tilde{\theta}) = r$ in the first auction or submitting no bid and bidding $\beta_2(\tilde{\theta})$ in the second auction when the first auction fails. Thus a necessary condition for $\tilde{\theta}$ to be an equilibrium cutoff is

$$\pi^b_{\tilde{\theta}}(\beta_1(\tilde{\theta})) = (v(\tilde{\theta}) - r)G(\tilde{\theta}) = \pi^w_{\tilde{\theta}}(\beta_2(\tilde{\theta})) = (v(\tilde{\theta}) - \beta_2(\tilde{\theta}))G(\tilde{\theta}), \tag{38}$$

which can be expressed as

$$r = \beta_2(\tilde{\theta}) = v(\tilde{\theta}) - \frac{\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)udy}{G(\tilde{\theta})},$$

or

$$v(\tilde{\theta}) = r + \frac{\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)udy}{G(\tilde{\theta})}.$$

This is (4), the condition given in part (ii) of Theorem 1.

Suppose that all other bidders play according to the proposed strategy where the cutoff value $\tilde{\theta}$ is determined by (4). We show now that any bidder with θ, $\theta(r) \leq \theta < \tilde{\theta}$, has no incentive to submit a bid $b = \beta_1(\theta')$ (where $r \leq \beta_1(\theta') \leq v(\theta)$) in the first auction . By doing this, his payoff is $(v - \beta_1(\theta'))G(\theta')$, that is equal to

$$\begin{aligned}
(v(\theta) - \beta_1(\theta'))G(\theta') &= [v(\theta) - v(\theta') + \frac{G(\tilde{\theta})(v(\tilde{\theta}) - r)}{G(\theta')} + \frac{\int_{\tilde{\theta}}^{\theta'} G(y)udy}{G(\theta')}]G(\theta') \\
&= (v(\theta) - v(\theta'))G(\theta') + \int_{\theta(\delta r)}^{v(\theta)} G(y)udy + \int_{v(\tilde{\theta})}^{\theta'} G(y)udy \\
&= -\int_{\theta}^{\theta'} G(z)udy + \int_{\theta(\delta r)}^{\theta'} G(y)udy \\
&< -\int_{\theta}^{\theta'} G(y)udy + \int_{\theta(\delta r)}^{\theta} G(y)udy \\
&= \int_{\theta(\delta r)}^{\theta} G(y)udy = (v(\theta) - \beta_2(\theta))G(\theta).
\end{aligned}$$

Let us show next that given $\tilde{\theta}$ determined by (4), a bidder with $\theta \geq \tilde{\theta}$ has a greater expected payoff from bidding $\beta_1(\theta)$ in the first auction than waiting and then bidding $b = \beta_2(\theta')$, $\theta(\delta r) \leq \beta_2(\theta') \leq v(\theta)$, in the second auction. By doing this, his payoff is $(v(\theta) - \beta_2(\theta'))G(\theta')$ and

$$\begin{aligned}
(v - \beta_2(\theta'))G(\theta') &= [v(\theta) - v(\theta') + \frac{\int_{\theta(\delta r)}^{\theta'} G(y)udy}{G(\theta')}]G(\theta') \\
&= \int_{\theta'}^{\theta} G(\theta')udy + \int_{\theta(\delta r)}^{\theta'} G(y)udy \\
&< \int_{\theta'}^{\theta} G(y)udy + \int_{\theta(\delta r)}^{\theta'} G(y)udy \\
&= \int_{\theta(\delta r)}^{\theta} G(y)udy = (v(\theta) - \beta_1(\theta))G(v).
\end{aligned} \tag{39}$$

□.

Proof of Corollary 1. Take total derivative of (4), we have

$$u d\tilde{\theta} - dc = dr + \frac{G(\tilde{\theta}) d[\int_{\theta(\delta r)}^{\tilde{\theta}} G(y) u dy] - [\int_{\theta(\delta r)}^{\tilde{\theta}} G(y) u dy] dG(\tilde{\theta})}{G^2(\tilde{\theta})} \tag{40}$$

which is equivalent to

$$\begin{aligned} d\tilde{\theta} =& \frac{dr}{u} + \frac{dc}{u} + \frac{G(\tilde{\theta})[G(\tilde{\theta}) d\tilde{\theta} - G(\theta(\delta r)) d\theta(\delta r) + (\int_{\theta(\delta r)}^{\tilde{\theta}} \frac{\partial G(y)}{\partial n} dn dy)]}{G^2(\tilde{\theta})} \\ & - \frac{(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y) dy)[g(\tilde{\theta}) d\tilde{\theta} + \frac{\partial G(\tilde{\theta})}{\partial n} dn]}{G^2(\tilde{\theta})} \\ =& \frac{dr}{u} + \frac{dc}{u} + \frac{G(\tilde{\theta})[G(\tilde{\theta}) d\tilde{\theta} - G(\theta(\delta r)) \frac{\delta dr + r d\delta + dc}{u} + (\int_{\theta(\delta r)}^{\tilde{\theta}} \frac{\partial G(y)}{\partial n} dn dy)]}{G^2(\tilde{\theta})} \\ & - \frac{(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y) dy)[g(\tilde{\theta}) d\tilde{\theta} + \frac{\partial G(\tilde{\theta})}{\partial n} dn]}{G^2(\tilde{\theta})}. \end{aligned}$$

and thus

$$\begin{aligned} &(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y) dy) g(\tilde{\theta}) d\tilde{\theta} = G^2(\tilde{\theta}) \frac{dr + dc}{u} - G(\tilde{\theta}) G(\theta(\delta r)) \frac{r d\delta + \delta dr + dc}{u} \\ &+ [G(\tilde{\theta})(\int_{\theta(\delta r)}^{\tilde{\theta}} \frac{\partial G(y)}{\partial n} dy) - (\int_{\theta(\delta r)}^{\tilde{\theta}} G(y) dy) \frac{\partial G(\tilde{\theta})}{\partial n}] dn. \end{aligned} \tag{41}$$

Let $dr = d\delta = dc = 0 < dn$ in (41) and note that $lnF(y) < lnF(\tilde{\theta})$ for any $\theta \in [\theta(\delta r), \tilde{\theta})$, we have

$$
\begin{aligned}
\frac{\partial\tilde{\theta}}{\partial n} &= \frac{G(\tilde{\theta})(\int_{\theta(\delta r)}^{\tilde{\theta}} \frac{\partial G(y)}{\partial n} dy) - (\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)dy)\frac{\partial G(\tilde{\theta})}{\partial n}}{(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)dy)g(\tilde{\theta})}. \\
&= \frac{\int_{\theta(\delta r)}^{\tilde{\theta}} G(\tilde{\theta})F(y)^{n-1}lnF(y)dy - \int_{\theta(\delta r)}^{\tilde{\theta}} G(y)F(\tilde{\theta})^{n-1}lnF(\tilde{\theta})dy}{(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)dy)g(\tilde{\theta})}. \\
&= \frac{\int_{\theta(\delta r)}^{\tilde{\theta}} G(\tilde{\theta})G(y)lnF(y)dy - \int_{\theta(\delta r)}^{\tilde{\theta}} G(y)G(\tilde{\theta})lnF(\tilde{\theta})dy}{(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)dy)g(\tilde{\theta})} \\
&= \frac{\int_{\theta(\delta r)}^{\tilde{\theta}} G(\tilde{\theta})G(y)(lnF(y) - lnF(\tilde{\theta}))dy}{(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)dy)g(\tilde{\theta})} < 0.
\end{aligned}
\tag{42}
$$

Let next $d\delta = dn = dc = 0 < dr$, $dr = d\delta = dn = 0 < dc$ and $dr = dn = dc = 0 < d\delta$ respectively, we have

$$
\frac{d\tilde{\theta}}{dr} = \frac{G(\tilde{\theta})[G(\tilde{\theta}) - G(\theta(\delta r))\delta)]}{u(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)dy)G(\tilde{\theta})} > 0, \tag{43}
$$

$$
\frac{d\tilde{\theta}}{dc} = \frac{G(\tilde{\theta})[G(\tilde{\theta}) - G(\theta(\delta r))]}{u(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)dy)G(\tilde{\theta})} > 0, \tag{44}
$$

and

$$
\frac{d\tilde{\theta}}{d\delta} = \frac{-G(\tilde{\theta})G(\theta(\delta r))r}{u(\int_{\theta(\delta r)}^{\tilde{\theta}} G(y)dy)G(\tilde{\theta})} < 0.
$$

□.

Proof of corollary 2. We show (i) first. By (6) and note that $lnF(\theta) < 0$ for any $\theta < \bar{\theta}$, we have

$$
\frac{\partial P_1}{\partial n} = -F(\tilde{\theta})^n lnF(\tilde{\theta}) - nF(\tilde{\theta})^{n-1} f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial n} > 0,
$$

and

$$
\frac{\partial P_3}{\partial n} = F(\theta(\delta r))^n lnF(\theta(\delta r)) < 0.
$$

The sign of $\frac{\partial P_2}{\partial n}$ is indeterminate since

$$
\begin{aligned}
\frac{\partial P_2}{\partial n} &= F(\tilde{\theta})^n lnF(\tilde{\theta}) + nF(\tilde{\theta})^{n-1} f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial n} - F(\theta(\delta r))^n lnF(\theta(\delta r)) \\
&> F(\tilde{\theta})^n lnF(\tilde{\theta}) + nF(\tilde{\theta})^{n-1} f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial n} - F(\theta(\delta r))^n lnF(\tilde{\theta}) \\
&= lnF(\tilde{\theta})\{F(\tilde{\theta})^n - F(\theta(\delta r))^n\} + nF(\tilde{\theta})^{n-1} f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial n} < 0.
\end{aligned}
$$

By (7), (8) and (9), we have

$$\frac{\partial P_1}{\partial r} = -nF(\tilde{\theta})^{n-1} f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial r} < 0,$$

$$\frac{\partial P_1}{\partial c} = -nF(\tilde{\theta})^{n-1} f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial c} < 0,$$

and

$$\frac{\partial P_1}{\partial \delta} = -nF(\tilde{\theta})^{n-1} f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial \delta} > 0.$$

We have also

$$\frac{\partial P_3}{\partial r} = nF(\delta r)^{n-1} f(\delta r) \frac{\delta}{u} > 0, \tag{45}$$

$$\frac{\partial P_3}{\partial r} = nF(\delta r)^{n-1} f(\delta r) \frac{\delta}{u} > 0, \tag{46}$$

$$\frac{\partial P_3}{\partial \delta} = nF(\delta r)^{n-1} f(\delta r) \frac{r}{u} > 0,$$

and

$$\frac{\partial P_2}{\partial \delta} = nF(\tilde{\theta})^{n-1} f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial \delta} - nF(\theta(\delta r))^{n-1} f(\theta(\delta r)) \frac{\delta}{u} < 0. \tag{47}$$

However, the signs of

$$\frac{\partial P_2}{\partial r} = nF(\tilde{\theta})^{n-1} f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial r} - nF(\theta(\delta r))^{n-1} f(\theta(\delta r)) \frac{\delta}{u},$$

$$\frac{\partial P_2}{\partial c} = nF(\tilde{\theta})^{n-1} f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial c} - nF(\theta(\delta r))^{n-1} f(\theta(\delta r)) \frac{1}{u}$$

are indeterminate. □.

Proof of Corollary 3 . We show first (i) and (ii).

$$\begin{aligned}
E(n_1) &= \sum_{k=0}^{n} k[1 - F(\tilde{\theta})]^k F(\tilde{\theta})^{n-k} \begin{pmatrix} n \\ k \end{pmatrix} \\
&= \sum_{k=1}^{n} k[1 - F(\tilde{\theta})]^k F(\tilde{\theta})^{n-k} \begin{pmatrix} n \\ k \end{pmatrix} \\
&= \sum_{k=1}^{n} [1 - F(\tilde{\theta})]^k F(\tilde{\theta})^{n-k} \frac{n!}{(n-k)!(k-1)!} \\
&= [1 - F(\tilde{\theta})] n \sum_{k=1}^{n} [1 - F(\tilde{\theta})]^{k-1} F(\tilde{\theta})^{n-k} \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} \\
&= [1 - F(\tilde{\theta})] n \sum_{i=0}^{n-1} [1 - F(\tilde{\theta})]^{i} F(\tilde{\theta})^{n-1-i} \begin{pmatrix} n-1 \\ i \end{pmatrix}
\end{aligned}$$

Newton's Binomial Formula can be stated as $\sum_{i=0}^{n-1} \frac{a^{n-1-i}b^i(n-1)!}{i!(n-1-i)!} = (a+b)^{n-1}$. Letting $a = F(\tilde{\theta})$ and $b = 1 - F(\tilde{\theta})$, we have

$$E(n_1) = n[1 - F(\tilde{\theta})]. \tag{48}$$

Similarly, we can show that

$$\begin{aligned}
E(n_2) &= \sum_{k=0}^{n} k[F(\tilde{\theta}) - F(\theta(\delta r))]^k F(\theta(\delta r))^{n-k} \begin{pmatrix} n \\ k \end{pmatrix} \\
&= \sum_{k=1}^{n} [F(\tilde{\theta}) - F(\theta(\delta r))]^k F(\theta(\delta r))^{n-k} \frac{n!}{(n-k)!(k-1)!} \\
&= [F(\tilde{\theta}) - F(\theta(\delta r))]n \sum_{k=1}^{n} [F(\tilde{\theta}) - F(\theta(\delta r))]^{k-1} F(\theta(\delta R))^{n-k} \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} \\
&= [F(\tilde{\theta}) - F(\theta(\delta r))]n \sum_{i=0}^{n-1} [F(\tilde{\theta}) - F(\theta(\delta r))]^{i} F(\theta(\delta R))^{n-1-i} \begin{pmatrix} n-1 \\ i \end{pmatrix}.
\end{aligned}$$

Letting $a = F(\tilde{\theta}) - F(\theta(\delta r))$ and $b = F(\theta(\delta r))$, we have

$$E(n_2) = n[F(\tilde{\theta}) - F(\theta(\delta r))]F(\tilde{\theta})^{n-1}. \tag{49}$$

Next we show (iv). Note that

$$\begin{aligned}
\frac{\partial E(n_2)}{\partial n} =& [F(\tilde{\theta})) - F(\theta(\delta r)]F(\tilde{\theta})^{n-1} + nf(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial n}F(\tilde{\theta})^{n-1} \\
&+ n[F(\tilde{\theta}) - F(\theta(\delta r))][F(\tilde{\theta})^{n-1} * lnF(\tilde{\theta}) + (n-1) * F(\tilde{\theta})^{n-2} f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial n}].
\end{aligned} \tag{50}$$

The sign of $\frac{\partial E(n_2)}{\partial n}$ is indeterminate since the first term of the right-hand side of equation (50) is positive although its last two terms are negative. The effect of increasing r and c on the number of actual bidders in the second auction is,

$$\begin{aligned}
\frac{\partial E(n_2)}{\partial r} =& nF(\tilde{\theta})^{n-1}[f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial r} - \frac{\delta}{u} f(\theta(\delta r))] \\
&+ n[F(\tilde{\theta}) - F(\theta(\delta r))] * (n-1) * F(\tilde{\theta})^{n-2} f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial r},
\end{aligned} \tag{51}$$

$$\begin{aligned}
\frac{\partial E(n_2)}{\partial c} =& nF(\tilde{\theta})^{n-1}[f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c} - \frac{1}{u} f(\theta(\delta r))] \\
&+ n[F(\tilde{\theta}) - F(\theta(\delta r))] * (n-1) * F(\tilde{\theta})^{n-2} f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c},
\end{aligned} \tag{52}$$

repsectively. The signs of $\frac{\partial E(n_2)}{\partial r}$ and $\frac{\partial E(n_2)}{\partial c}$ are also indeterminate. However,

$$\begin{aligned}\frac{\partial E(n_2)}{\partial \delta} =& nF(\tilde{\theta})^{n-1}[f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial \delta} - \frac{r}{u}f(\theta(\delta r))] \\ &+n[F(\tilde{\theta}) - F(\theta(\delta r))] * (n-1) * F(\tilde{\theta})^{n-2}f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial \delta} < 0.\end{aligned} \tag{53}$$

□.

Proof of Proposition 1: By (17) and differentiating $E(\beta_1(\theta_1^n))$ with respect to c, we show first part (i)

$$\begin{aligned}&\frac{\partial E(\beta_1(\theta_1^n))}{\partial c} \\ =& n\{rg(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c}[1-F(\tilde{\theta})] - rG(\tilde{\theta})f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c} \\ &-[1-F(\tilde{\theta})]v(\tilde{\theta})g(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c} - \int_{\tilde{\theta}}^{\bar{\theta}}[1-F(\theta)]g(\theta)d\theta\} \\ =& n\{\frac{\partial \tilde{\theta}}{\partial c}g(\tilde{\theta})[1-F(\tilde{\theta})][r - v(\tilde{\theta})] - \frac{\partial \tilde{\theta}}{\partial c}rG(\tilde{\theta})f(\theta) - \int_{\tilde{\theta}}^{\bar{\theta}}[1-F(\theta)]g(\theta)d\theta\} < 0.\end{aligned} \tag{54}$$

By (18) and differentiating $E(\beta_2(\theta_1^n))$ with respect to c, we show part (ii).

$$\begin{aligned}\frac{\partial E(\beta_2(\theta_1^n))}{\partial c} =& n\{\frac{\delta rg(\theta(\delta r))[F(\tilde{\theta}) - F(\theta(\delta r))]}{u} + \delta rG(\theta(\delta r))[f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c} - \frac{f(\theta(\delta r))}{u}] \\ &+ \frac{\partial \tilde{\theta}}{\partial c}[F(\tilde{\theta}) - F(\tilde{\theta}))]v(\tilde{\theta}))g(\tilde{\theta}) - \frac{[F(\tilde{\theta}) - F(\theta(\delta r))]v(\theta(\delta r))g(\theta(\delta r))}{u} \\ &+ \frac{\partial \tilde{\theta}}{\partial c} * \int_{\theta(\delta r)}^{\bar{\theta}} f(\tilde{\theta})v(\theta)g(\theta)d\theta - \int_{\theta(\delta r)}^{\bar{\theta}}[F(\tilde{\theta}) - F(\theta)]g(\theta)d\theta\} \\ =& n\{\delta rG(\theta(\delta r))[f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c} - \frac{f(\theta(\delta r))}{u}] \\ &+ \frac{\partial \tilde{\theta}}{\partial c} * \int_{\theta(\delta r)}^{\bar{\theta}} f(\tilde{\theta})v(\theta)g(\theta)d\theta - \int_{\theta(\delta r)}^{\bar{\theta}}[F(\tilde{\theta}) - F(\theta)]g(\theta)d\theta\} \\ =& n\{f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c}[\delta rG(\theta(\delta r)) + \int_{\theta(\delta r)}^{\bar{\theta}} v(\theta)g(\theta)d\theta] \\ &- \delta G(\theta(\delta r))\frac{f(\theta(\delta r))}{u} - \int_{\theta(\delta r)}^{\bar{\theta}}[F(\tilde{\theta}) - F(\theta)]g(\theta)d\theta\}. \\ =& f(\tilde{\theta})\frac{\partial \tilde{\theta}}{\partial c}r - \delta G(\theta(\delta r))\frac{f(\theta(\delta r))}{u} - \int_{\theta(\delta r)}^{\bar{\theta}}[F(\tilde{\theta}) - F(\theta)]g(\theta)d\theta\}\end{aligned} \tag{55}$$

the sing of which is indeterminate since $\frac{\partial \tilde{\theta}}{\partial c} > 0$.

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